Camera Calibration and Stereo via a Single Image of a Spherical Mirror

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Majoring in **Computer Vision** under the supervision of **Prof. Michael Werman**.
His research focuses on the intersection of **computational imaging, classic computer vision techniques, artificial intelligence (particularly deep learning), and sensor technologies**.

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Aims and Contributions of Our Paper

In our paper, we aimed at:

1. Developing a **camera calibration method** that works with a **single image of a spherical mirror**.
2. Enabling **depth estimation and stereo reconstruction** from a single reflection.
3. Providing a solution for cases where **only one mirrored sphere is visible** in the scene.

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Aims and Contributions of Our Paper

Contributions of our study are threefold:

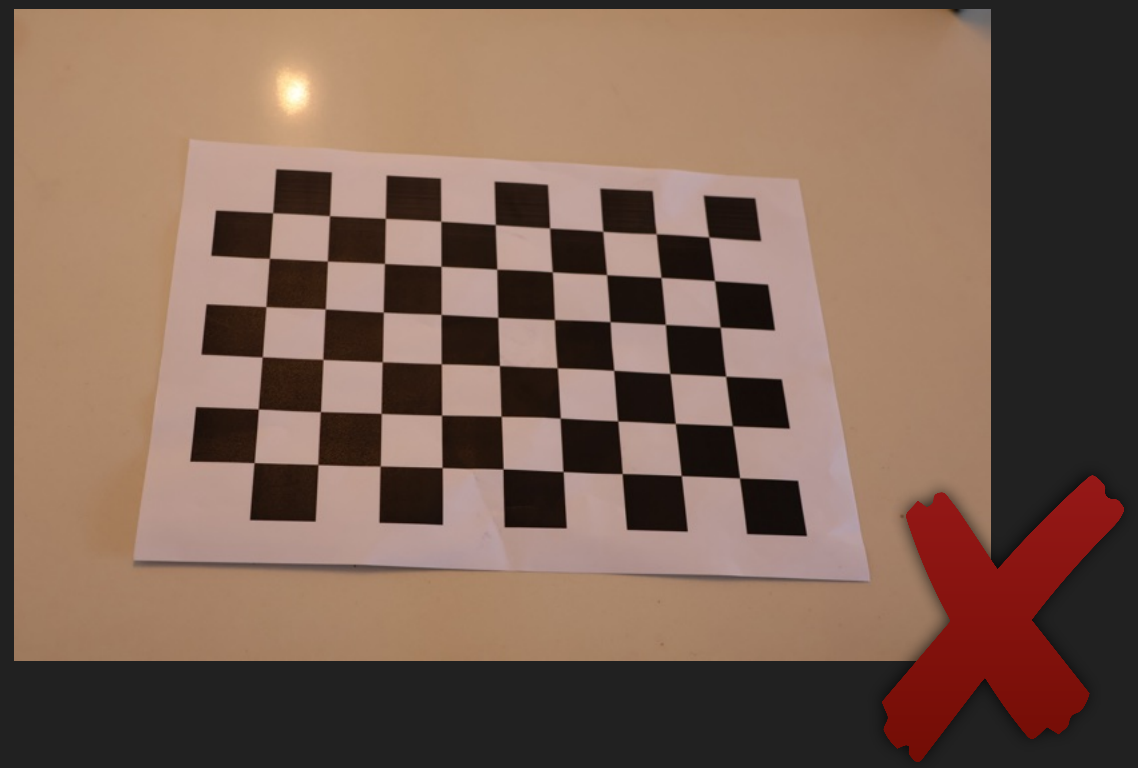
1. We introduced a **mathematical formulation** for extracting the **camera matrix** from a single sphere's reflection in real time.
2. We demonstrated **a novel method for computing depth information** from a single viewpoint using geometric constraints.
3. We validated our approach with **synthetic and real-world experiments**, achieving accuracy comparable to traditional multi-view calibration methods.

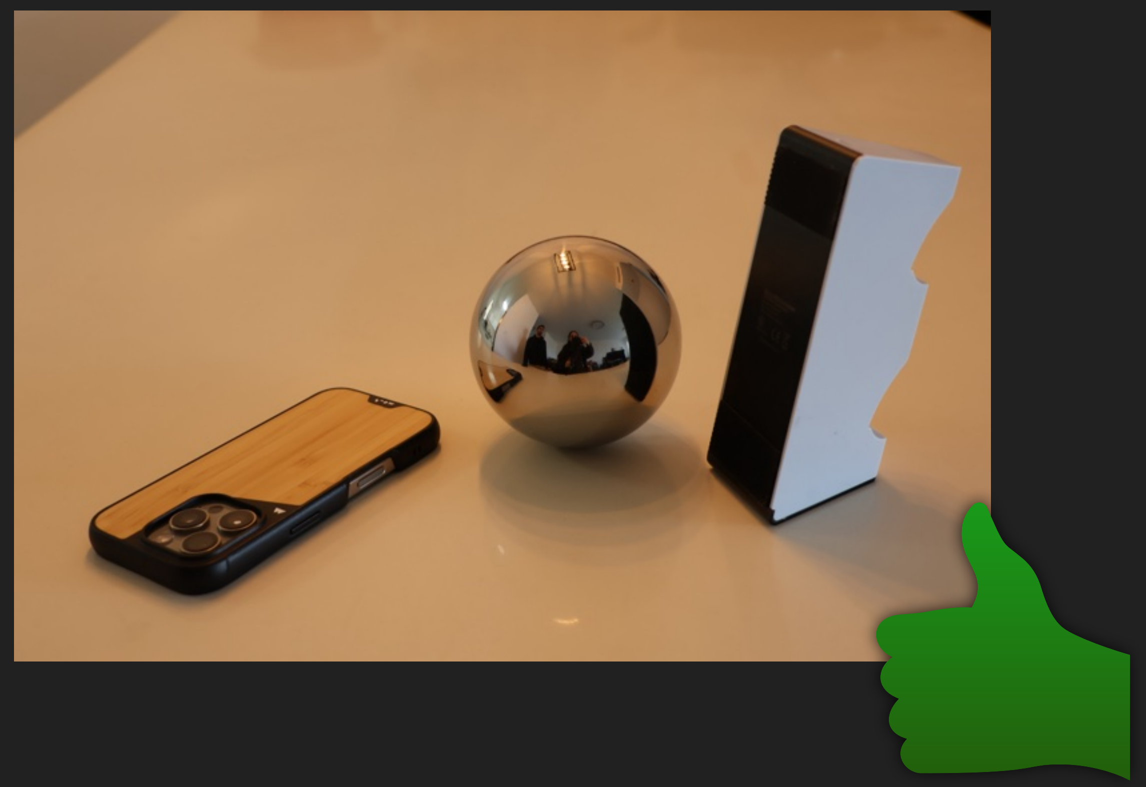
In this paper, we assume:

- A projective camera with no skew.
- The image contains a spherical mirror.
- The unit is defined by the sphere's radius.

$$P := \begin{bmatrix} f_x & 0 & t_x & 0 \\ 0 & f_y & t_y & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} \mathbf{K} & \begin{matrix} 0 \\ 0 \\ 0 \end{matrix} \end{bmatrix}$$

$$B = \begin{bmatrix} b_x & b_y & b_z \end{bmatrix}^T$$



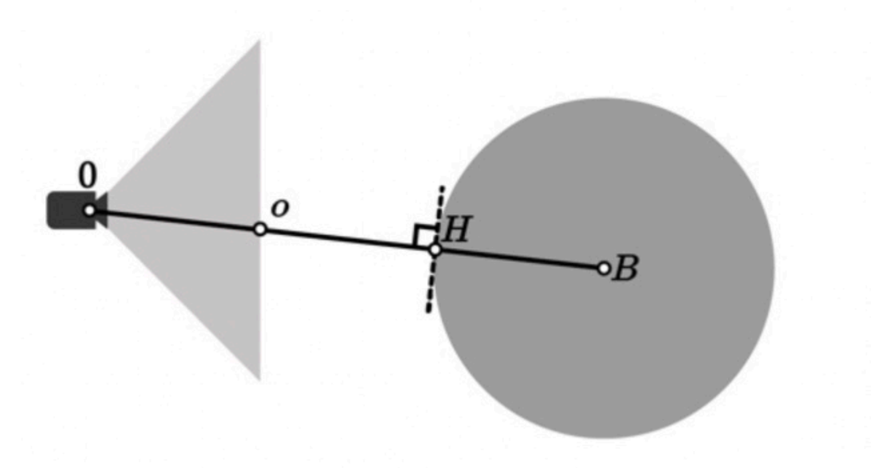


Define v is a homogeneous coordinates of a point on the conic,

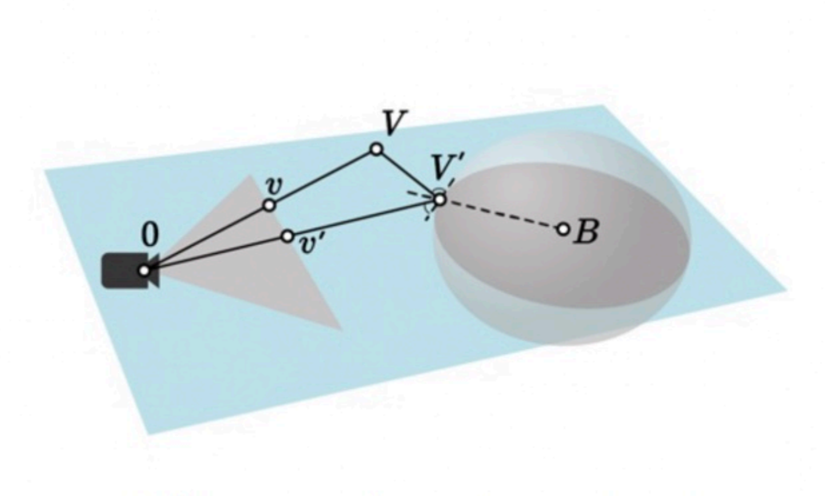
and C is the 3→3 symmetric matrix represent the conic matrix then $v^T C v = 0$

Define O = [ox, oy, 1] the sphere's center in the image

and tangent



(a) The rays from the camera to the mirror $0 \to H$, from the mirror to the camera $H \to 0$, and the normal at the mirror coincide.



(a) A 2D cross-section of a pair of correspondence points.

C is a 3×3 symmetric matrix which represent the contour of the sphere projects as an **ellipse:**
A general **conic equation** in Cartesian coordinates is:

$$C = C_1x^2 + C_2xy + C_3y^2 + C_4x + C_5y + C_6 = 0$$

$$v^T C v = 0$$

$$\begin{bmatrix} x & y & 1 \end{bmatrix} \begin{bmatrix} C_1 & C_2/2 & C_4/2 \\ C_2/2 & C_3 & C_5/2 \\ C_4/2 & C_5/2 & C_6 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = 0$$

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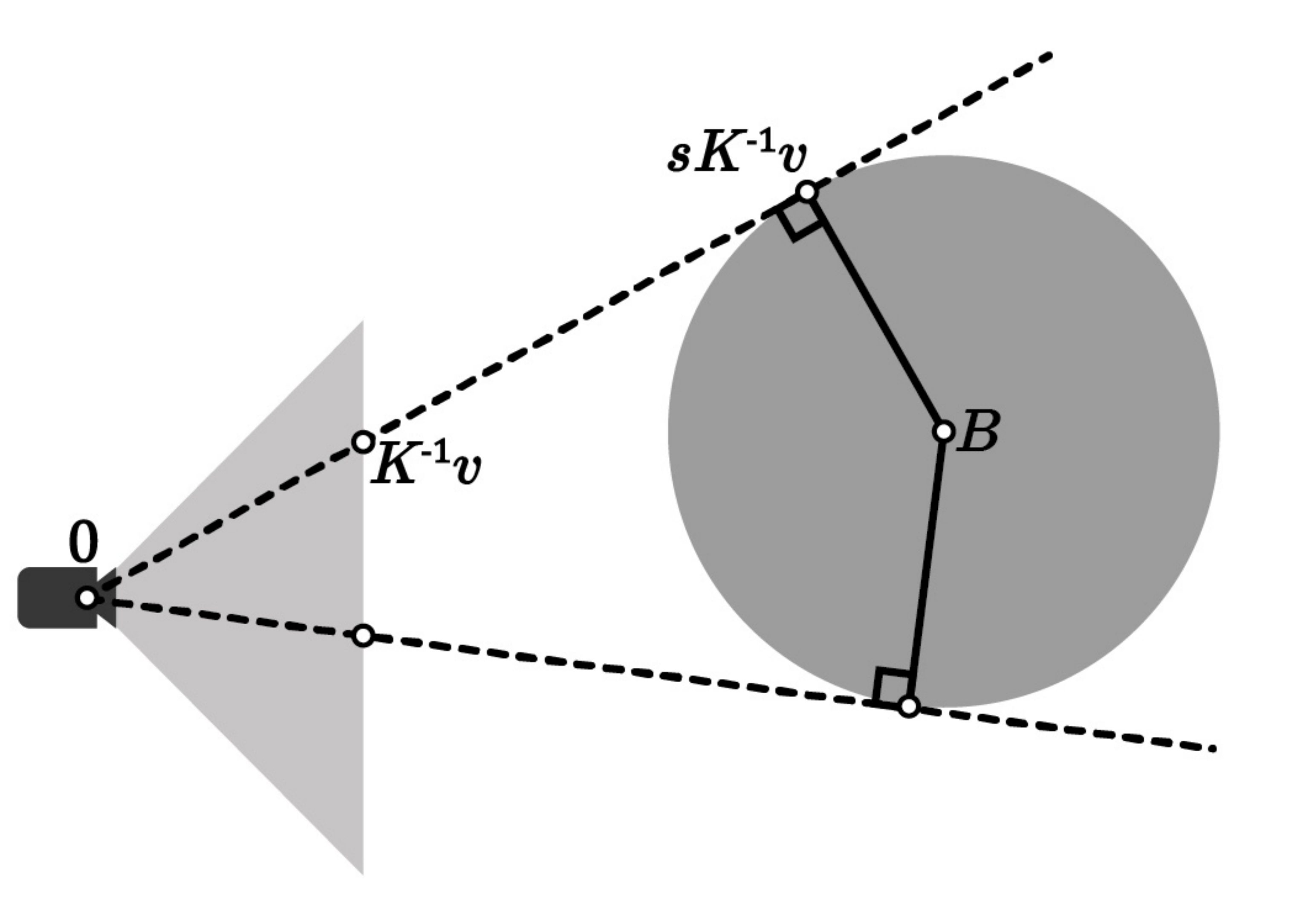
Finding the conic matrix C from the image helps identify the sphere's contour.

Right Triangle Constraint:

equation (1) $\langle sK^{-1}V, B - sK^{-1}V \rangle = 0$

$$s\langle K^{-1}V, K^{-1}V \rangle = \langle K^{-1}V, B \rangle$$

$$s = \frac{\langle K^{-1}V, B \rangle}{|K^{-1}V|^2}$$



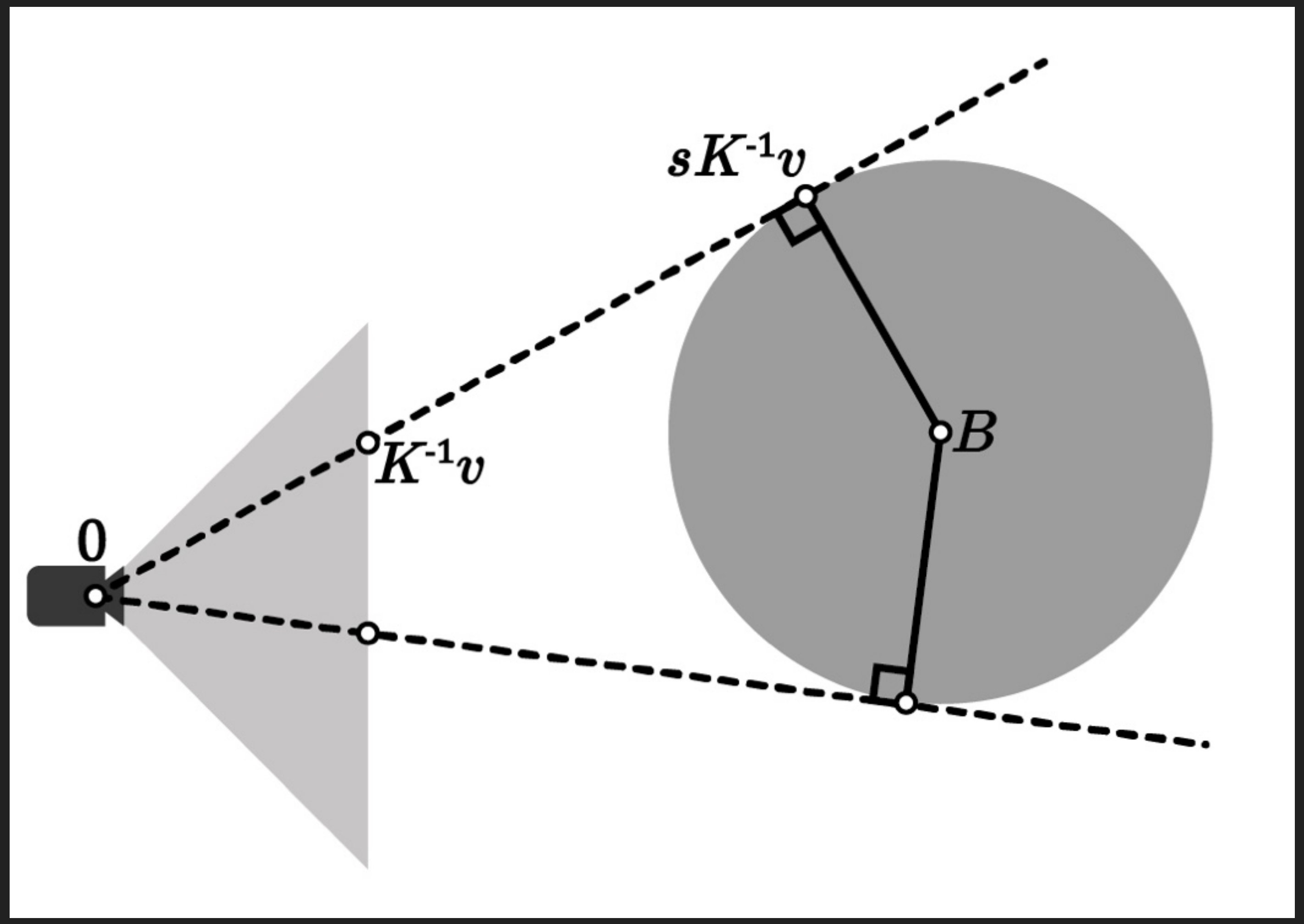
Distance Constraint:

equation (2) $|sK^{-1}V - B| = 1$

$$s^2|K^{-1}V|^2 - 2s\langle K^{-1}V, B\rangle + |B|^2 = 1$$

$$\left(\frac{\langle K^{-1}V, B\rangle}{|K^{-1}V|^2}\right)^2 |K^{-1}V|^2 - 2\frac{\langle K^{-1}V, B\rangle}{|K^{-1}V|^2}\langle K^{-1}V, B\rangle + |B|^2 = 1$$

$$\langle K^{-1}V, B\rangle^2 + (1 - |B|^2)|K^{-1}V|^2 = 0$$



Camera Calibration

equation (3) $\langle K^{-1}V, B\rangle^2 + (1 - |B|^2)|K^{-1}V|^2 = 0$

$$\langle K^{-1}V, B\rangle^2 = V^T K^{-T} B B^T K^{-1} V$$

$$(1 - |B|^2)|K^{-1}V|^2 = (1 - |B|^2) V^T K^{-T} I K^{-1} V$$

Now, rewriting equation (3)

$$V^T K^{-T} B B^T K^{-1} V + (1 - |B|^2) V^T K^{-T} I K^{-1} V$$

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Now, rewriting equation (3)

$$V^T K^{-T} B B^T K^{-1} V + (1 - |B|^2) V^T K^{-T} I K^{-1} V$$

Factoring out the common term: $V^T K^{-T} (\cdot) K^{-1} V$

$$V^T K^{-T} (B B^T + (1 - |B|^2) I) K^{-1} V = 0$$

Define r scalar such that:

$$C = r K^{-T} (B B^T + (1 - |B|^2) I) K^{-1}$$

$$C = rK^{-T}(BB^T + (1 - |B|^2)I)K^{-1}$$

$$C = \begin{bmatrix} C_1 & C_2/2 & C_4/2 \\ C_2/2 & C_3 & C_5/2 \\ C_4/2 & C_5/2 & C_6 \end{bmatrix}$$

We currently have 8 unknowns:

$$r, b_x, b_y, b_z, f_x, f_y, t_x, t_y$$

Redefine the origin to be the sphere's center, then tx and ty become zero because they are now incorporated into the coordinate system shift :

Define shift matrix:

$$S := \begin{bmatrix} 1 & 0 & o_x \\ 0 & 1 & o_y \\ 0 & 0 & 1 \end{bmatrix}$$

The new transformed coordinates are:

$$v = Sv'$$

Substituting into the original conic equation:

$$(Sv')^T C (Sv') = 0$$

$$v'^T S^T C S v' = 0$$

This shows that the **new conic matrix** M in the shifted coordinate system is:

$$M := S^T C S$$

$$Q := b_z K^{-1} S = \begin{bmatrix} b_z f_x^{-1} & 0 & b_x \\ 0 & b_z f_y^{-1} & b_y \\ 0 & 0 & b_z \end{bmatrix}$$

$$p = \frac{r}{b_z^2} \qquad C = rK^{-T}(BB^T + (1 - |B|^2)I)K^{-1}$$

$$M = pQ^T(BB^T + (1 - |B|^2)I)Q$$

$$M := \begin{cases} m_{11} = pf_x^{-2}b_z^2(b_x^2 + 1 - |B|^2) \\ m_{22} = pf_y^{-2}b_z^2(b_y^2 + 1 - |B|^2) \\ m_{33} = p|B|^2 \\ m_{12} = pf_x^{-1}f_y^{-1}b_xb_yb_z^2 \\ m_{13} = pf_x^{-1}b_xb_z \\ m_{23} = pf_y^{-1}b_yb_z \end{cases} \qquad p = \frac{m_{13}m_{23}}{m_{12}}$$

Synthetic Data

2048*2048 px

TABLE I. COMPARISON OF REAL VALUES AND OUR ALGORITHM'S

	Parameters	
Ground Truth	$b_x = 3$	$b_y = -4,$
	$b_z = 7$	$f_x = 1024$
	$f_y = 1024$	$t_x = 1024$
	$t_y = 1024$	
Result	$b_x = 3.00$	$b_y = -3.94$
	$b_z = 7.03$	$f_x = 1027.99$
	$f_y = 1032.84$	$t_x = 1024.34$.
	$t_y = 1016.94$	
Error Range	Less than 1.5%	

Synthetic Data
1920*1080 px

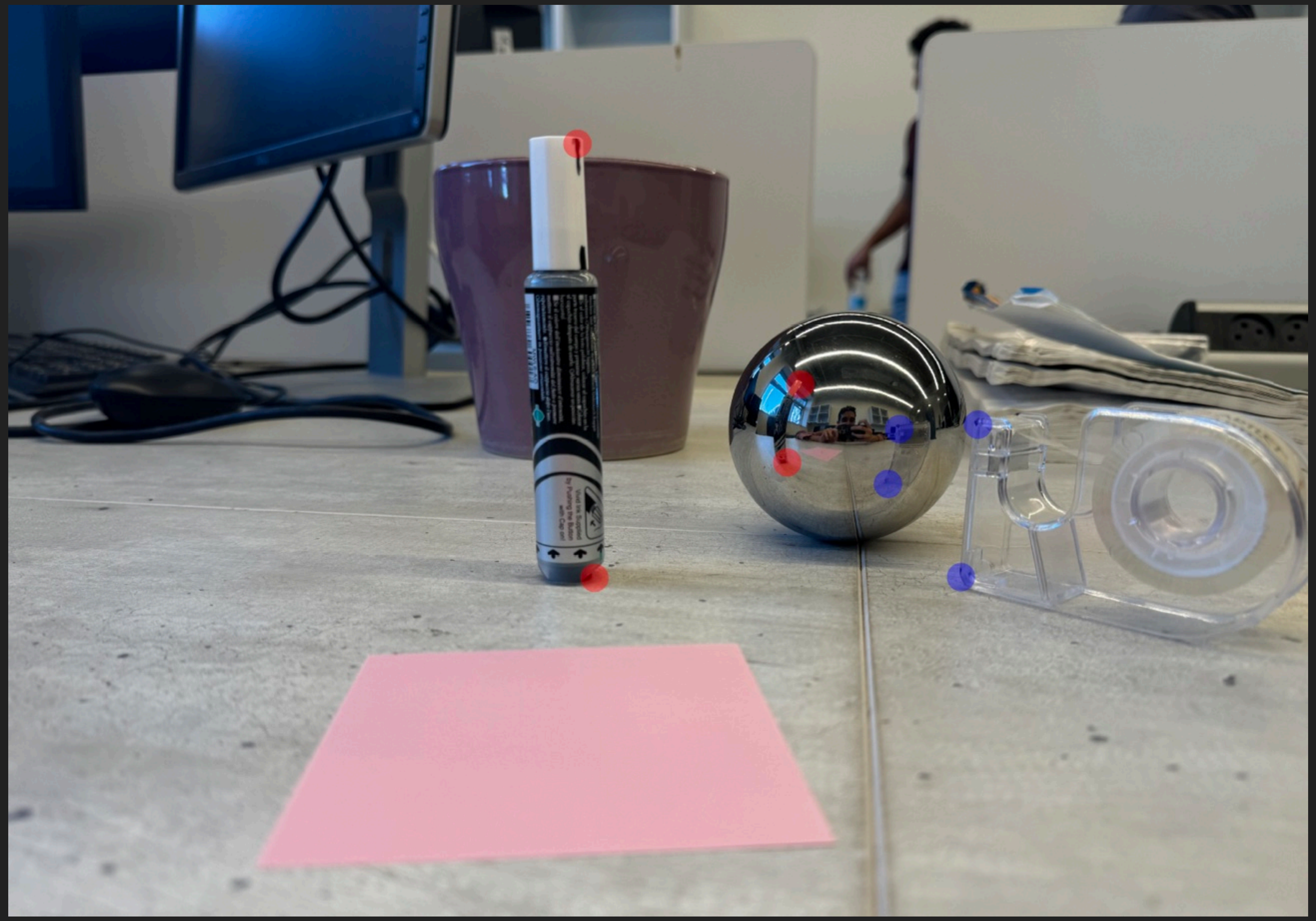
TABLE II. Comparison of real values and our algorithm's

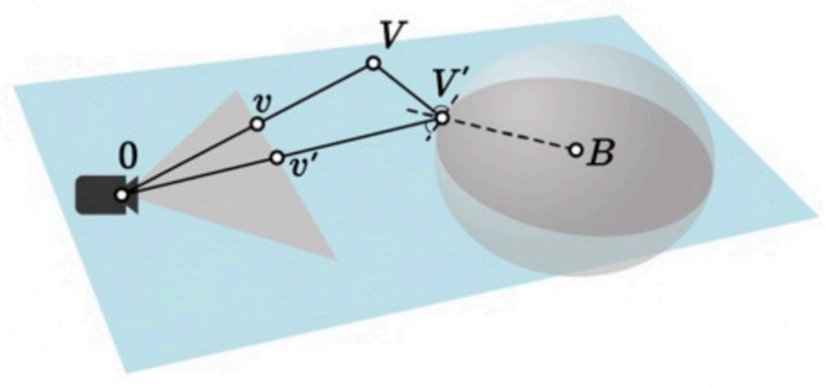
	Parameters	
Ground Truth	$b_x = -1.5$	$b_y = 3,$
	$b_z = 1$	$f_x = 1144$
	$f_y = 1144$	$t_x = 960$
	$t_y = 540$	
Result	$b_x = -1.47$	$b_y = 3.07$
	$b_z = 1$	$f_x = 1179$
	$f_y = 1167$	$t_x = 949$.
	$t_y = 535$	
Error Range	Less than 3.1%	

Real Data

The real height of tape: 5cm
Our: 5.05cm

The real height of marker: 13cm
Our: 14cm





(a) A 2D cross-section of a pair of correspondence points.

Real Data

6000*4000 px

TABLE III. Comparison of Zhang evaluation for over more then 20 images and our algorithm's on a single image result

		Parameters		
9.	Zhang Calibration	$f_x = 8146$		
		$f_y = 8286$	$t_x = 3143$	$t_y = 2397$
	Result	$f_x = 8139$		
		$f_y = 8175$	$t_x = 3432$	$t_y = 2044$

We presented a novel approach for calibrating the camera matrix using a single-view image. Our findings help reduce the requirements for achieving this calibration. Using our method, further image analysis is possible, such as determining the 3D location of a point from a pair of corresponding points or estimating an omnidirectional image centered at the sphere's origin.

Additionally, since a spherical mirror distorts the scene by projecting it onto a curved surface, we aim to leverage our findings to correct this distortion and reconstruct the scene as if it were reflected in a planar mirror in future work.

Future Work: