# An Integrative Strategy for Solving the EEG Inverse Problem and the Estimation of Brain Effective Connectivity in Epilepsy. A Proof-of-Concept Study

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*IARIA - Signal 2025*

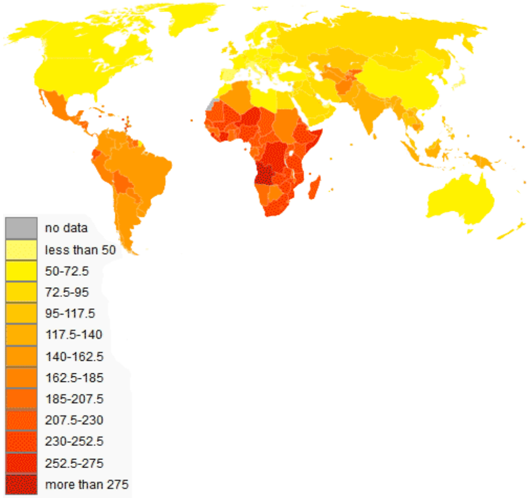
*March 12, 2025*

# Epilepsy

- **Epilepsy:** A brain disorder characterized predominantly by recurrent and unpredictable interruptions of normal brain function, called epileptic seizures.

- **Epileptic seizure:** induced by abnormal excessive or synchronous neural activity in the brain

- Around 50 million people worldwide have epilepsy




globometer.com/maladies-epilepsie.php
who.int/news-room/fact-sheets/detail/epilepsy

National Institute of Neurological Disorders and Strokes (https://www.ninds.nih.gov)

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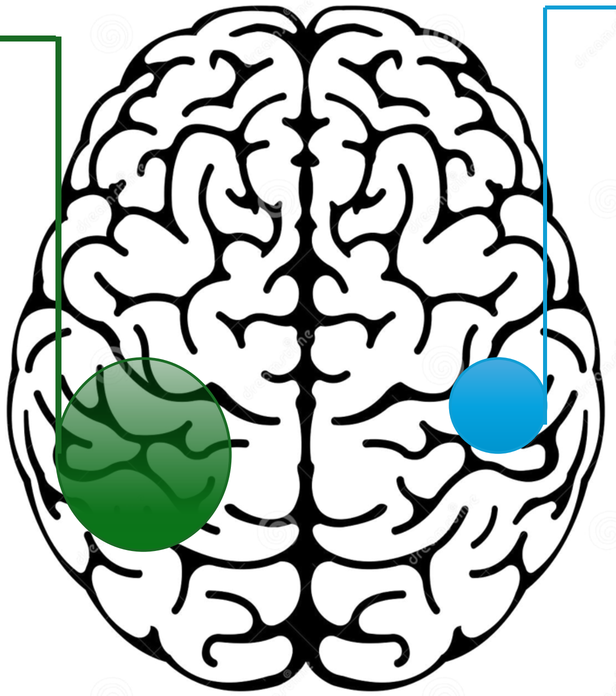
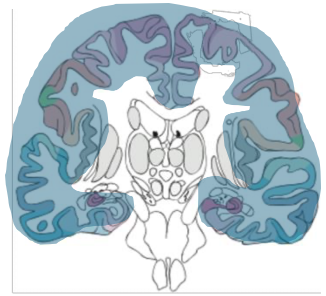
# Epilepsy

**60%** [1]
of epileptic seizures
**are focal seizures**

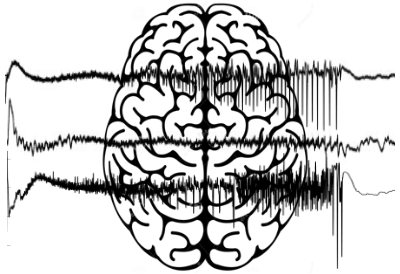


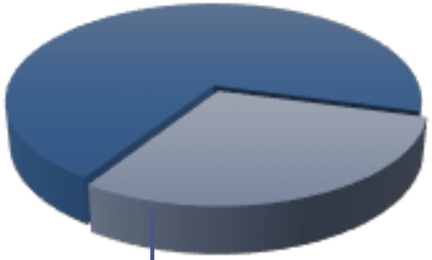
**30%** of people
with epilepsy experience
**generalized seizures**



National Institute of Neurological Disorders and Strokes (https://www.ninds.nih.gov)

# Epilepsy

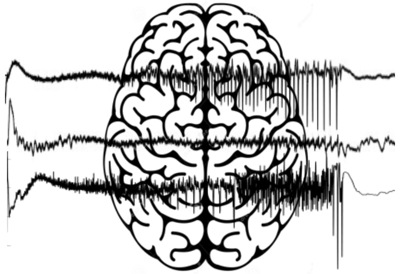


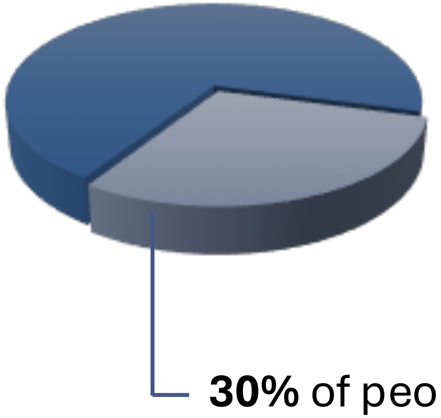


**30%** of people with epilepsy are **drug-resistant**

N. Child, M. Stead, E. C. Wirrell et al., Chronic subthreshold subdural cortical stimulation for the treatment of focal epilepsy originating from eloquent cortex, Epilepsia, vol. 55, no. 3, pp. 18-21, 2014.

# Epilepsy



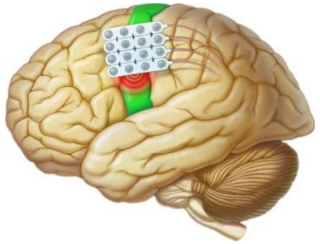


**30%** of people
with epilepsy are
**drug-resistant**

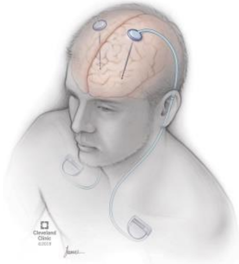
Vagus Nerve Stimulation (VNS)

© Epilepsy society

Cortical Stimulation



Deep brain Stimulation (DBS)



Surgery

© Vitals

N. Child, M. Stead, E. C. Wirrell et al., Chronic subthreshold subdural cortical stimulation for the treatment of focal epilepsy originating from eloquent cortex, Epilepsia, vol. 55, no. 3, pp. 18-21, 2014.

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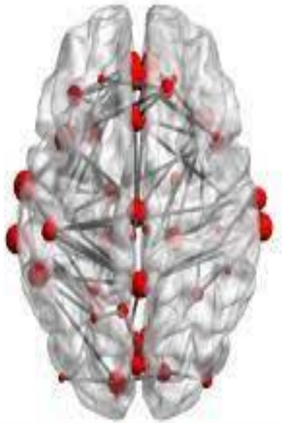
# Ultimate goal

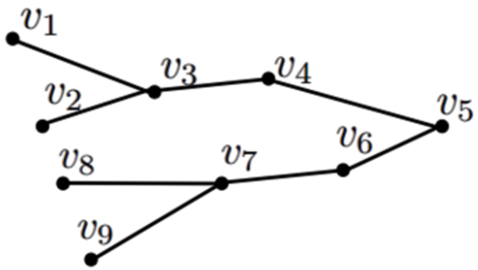
- Inferring **brain epileptic network** underlying the initiation and/or the propagation of epileptic seizure using **surface Electroencephalographic (EEG)** signals.

**EEG recordings**



**A graph** $\mathcal{G}(\mathbb{V}, \mathbb{E}, \boldsymbol{A})$ is a set $\mathbb{V}$ of vertices linked using a set $\mathbb{E}$ of edges with weights defined in the adjacency matrix $\boldsymbol{A}$

$$\boldsymbol{A} = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$




D. Shuman *et al.*, "The emerging field of signal processing on graphs: Extending high-dimensional data analysis to network and other irregular domains" , IEEE Signal Processing Magazine, (30)3, 2013.

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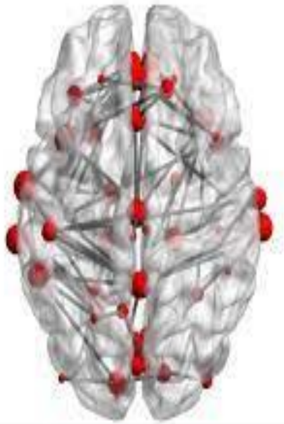

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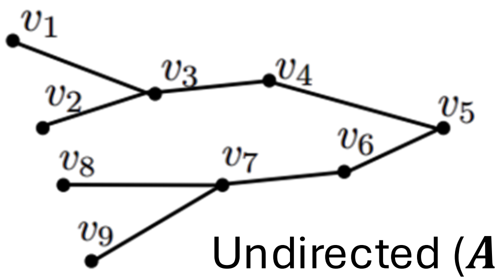
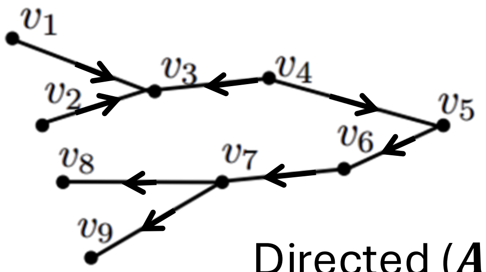


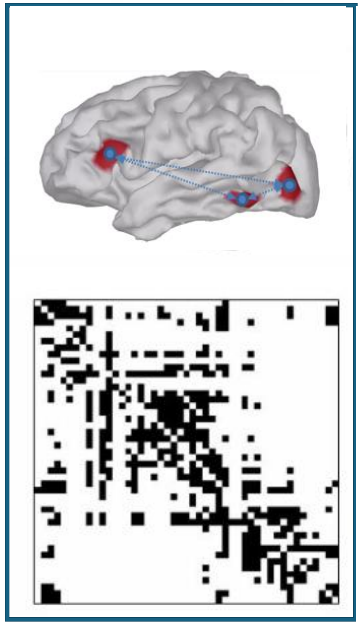

Undirected ($\boldsymbol{A}$: symmetric)




Directed ($\boldsymbol{A}$: asymmetric)

D. Shuman *et al.*, "The emerging field of signal processing on graphs: Extending high-dimensional data analysis to network and other irregular domains" , IEEE Signal Processing Magazine, (30)3, 2013.

# Brain connectivity

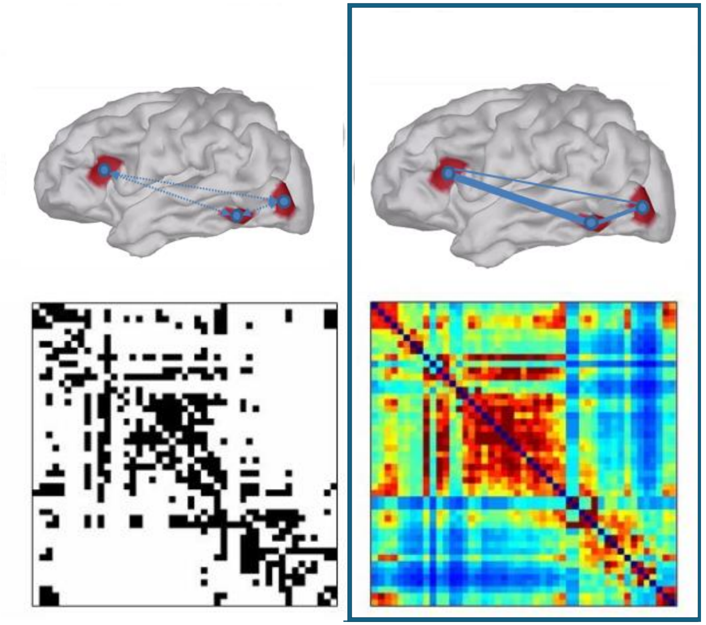


**Structural**

- Structural connectivity also called "**anatomical connectivity**"
- A network of **physical** or **structural links** (**synaptic connections**) between pairs of brain regions

O. Sporns, Brain connectivity. Scholarpedia, Vol. 2 (10): 4695, 2007.

# Brain connectivity

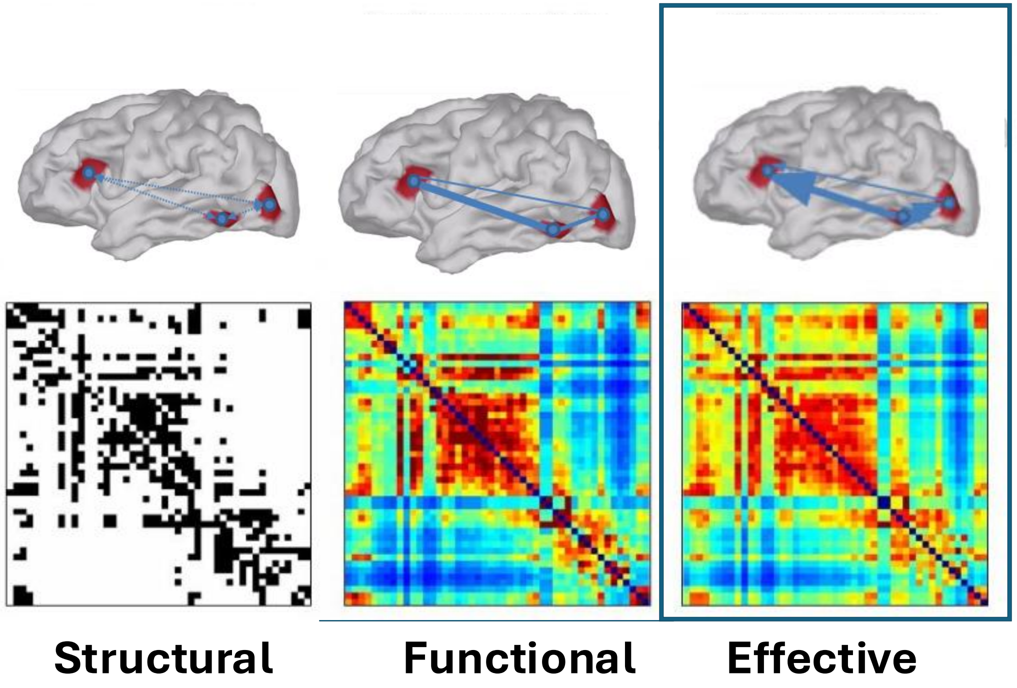


Structural Functional

- **Statistical dependencies** among **remote** neurophysiological events
- **Temporal correlation** among the activities of neural assemblies
- **Undirected graph**
- Connectivity matrix (**symmetric**)

O. Sporns, Brain connectivity. Scholarpedia, Vol. 2 (10): 4695, 2007.

# Brain connectivity




- Completes the structural and functional connectivities
- **Causal influences** between different neurons or neuronal populations
- **Directed graph**
- Connectivity matrix (**asymmetric**)

O. Sporns, Brain connectivity. Scholarpedia, Vol. 2 (10): 4695, 2007.

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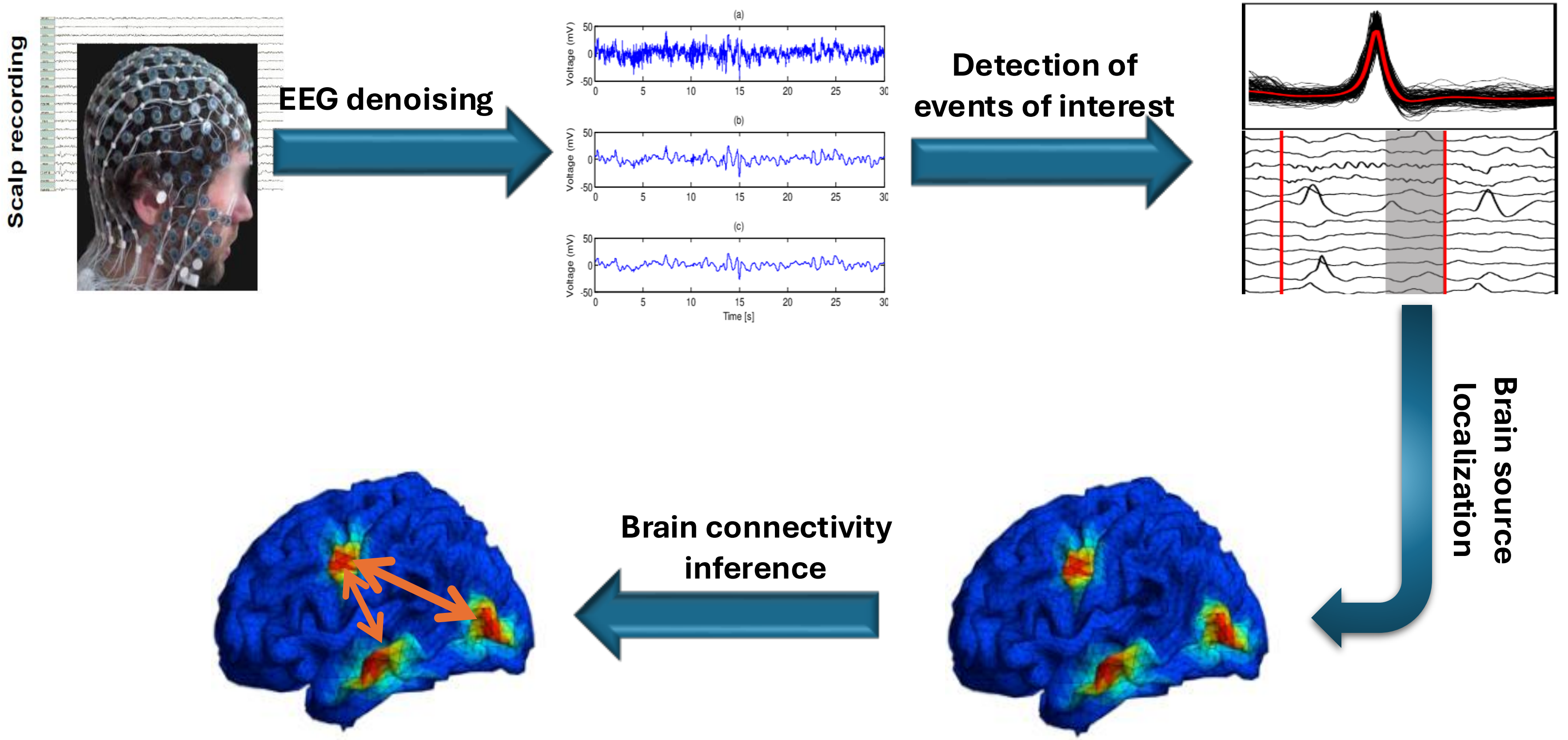
# Conventional strategy

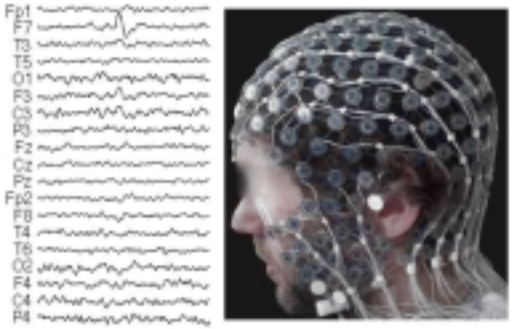


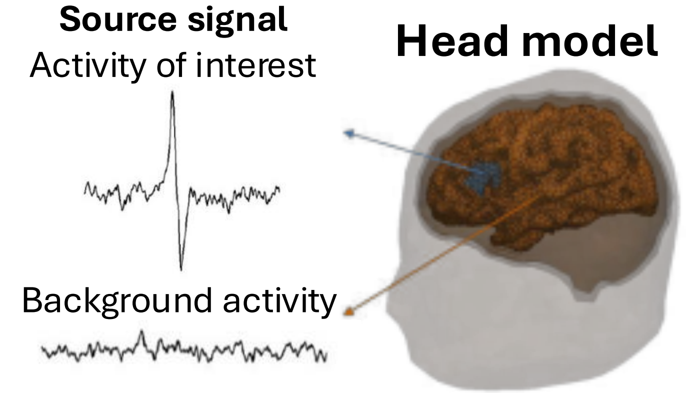
# Background

## ❑ Brain source localization

- **Observations model :** $$\boldsymbol{X} = \boldsymbol{G}\boldsymbol{Y} + \boldsymbol{X}_{art} + \boldsymbol{X}_{inst}$$
  - $\boldsymbol{X} \in \mathbb{R}^{N \times T}$ : observations
  - $\boldsymbol{G} \in \mathbb{R}^{N \times D}$ : Leadfield matrix
  - $\boldsymbol{Y} \in \mathbb{R}^{D \times T}$ : source matrix
  - $\boldsymbol{X}_{art} \in \mathbb{R}^{N \times T}$ : contributions of physiological artefacts
  - $\boldsymbol{X}_{inst} \in \mathbb{R}^{N \times T}$ : instrumental noise

**EEG**

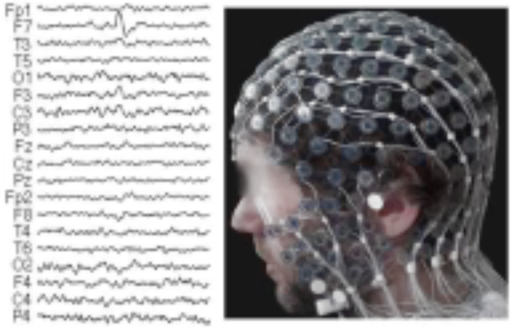





$N$ : Number of sensors
$D$ : Number de dipoles
$T$ : Number of temporal samples

# Background

EEG



## ❑ Brain source localization

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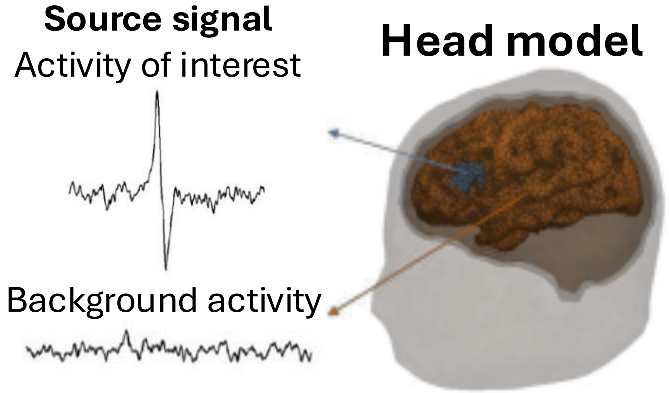
- **Denoised model :** $$\begin{aligned}\boldsymbol{X}_d = \boldsymbol{G}\boldsymbol{Y} &= \boldsymbol{G}\boldsymbol{Y}^{(e)} + \boldsymbol{G}\boldsymbol{Y}^{(b)} \\ &= \boldsymbol{G}\boldsymbol{Y}^{(e)} + \boldsymbol{X}^{(b)}\end{aligned}$$
  - $\boldsymbol{X}_d \in \mathbb{R}^{N\times T}$ : denoised EEG signals
  - $\boldsymbol{Y}^{(e)} \in \mathbb{R}^{D\times T}$ : epileptic activities
  - $\boldsymbol{Y}^{(b)} \in \mathbb{R}^{D\times T}$ : background activity
  - $\boldsymbol{X}^{(b)} \in \mathbb{R}^{N\times T}$ : noise related to the denoised observations




$N$ : Number of sensors
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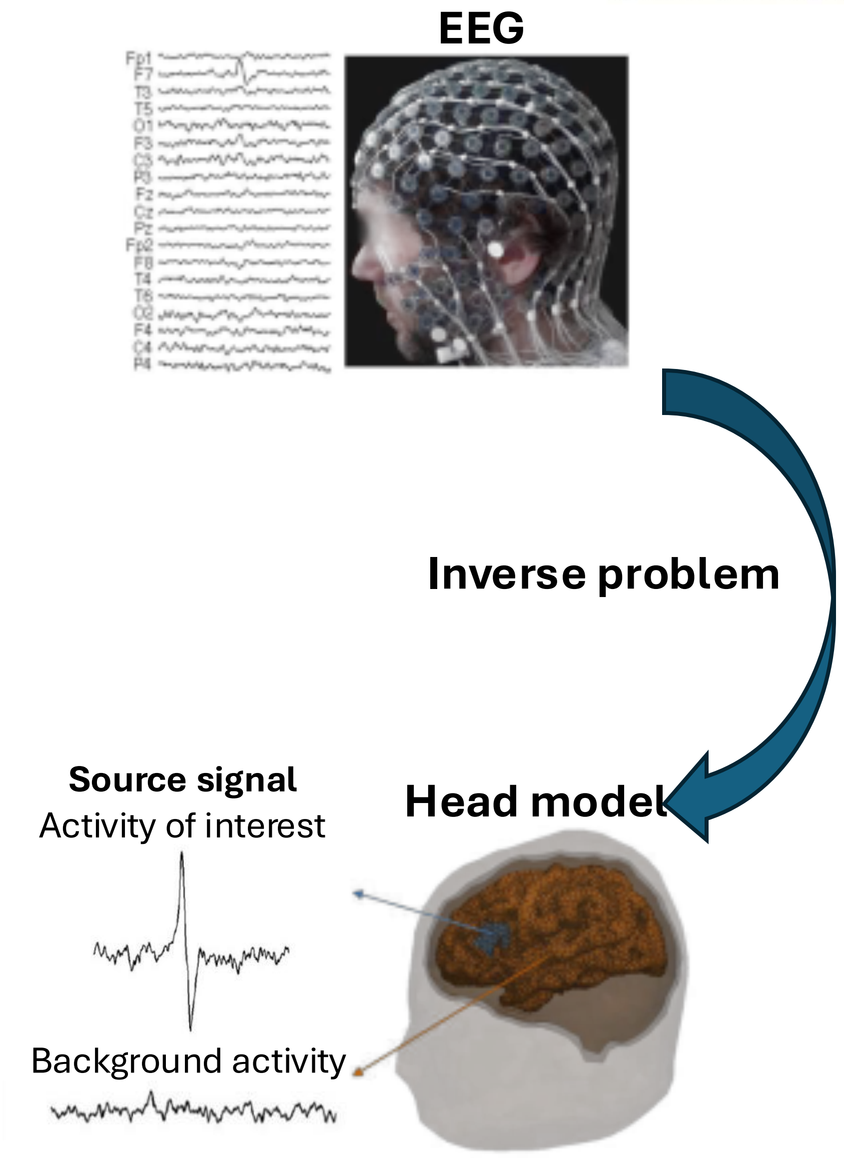
# Background

## ❑ Brain source localization

- Locating epileptic sources on the cortical surface
- Reconstruction of neural activity of each localized sources

$$\boldsymbol{Y}^{(e)^*} = \underset{\boldsymbol{Y}^{(e)}}{\operatorname{argmin}} \left\| \boldsymbol{X}_d - \boldsymbol{G}\boldsymbol{Y}^{(e)} \right\|_{\mathrm{F}}^2$$




$\|.\|_{\mathrm{F}}$ : Frobenius norm

# Background

## ❑ Brain source localization

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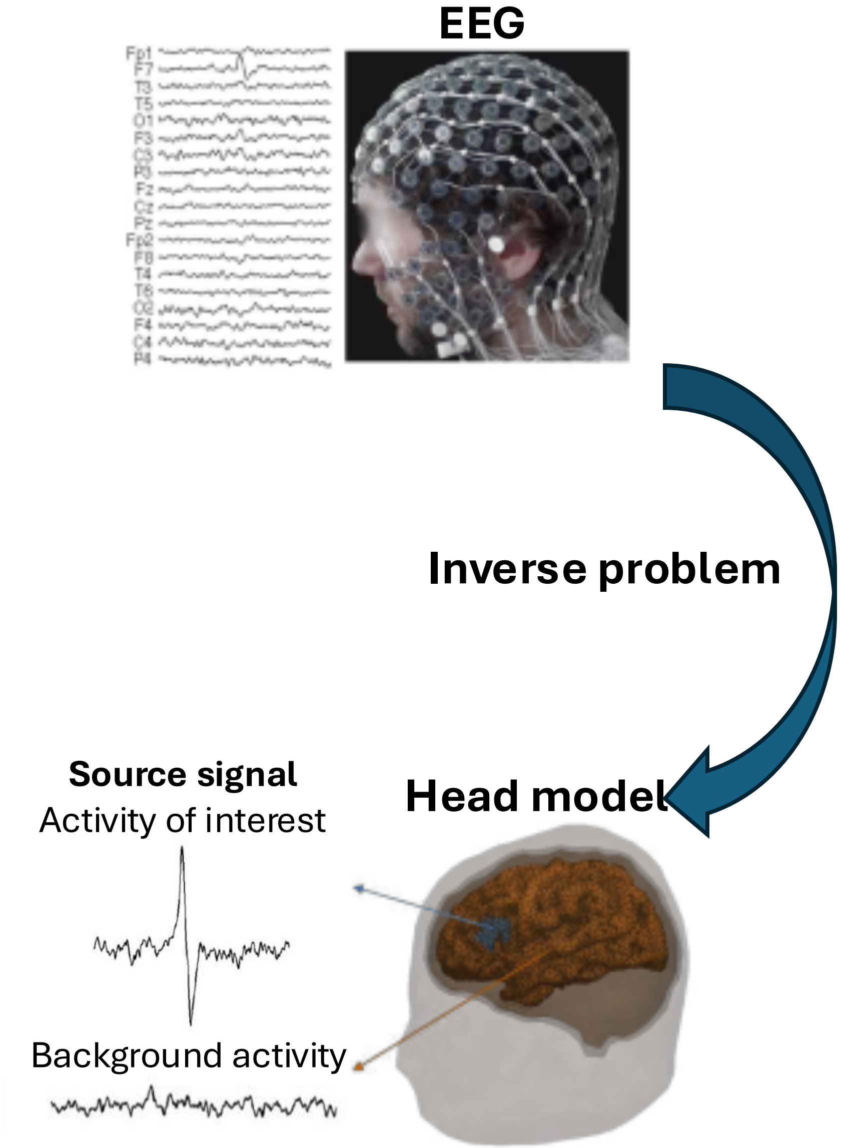
$$\boldsymbol{Y}^{(e)^*} = \underset{\boldsymbol{Y}^{(e)}}{\operatorname{argmin}} \left\| \boldsymbol{X}_d - \boldsymbol{G}\boldsymbol{Y}^{(e)} \right\|_{\mathrm{F}}^2$$

Ill-posed problem




$\|.\|_{\mathrm{F}}$ : Frobenius norm

# Background

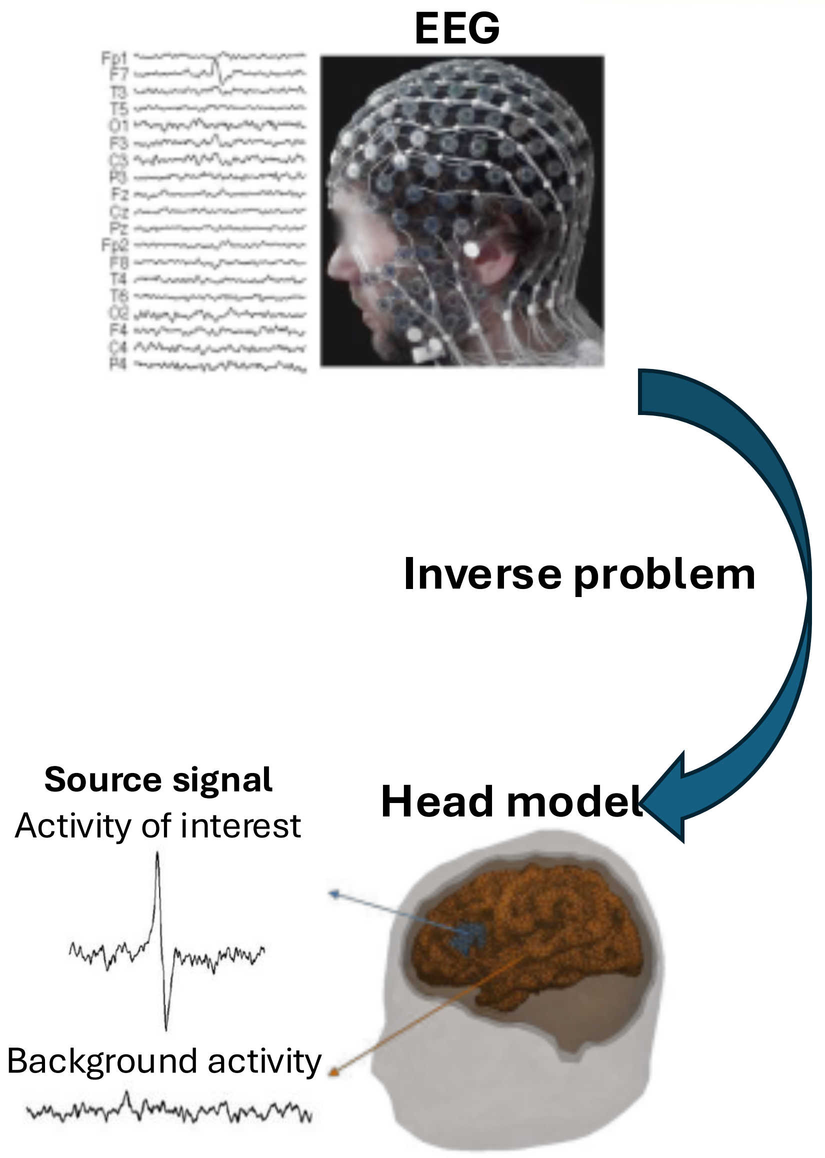
## ❑ Brain source localization

- Locating epileptic sources on the cortical surface
- Reconstruction of neural activity of each localized sources

$$\boldsymbol{Y}^{(e)^*} = \underset{\boldsymbol{Y}^{(e)}}{\operatorname{argmin}} \left\| \boldsymbol{X}_d - \boldsymbol{G}\boldsymbol{Y}^{(e)} \right\|_{\mathrm{F}}^2$$

**Regularization**

$$\boldsymbol{Y}^{(e)^*} = \underset{\boldsymbol{Y}^{(e)}}{\operatorname{argmin}} \left\| \boldsymbol{X}_d - \boldsymbol{G}\boldsymbol{Y}^{(e)} \right\|_{\mathrm{F}}^2 + \sum_{c=1}^{C} \lambda_c f_c\left(\boldsymbol{Y}^{(e)}\right)$$




$\lambda_c$ : $c^{\text{ème}}$ regularization parameter

$f_c\left(\boldsymbol{Y}^{(e)}\right)$ : $c^{\text{th}}$ regularization term

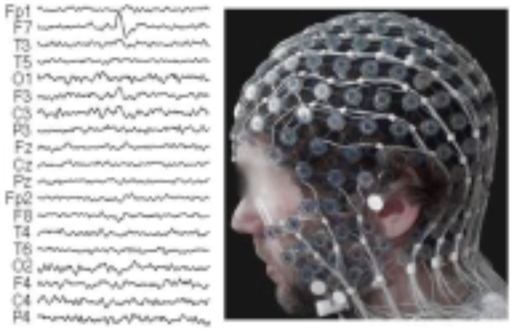
$\|.\|_{\mathrm{F}}$ : Frobenius norm

# Background

## ❑ Brain source localization

- Locating epileptic sources on the cortical surface
- Reconstruction of neural activity of each localized sources

EEG



$$\boldsymbol{Y}^{(e)^*} = \underset{\boldsymbol{Y}^{(e)}}{\operatorname{argmin}} \left\| \boldsymbol{X}_d - \boldsymbol{G}\boldsymbol{Y}^{(e)} \right\|_{\mathrm{F}}^2$$

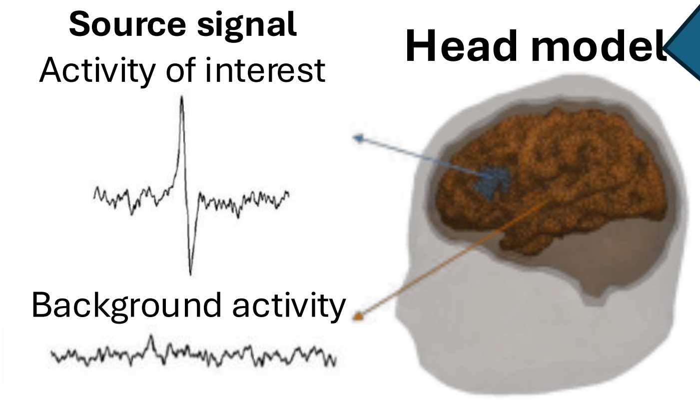
**Regularization**

**Inverse problem**

Prior
(spatial, temporal, spatio-temporal, etc.)

$$\boldsymbol{Y}^{(e)^*} = \underset{\boldsymbol{Y}^{(e)}}{\operatorname{argmin}} \left\| \boldsymbol{X}_d - \boldsymbol{G}\boldsymbol{Y}^{(e)} \right\|_{\mathrm{F}}^2 + \sum_{c=1}^{C} \lambda_c f_c\left(\boldsymbol{Y}^{(e)}\right)$$

**Head model**



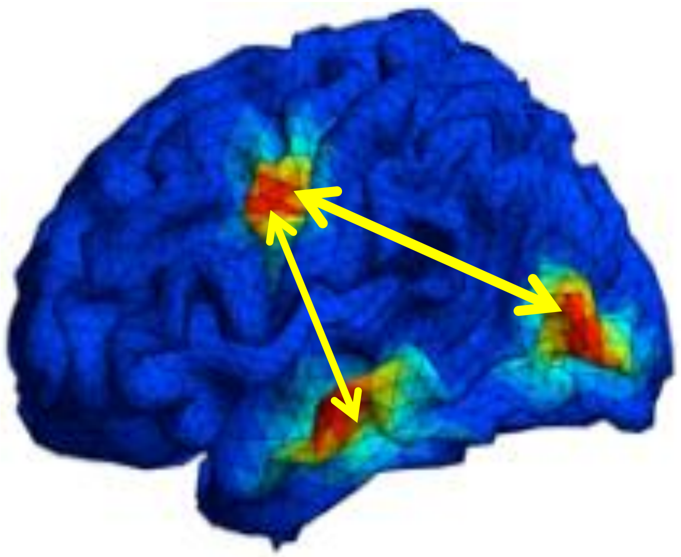

$\lambda_c$ : $c^{\text{ème}}$ regularization parameter
$f_c\left(\boldsymbol{Y}^{(e)}\right)$ : $c^{\text{th}}$ regularization term
$\|.\|_{\mathrm{F}}$ : Frobenius norm

# Background

## ❑ Effective connectivity inference

- **Data-driven techniques**
  - Directed coherence
  - Mutual information
  - Transfer entropy
  - Granger measure
  - Directed transfer function (DTF)
  - Partial directed coherence (PDC)

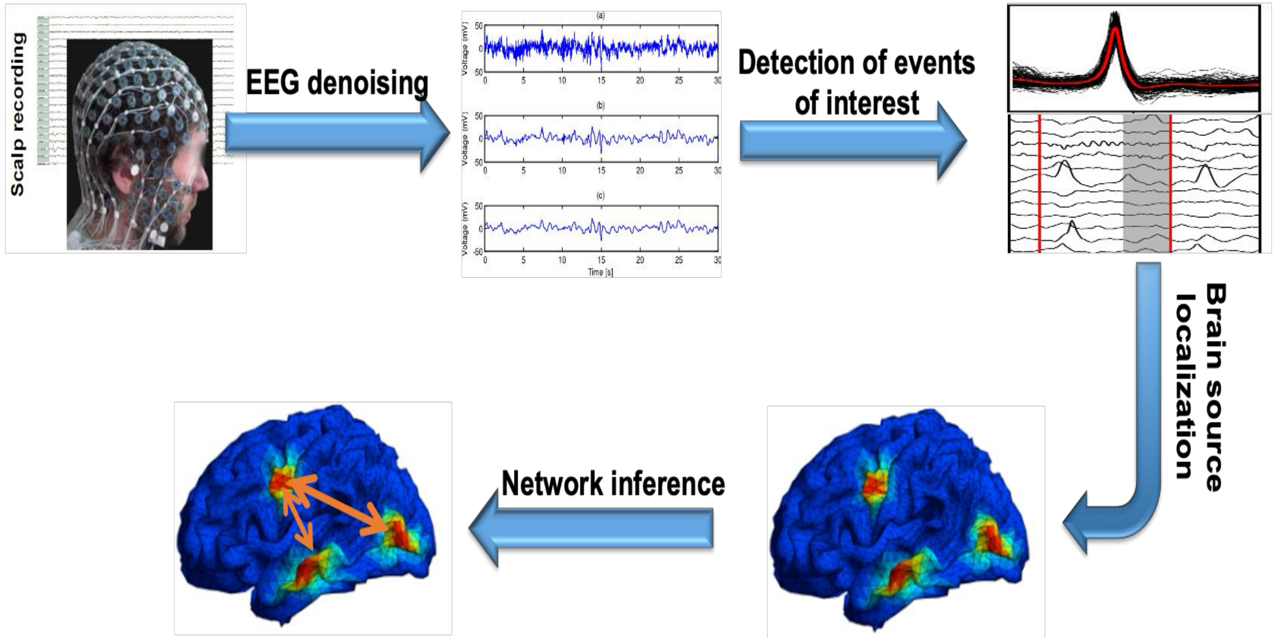
- **Model-based techniques**
  - Structural equation modeling (SEM)
  - Dynamic causal modeling (DCM)



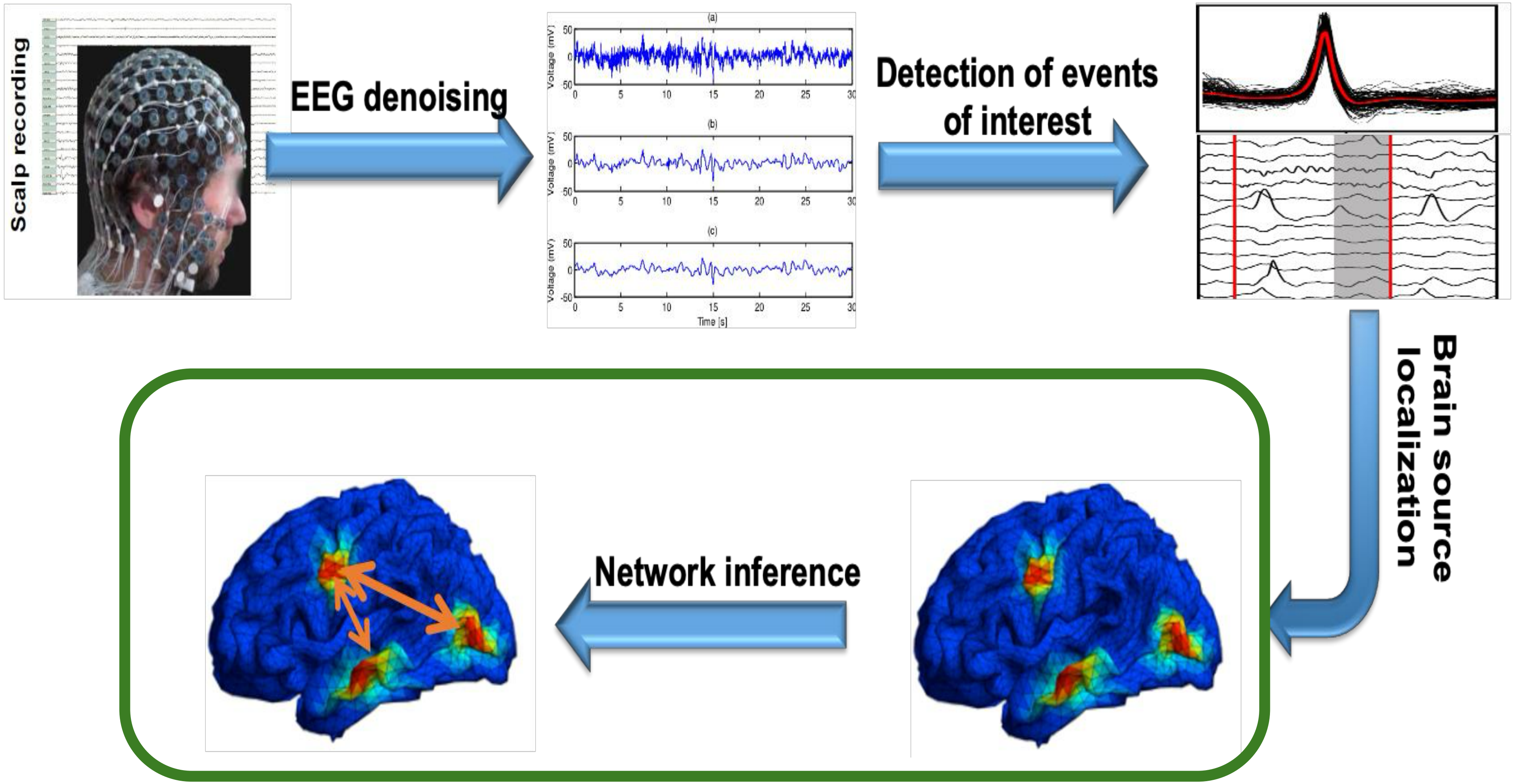
# Conventional strategy

## ❑ Limitations

- Network inference quality is subject to the source localization’s one
- Lack of the best combination **brain source localization method - brain connectivity measure**



# Motivation


Scalp recording
EEG denoising
Detection of events of interest
Brain source localization
Network inference

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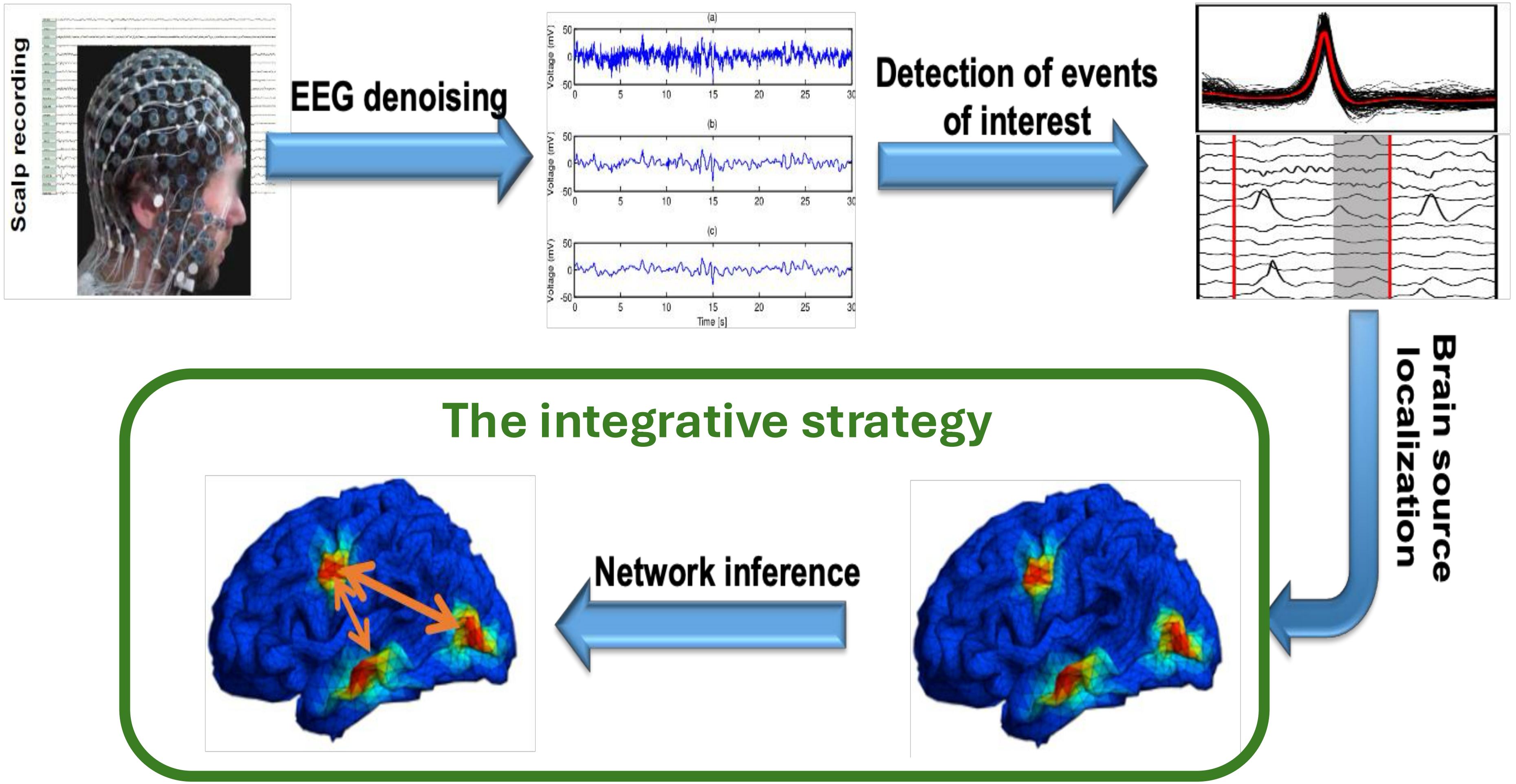
# Motivation



# The proposed integrative strategy

- **Physiological assumptions:**
  - Epileptic sources of minimal spatial energy (ex. MNE, wMNE algorithms)
  - Synchronized neural activities:
    - ✓ among current dipoles inside each extended epileptic sources

M. S. Hämäläinen, R. J. Ilmoneimi, Interpreting measured magnetic fields of the brain : minimum norm estimates of current distributions, Technical Report TKK-F-A559, Helsinki University of Technology, 1984.
R. D. Pascual-Marqui, C. M. Michel, D. Lehmann, Low resolution electromagnetic tomography: A new method for localizing electrical activity in the brain, International Journal of Psychophysiology, vol. 18, no. 1, pp. 49-65, 1994.

# The proposed integrative strategy

Inverse problem ➕ Connectivity

$$\boldsymbol{Y}^{(e)*}, \boldsymbol{\Theta}^* = \underset{\boldsymbol{Y}^{(e)}, \boldsymbol{\Theta}}{\operatorname{argmin}} \left\| \boldsymbol{X}_d - \boldsymbol{G}\boldsymbol{Y}^{(e)} \right\|_{\mathrm{F}}^2 + \sum_{c=1}^{C} \lambda_c f_c\left(\boldsymbol{Y}^{(e)}\right) + \gamma \underbrace{g(\boldsymbol{Y}^{(e)}, \boldsymbol{\Theta})}_{\text{Source model}}$$

$\lambda_c, \beta > 0$ : paramètres de régularisation ($1 \leq c \leq C$)
$\boldsymbol{\Theta}$: matrix of model parameters

# The proposed integrative strategy

Inverse problem

Connectivity

$$\boldsymbol{Y}^{(e)*}, \boldsymbol{\Theta}^* = \underset{\boldsymbol{Y}^{(e)}, \boldsymbol{\Theta}}{\operatorname{argmin}} \left\| \boldsymbol{X}_d - \boldsymbol{G}\boldsymbol{Y}^{(e)} \right\|_{\mathrm{F}}^2 + \sum_{c=1}^{C} \lambda_c f_c\left(\boldsymbol{Y}^{(e)}\right) + \underbrace{\gamma g(\boldsymbol{Y}^{(e)}, \boldsymbol{\Theta})}_{\text{Source model}}$$

Source model

- MVAR modelling of the pre-ictal epileptic spikes

$$\boldsymbol{Y}^{(e)} = \sum_{l=1}^{L} \boldsymbol{\Theta}^l \boldsymbol{Y}_l^{(e)} + \boldsymbol{W}$$

$\lambda_c, \beta > 0$ : Régularisation parameters ($1 \leq c \leq C$)
$\boldsymbol{\Theta} \in \mathbb{R}^{D \times D}$: Matrix of model parameters
$\boldsymbol{Y}^{(e)} \in \mathbb{R}^{D \times T}$: Spatio-temporal depth source matrix
$\boldsymbol{Y}_l^{(e)} \in \mathbb{R}^{D \times T}$: Delayed spatio-temporal depth source matrix
$\boldsymbol{\Theta}^l \in \mathbb{R}^{D \times D}$: Matrix of model parameters associated with $l^{th}$ time lag
$\boldsymbol{W} \in \mathbb{R}^{D \times T}$: Model error

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# The proposed integrative strategy

Inverse problem + Connectivity

$$\boldsymbol{Y}^{(e)*}, \boldsymbol{\Theta}^* = \underset{\boldsymbol{Y}^{(e)},\boldsymbol{\Theta}}{\operatorname{argmin}} \underbrace{\left\|\boldsymbol{X}_d - \boldsymbol{G}\boldsymbol{Y}^{(e)}\right\|_{\mathrm{F}}^2 + \sum_{c=1}^{C} \lambda_c f_c\left(\boldsymbol{Y}^{(e)}\right)} \; + \; \gamma \underbrace{g(\boldsymbol{Y}^{(e)}, \boldsymbol{\Theta})}$$

wMNE : $\underset{\boldsymbol{Y}^{(e)},\boldsymbol{\Theta}}{\operatorname{argmin}} \left\|\boldsymbol{X}_d - \boldsymbol{G}\boldsymbol{Y}^{(e)}\right\|_{\mathrm{F}}^2 + \lambda \left\|\boldsymbol{B}\boldsymbol{Y}^{(e)}\right\|_{\mathrm{F}}^2$

Source model

- MVAR modelling of the pre-ictal epileptic spikes

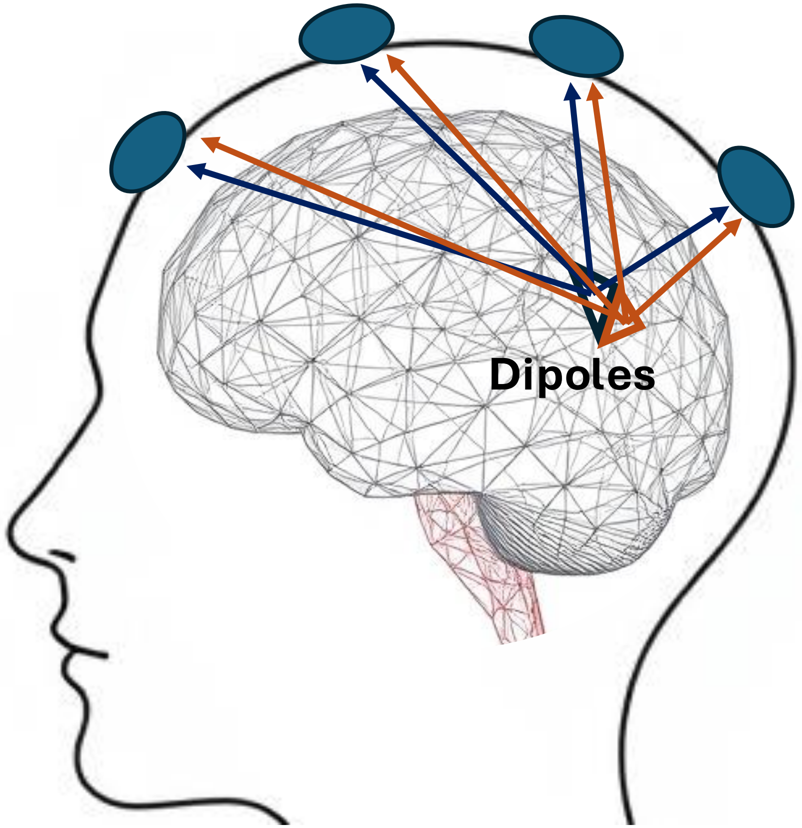
$$\boldsymbol{Y}^{(e)} = \sum_{l=1}^{L} \boldsymbol{\Theta}^l \boldsymbol{Y}_l^{(e)} + \boldsymbol{W}$$

$\lambda_c, \gamma > 0$ : Regularization parameters ($1 \leq c \leq C$)
$\boldsymbol{G} = [\boldsymbol{g_1}, \cdots, \boldsymbol{g_D}] \in \mathbb{R}^{N\times D}$ : Leadfield matrix
$\boldsymbol{Y}^{(e)} \in \mathbb{R}^{D\times T}$ : Delayed version of $\boldsymbol{Y}$
$\boldsymbol{\Theta}^l \in \mathbb{R}^{D\times D}$: Connectivity matrix related to à $\boldsymbol{Y}^l$
MVAR : MultiVariate AutoregRessive
$L$ : MVAR model order
$D$ : Number of dipoles

$$\boldsymbol{B} = \begin{pmatrix} \frac{\mathbf{1}}{\|\boldsymbol{g_1}\|_2} & & 0 \\ & \ddots & \\ 0 & & \frac{\mathbf{1}}{\|\boldsymbol{g_D}\|_2} \end{pmatrix}$$

# The proposed integrative strategy

- MVAR model for each current electrical dipole ➡ Computational burden



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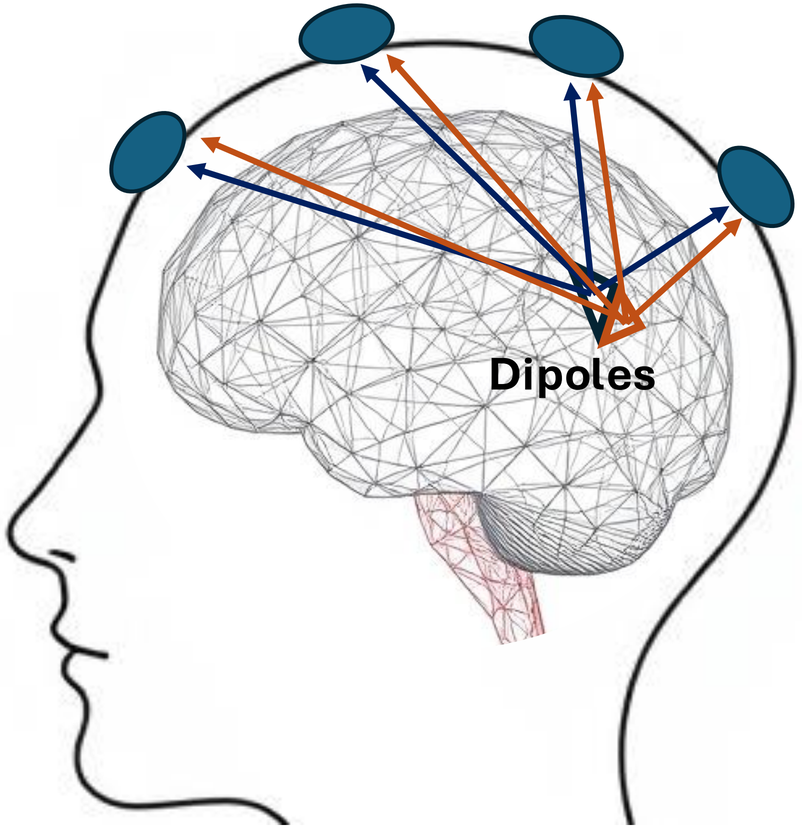
# The proposed integrative strategy

- MVAR model for each current electrical dipole ➡ Computational burden

- **Solution:** Region-wise MVAR modelling



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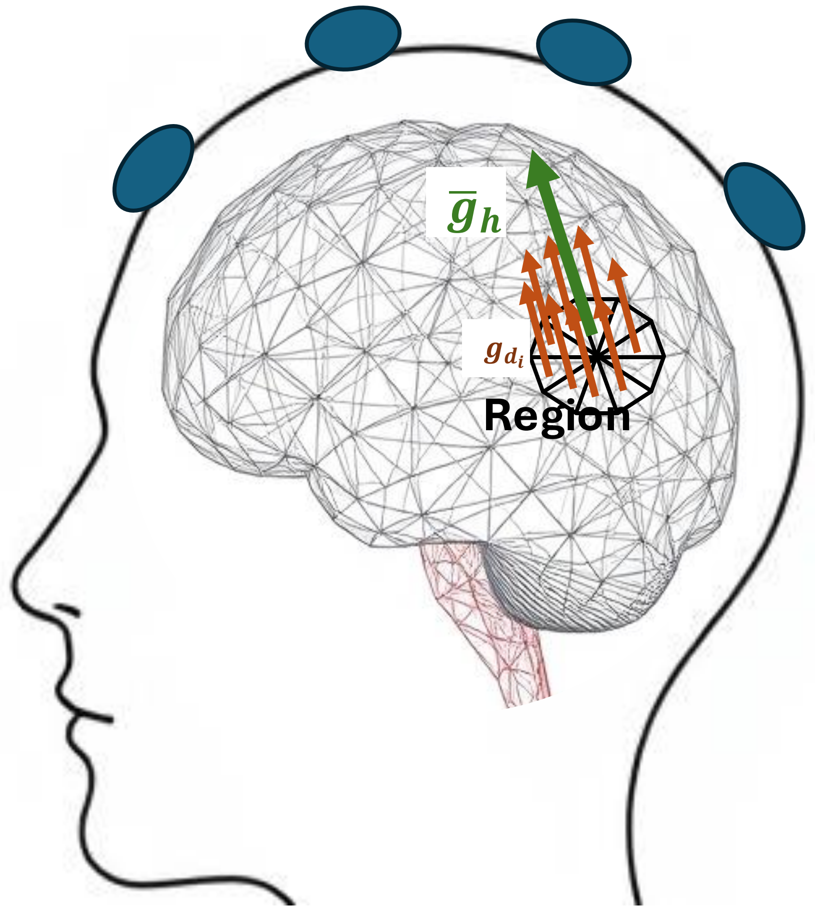
# The proposed integrative strategy

## ❑ Dipole-wise ⟶ Region-wise manner

- ✓ Reformulating the problem at the level of brain regions
- ✓ Dipoles belonging to the same region → Region's steering vector as a linear combination of dipoles' steering vectors

$$
\begin{aligned}
\boldsymbol{X}_d &= \overline{\boldsymbol{G}}\overline{\boldsymbol{Y}}^{(e)} + \overline{\boldsymbol{G}}\overline{\boldsymbol{Y}}^{(b)} \\
&= \overline{\boldsymbol{G}}\overline{\boldsymbol{Y}}^{(e)} + \overline{\boldsymbol{X}}^{(b)}
\end{aligned}
$$




$N$ : Number of sensors
$H$ : Number of regions
$T$ : Number of time samples
$\overline{\boldsymbol{G}} = [\overline{\boldsymbol{g}}_1, \cdots, \overline{\boldsymbol{g}}_H] \in \mathbb{R}^{N\times H}$ : Region-wise Leadfield matrix
$\overline{\boldsymbol{Y}}^{(e)} \in \mathbb{R}^{H\times T}$ : Region-wise sources matrix

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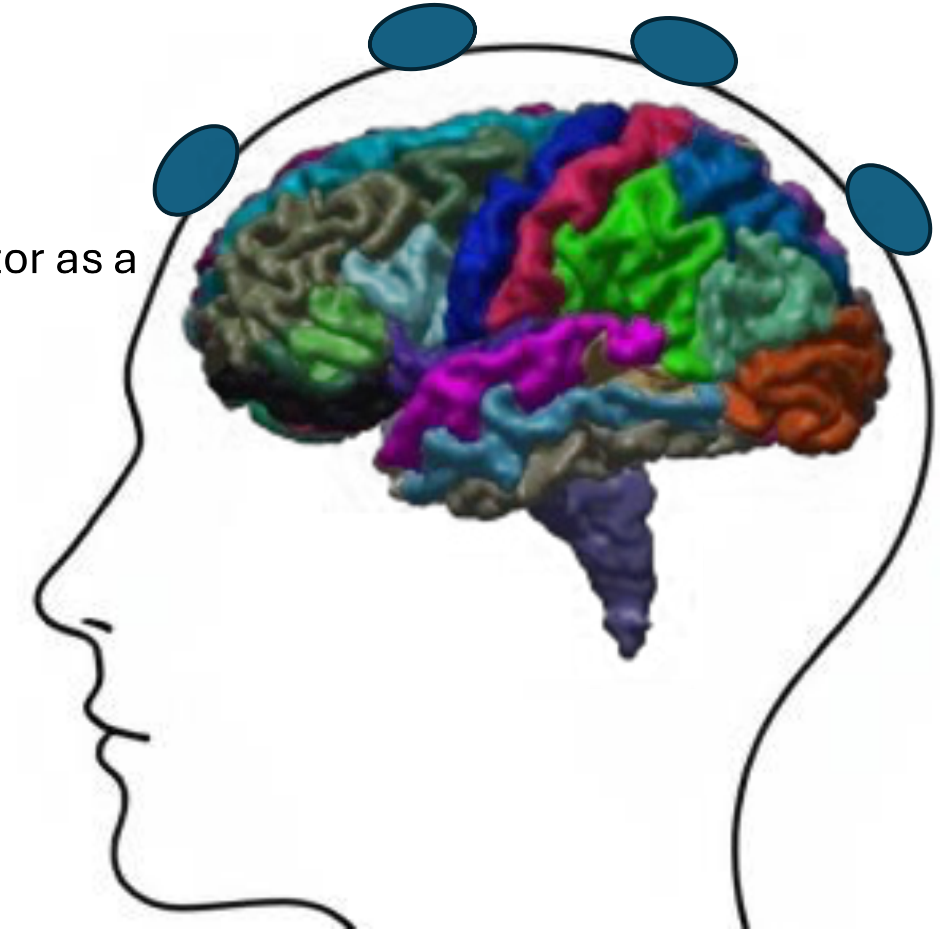
# The proposed integrative strategy

## ❑ Dipole-wise ⟶ Region-wise manner

- ✓ Reformulating the problem at the level of brain regions.
- ✓ Dipoles belonging to the same region → Region's steering vector as a linear combination of dipoles' steering vectors



$$\begin{aligned} \boldsymbol{X}_d &= \overline{\boldsymbol{G}}\overline{\boldsymbol{Y}}^{(e)} + \overline{\boldsymbol{G}}\overline{\boldsymbol{Y}}^{(b)} \\ &= \overline{\boldsymbol{G}}\overline{\boldsymbol{Y}}^{(e)} + \overline{\boldsymbol{X}}^{(b)} \end{aligned}$$

$N$ : Number of sensors
$H$ : Number of regions
$T$ : Number of time samples
$\overline{\boldsymbol{G}} = [\overline{\boldsymbol{g}}_{\boldsymbol{1}}, \cdots, \overline{\boldsymbol{g}}_H] \in \mathbb{R}^{N\times H}$ : Region-wise Leadfield matrix
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# The proposed integrative strategy

## ❑ Problem formulation

$$\overline{\boldsymbol{Y}}^{(e)*}, \left\{\overline{\boldsymbol{Y}}_l^{(e)}\right\}_{1\le l\le L}, \left\{\overline{\boldsymbol{\Theta}}^{l*}\right\}_{1\le l\le L},$$

$$= \operatorname{argmin}_{\overline{\boldsymbol{Y}}^{(e)}, \left\{\overline{\boldsymbol{Y}}_l^{(e)}\right\}_{1\le l\le L}, \left\{\overline{\boldsymbol{\Theta}}^{l}\right\}_{1\le l\le L},} \left\|\boldsymbol{X}_d - \overline{\boldsymbol{G}}\overline{\boldsymbol{Y}}_l^{(e)}\right\|_{\mathrm{F}}^2 + \lambda\left\|\overline{\boldsymbol{B}}\overline{\boldsymbol{Y}}^{(\boldsymbol{e})}\right\|_{\mathrm{F}}^2 + \xi\sum_{l=1}^{L}\left\|\overline{\boldsymbol{B}}\overline{\boldsymbol{Y}}_l^{(e)}\right\|_{\mathrm{F}}^2 + \gamma\left\|\overline{\boldsymbol{Y}}^{(e)} - \sum_{l=1}^{L}\overline{\boldsymbol{\Theta}}^{l}\overline{\boldsymbol{Y}}_l^{(e)}\right\|_{\mathrm{F}}^2 + \beta\sum_{l=1}^{L}\left\|\overline{\boldsymbol{\Theta}}^{l}\right\|_1$$

$$\forall l \in \{1, \dots, L\}$$

Weighting matrix : $\overline{\boldsymbol{B}} = \begin{pmatrix} \frac{\mathbf{1}}{\|\overline{\boldsymbol{g}}_{\mathbf{1}}\|_{\mathbf{2}}} & & 0 \\ & \ddots & \\ 0 & & \frac{\mathbf{1}}{\|\overline{\boldsymbol{g}}_{\boldsymbol{D}}\|_{\mathbf{2}}} \end{pmatrix}$

# The proposed integrative strategy

## ❑ Problem formulation

$$\overline{\boldsymbol{Y}}^{(e)*}, \left\{\overline{\boldsymbol{Y}}_l^{(e)}\right\}_{1\leq l\leq L}, \left\{\overline{\boldsymbol{\Theta}}^{l*}\right\}_{1\leq l\leq L},$$

$$= \operatorname{argmin}_{\overline{\boldsymbol{Y}}^{(e)},\left\{\overline{\boldsymbol{Y}}_l^{(e)}\right\}_{1\leq l\leq L},\left\{\overline{\boldsymbol{\Theta}}^{l}\right\}_{1\leq l\leq L}} \underbrace{\boxed{\left\|\boldsymbol{X}_d - \overline{\boldsymbol{G}}\overline{\boldsymbol{Y}}_l^{(e)}\right\|_{\mathrm{F}}^2 + \lambda\left\|\overline{\boldsymbol{B}}\overline{\boldsymbol{Y}}^{(e)}\right\|_{\mathrm{F}}^2}}_{\text{wMNE}} + \xi\sum_{l=1}^{L}\left\|\overline{\boldsymbol{B}}\overline{\boldsymbol{Y}}_l^{(e)}\right\|_{\mathrm{F}}^2 + \gamma\underbrace{\boxed{\left\|\overline{\boldsymbol{Y}}^{(e)} - \sum_{l=1}^{L}\overline{\boldsymbol{\Theta}}^{l}\overline{\boldsymbol{Y}}_l^{(e)}\right\|_{\mathrm{F}}^2}}_{\text{MVAR}} + \beta\sum_{l=1}^{L}\left\|\overline{\boldsymbol{\Theta}}^{l}\right\|_1$$

$$\forall l \in \{1, \dots, L\}$$

- ➢ Solution within the framework of proximal optimisation
  - ➢ Iterative approach
  - ➢ The PALM method as a solver

$$\overline{\boldsymbol{B}} = \begin{pmatrix} \dfrac{\mathbf{1}}{\|\overline{\boldsymbol{g}}_{\mathbf{1}}\|_{\mathbf{2}}} & & 0 \\ & \ddots & \\ 0 & & \dfrac{\mathbf{1}}{\|\overline{\boldsymbol{g}}_{\boldsymbol{D}}\|_{\mathbf{2}}} \end{pmatrix}$$

$\lambda, \gamma, \beta > 0$ : Regularization parameters
$\overline{\boldsymbol{G}} = [\overline{\boldsymbol{g}}_{\mathbf{1}}, \cdots, \overline{\boldsymbol{g}}_H] \in \mathbb{R}^{N\times H}$ : Region-wise Leadfield matrix
$\overline{\boldsymbol{Y}}^l \in \mathbb{R}^{D\times T}$ : Delayed version of $\overline{\boldsymbol{Y}}$
$\overline{\boldsymbol{\Theta}}^l \in \mathbb{R}^{D\times D}$: region-wise connectivity matrix related to the $l^{th}$ time lag
MVAR : MultiVariate AutoregRessive
$L$ : MVAR model order

J. Bolte *et al*. “Proximal alternating linearized minimization for nonconvex and nonsmooth problems”, Mathematical Programming, (146), 2014.

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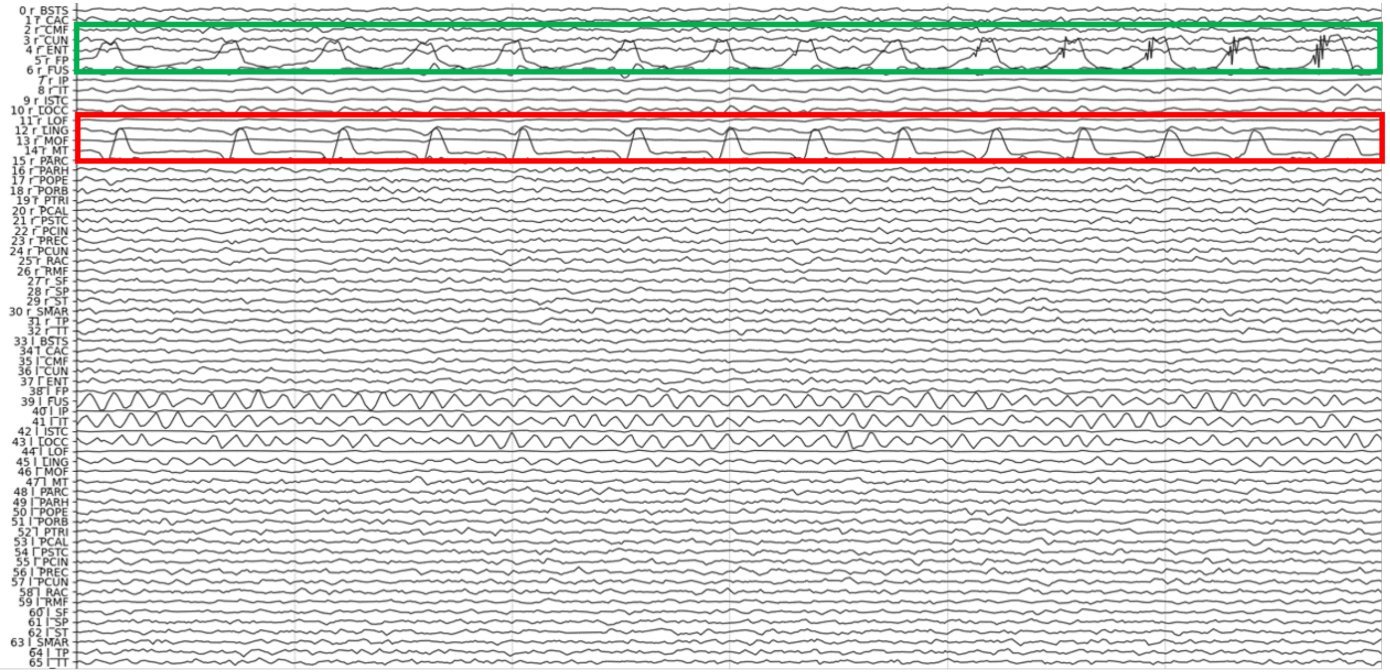
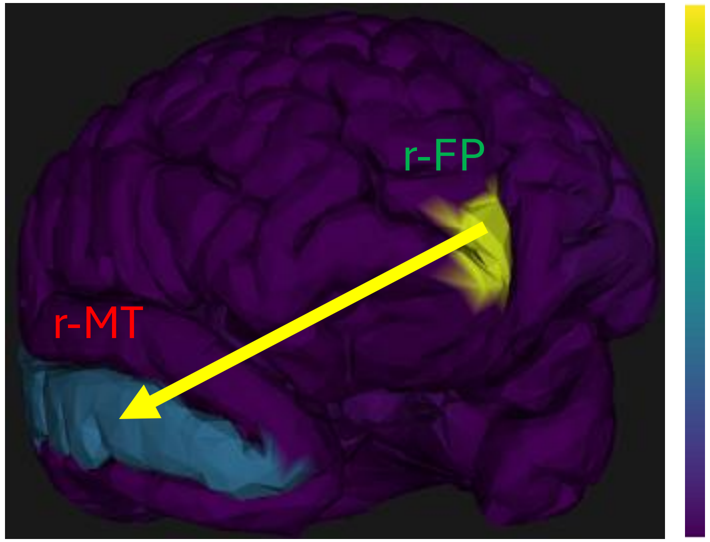
# Simulation setup

- Desikan-Killiany atlas (66 regions)
- Two connected epileptic regions with information flow ''r-FP'' →''r-MT''
- 257 High-Resolution electroencephalography recordings, 60 sec length
- Samplng frequency 1024 Hz
- Comparison with the conventional strategy :
  - wMNE method for source localization
  - Granger causality for effective connectivity inference

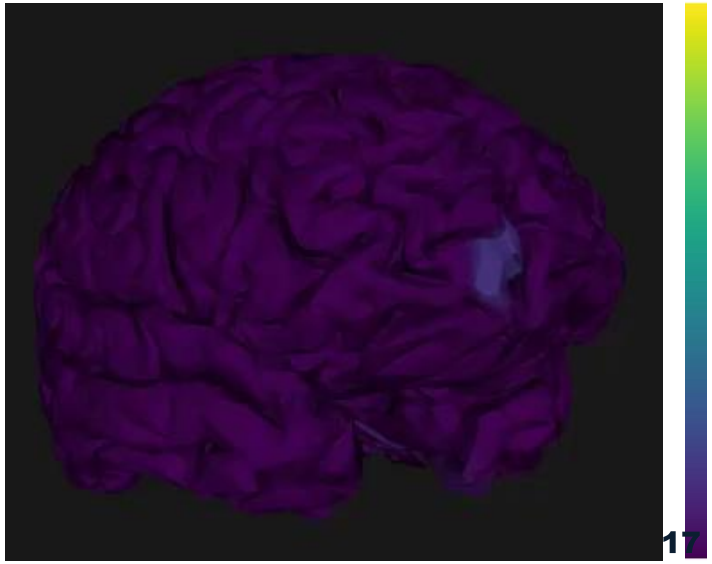


Ground truth: Pre-ictal phase

R- : Right hemisphere
r-FP : Right Frontal Pole
r-MT : Right Middle Temporal




Ground truth: Pre-ictal phase



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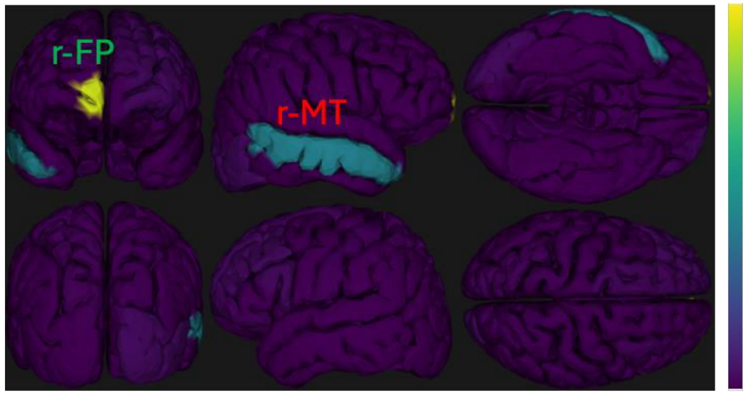
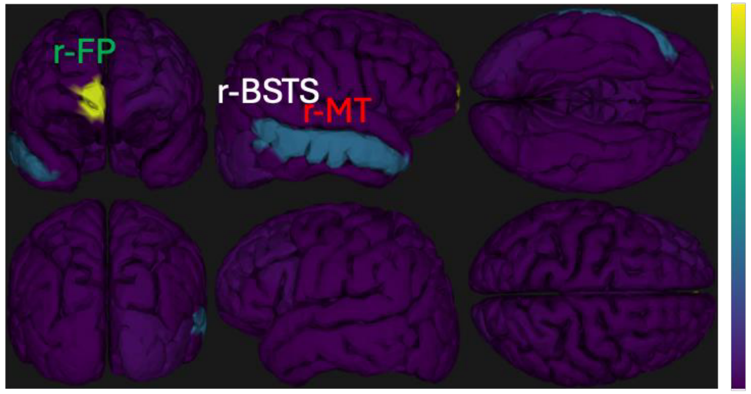
# Numerical results

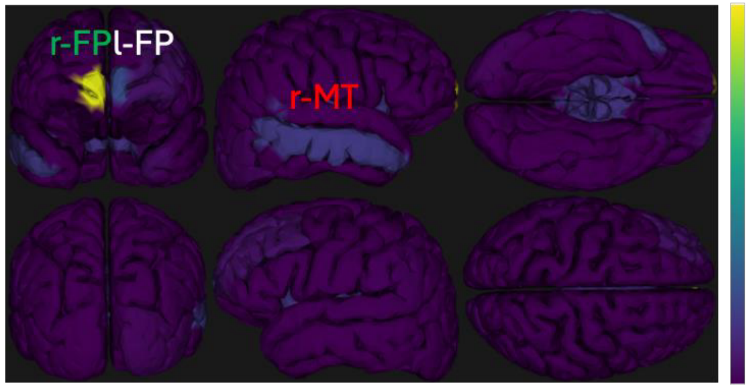
## ❑ Brain source localization




Ground truth




Integrative strategy

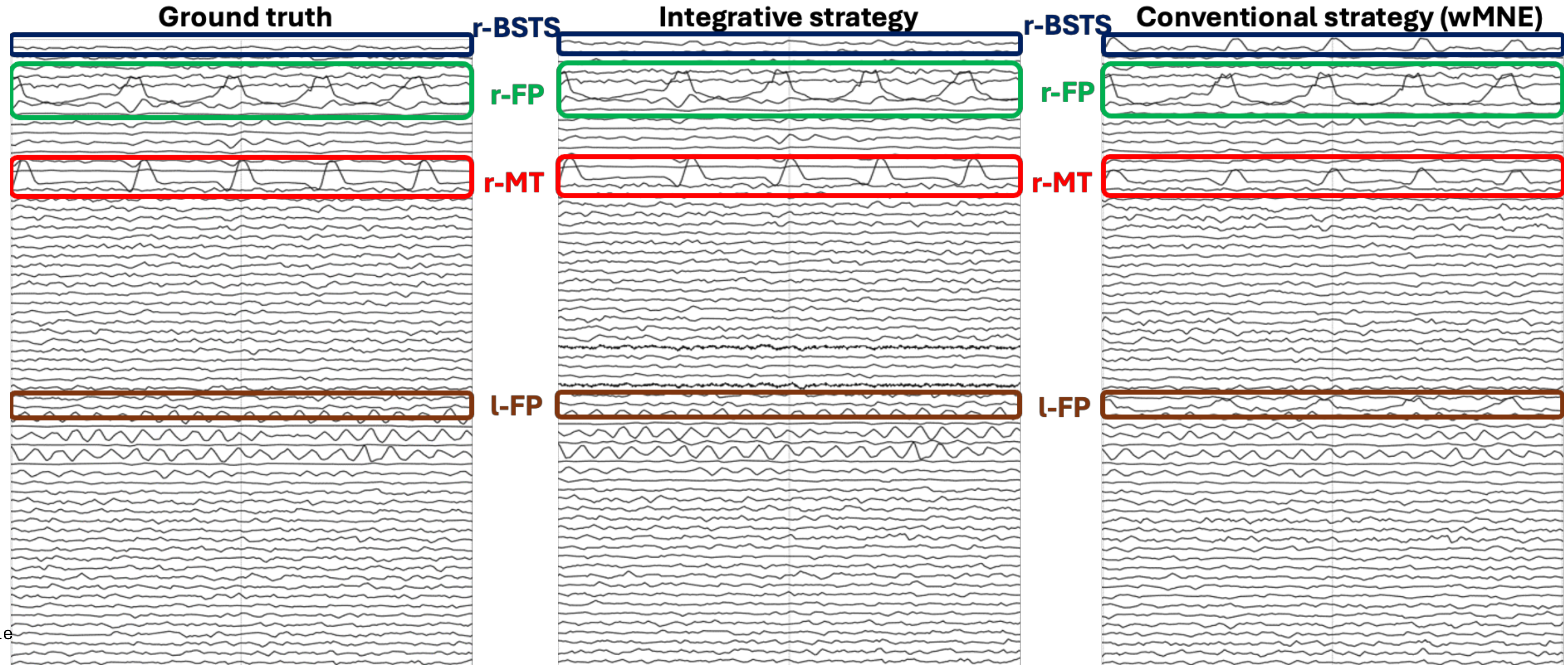



Conventional strategy

r- : right hemisphere
l- : left hemisphere
r-FP : right Frontal Pole
l-FP : left Frontal Pole
r-MT : right Middle Temporal
r-BSTS : right Banks of the Superior Temporal Sulcus
l-BSTS : left Banks of the Superior Temporal Sulcus
wMNE : weighted Minimum Norm Estimate

# Numerical results

- Brain source localization – reconstructed neural activities




r- : right hemisphere
l- : left hemisphere
r-FP : right Frontal Pole
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r-MT : right Middle Temporal
r-BSTS : right Banks of the Superior Temporal Sulcus
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wMNE : weighted Minimum Norm Estimate

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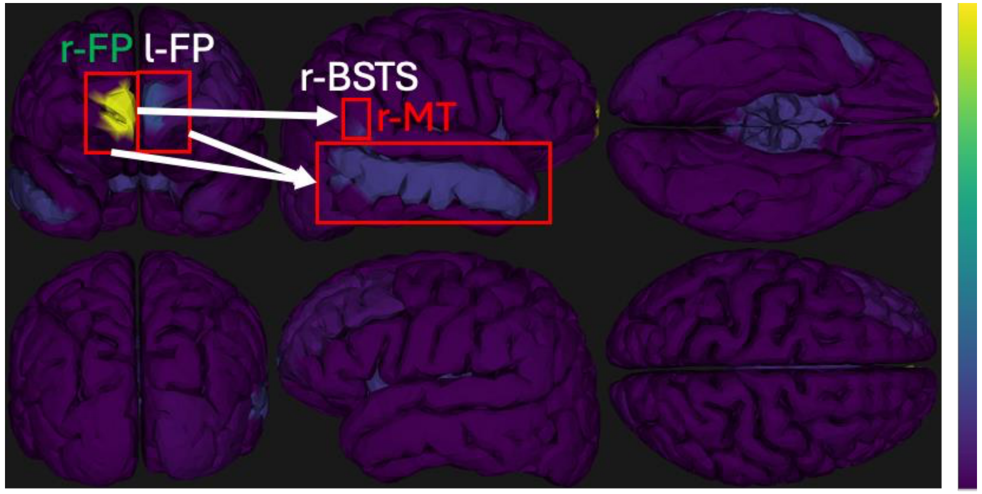
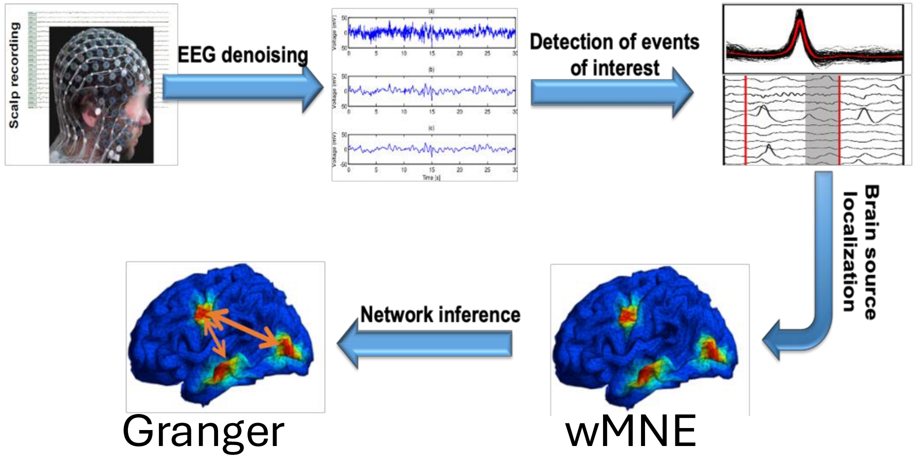
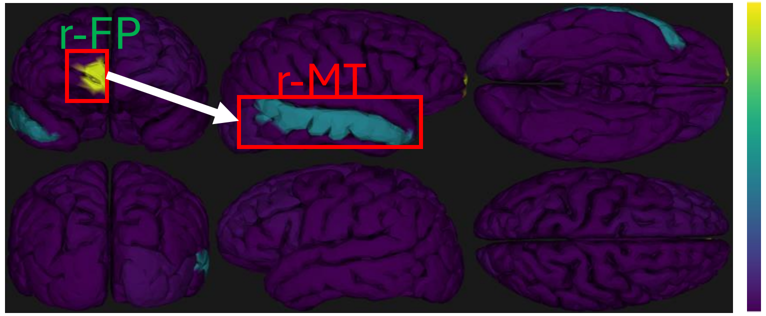
# Numerical results

## ❑ Effective connectivity inference: Conventional strategy




**Conventional strategy (90% thresholding)**




**Ground truth**

| Interaction 1 | Interaction 2 | Interaction 3 | Interaction 4 |
|---|---|---|---|
| l-FP → r-MT | r-FP → r-MT | r-FP → r-BSTS | l-FP → r-BSTS |

**Information flow (90% thresholding)**

r- : right hemisphere
l- : left hemisphere
r-FP : right Frontal Pole
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r-MT : right Middle Temporal
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wMNE : weighted Minimum Norm Estimate

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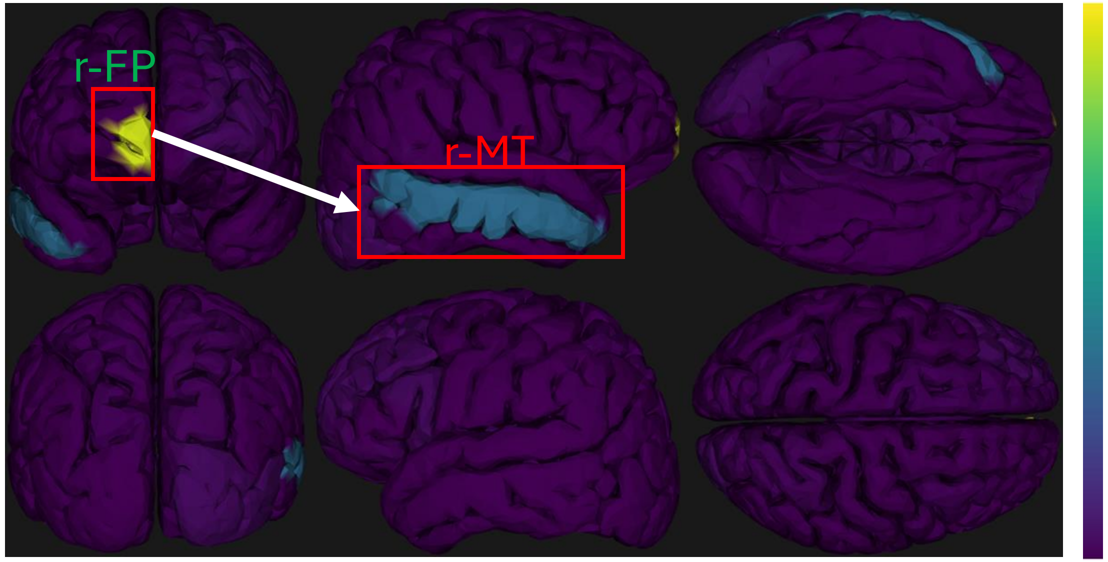
# Numerical results

- Effective connectivity inference: Integrative strategy



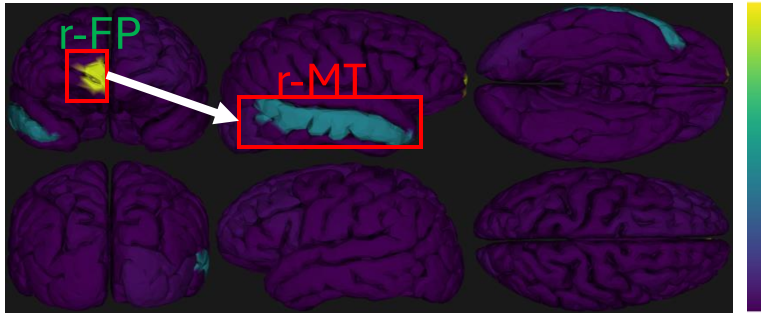

Integrative strategy (90% thresholding)

$\Theta^l$

$1^{st}$ time lag

r-LOF → l-FP

l-SP → r-CMF




Ground truth

r- : right hemisphere
l- : left hemisphere

LOF : Lateral OrbitoFrontal gyrus
FP : Frontal Pole
SP : Superior Parietal gyrus
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MT : Middle Temporal gyrus
SF : Superior Frontal gyrus
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PREC : PRECentral gyrus
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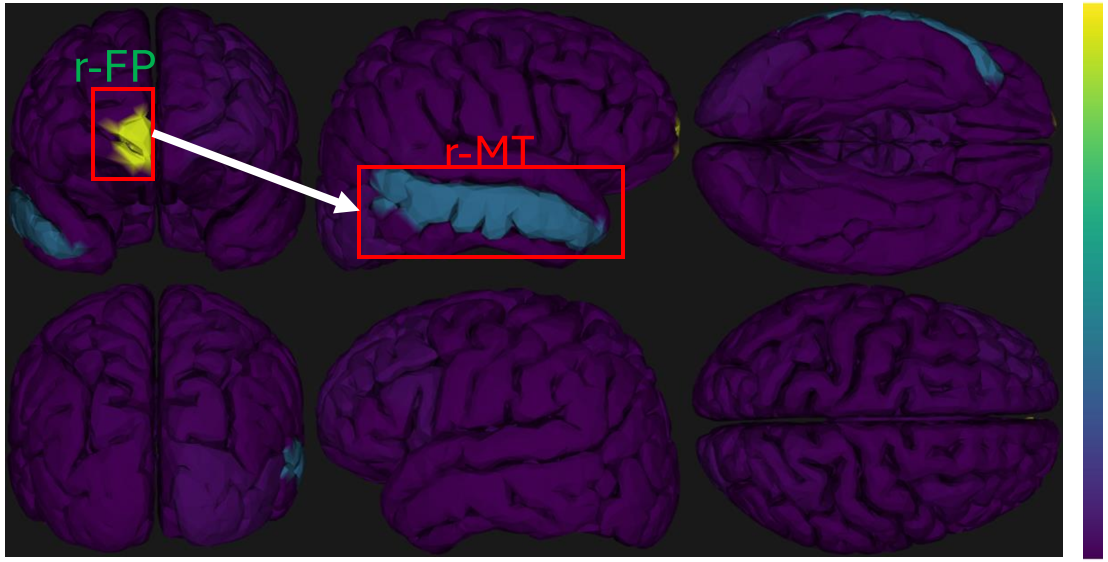
PCAL : Pericalcarine Cortex

# Numerical results

- Effective connectivity inference: Integrative strategy

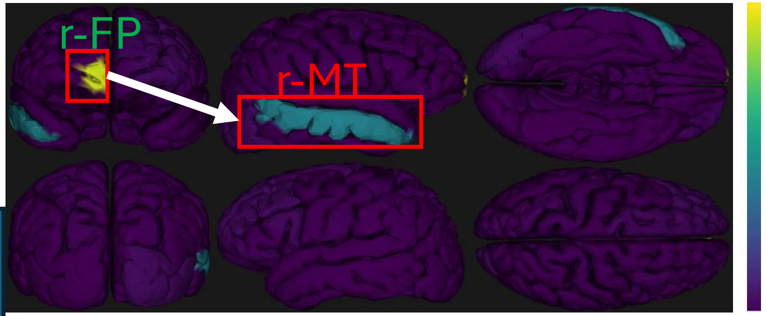



Integrative strategy (90% thresholding)

$\Theta^l$

$2^{nd}$ time lag

r-LOF → l-FP




Ground truth

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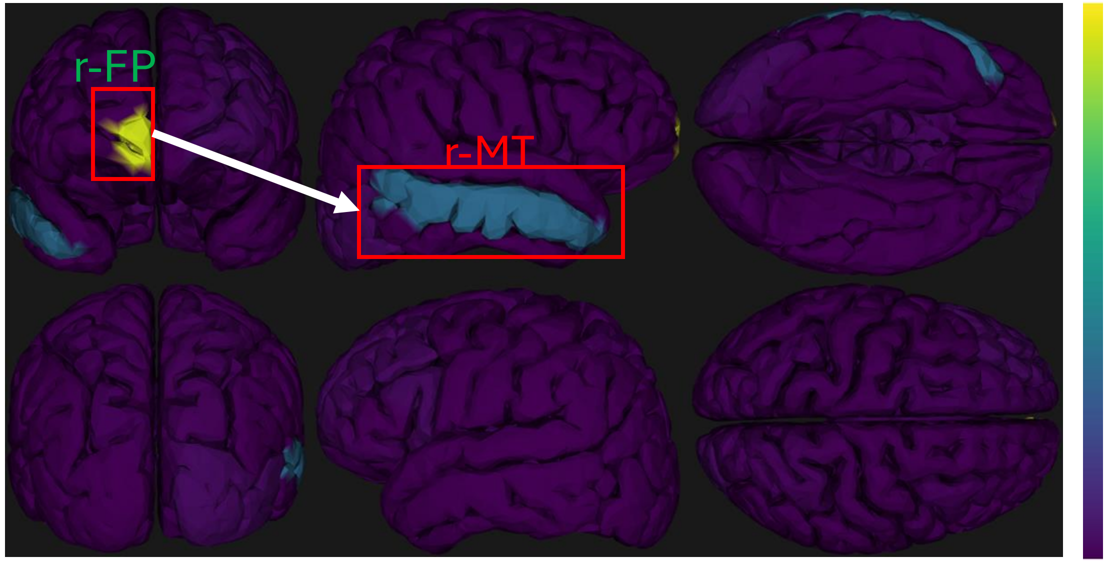
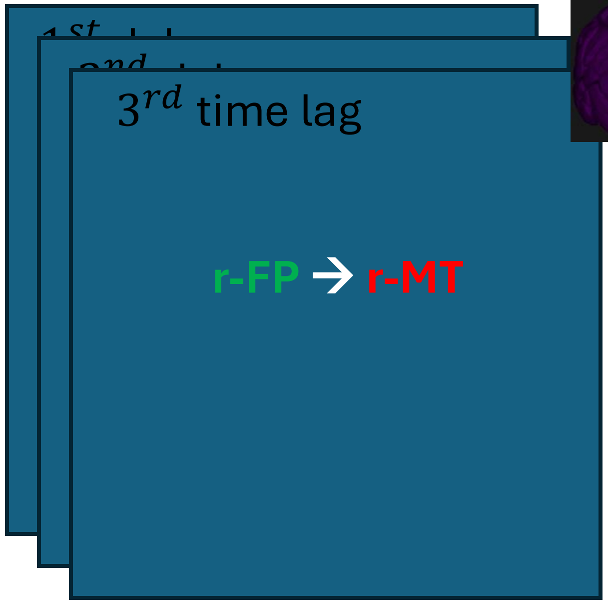
# Numerical results

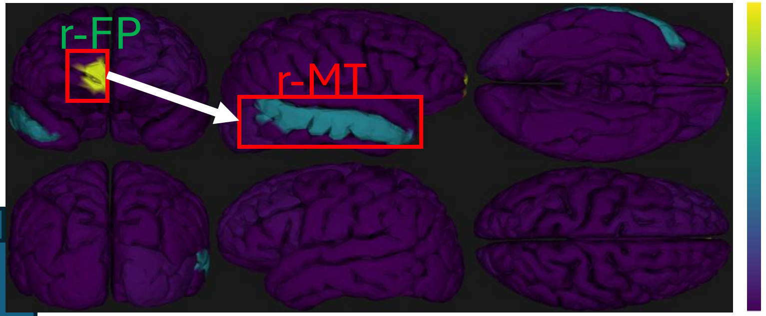
- Effective connectivity inference: Integrative strategy




Integrative strategy (90% thresholding)

$\Theta^l$

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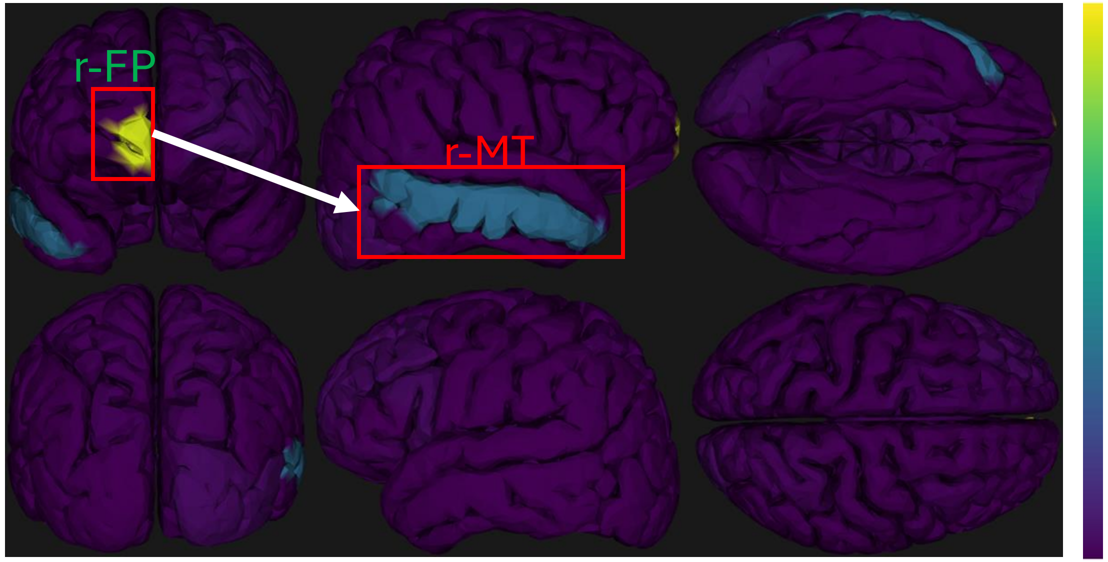
# Numerical results

- Effective connectivity inference: Integrative strategy

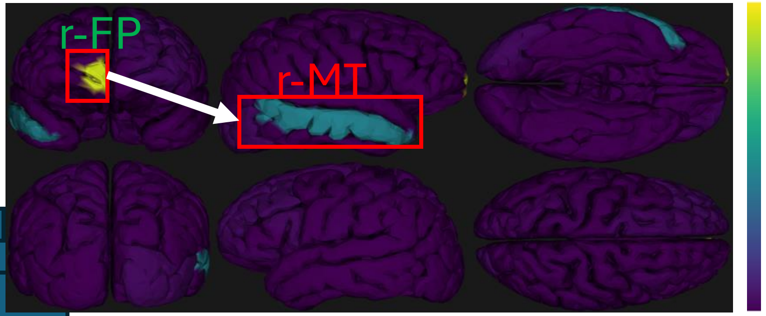



Integrative strategy (90% thresholding)

$\Theta^l$

$4^{th}$ time lag

r-CUN → r-SF




Ground truth

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l- : left hemisphere

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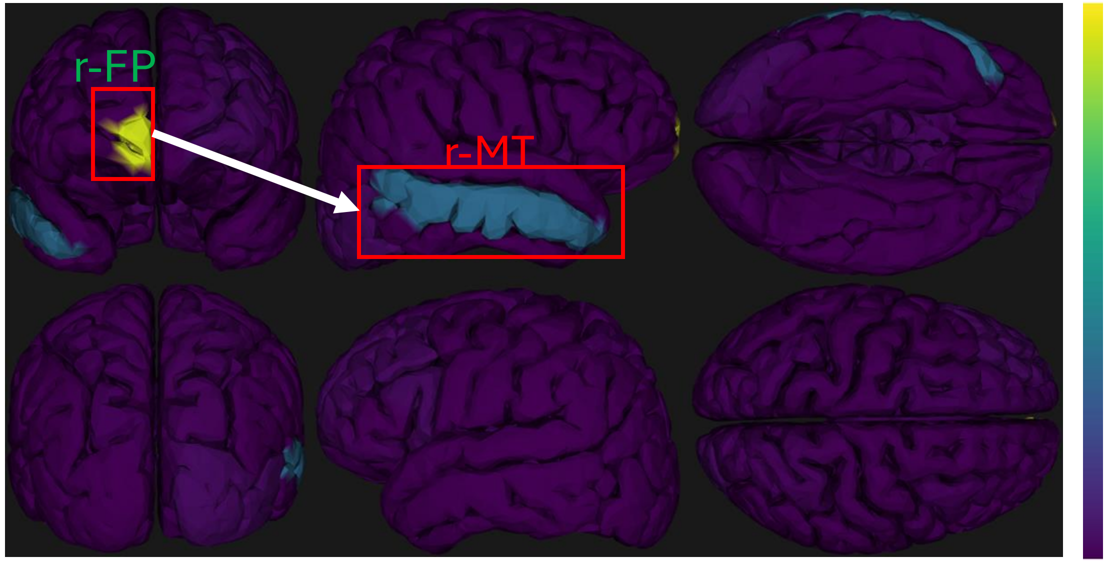
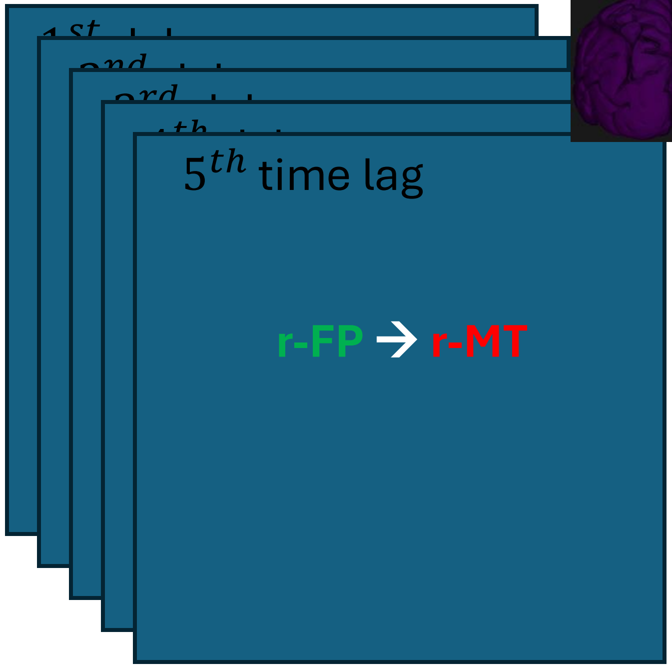
# Numerical results

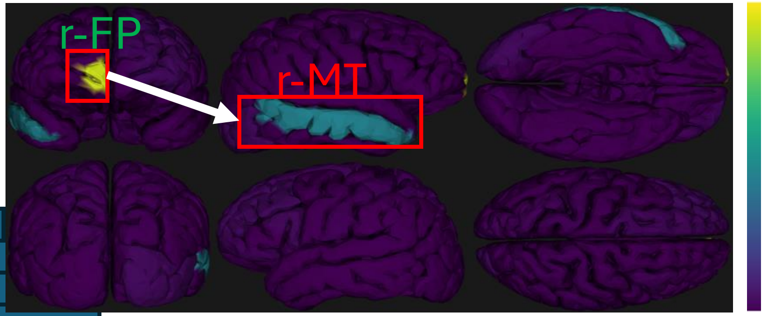
## ❑ Effective connectivity inference: Integrative strategy




**Integrative strategy (90% thresholding)**

$\Theta^l$




**Ground truth**

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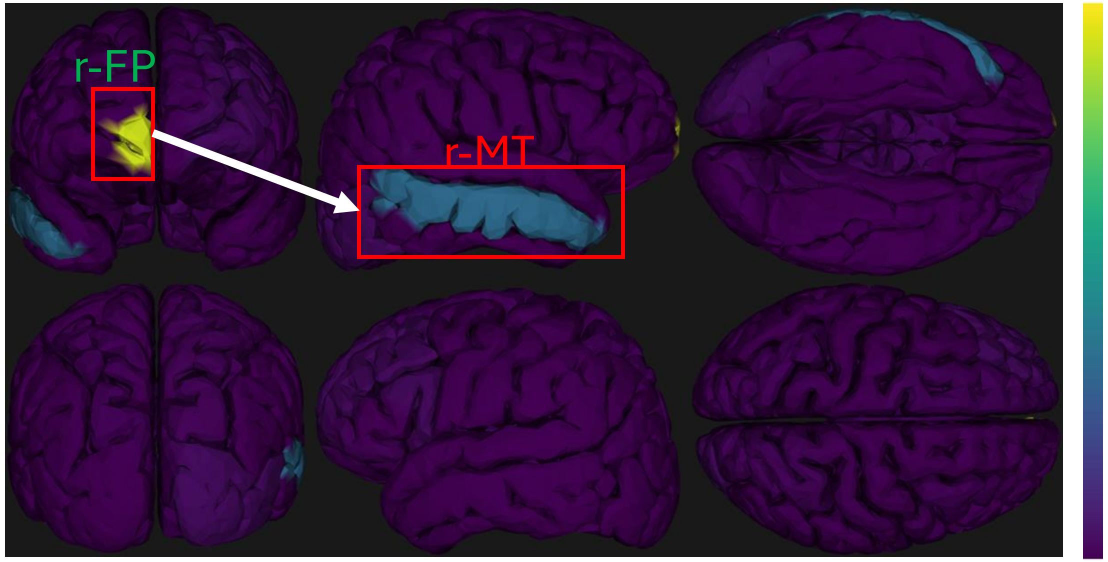
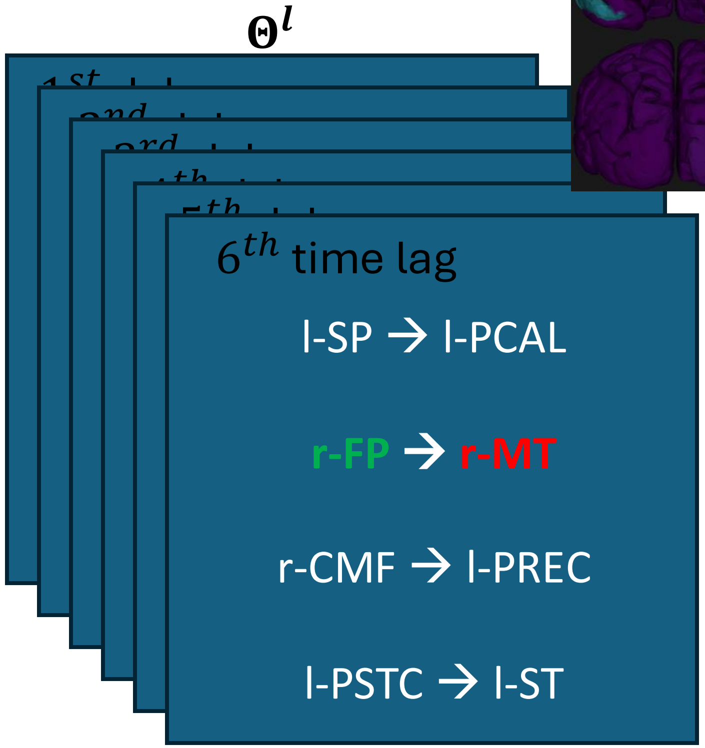
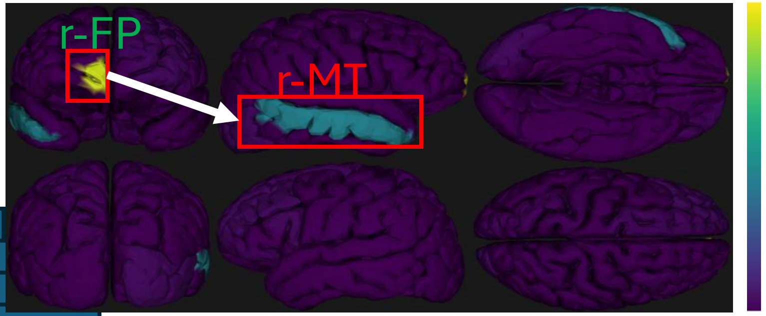
# Numerical results

## ❑ Effective connectivity inference: Integrative strategy




Integrative strategy (90% thresholding)

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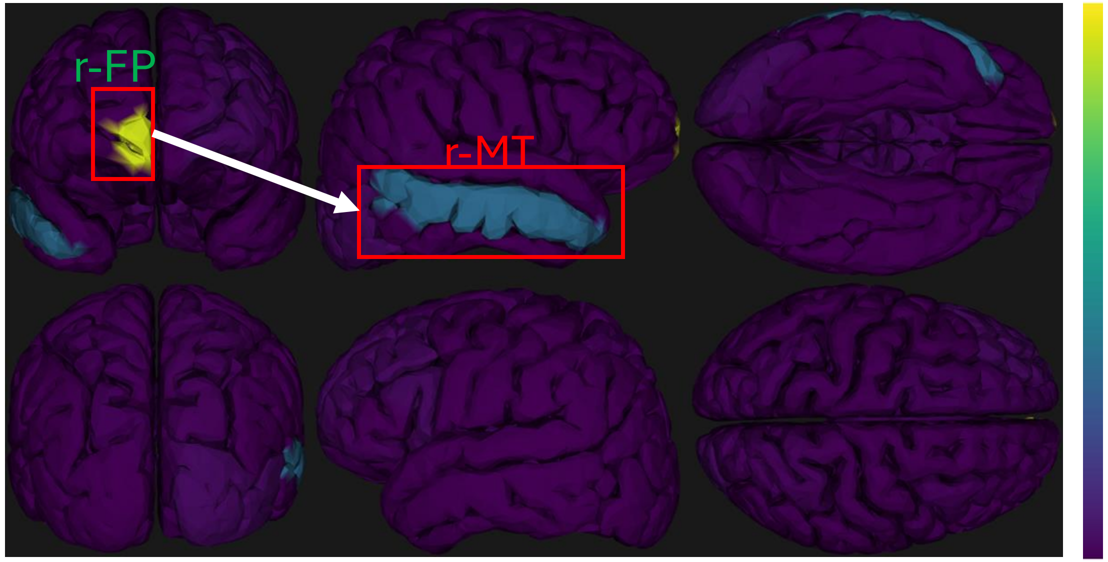
ST : Superior Temporal gyrus
PCAL : Pericalcarine Cortex

# Numerical results

❑ **Effective connectivity inference: Integrative strategy**




**Integrative strategy (90% thresholding)**

$\Theta^l$

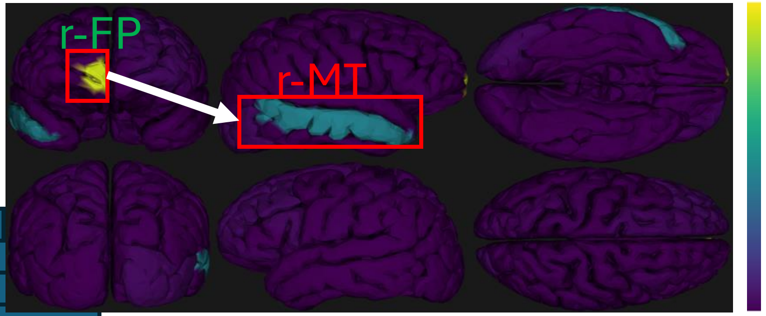
$6^{th}$ time lag

l-SP → l-PCAL

r-FP → r-MT

r-CMF → l-PREC

l-PSTC → l-ST




**Ground truth**

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PREC : PRECentral gyrus
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ST : Superior Temporal gyrus
PCAL : Pericalcarine Cortex

**All connections are occurring with delays in the range [4 – 12] ms**

# Take-home points

- A feasibility of an integrative strategy for brain source localization & effective connectivity inference
- Physiological assumptions
- MVAR modelling of pre-ictal epileptic spikes as a regularizer
- More focal brain source localization
- Inference and dynamic analysis of brain effective connectivity

# THANK YOU!