Genetic Algorithm Optimization of Bi-phase Codes: Surpassing Barker Code Performance in ISAC Applications

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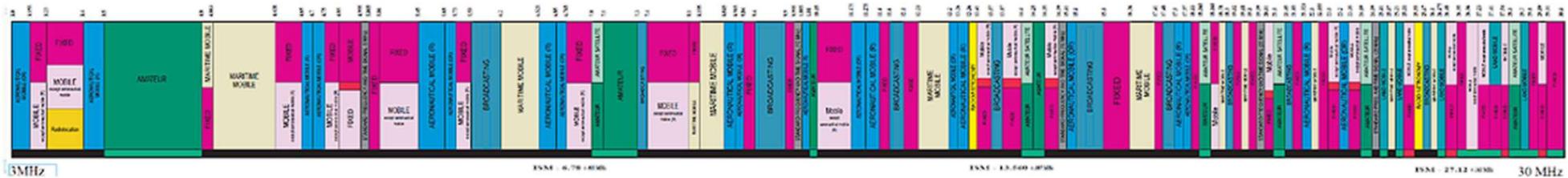Prof. Tolga Girici tgirici@mdx.ac.uk
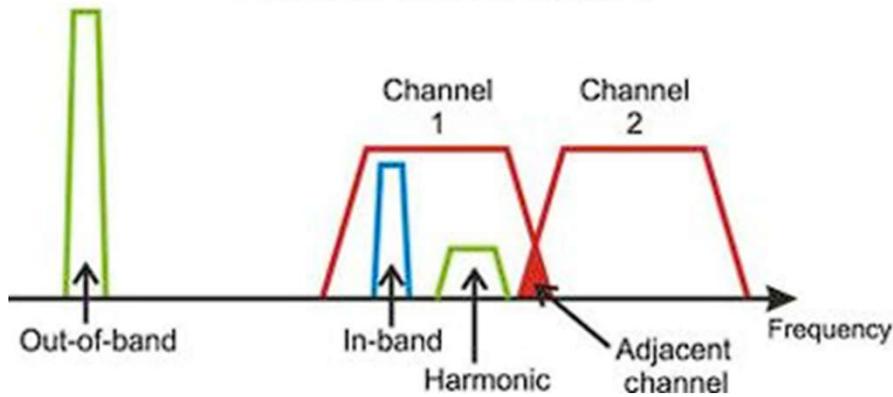
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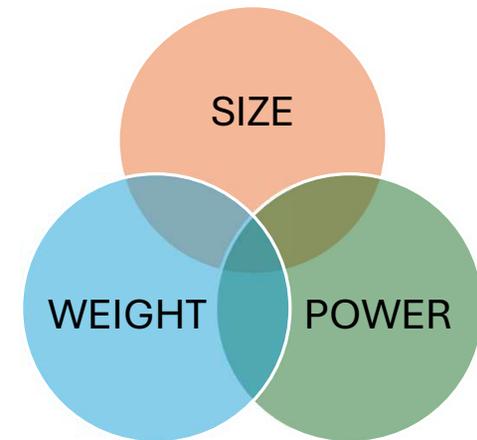- ➢ Numerical Analysis
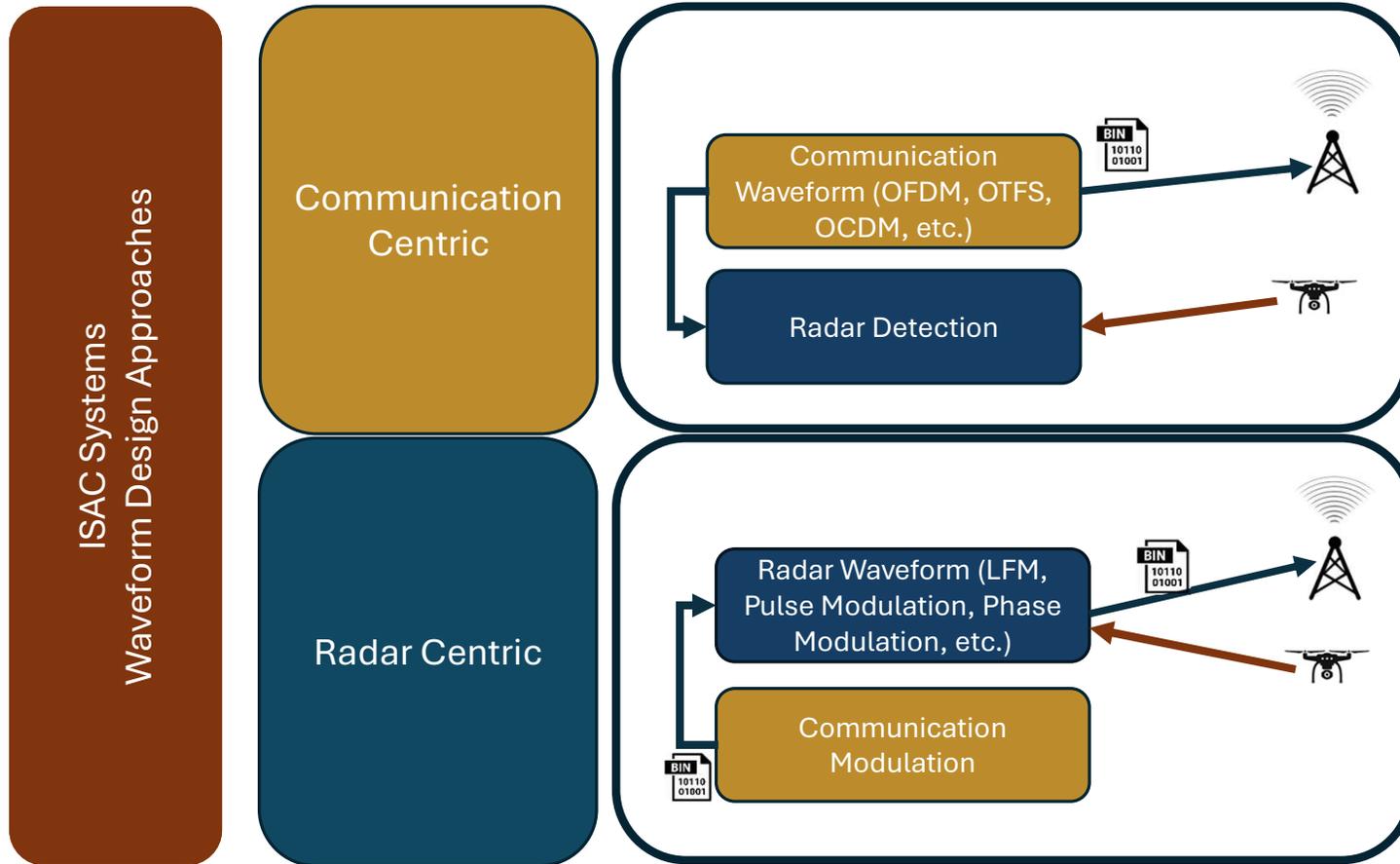- ➢ Conclusion

Motivation

Congestion in the spectrum

Interference between systems

Size, weight and power limitations on the platform

ISAC Waveform Design

ISAC Systems Waveform Design Approaches

Communication Centric

Communication Waveform (OFDM, OTFS, OCDM, etc.)

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Radar Detection

Radar Centric

Radar Waveform (LFM, Pulse Modulation, Phase Modulation, etc.)

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Communication Modulation

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Barker Coded Radar Waveform

Radar Tutorial

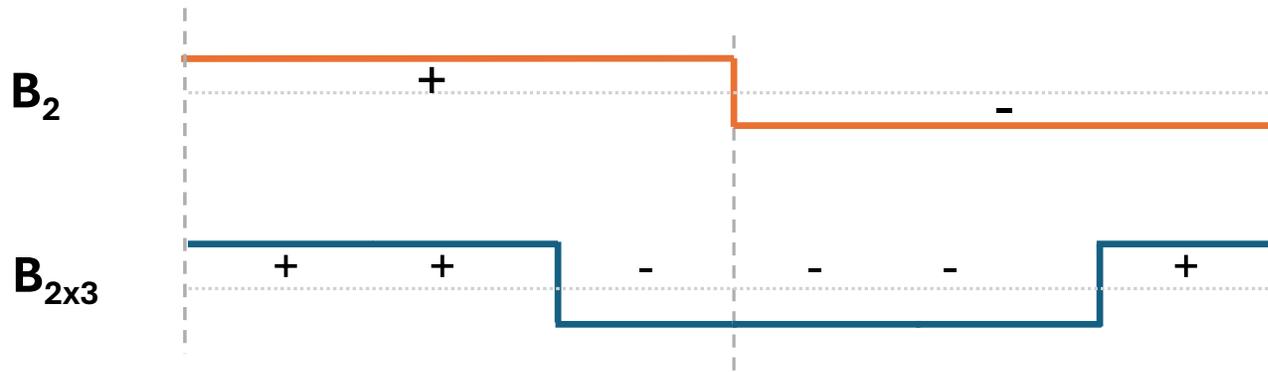| Code	Length	Code Elements	PSL (dB)
B2	2	+-,++	6.0
B3	3	++-	9.5
B4	4	++-+, +++-	12.0
B5	5	+++-+	14.0
B7	7	+++-+-	16.9
B11	11	+++---+--+-	20.8
B13	13	+++++--++-+-+	22.3
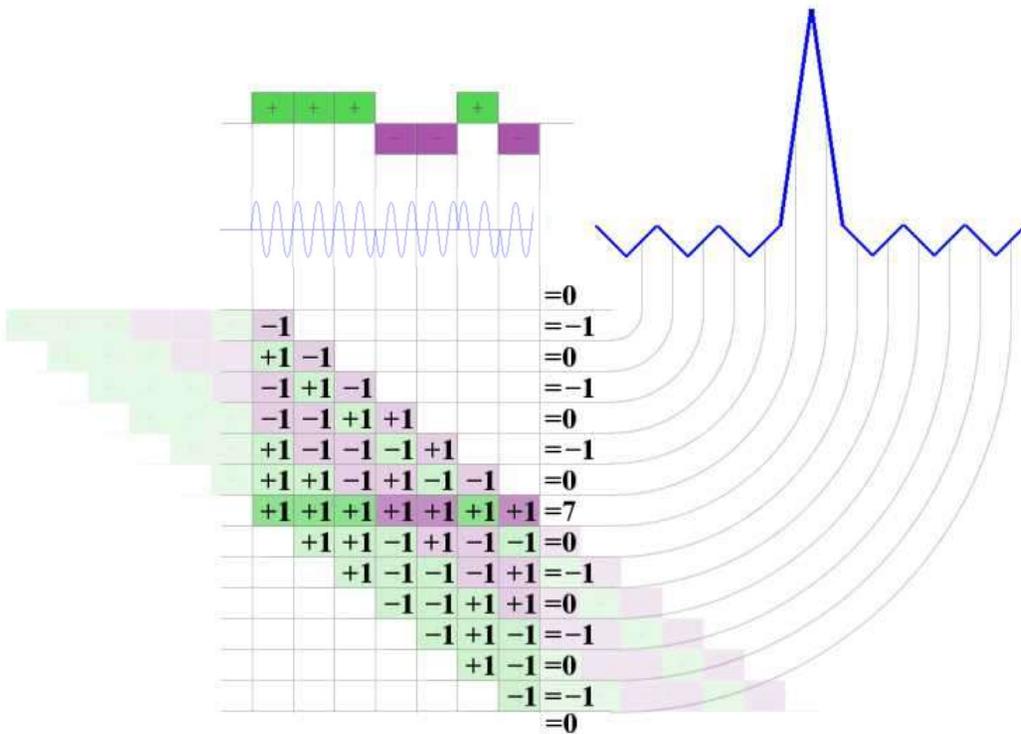
Nested Barker Codes and ISAC Waveform Generation

$$\begin{bmatrix} B4_1 \\ B4_2 \\ B4_3 \\ B4_4 \end{bmatrix} = \begin{bmatrix} + & + & + & - \\ + & + & - & + \\ + & - & + & + \\ - & + & + & + \end{bmatrix} \qquad \begin{bmatrix} m_1 \\ m_2 \\ m_3 \\ m_4 \end{bmatrix} = \begin{bmatrix} B4_1 \otimes B11 \\ B4_2 \otimes B11 \\ B4_3 \otimes B11 \\ B4_4 \otimes B11 \end{bmatrix} \qquad \begin{bmatrix} m_1 \\ m_2 \\ m_3 \\ m_4 \end{bmatrix} = \begin{bmatrix} (B4_1 \otimes B11) \otimes B5 \\ (B4_2 \otimes B11) \otimes B5 \\ (B4_1 \otimes B11) \otimes B5 \\ (B4_1 \otimes B11) \otimes B5 \end{bmatrix}$$

Receiver Processing

Communication Receiver

➤ Received signal at the communication receiver:

$$y_k(t) = S_{RadCom}(t) + w_k(t)$$

➤ The demodulation of this signal at the receiver by using matched filtering.

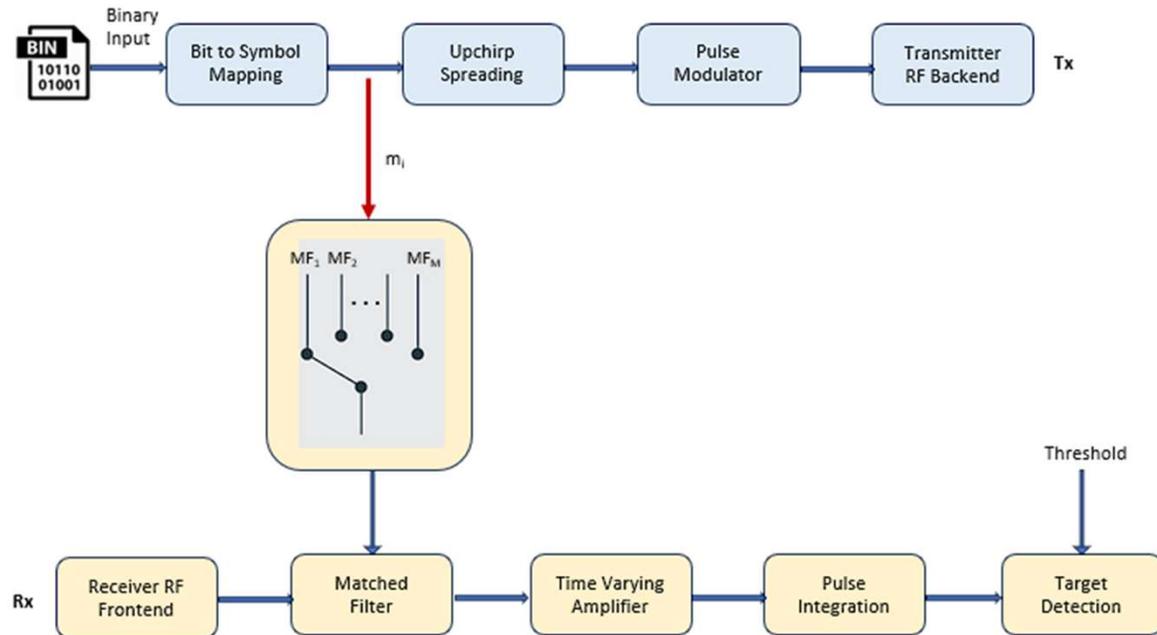$$z_i(t) = corr\big(y_k(t), S_i(t)\big) \qquad i = 1, \dots, M$$

➤ In cross-correlation, the signature signal that gives the result with the highest peak value is accepted as the sent symbol.

$$i_{est} = argmax_{i=1,\dots,M}\big(max\{|z_i(t)|\}\big)$$

Receiver Processing

Radar Receiver

Ambiguity Function (AF)

➢ Output of the radar matched filter

➢ Measures response vs. **delay (τ)** and **Doppler (f_n)**

➢ Peak at **(τ = 0, f_n = 0)** → perfectly matched target

➢ Off-peak values → interference from other ranges / velocities

➢ Shape of AF → range & Doppler resolution

➢ Sidelobes → risk of false or ambiguous detections

Ambiguity Function (AF)

➢ Matched filter output

$$\chi(\tau, f_d) = \int_{-\infty}^{\infty} \tilde{x}(t)\, \tilde{x}^*(t - \tau)e^{j\pi f_d t} dt$$

➢ Ambiguity function

$$|\chi(\tau, f_d)|^2 = \left| \int_{-\infty}^{\infty} \tilde{x}(t)\, \tilde{x}^*(t - \tau)e^{j\pi \ _d t} dt \right|^2$$

ISAC-Barker Ambiguity Function

➢ Phase coded radar signal

$$x(t) = e^{jw_0 t} \sum_{n=1}^{N} D_n(t) e^{j\theta_n}$$

➢ Where w_0 denotes the carrier angular frequency of the pulse

➢ For a binary phase code $\theta_n \rightarrow 0$ or 1

➢ $D_n = e^{j\theta_n} = \pm 1$

➢ For the joint waveform, D_n changes in accordance with the transmitted communication symbol.

ISAC-Barker Ambiguity Function

$$\chi(\tau, f_d) = \chi_0(\tau', f_d) \sum_{n=1}^{N-k} D_n D_{n+k} e^{-j2\pi f_d(n-1)\tau_0}$$

$$+\chi_0(\tau_0 - \tau', f_d) \sum_{n=1}^{N-k} D_n D_{n+k+1} e^{-j2\pi f_d n \tau_0}$$

$$\tau = k\tau_0 + \tau' \begin{cases} 0 < \tau' < \tau_0 \\ k = 0,1,2,\dots,N \end{cases}$$

$$\chi_0(\tau', f_d) = \int_0^{\tau_0 - \tau'} e^{-j2\pi f_d t} dt \quad 0 < \tau' < \tau_0$$

➢ Zero Doppler Cut for L periodic pulse

$$|\chi(\tau; 0)| = \lim_{f_d \to 0} \frac{1}{L} \sum_{q=-(L-1)}^{L-1} |\chi_{ISAC-Barke}\ (\tau - qT; f_d)| \times \left| \frac{\sin[\pi f_d LT]}{L \sin(\pi f_d T)} \right|$$

$$= \frac{1}{L} \sum_{q=-(L-1)}^{L-1} |\chi_{ISAC-Barker}(\tau - qT; 0)|$$

$$= \frac{1}{L} \sum_{q=-(L-1)}^{L-1} \left((\tau_0 - \tau' + qT) \sum_{n=1}^{N-k} D_n D_{n+k} + (\tau' + qT) \sum_{n=1}^{N-k} D_n D_{n+k+1} \right)$$
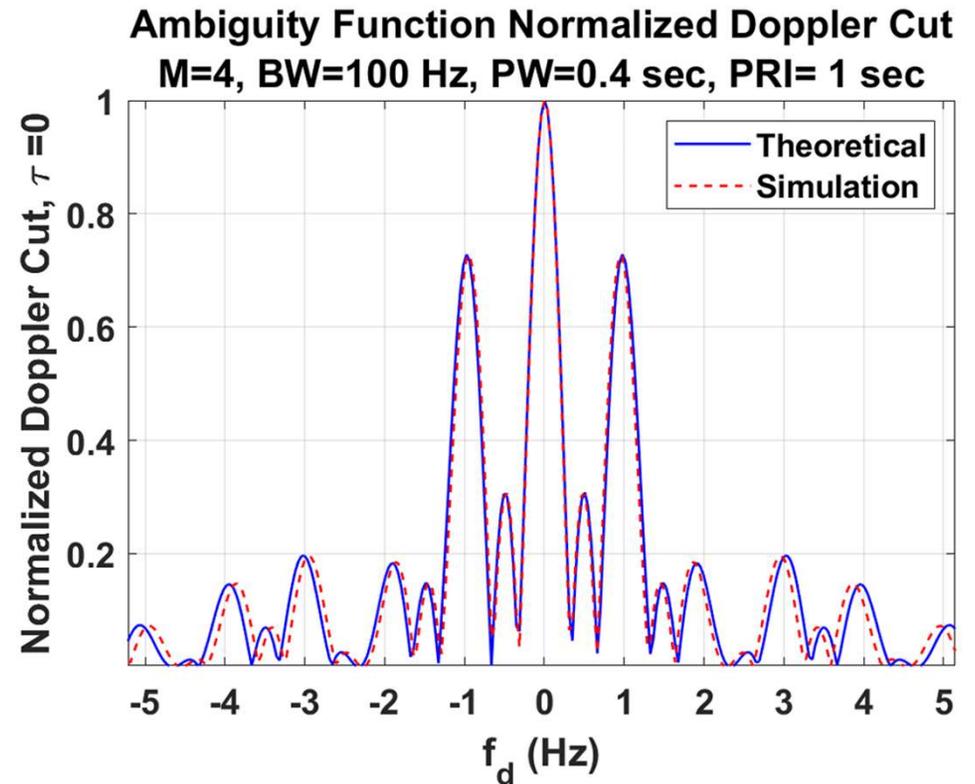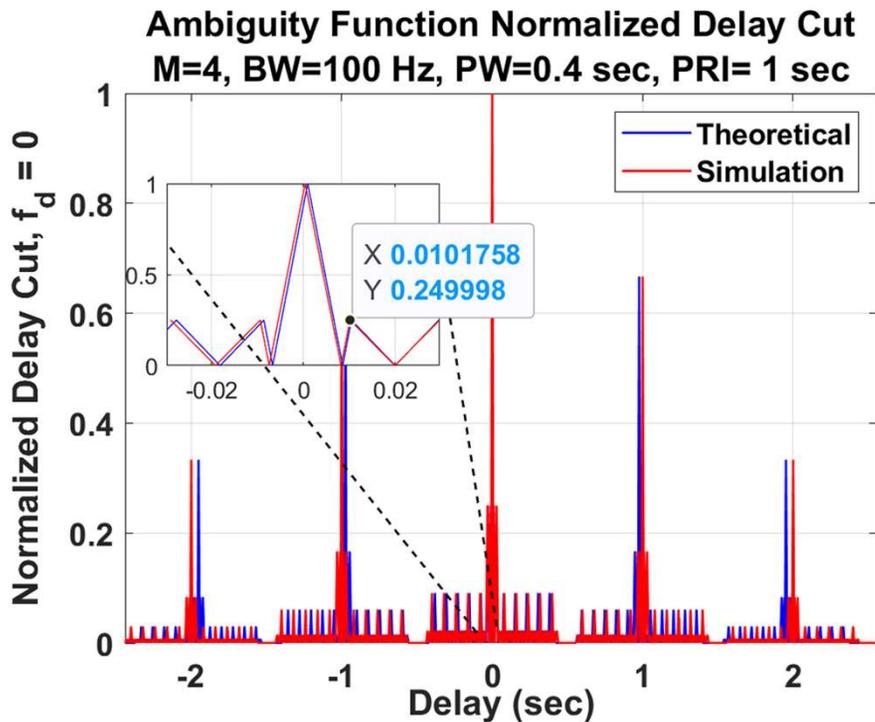
ISAC-Barker Ambiguity Function

➢ Zero Delay Cut for L periodic pulse

$$\chi(0, f_d)$$

$$= \left(\chi_0(0, f_d) \sum_{n=1}^{N-k} D_n D_{n+k} e^{-j2\pi \ d(n-1)\tau_0} \right.$$

$$\left. + \chi_0(\tau_0, f_d) \sum_{n=1}^{N-k} D_n D_{n+k+1} e^{-j2\pi f_d n \tau_0} \right) \times \left| \frac{\sin[\pi f_d (L - |q|) T]}{\sin(\pi f_d T)} \right|$$

ISAC-Barker Ambiguity Function

Genetic Algorithm

➤ Derived ambiguity function applies to all biphase codes

➤ Enables optimization beyond Barker sequences

➤ Design space is discrete, non-convex, and combinatorial

➤ Multiple conflicting goals:
- Radar: low sidelobes (high PSLR)
- Communication: large Hamming distance

Genetic Algorithm

➢ Objective functions

$$f_{ZeroCut} = \frac{w_1 M}{N} PSLR_{Zdel} - \frac{w_2 M}{N} PSLR_{Zdopp} + \frac{w_3}{N} d_{min}$$

$$f_{Global} = \frac{w_1 M}{N} PR_{Global} + \frac{w_3}{N} d_{min}$$

Genetic Algorithm

➢ Genetic algorithm parameters

Parameter	N44M4	N88M8
Population Size	40	140
Number of Generations	100	100
Crossover Fraction	0.8	0.8
Elite Count	5	5
Population Type	bitstring	bitstring

Numerical Analysis

- ➢ Sequence length: N = 44
- ➢ Communication symbols: M = 4; 2 bits per PRI
- ➢ Genetic algorithm–optimized biphase codes
- ➢ Clear performance improvement over Barker code
- ➢ Key Observations
  - Lower sidelobes in zero-Doppler cut→ higher PSLR
  - Hamming distance preserved→ reliable communication performance

Numerical Analysis

➢ Sequence length: N = 88
➢ Communication symbols: M = 8
  ▪ 3 bits per PRI
➢ Analysis extended to longer sequences
➢ Goal: verify scalability of the optimization approach
➢ Key Findings
  ▪ Stronger PSLR improvement in zero-Doppler cut
  ▪ Optimized codes outperform Barker sequences
  ▪ Minimum Hamming distance maintained or improved
  ▪ Objective function weights enable flexible trade-offs
  ▪ Radar sidelobe suppression
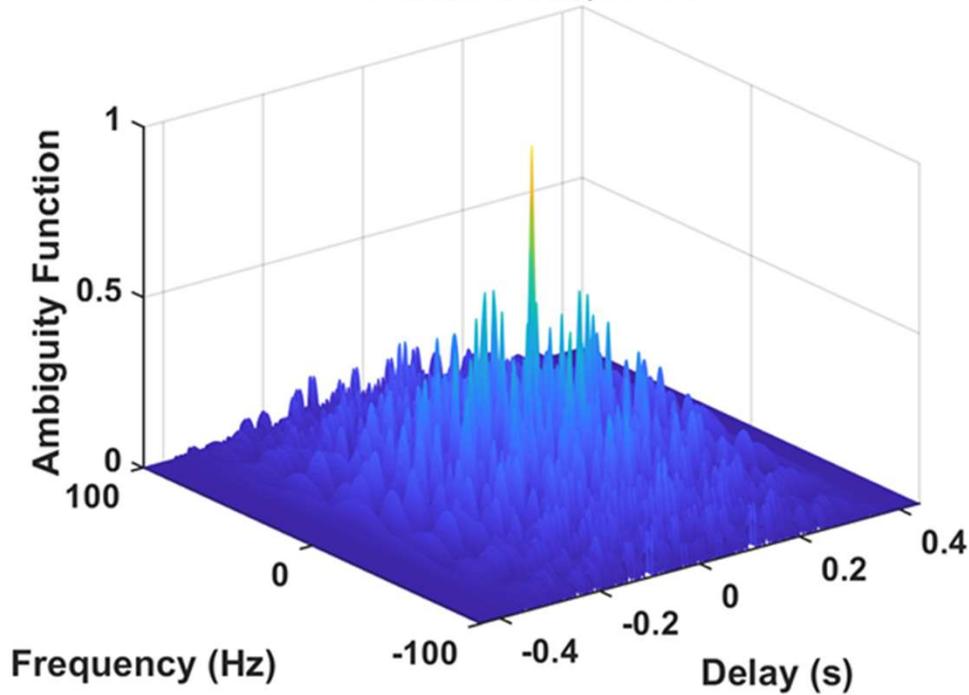 ▪ Communication reliability.

Numerical Analysis

| Code	PSLR Zero Doppler	PSLR Zero Delay	Hamming Distance
Barker44M4	4.0000	4.6031	22
N44M4W111ZC	4.8889	4.6031	22
N44M4W110ZC	5.5000	4.6031	20
N44M4W111G	4.0000	4.6031	25
Barker88M8	4.0000	4.6040	44
N88M8W111ZC	6.2857	4.6040	44
N88M8W110ZC	6.7692	4.6040	37
N88M8W111G	5.5000	4.6040	46

Numerical Analysis

Barker Code, N=44

Optimum Biphase Code
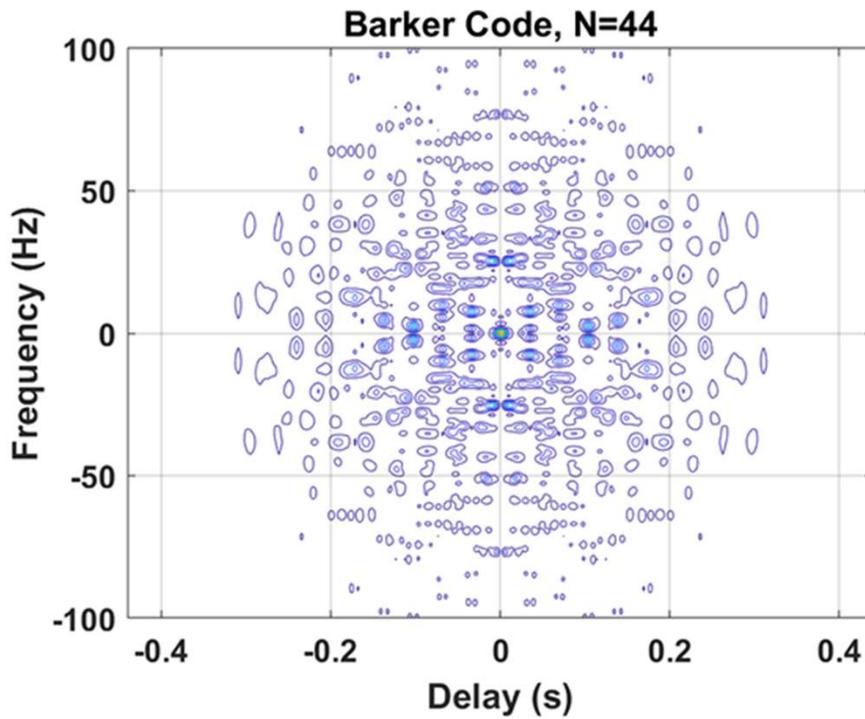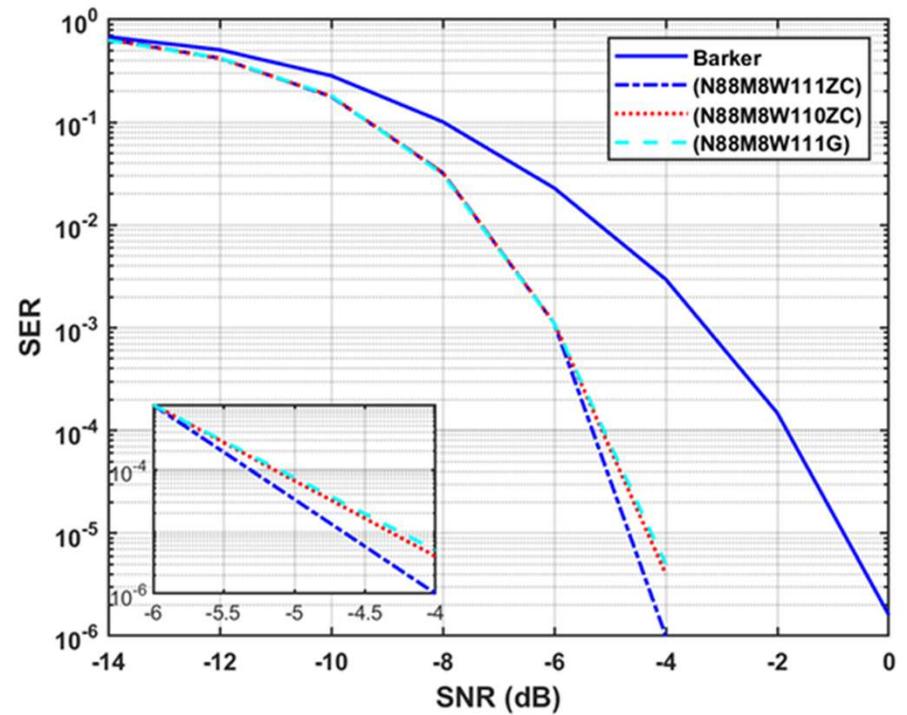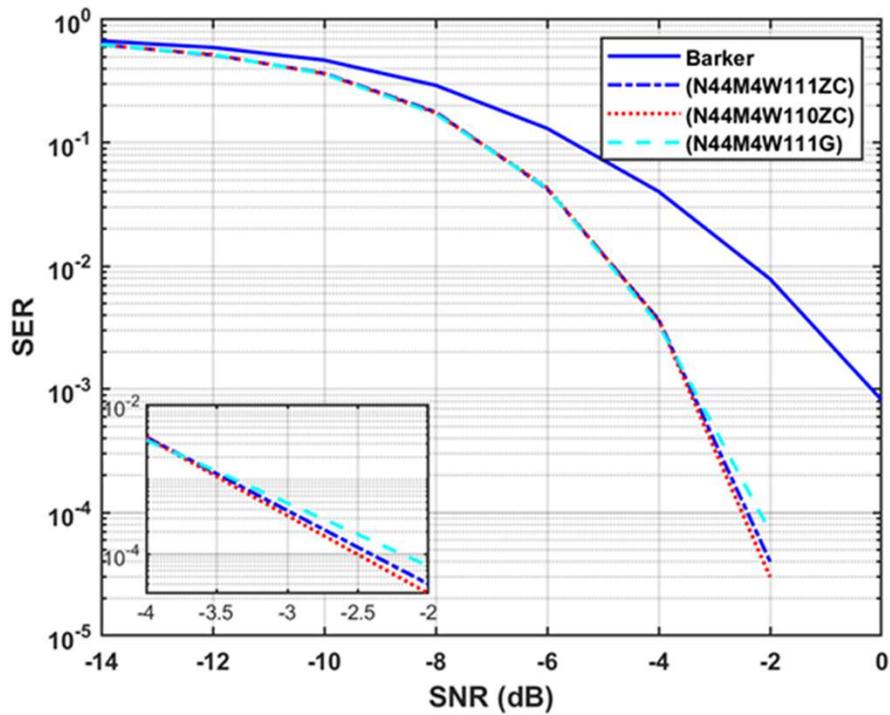(N44M4W111ZC), N=44, M=4, w_1=1, w_2=1, w_3=1

Numerical Analysis

Barker Code, N=44

Optimum Biphase Code
(N44M4W111ZC), N=44, M=4, w_1=1, w_2=1, w_3=1
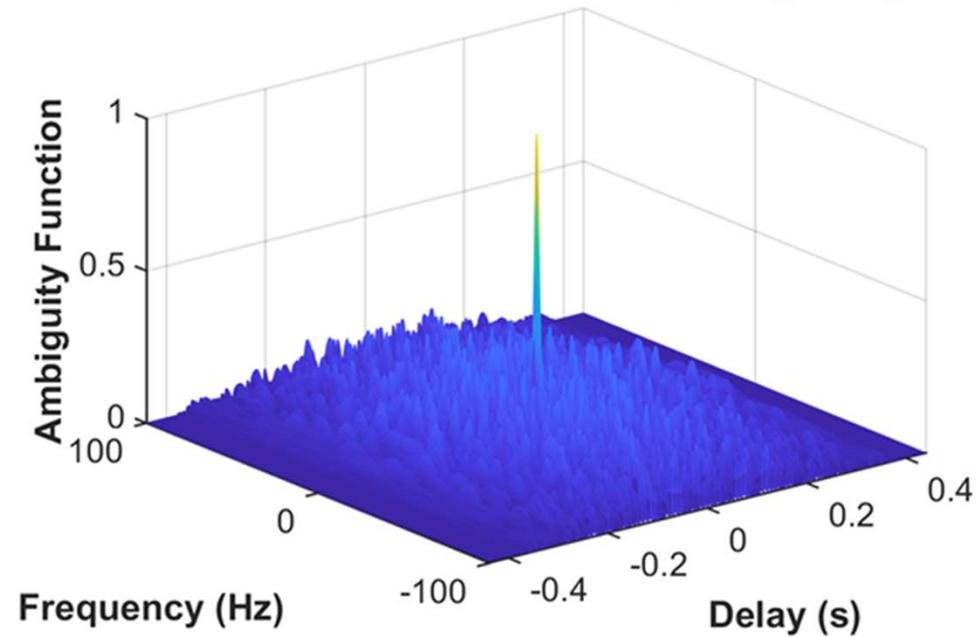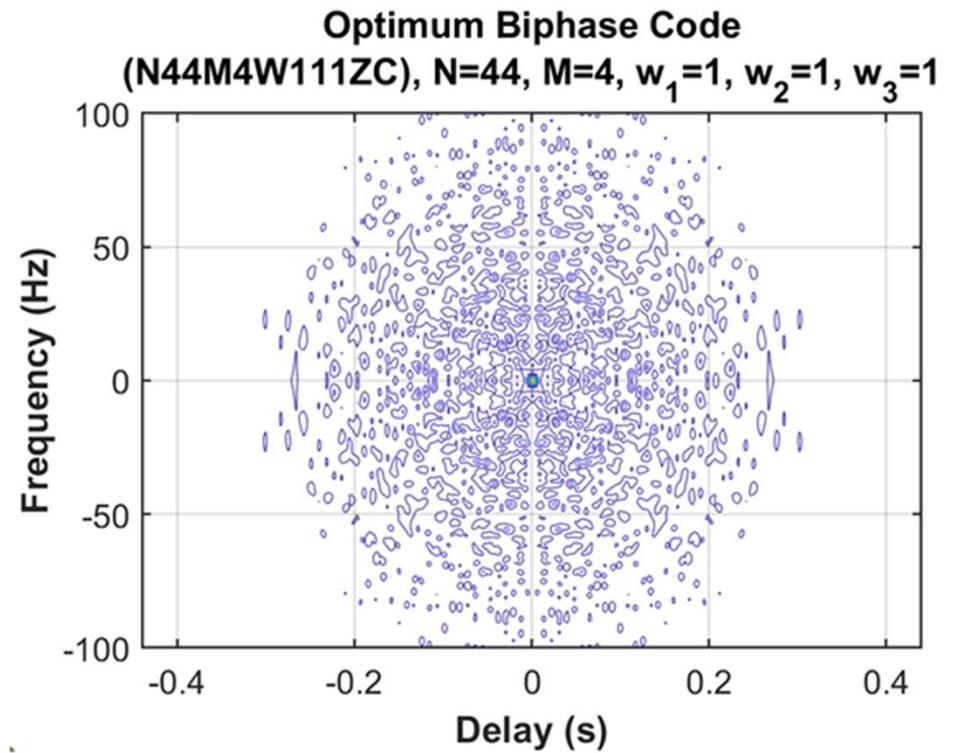
Numerical Analysis

Conclusion

➢ Joint ISAC waveform with Barker intra-pulse modulation analyzed

➢ Ambiguity function derived and evaluated (PSLR & SER)

➢ Genetic algorithm–optimized biphase code
- Lower sidelobes (higher PSLR)
- Improved SER performance

➢ Consistent gains for N=44, M=4 and N=88, M=8

➢ Suitable for sensing-oriented low-rate communication(2–3 bits per PRI)

➢ GA-based biphase optimization enhances both radar and communication performance in ISAC systems.