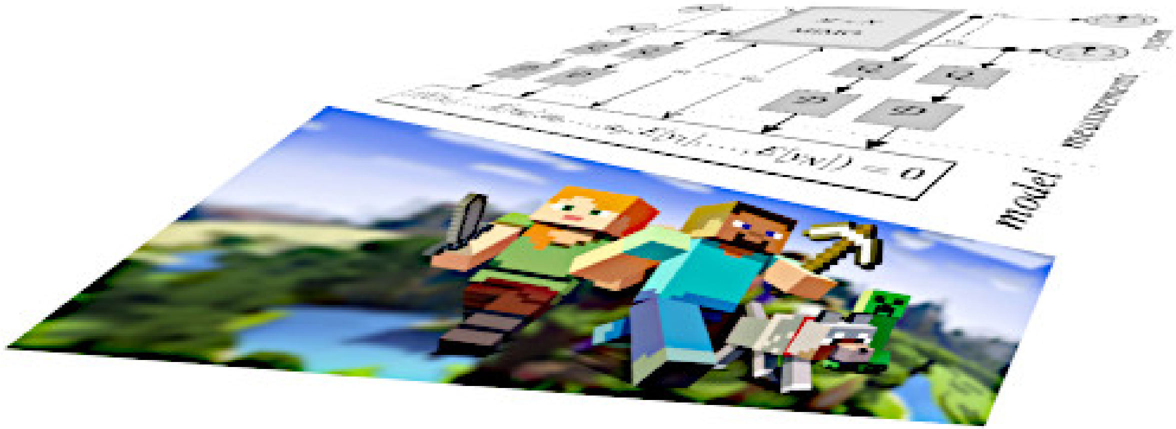
Beyond Calculus: Modernizing System Modeling and Computational Methods



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About Me

Pavel Loskot joined the ZJU-UIUC Institute as Associate Professor in January 2021. He received his PhD degree in Wireless Communications from the University of Alberta in Canada, and the MSc and BSc degrees in Radioelectronics and Biomedical Electronics, respectively, from the Czech Technical University of Prague. He is the Senior Member of the IEEE, Member of ACM and APSIPA, Fellow of the UKHEA, and the Recognized Research Supervisor of the UKCGE. He was elected the IARIA 2025 Fellow.

In the past nearly 30 years, he was involved in numerous industrial and academic collaborative projects in the Czech Republic, Finland, Canada, the UK, Turkey, and in China. These projects concerned wireless and optical telecommunication networks, genetic regulatory circuits, air transport services and renewable energy systems. This experience allowed him to truly understand the interdisciplinary workings, and crossing the disciplines boundaries.

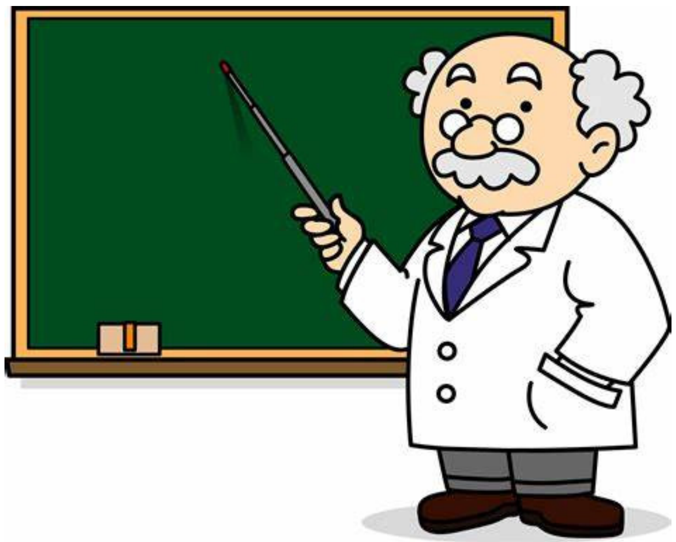
His current research focuses on mathematical and probabilistic modeling, statistical signal processing and machine learning for multi-sensor and longitudinal data in biomedicine, computational molecular biology, and wireless communications.

Objectives

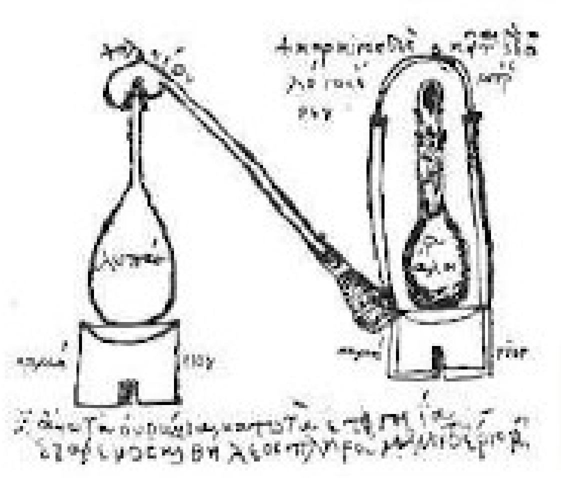
- modern intelligent and autonomous systems are much more complex
  → are well-established modeling methods in engineering sufficient?
  → how to model abstractions, semantics, and information granularity?
- there are many results in pure and applied mathematics
  → which of these results may be useful in engineering?
  → how to find such results, and teach them to engineering students?
- let's start by reviewing fundamental mathematical objects and topics
  → they are basic knowledge for undergraduate students in mathematics
  → but not included in engineering curricula

Outline

- A bit of history of mathematical modeling
- Current challenges in modeling engineering systems
- Topics in advanced mathematical modeling
- Future outlook



Evolution of Research Methods



Problems of simplicity (1600-1800)

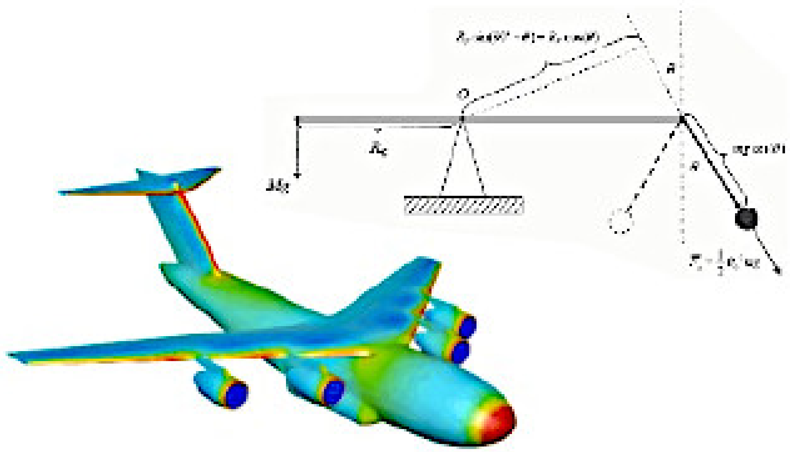
- simple physical models of measurable quantities

Problems of disorganized complexity (1900-1950)

- more complex but with well defined behavior

Problems of organized complexity (since 1950's)

- embracing all factors influencing whole system



Experimental science (until 1700's)

- observations and experiments

Theoretical science (until 1940's)

- mathematical analysis

Computational science (until 2000)

- computer simulations

Big data science (after 2000)

- data collection and knowledge mining

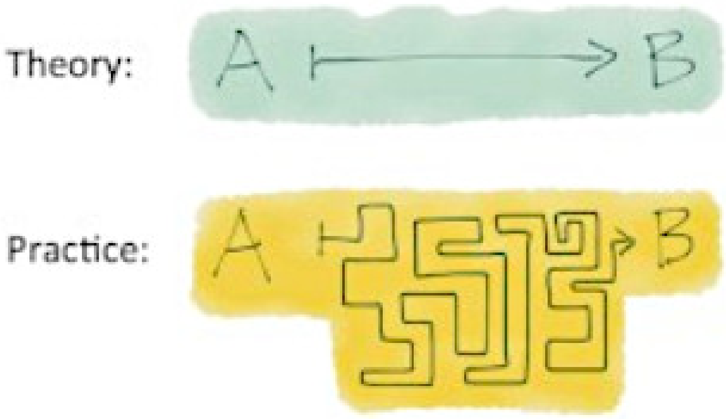
Mathematical abstractions (past decade)

- information granularization and modularization

Current Modeling Methods and Challenges

Computational models

- vectors and matrices, linear algebra, calculus
- signal and systems, software, optimization
  → well understood, optimum, interpretable

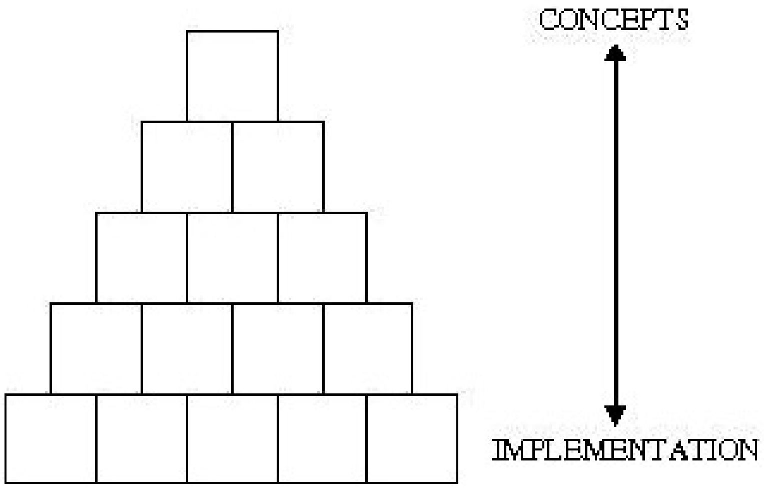


Universal models

- deep neural networks
- challenges
  → no design guidelines
  → poor numerical efficiency
  → difficult to obtain

Top-down evolution

- start from theoretical limits (best possible)
- break down complexity into smaller tasks



Bottom-up evolution

- heuristically improve existing solutions
- integrate tasks into more complex system

Beyond Numbers

Mathematical objects

- defined structure
- induced properties
- defined operators and transformations
- can be assigned numerical attributes

Examples

- matrices, tensors, sets, classes, functions, maps
- vector and other spaces, graphs, categories

Structural operators

- binary: sums and products $O_3 = O_1 \,\square\, O_2$
- unary: inverse $O_2 = \triangle\, O_1$

Numerical operators

- functions, $\{v_i'\} = T_O[\{v_i\}]$

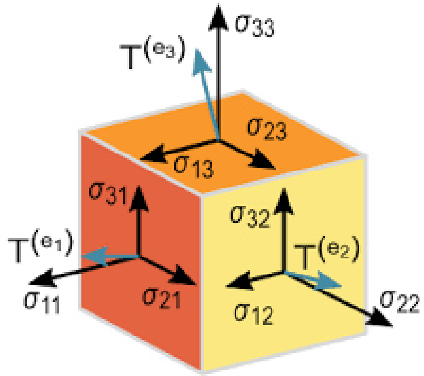
Example

- graphs with numerical attributes
  $\rightarrow$ numerical values assigned to vertices, edges, ...
- sum or product of two graphs $\longrightarrow$ another graph or other structure
- need also rules for computing numerical attributes of the new structure

Advanced Objects: Tensors and Manifolds

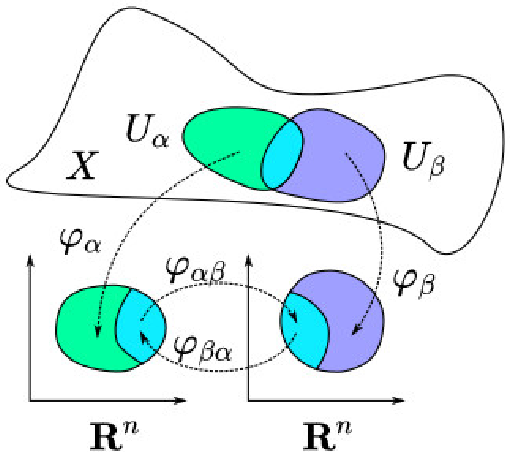
Tensors

- collections of multi-dimensional linear transformations
  → covariant and contravariant components
- tensors are key objects in multi-linear algebras
- they are not multi-dimensional matrix data structures



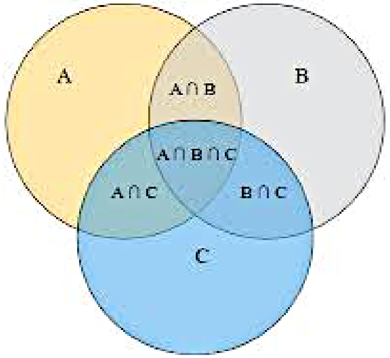
Manifolds

- describe complicated structures as topological properties of simpler spaces
- collections of locally Euclidean spaces with bijective maps
- special cases: tangent manifold, manifolds in metric spaces
- manifold hypothesis



ADVANCED OBJECTS: SET THEORY AND LOGIC

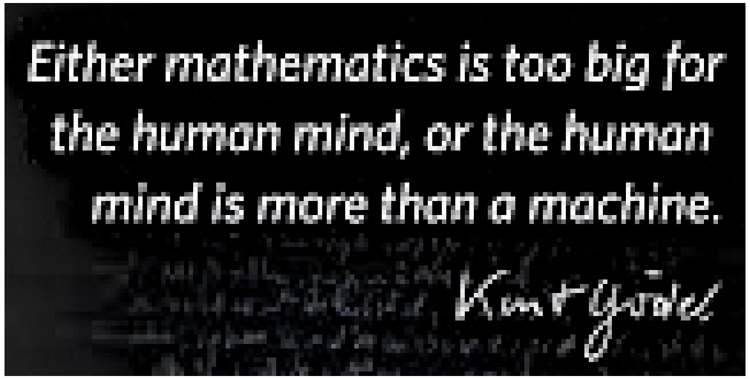
Set theory



- a very general formal approach to ordinary mathematics
  → everything is a set
  → Zermelo-Fraenkel axioms and Axiom of Choice
- many obvious things cannot be proven

Mathematical logic

- formal, unambiguous language of statements and their proofs
- propositional logic formalizes deductions
  → logical formulas and Boolean functions
- first-order logic extends propositional logic
  → quantifiers, predicates, relations, functions
- many practical applications
  → reasoning and decision making systems
  → Gödel's (in-) completeness theorems
  → discovering new knowledge
  → explanations



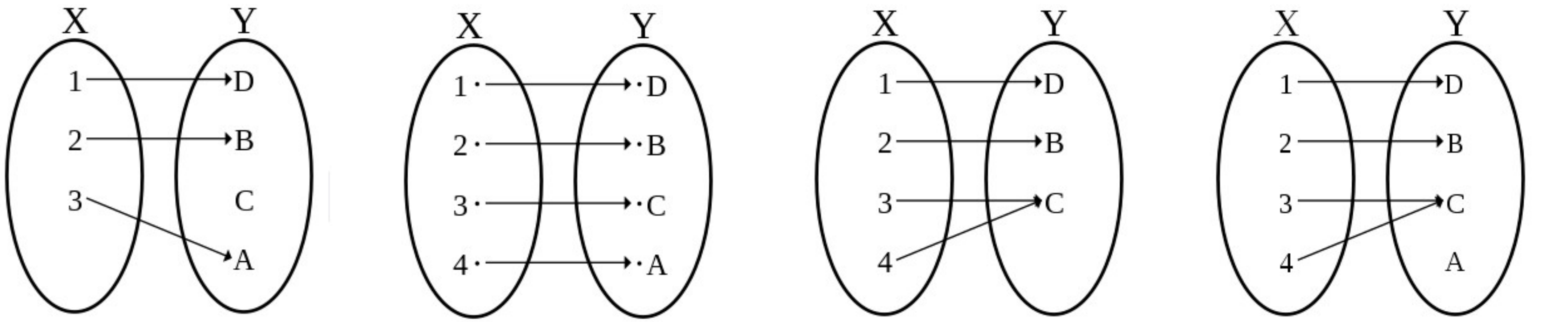
Abstract Algebra

Define abstract mathematical structures

- work with sets of object representations
- attach operations and closure properties to these sets

Maps and morphisms

injection	**bijection**	**surjection**	**–**
(injective & non-surjective)	(injective & surjective)	(non-injective & surjective)	(non-injective & non-surjective)



Group $(S,+)$

- closure, associativity
- inverse element

Ring $(S,+,*)$

- combine group $(S,+)$
- and semi-group $(S,*)$

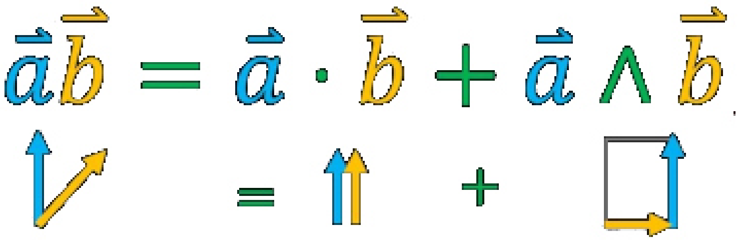
Field $(S,+,*)$

- ring with $(S,*)$
- being a group

Algebraic Geometry and Topology

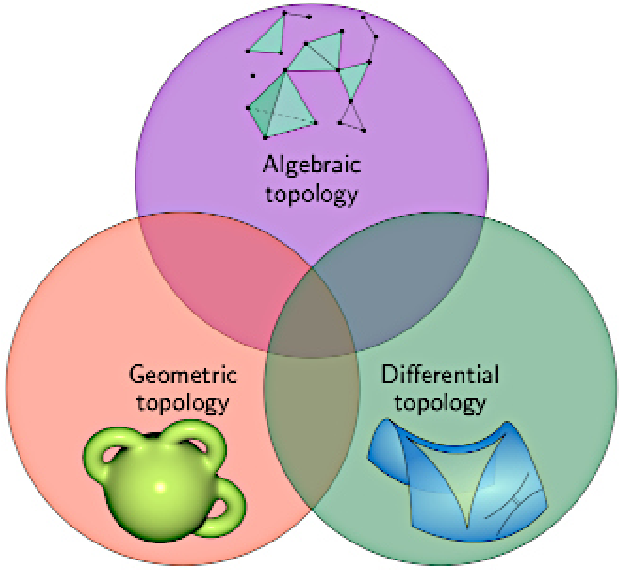
Algebraic geometry

- generalize ordinary vectors to arbitrary number of dimensions
  → k-vectors with magnitude, direction, and orientation
- geometric product combines inner and outer products
- applications
  → generalize complex numbers to 4D
  → greatly simplify notations
  → connects geometry and calculus



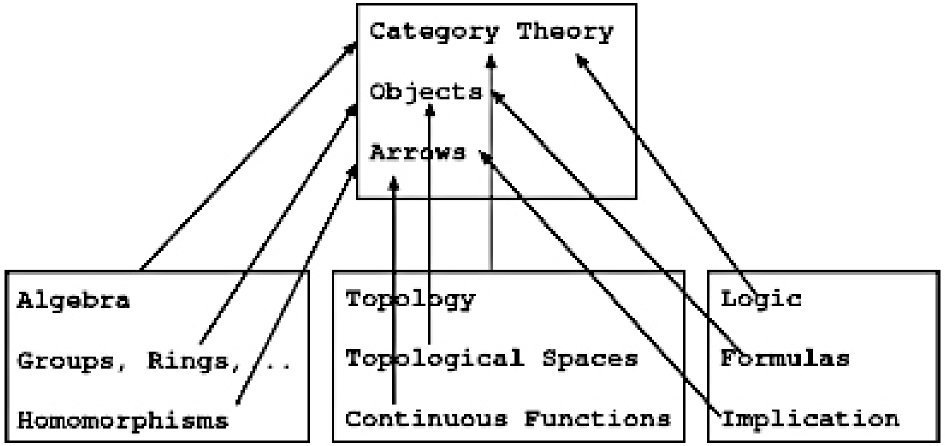
Topology

- the goal is to compare shapes of geometrical objects
  → topologist cannot tell difference between circle & square
- re-shape objects using homeomorphisms
  → look for invariants
- homotopy to map topological spaces
- topological algebra
  → computational persistent homology



Category Theory

World of abstractions ... and unifying views



- abstracts high-order theory for diverse mathematical objects
  $\longrightarrow$ governed by laws of free (general) algebra
  $\longrightarrow$ focus on external relations (morphisms) between objects
- identify properties that are
  $\longrightarrow$ transferable between objects
  $\longrightarrow$ universal to objects

Examples

- category of vector spaces and linear transformations
- category of metric spaces and continuous maps

Mathematics in Engineering

Using mathematics in engineering

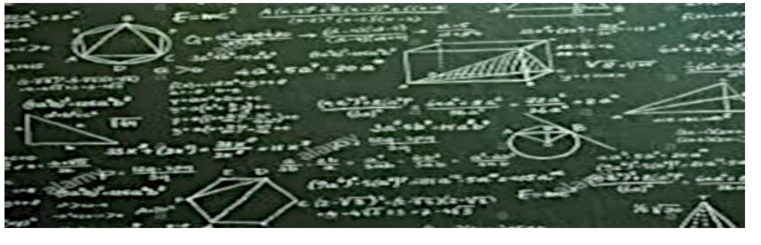
- excessive focus on calculus and numerical computations
- assuming more abstract modeling methods can be very useful
- unaware of many results in mathematics that might be useful

Teaching mathematics in engineering

- more challenging over time for younger generations
- need plenty of examples to gain intuition
- there has to be a hope for practical applications

Mathematics can be difficult

- mathematics can be easily difficult for mathematicians too
- mindset of mathematicians and engineers are totally different
- imminent challenge is obscure mathematical notations (not concepts)



A Few Suggestions

Revise mathematical curriculum in engineering

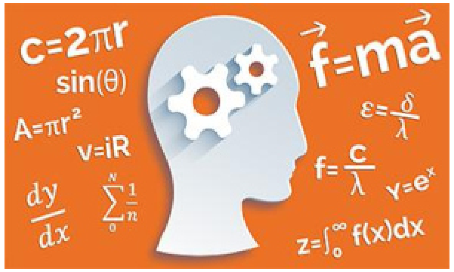
- current systems are much more complex
- be able to communicate with mathematicians
- be able to read mathematical literature

Current challenges

- simulations (i.e., also machine learning) are very time consuming
- lack of guided designs for complex systems
- methods are neither explainable, nor optimum

Disruptive impact of AI

- both in research and in teaching
- AI cannot be ignored or forbidden
- a simple and powerful use case: use LLM as Wikipedia replacement



Thank you!

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