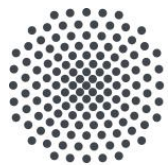


Warm-Starting Patterns for Quantum Algorithms



University of Stuttgart

**Felix Truger, Johanna Barzen, Martin Beisel,
Frank Leymann, and Vladimir Yussupov**

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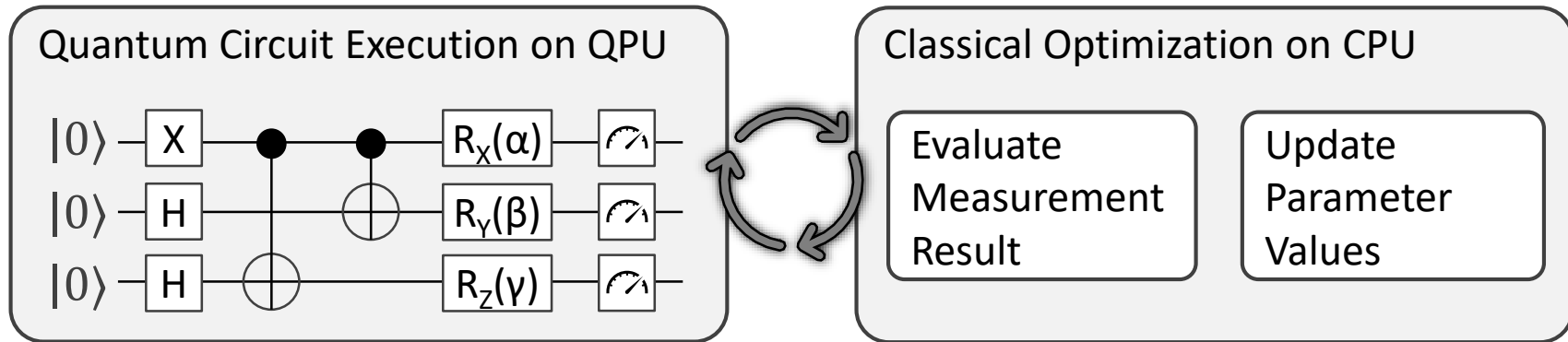
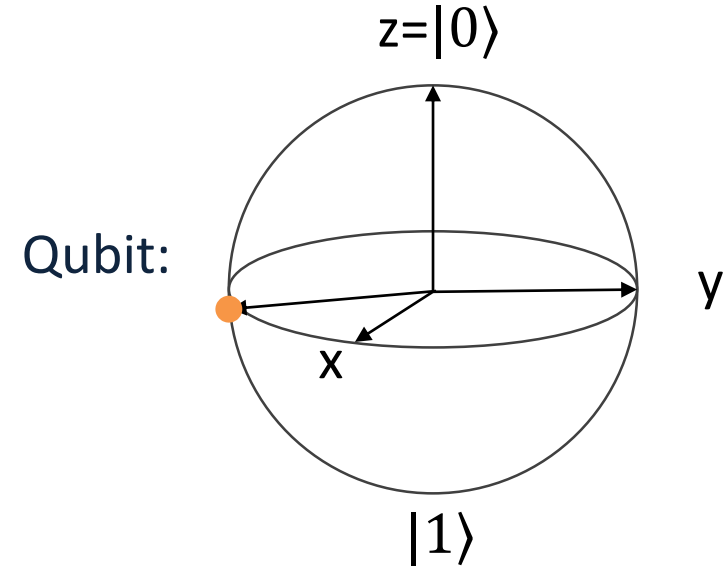
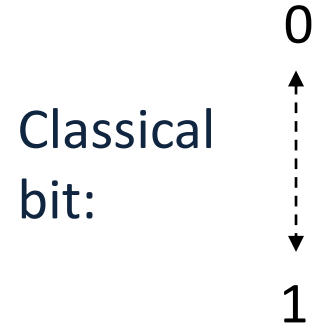


Agenda

- Fundamentals
- Motivation
- Pattern Authoring Method & Format
- Warm-Starting Patterns for Quantum Algorithms
- Integration into Pattern Atlas
- Summary & Outlook

Fundamentals: Quantum Algorithms

1. Quantum Bits (Qubits)
2. Quantum Circuits
3. Variational Quantum Algorithms (VQAs)



Motivation: Warm-Starts in Quantum Computing

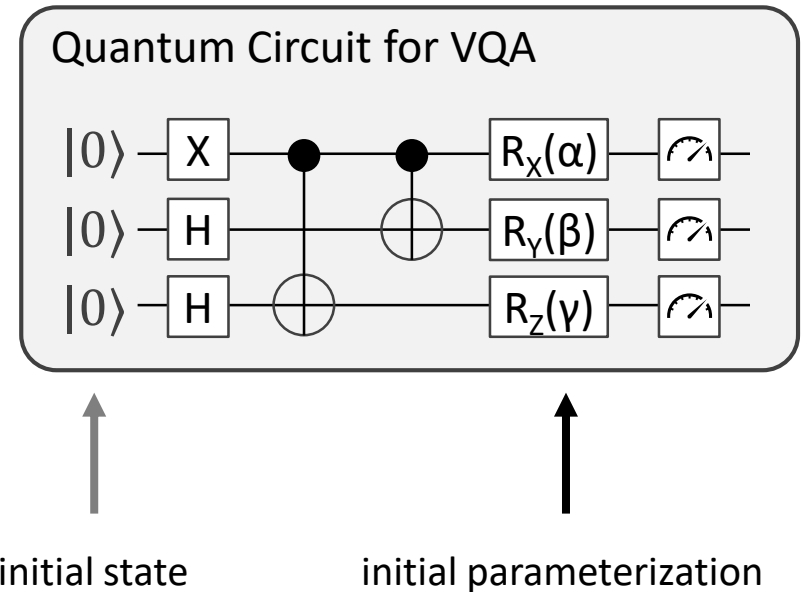
Warm-Starting:

≈ Utilization of existing or efficiently obtained information instead of starting from scratch.

... in Quantum Computing:

- Two points of initialization in circuits for VQAs.

- Classical approximation algorithms for many problems
- Similarities among problem instances
- „Learn“ from past results



Pattern Language for Quantum Algorithms [1]

- **Pattern:** structured document containing an abstract description of a proven solution of a recurring problem
 - points to other patterns that may jointly contribute to an encompassing solution of a complex problem
- **Pattern Language:** network of related patterns

Towards a Pattern Language for Quantum Algorithms

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Universitätsstr. 38, 70569 Stuttgart, Germany
Frank.Leymann@iaas.uni-stuttgart.de

Abstract. Creating quantum algorithms is a difficult task, especially for computer scientist not used to quantum computing. But quantum algorithms often use similar elements. Thus, these elements provide proven solutions to recurring problems, i.e. a pattern language. Sketching such a language is a step towards establishing a software engineering discipline of quantum algorithms.

Keywords: Quantum algorithms, Pattern Languages, Software Engineering.

1. Introduction

1.1. Patterns and Pattern Languages

There is a significant difference in how quantum algorithms are presented and invented, and the way how traditional algorithms are build. Thus, computer scientists and software developers used to solve classical problems need a lot of assistance when being assigned to build quantum algorithms.

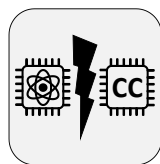
To support and guide people in creating solutions in various domains, pattern languages are established. A *pattern* is a structured document containing an abstract description of a proven solution of a recurring problem. Furthermore, a pattern points to other patterns that may jointly contribute to an encompassing solution of a complex problem. This way, a network of related patterns, i.e. a *pattern language*, results.

This notion of pattern and pattern language has its origin in [1]. Although invented to support architects in building houses and planning cities, it has been accepted in several other domains like pedagogy, manufacturing, and especially in software architecture (e.g. [13]).

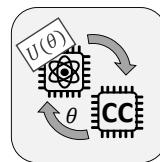
In this paper, we lay the foundation for a pattern language for quantum algorithms. The need for documenting solutions for recurring problems in this domain can be observed in text books like [19, 21] that contain unsystematic explanations of basic „tricks“ used in quantum algorithms. Our contribution is to systematize this to become a subject of a software engineering discipline for quantum algorithms.

Pattern Language for Quantum Algorithms [1]

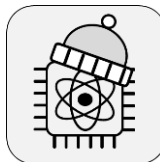
Program Flow



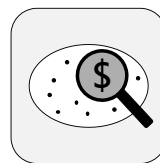
Quantum-
Classic Split



VQA



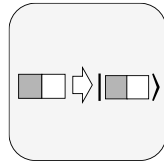
Warm-Start



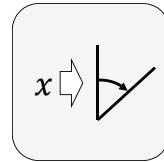
QAOA

...

State Preparation

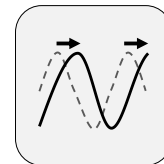


Basis
Encoding

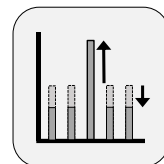


Angle
Encoding ...

Unitary Transformations

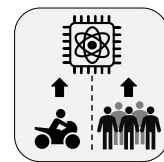


Phase
Shift

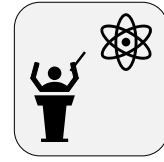


Amplitude
Amplification ...

Execution

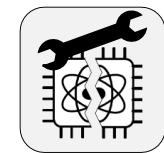


Prioritized
Execution

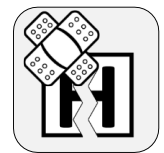


Orchestrated
Execution ...

Error Handling

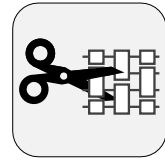


Error
Correction



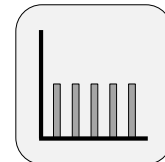
Gate Error
Mitigation ...

Cutting

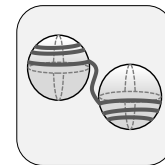


Circuit
Cutting ...

Quantum States

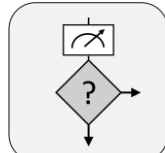


Uniform
Super-
position



Creating
Entangle-
ment

Measurement



Post-selective
Measurement

[1] F. Leymann, "Towards a Pattern Language for Quantum Algorithms," *QTOP*, Springer, 2019.

[2] M. Weigold et al., "Expanding Data Encoding Patterns For Quantum Algorithms," *ICSA-C*, IEEE, 2021.

[3] M. Weigold et al., "Patterns for Hybrid Quantum Algorithms," *SummerSoC*, Springer, 2021.

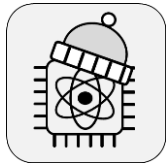
[4] M. Beisel et al., "Patterns for Quantum Error Handling," *PATTERNS*, XPS, 2022.

[5] F. Bühler et al., "Patterns for Quantum Software Development," *PATTERNS*, XPS, 2023.


[6] M. Bechtold et al., "Patterns for Quantum Circuit Cutting," *PLoP*, Hillside, 2023.

[7] D. Georg et al., "Execution Patterns for Quantum Applications," *ICSOFT*, SciTePress, 2023.

Refined Warm-Starting Patterns: Overview & Pattern Format



WARM-START



Warm-Starting and Quantum Computing: A Systematic Mapping Study

FELIX TRUGER, JOHANNA BARZEN, MARVIN BECHTOLD, MARTIN BEISEL, FRANK LEYMANN, ALEXANDER MANDL, and VLADIMIR YUSSUPOV, Institute of Architecture of Application Systems, University of Stuttgart, 70569 Stuttgart, Germany

Due to low numbers of qubits and their error-proneness, Noisy Intermediate-Scale Quantum (NISQ) computers impose constraints on the size of quantum algorithms they can successfully execute. State-of-the-art research introduces various techniques addressing these limitations by utilizing known or inexpensively generated approximations, solutions, or models as a starting point to approach a task instead of starting from scratch. These so-called warm-starting techniques aim to reduce quantum resource consumption, thus facilitating the design of algorithms suiting the capabilities of NISQ computers. In this work, we collect and analyze scientific literature on warm-starting techniques in the quantum computing domain. In particular, we (i) create a systematic map of state-of-the-art research on warm-starting techniques using established guidelines for systematic mapping studies, (ii) identify relevant properties of such techniques, and (iii) based on these properties classify the techniques identified in the literature in an extensible classification scheme. Our results provide insights into the research field and aim to help quantum software engineers to categorize warm-starting techniques and apply them in practice. Moreover, our contributions may serve as a starting point for further research on the warm-starting topic since they provide an overview of existing work and facilitate the identification of research gaps.

CCS Concepts: • General and reference → Surveys and overviews; • Computer systems organization → Quantum computing; • Software and its engineering; • Hardware → Quantum computation.

Additional Key Words and Phrases: Warm-Start, Quantum Software Engineering, Quantum Algorithm, Systematic Mapping Study.

1 INTRODUCTION

Quantum computers promise to solve a number of problems that are intractable on classical hardware [16, 41]. However, contemporary Noisy Intermediate-Scale Quantum (NISQ) devices are error-prone and impose restrictions on the depth of quantum circuits they can successfully execute [30, 41]. Such limitations and the scarcity of contemporary NISQ devices encourage the development of specialized quantum algorithms and techniques, e.g., the Quantum Approximate Optimization Algorithm (QAOA) is able to address the circuit depth limitation by means of an adaptable circuit depth [13]. Furthermore, quantum computation on NISQ devices is currently possible only via cloud services from a limited number of vendors [31], thus, computational tasks on NISQ devices can be either scheduled via reserved time slots or queued for execution on the respective cloud offering [29]. In both cases, quantum devices may not be available as needed or only after an undesirable, significant waiting time. Hence, to increase the efficiency of quantum applications and reduce waiting times, the utilization of quantum devices should

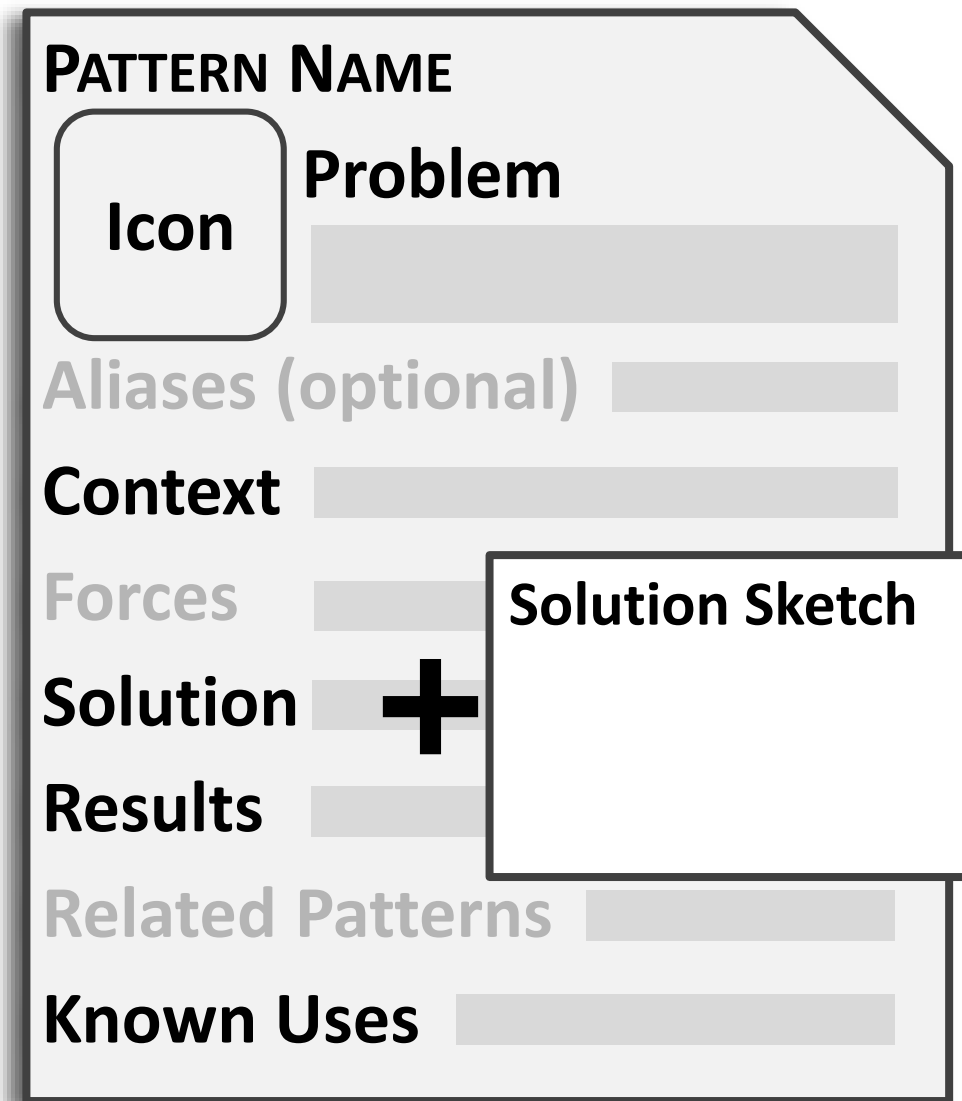
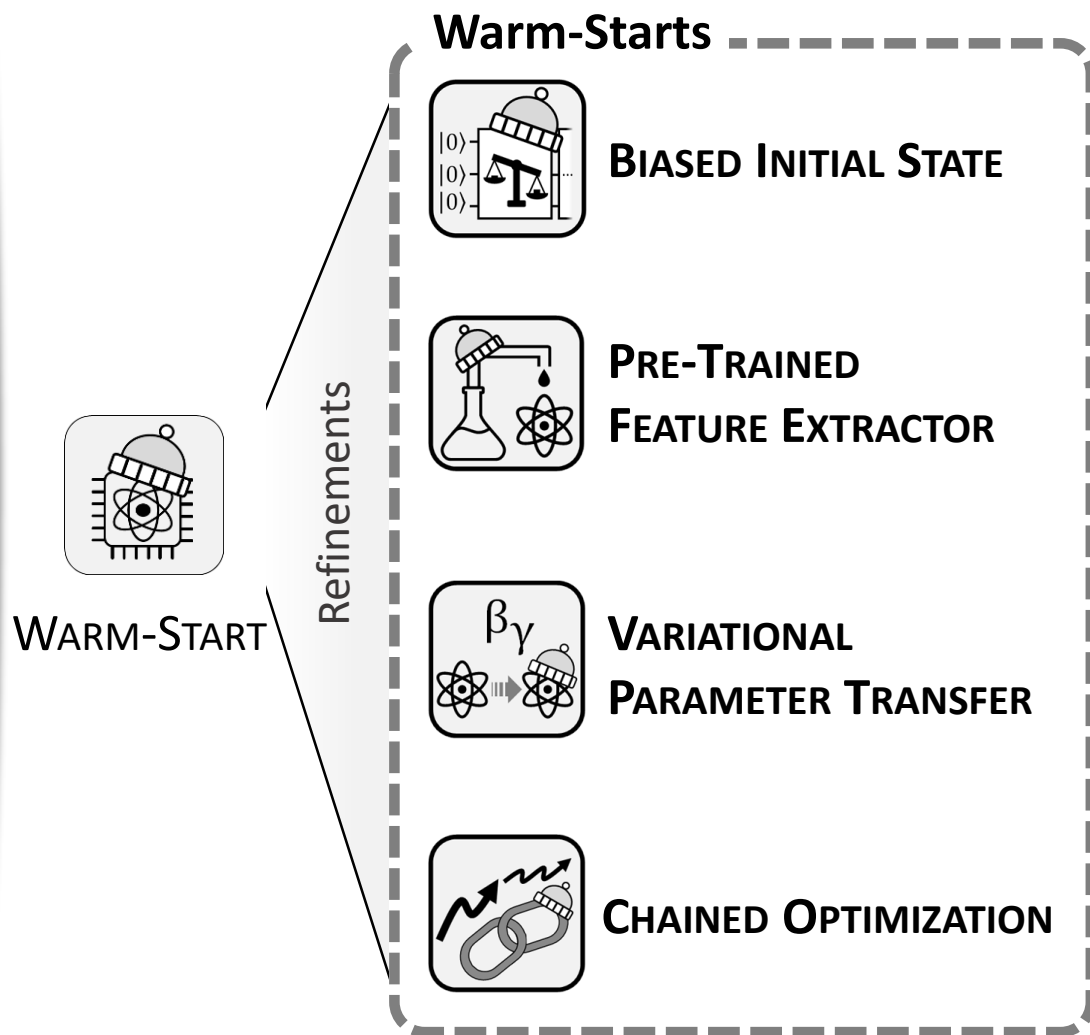
Authors' address: Felix Truger, truger@iaas.uni-stuttgart.de; Johanna Barzen, barzen@iaas.uni-stuttgart.de; Marvin Bechtold, bechtold@iaas.uni-stuttgart.de; Martin Beisel, beisel@iaas.uni-stuttgart.de; Frank Leymann, leymann@iaas.uni-stuttgart.de; Alexander Mandl, mandl@iaas.uni-stuttgart.de; Vladimir Yussupov, yussupov@iaas.uni-stuttgart.de, Institute of Architecture of Application Systems, University of Stuttgart, Universitätsstraße 38, 70569 Stuttgart, Germany.

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ACM 0360-0300/2024/3-ART
<https://doi.org/10.1145/3652510>

ACM Comput. Surv.

Refined Warm-Starting Patterns: Overview & Pattern Format



BIASED INIT



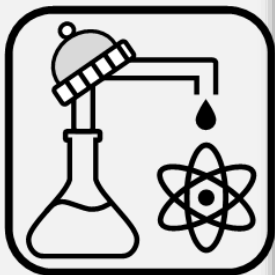
Context:

Solution

Results

Known Use

PRE-TRAINED



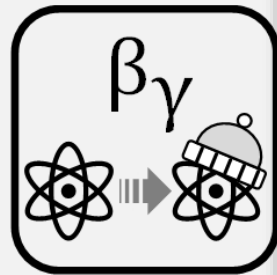
Context:

Solution:

Results:

Known Use

VARIATIONAL



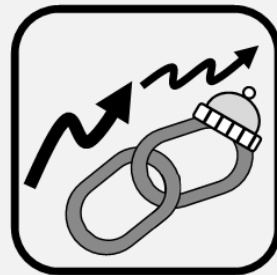
Context:

Solution:

Results:

Known Use

CHAINED OPTIMIZATION



Problem:

Context:

Solution:

Results:

Known Uses:

BIASED INITIAL STATE



Problem: Improve solution quality or speed up through efficient approximations.

Context: Valuable information from approximations often remains unused.

Solution: Encode approximations into the initial state of quantum circuits.

Results: Approximation utilized as starting point for quantum algorithm to improve on.

Known Uses: Warm-started QAOA for MaxCut and Knapsack problems [1][2], “classically-boosted” algorithm for Max3SAT and MaxBisect problems [3], ...

[1] D. J. Egger et al. “Warm-starting quantum optimization,” *Quantum*, 2021.

[2] W. van Dam et al, “Quantum Optimization Heuristics with an Application to Knapsack Problems,” *QCE*, IEEE, 2021.

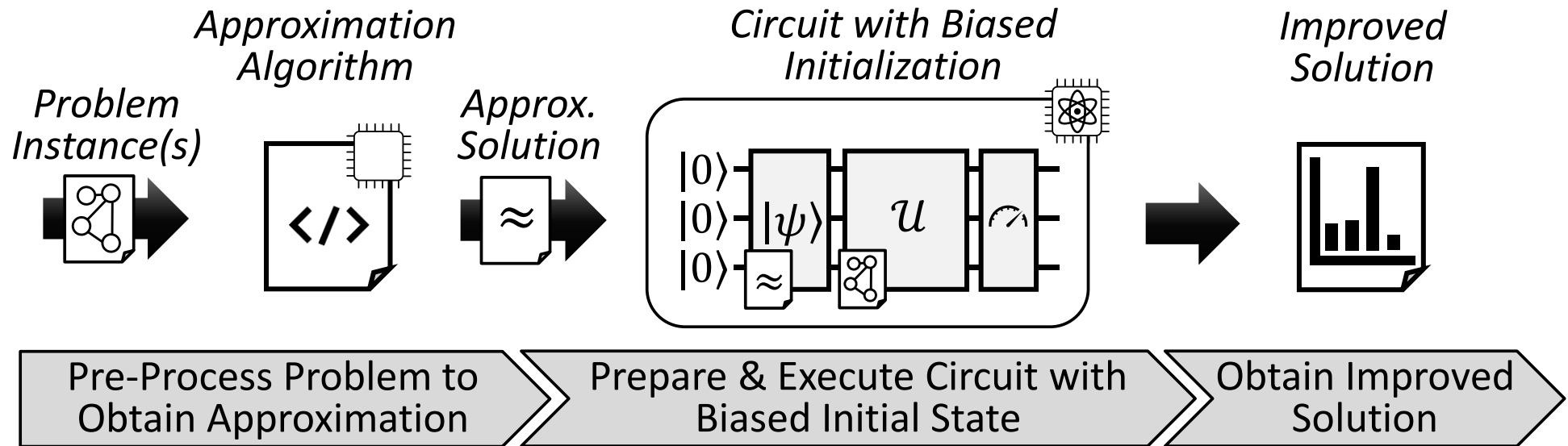
[3] G. Wang, “Classically-Boosted Quantum Optimization Algorithm,” *arXiv:2203.13936*, 2022.

BIASED INITIAL STATE



Problem: Improve solution quality or speed up through efficient approximations.

Solution Sketch:



[1] D. J. optimi
[2] W. Heuris
Proble
[3] G. Wang, "Classically-boosted Quantum Optimization Algorithm," *arXiv:2203.13936*, 2022.

PRE-TRAINED FEATURE EXTRACTOR



Problem: Process large data items through QNNs with low number of qubits.

Context: Lower number of qubits available than required to directly load data items.

Solution: Use pre-trained classical model to reduce dimensions of data items.

Results: Compressed representation enables processing data through QNN.

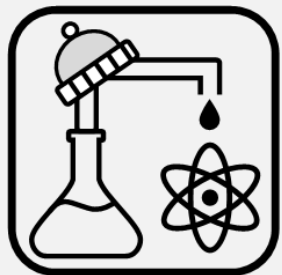
Known Uses: Image processing and classification [1], text classification [2], auto-encoders [3] as a special case of pre-trained feature extractors, ...

[1] A. Mari et al., "Transfer learning in hybrid classical-quantum neural networks," *Quantum*, 2020.

[2] C.-H. H. Yang et al., "When BERT Meets Quantum Temporal Convolution Learning for Text Classification in Heterogeneous Computing," *ICASSP*, IEEE, 2022.

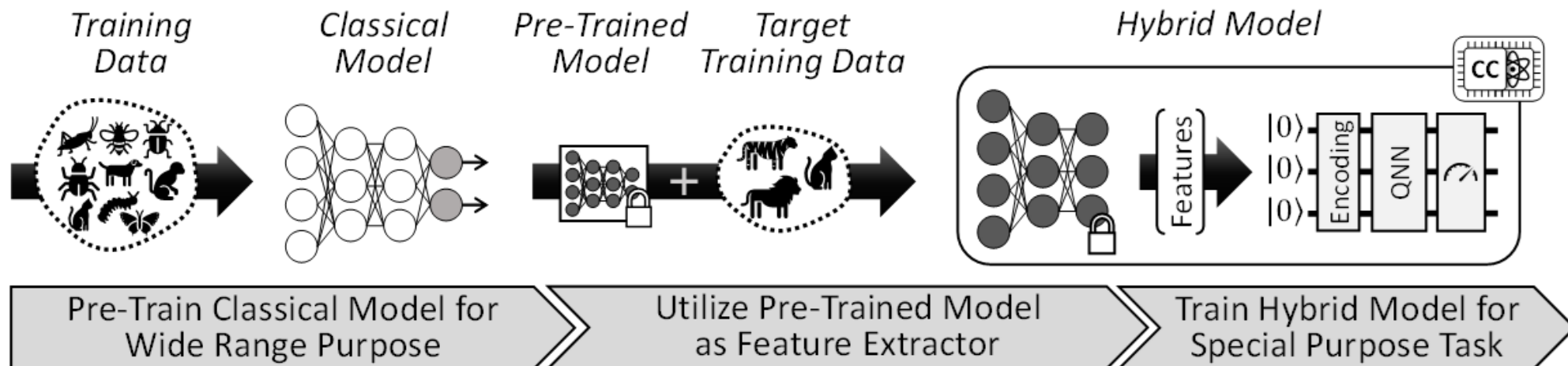
[3] M. A. Kramer, "Nonlinear principal component analysis using autoassociative neural networks," *AIChE Journal*, Wiley, 1991.

PRE-TRAINED FEATURE EXTRACTOR



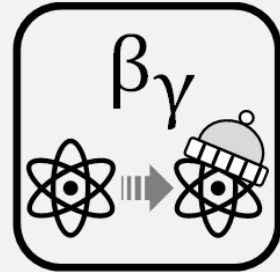
Problem: Process large data items through QNNs with low number of qubits.

Solution Sketch:



[1] A. M. ...
 classica
 2020.
 [2] C.-H.
 Quant
 Text Cl
 Compu
 [3] M.
 compo
 neural networks," *AICHE Journal*, Wiley, 1991.

VARIATIONAL PARAMETER TRANSFER



Problem: Obtain problem-aware parameter initialization for VQA to save time.

Context: Repeated circuit execution needed to optimize the circuit parameters.

Solution: Transfer parameter values from related problem instances as an initialization.

Results: Reduced number of iterations in VQA and increased chance to find global optimum.

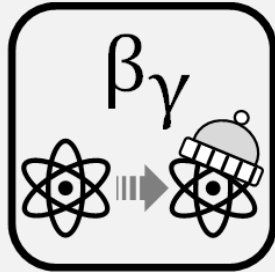
Known Uses: Parameter transfers for QAOA and MaxCut [1], Repository of pre-optimized parameter values [2], modelling constructs for integration in quantum workflows [3]

[1] A. Galda et al. "Transferability of optimal QAOA parameters between random graphs," *QCE*, IEEE, 2021

[2] R. Shaydulin et al., "QAOAKit: A Toolkit for Reproducible Study, Application, and Verification of the QAOA," *QCS*, IEEE, 2021.

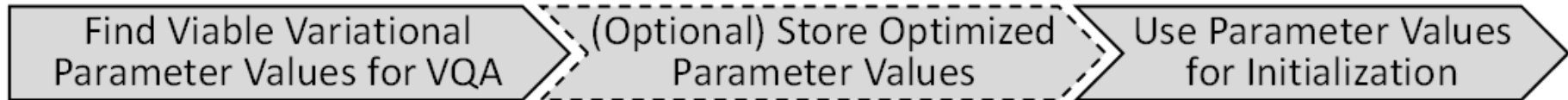
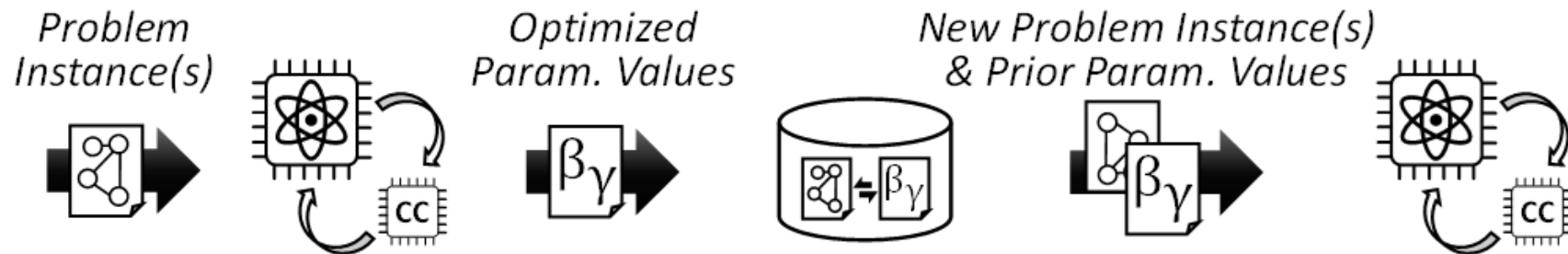
[3] M. Beisel et al., "QuantME4VQA: Modeling and Executing Variational Quantum Algorithms Using Workflows," *CLOSER*. SciTePress, 2023.

VARIATIONAL PARAMETER TRANSFER



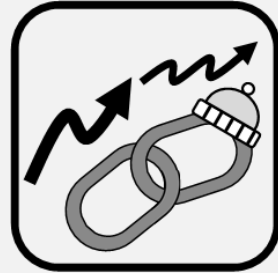
Problem: Obtain problem-aware parameter initialization for VQA to save time.

Solution Sketch:



[1] A. C. ...
 QAOA ...
 QCE, IB ...
 [2] R. S. ...
 Reprod ...
 Verific ...
 [3] M. ...
 and Executing Variational Quantum Algorithms ...
 Using Workflows,” CLOSER. SciTePress, 2023.

CHAINED OPTIMIZATION



Problem: Avoid local optima and improve convergence in VQA optimization.

Context: Globally optimal parameter values are required to obtain optimal solutions.

Solution: Chain optimizers with different scopes or strengths together.

Results: Subsequent optimizers warm-started to benefit from their respective strengths.

Known Uses: Avoid barren plateaus in VQAs, QNN optimization [1][2], reinforcement learning-based optimizer for QAOA combined with gradient-based local optimization [3]

[1] A. Rad et al., "Surviving The Barren Plateau in Variational Quantum Circuits with Bayesian Learning Initialization," arXiv:2203.02464, 2022.

[2] Z. Tao et al., "LAWS: Look Around and Warm-Start Natural Gradient Descent for Quantum Neural Networks," QSW, IEEE, 2023.

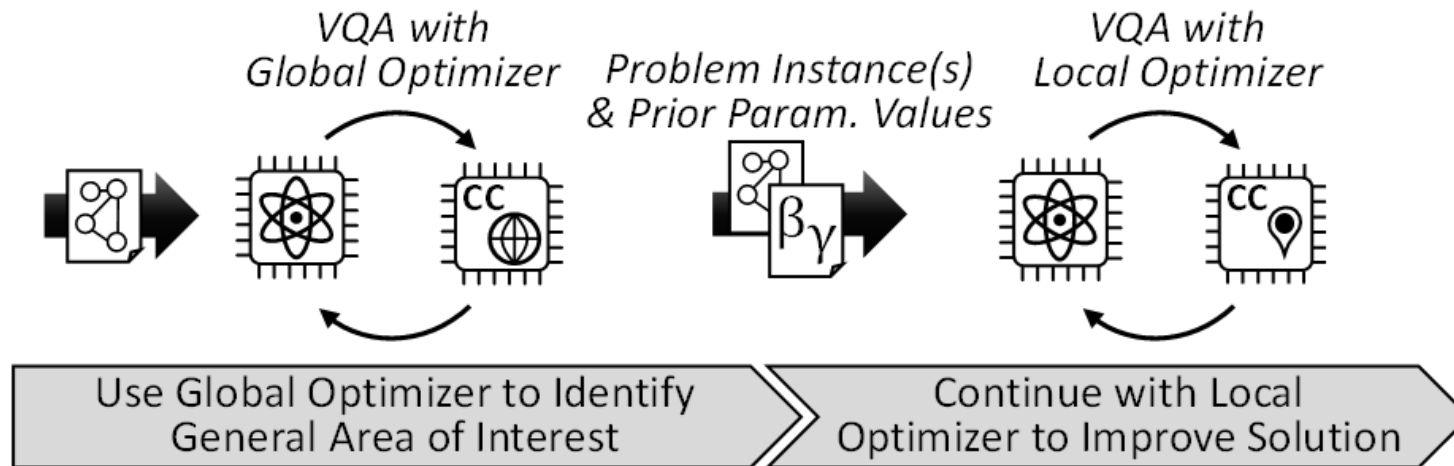
[3] M. M. Wauters et al., "Reinforcement-learning-assisted quantum optimization," Physical Review Research, 2020.

CHAINED OPTIMIZATION







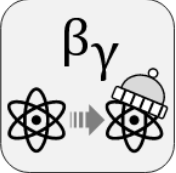

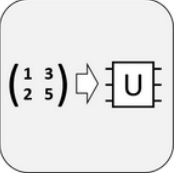
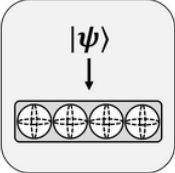
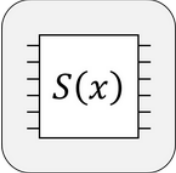

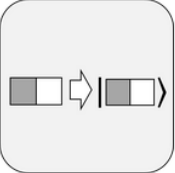

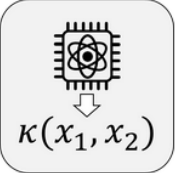



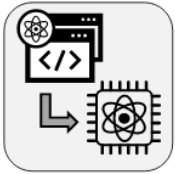
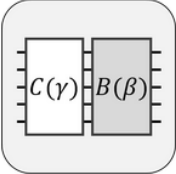
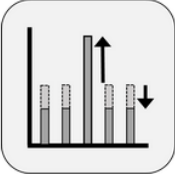
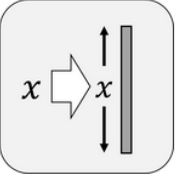
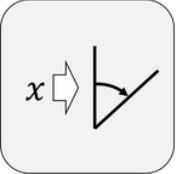
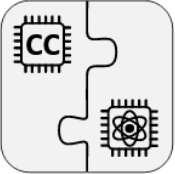





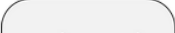


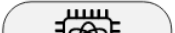

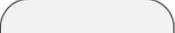



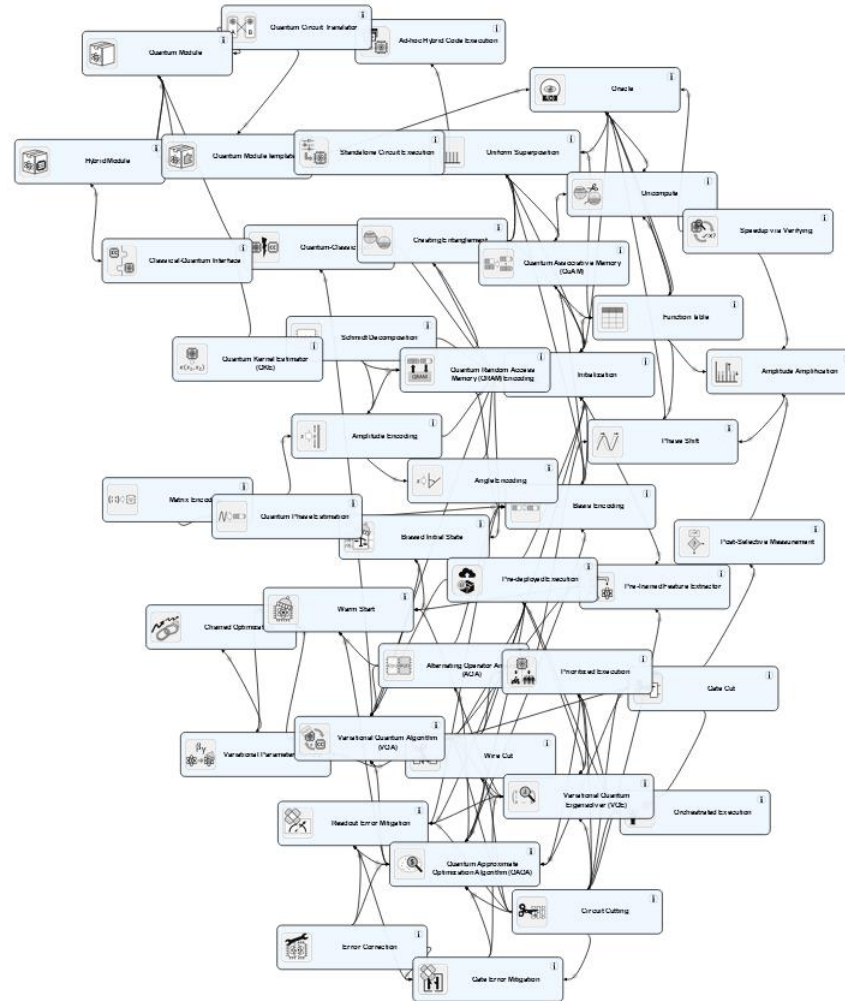
Problem: Avoid local optima and improve convergence in VQA optimization.

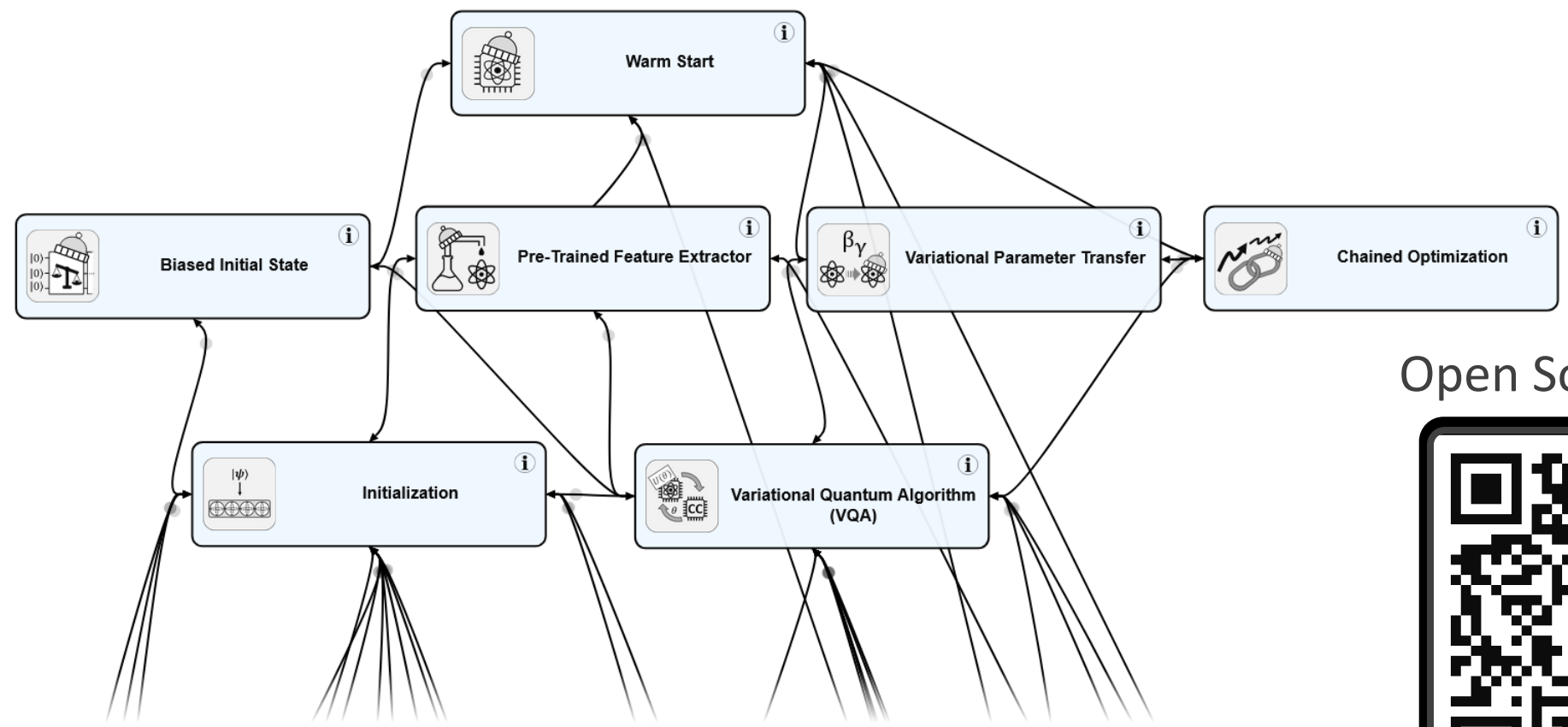
Solution Sketch:



[1] A. P. ...
 in Vari...
 Learni...
 2022.
 [2] Z. T...
 Warm-...
 Quant...
 [3] M. M. Wauters et al., "Reinforcement-learning-assisted quantum optimization," Physical Review Research, 2020.

| | | | | | | | | |
|---|---|---|--|--|---|--|---|--|
| <p>Uniform Superposition</p>  <p>Details</p> | <p>Wire Cut</p>  <p>Details</p> | <p>Gate Cut</p>  <p>Details</p> | <p>Circuit Cutting</p>  <p>Details</p> | <p>Biased Initial State</p>  <p>Details</p> | <p>Pre-Trained Feature Extractor</p>  <p>Details</p> | <p>Variational Parameter Transfer</p>  <p>Details</p> | <p>Chained Optimization</p>  <p>Details</p> | <p>Matrix Encoding</p>  <p>Details</p> |
| <p>Initialization</p>  <p>Details</p> | <p>Schmidt Decomposition</p>  <p>Details</p> | <p>Uncompute</p>  <p>Details</p> | <p>Basis Encoding</p>  <p>Details</p> | <p>Quantum-Classical Split</p>  <p>Details</p> | <p>Quantum Kernel Estimator (QKE)</p>  <p>$\kappa(x_1, x_2)$</p> <p>Details</p> | <p>Error Correction</p>  <p>Details</p> | <p>Gate Error Mitigation</p>  <p>Details</p> | <p>Readout Error Mitigation</p>  <p>Details</p> |
| <p>Ad-hoc Hybrid Code Execution</p>  <p>Details</p> | <p>Alternating Operator Ansatz (AOA)</p>  <p>Details</p> | <p>Amplitude Amplification</p>  <p>Details</p> | <p>Amplitude Encoding</p>  <p>x</p> <p>Details</p> | <p>Angle Encoding</p>  <p>x</p> <p>Details</p> | <p>Classical-Quantum Interface</p>  <p>Details</p> | <p>Creating Entanglement</p>  <p>Details</p> | <p>Function Table</p>  <p>Details</p> | <p>Hybrid Module</p>  <p>Details</p> |
| <p>Oracle</p>  | <p>Orchestrated Execution</p>  | <p>Phase Shift</p>  | <p>Post-Selective Measurement</p>  | <p>Pre-deployed Execution</p>  | <p>Prioritized Execution</p>  | <p>Quantum Approximate Optimization Algorithm (QAOA)</p>  | <p>Quantum Associative Memory (QuAM)</p>  | <p>Quantum Circuit Translator</p>  |





Open Source on 



Running Instance: patterns.platform.planqk.de

Summary & Outlook

- Introduced 4 new patterns for warm-starting quantum algorithms
- Facilitate efficient utilization of quantum algorithms in the NISQ era
- Future Work
 - Analyze additional warm-starting techniques
 - Further refinements and re-evaluation based on community feedback
 - Solution language: Connect patterns with concrete solutions

Thank you for your attention 😊