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INTELLI 2024 – Athens, Greece

„Adaptive Tracking Control for Biologically Inspired, Nonclassical Motion Systems “

(March 12th, 2024)

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Short Resume of the Presenter



Carsten Behn received the diploma degree in Mathematics (2001), the Ph.D. degree in Mechanical Engineering (2005) and his habilitation in Mechanical Engineering (2013) with *venia legendi* in “Technical Mechanics”, all from Technische Universität Ilmenau, Germany. Since 2019 he is a full professor at Schmalkalden University of Applied Sciences, Chair “Applied Mathematics, Mechanics and Dynamics of Machines”. His research interests include mechanical and mathematical modeling, adaptive and robust control of uncertain systems, with view to bio-inspired mechanical motion systems.

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Schmalkalden



city with half-timbered houses in its historic downtown district (founded 874 A.D.)





Campus: student residents (left), main lecturer hall (middle)



Campus: top view



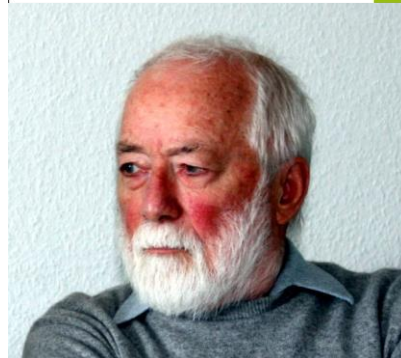
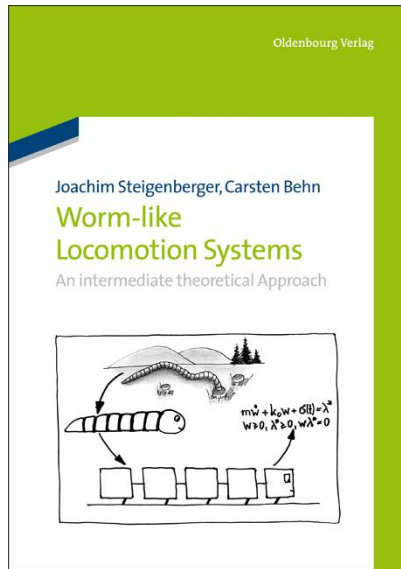
Lecture hall of Physics





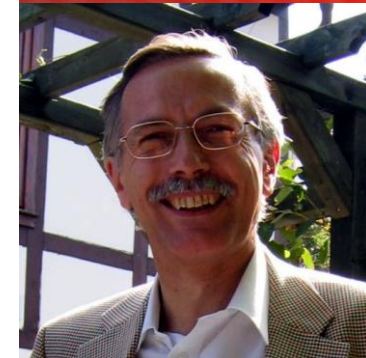
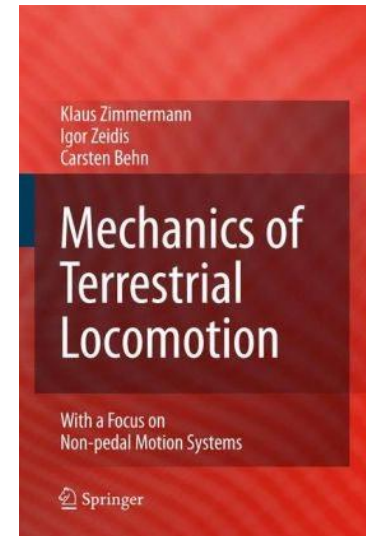
- founded July, 1st in 1902
- approx. 2500 students (Summer Semester 2022)
- 5 departments (Electrical Engineering, Mechanical Engineering, Business and Economics, Business Law, Computer Science)
- approx. 25% of the students at Dept. Of Mechanical Engineering
- 15 Bachelor- and 8 Master-courses
2 extra occupational Bachelor- and 7 Master-courses („Business Law“ and „Business Economics“)
- studies with a strong orientation / relation to practice and professional training

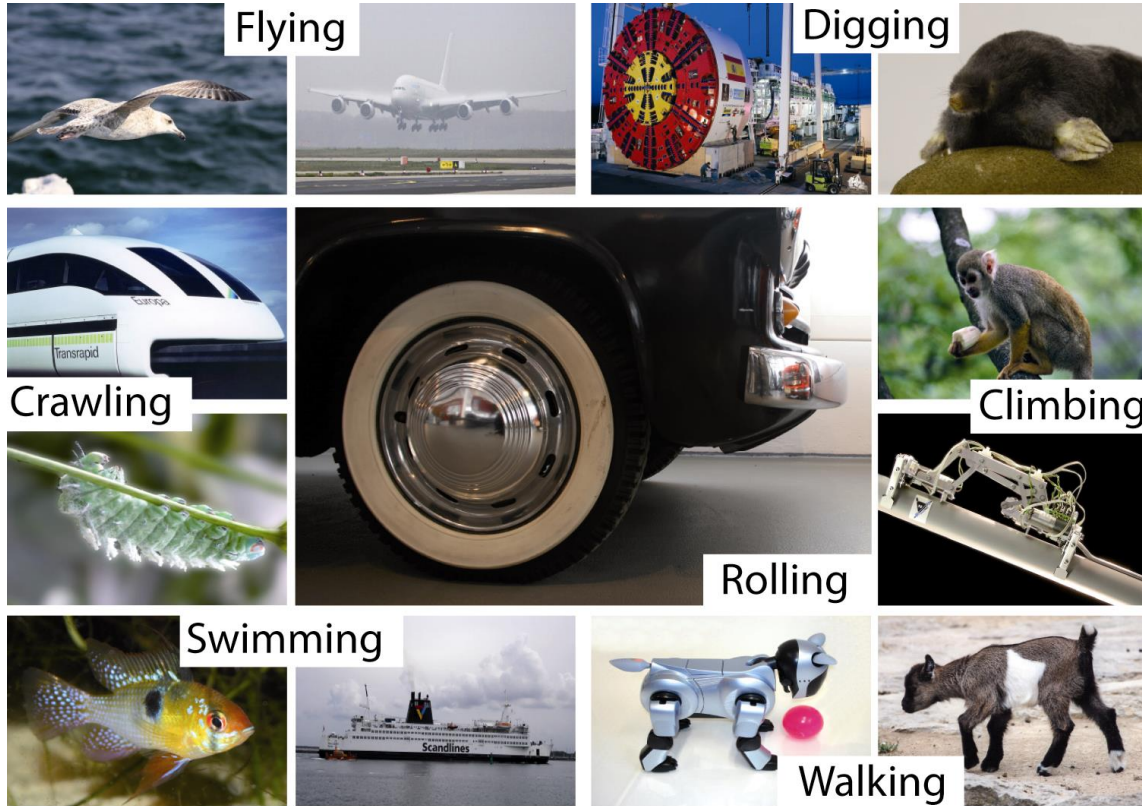
“Adaptive Tracking Control for Biologically Inspired, Nonclassical Motion Systems”



Worm-Like Locomotion Systems (WLLS)

1. Motivation & Introduction
2. Modeling
3. Kinematics
4. Gait Generation
5. Dynamics
6. Actuator Models
7. Adaptive Control
8. Simulations Part 1
9. Refinements
10. Simulations Part 2
11. Conclusions Part 1





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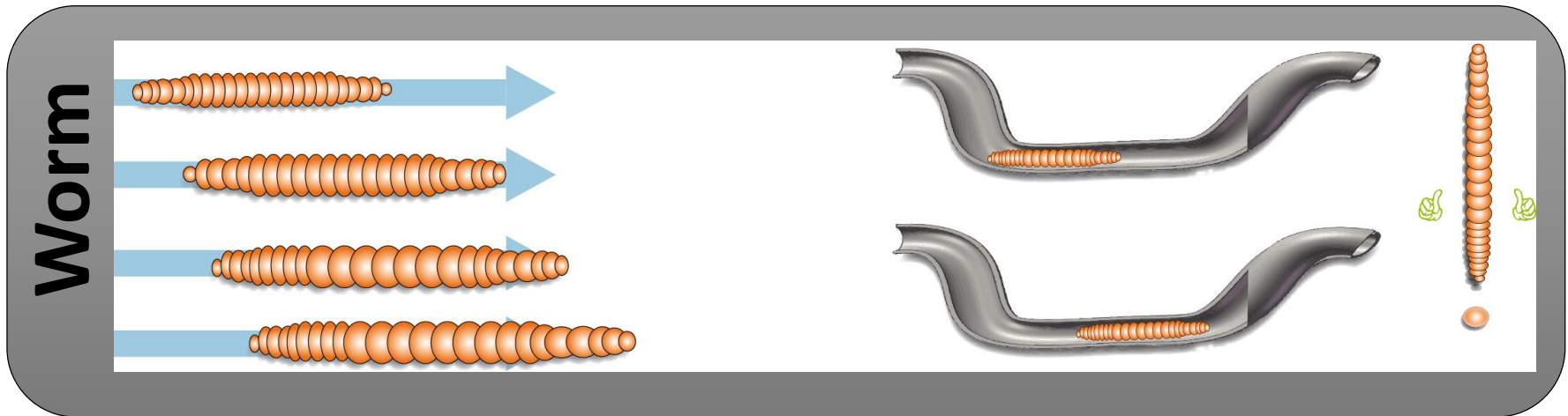
Focus on non-pedal / non-wheeled locomotion systems

biological example: the earthworm

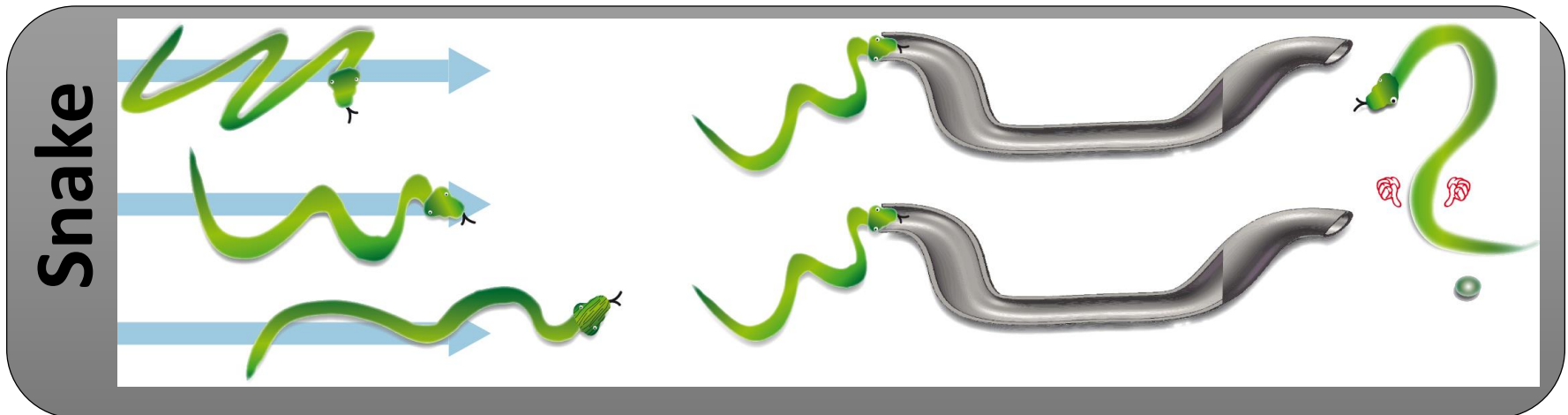


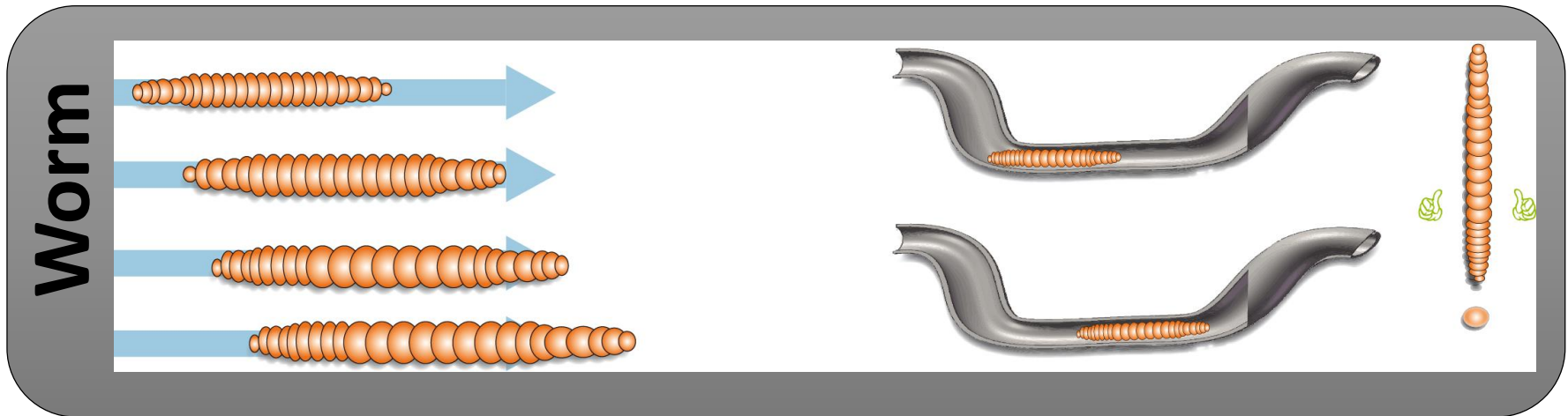
- peristaltic motion systems
- no concertina motion like snakes (sidewinder)

- possible applications / advantages in biologically inspired Robotics



Why focus on worms? Worm vs. Snake!





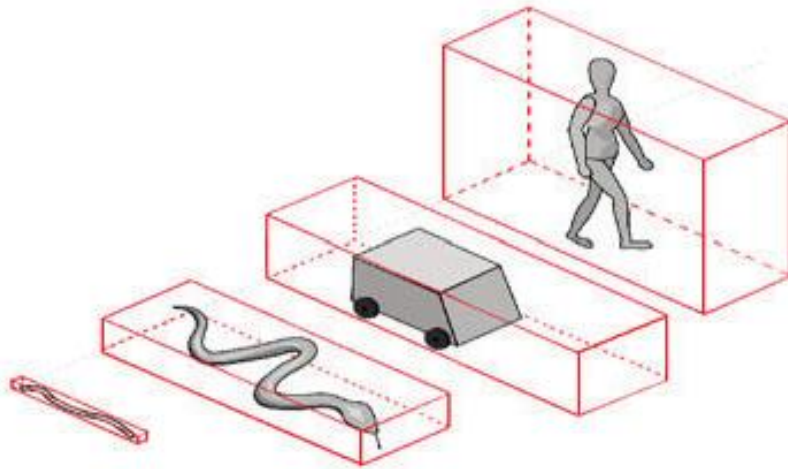
Why focus on worms? Worm vs. Snake!

Possible applications:

- inspection of cable and pipeline systems
- planned application in medical engineering (Minimal Invasive Surgery / Endoscopy)
- exploration in impassible regions (e.g., after earthquakes)



Necessity of motion systems with low space requirements



Space required for different forms of locomotion [Saga, 2004]



Search for survivors in a collapsed building [Radio912, 2009]

Goal: Development of locomotion systems with low space requirements

- Aim:**
- understand how such systems move
 - following analytical methods
 - well-founded mathematical framework !!! (control theory, e.g.)
 - and then build up prototypes to verify the theory

1. **analyzing** live biological systems, e.g. vibrissae,
2. **quantifying** the mechanical and environmental behavior: identifying and quantifying mechanosensitive responses (e.g., pressure, vibrations) and their mechanisms as adaptation,
3. **modeling** live paradigms with basic features developed before,
4. **exploiting** corresponding mathematical models in order to understand details of internal processes and,
5. **coming** to artificial prototypes (e.g., sensors in robotics), which exhibit features of the real paradigms.

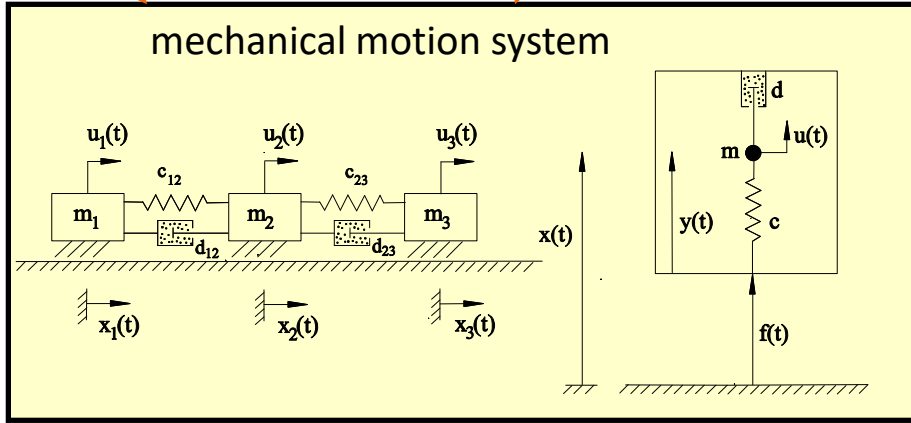
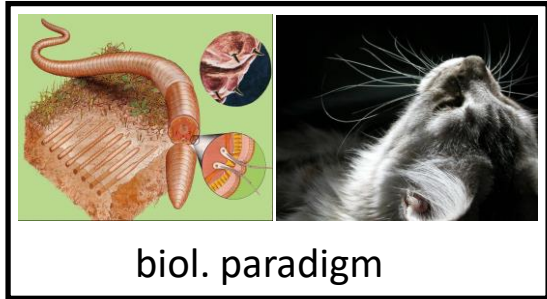
Important:

- focus is **not** on “copying” the solution from biology / animality
- **not** to construct prototypes with one-to-one properties of, e.g., a vibrissa

Different input possibilities
(open-loop/ closed-loop design)

Control goals
locomotion, track prescribed motion pattern, stability, robustness

Optimization criteria
(energy, time, ...)



mathem. model

Uncertainties

- system parameter
- unknown influences from environment (impulses, periodic excitations, ground interaction, ...)
- constraints, boundary limits (differential constraints)

Friction
(classifications, non-linear jump functions, approximations, num. aspects in simulations)

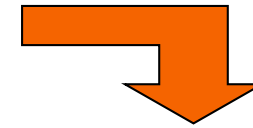
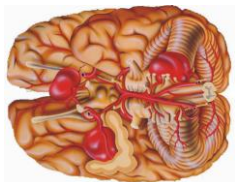
Tools
calculus, system theory, ODE theory (existence problems), control theory (adaptive)

Common problem to be solved in specific languages (live scientist / engineer)



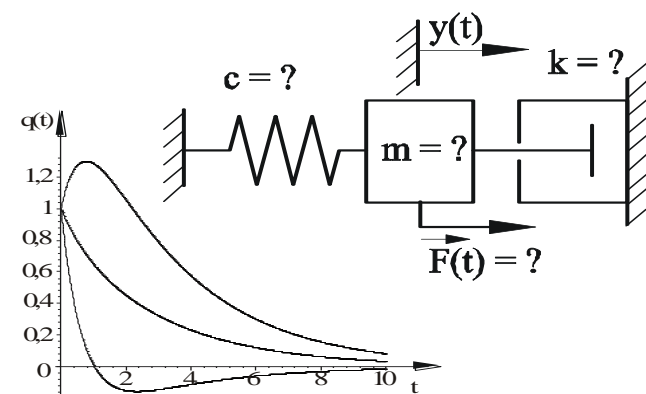
LIVE SCIENTIST

To what extent can a human brain be damaged without several lost of motor function of the owner?



ENGINEER

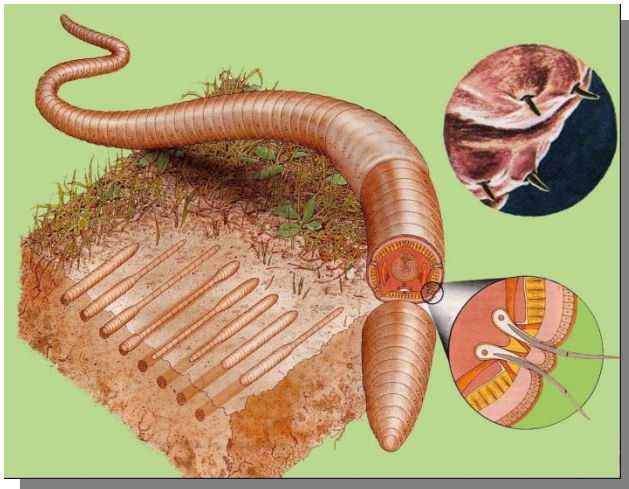
How many system information are needed to guarantee desired properties like stability of motion, controllability of the system or path following?



Basis of our theory:

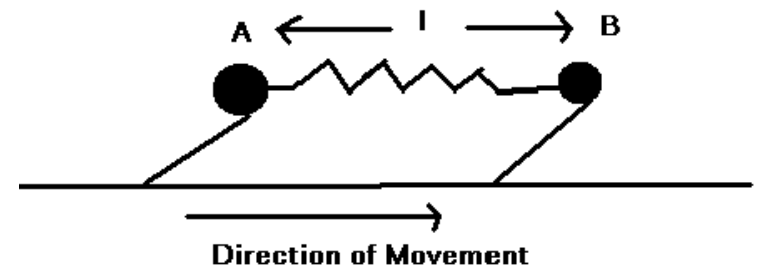
- i. A worm is a terrestrial locomotion system of one dominant linear dimension with no active legs nor wheels.
- ii. Global displacement is achieved by (periodic) **change of shape** and **interaction** with the environment (undulatory locomotion).

Change of shape: **peristaltic rectilinear locomotion**
- 'pumping' of worm segments



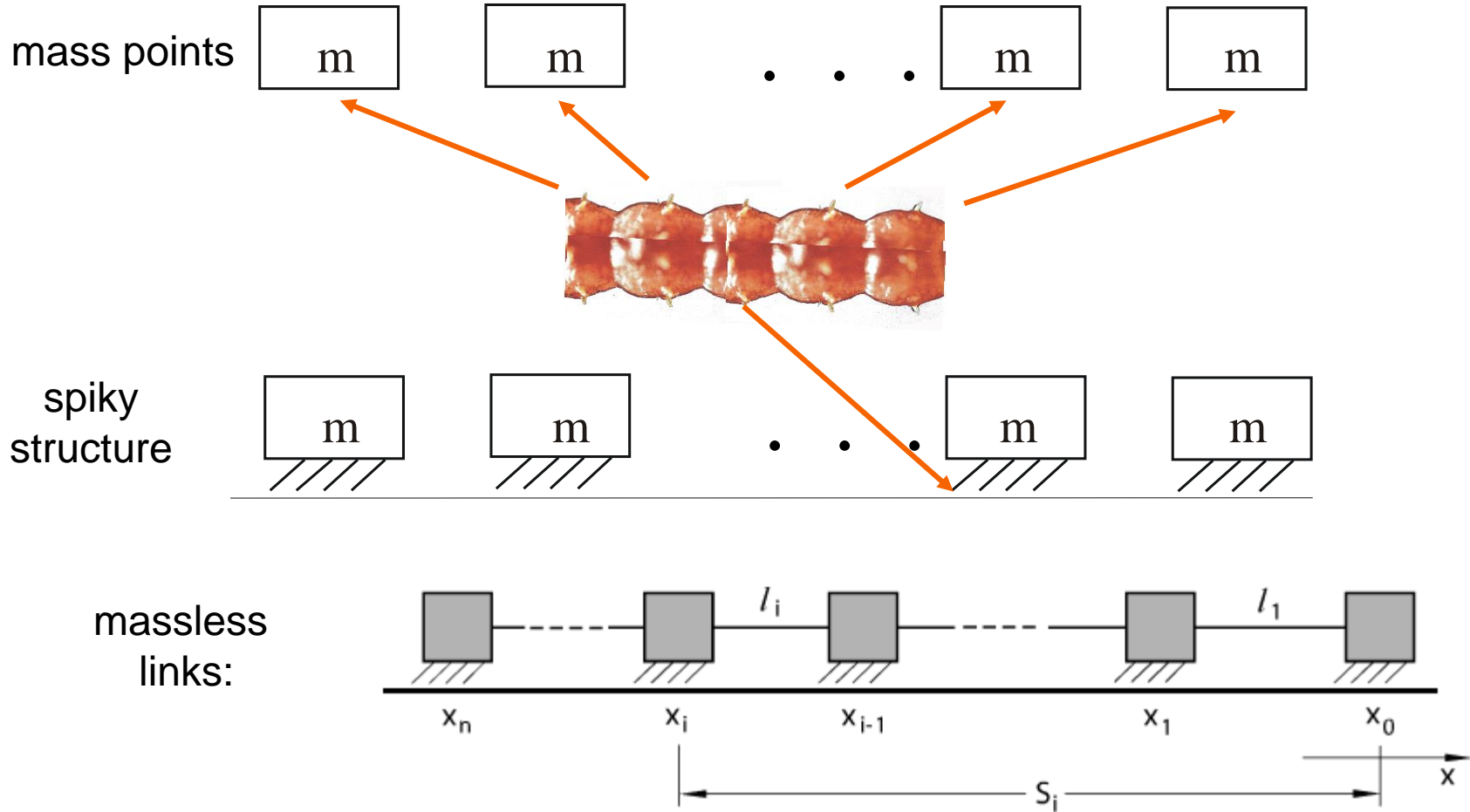
biological paradigm earthworm [Bailey, 93]

Interaction:
surface endowed with **spikes**
preventing velocities
from being negative



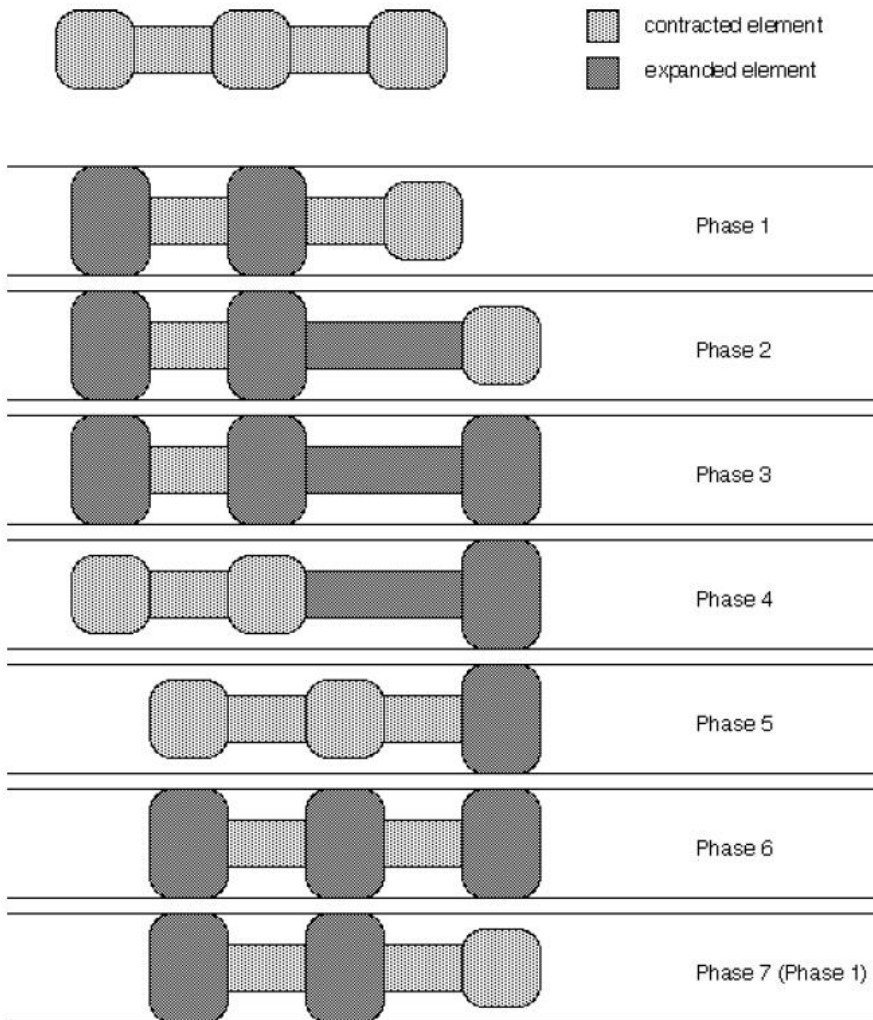
A first bristle model of Gavin Miller (1988)

Undulatory locomotion means a temporal process during which the internal actuators receive an activation signal and generate a (periodical in time) local deformation (contraction) which via interaction with the environment results in a global change of location. [Ostrowski, J.; Burdick, J.; Lewis, A.D.; Murray, R.M.: The mechanics of undulatory locomotion: the mixed kinematic and dynamic case. Proceedings of the IEEE International Conference on Robotics and Automation, 1995.]



First: surface endowed with **spikes** (later on friction) \longrightarrow thorough kinematic theory

GOAL: specific actual link lengths (gait) to achieve desired motion



[Slatkin, Burdick et. al. 1995]

Theory:

Slatkin, Burdick et. al. 1995

Chen, Yeo & Gao 1999 / 2001

Prototypes:

Chen & Yeo 2002

Nakamura et. al. 2006

Zimmermann et al. 2009

Soek et. al. 2010



view purely on kinematics to determine a specified gait

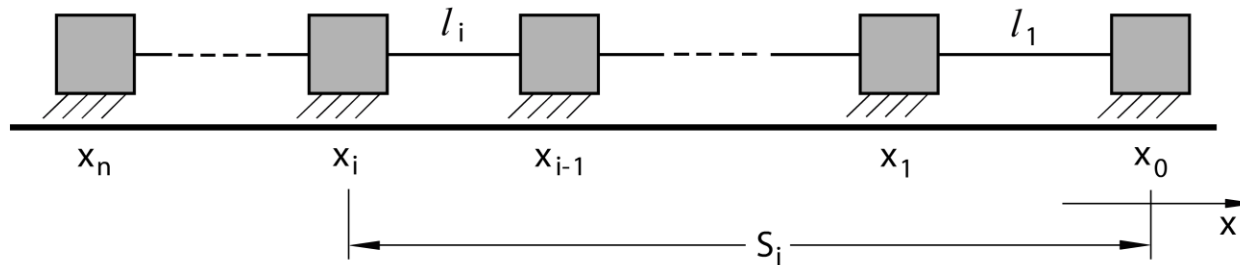
GOAL:

- not only to determine a specified gait in kinematics
- moreover: adjustment of desired speed
 - fast, i.e., only a few mass points should be at rest (active spikes)
 - realizable?
 - look at actuator and spikes forces:
 - need for high spikes forces?
 - increase number of active spikes
 - diminishes speed → start again
- What's the right number of active spikes?



Need for involving the external forces (viscous friction, spikes forces, e.g.) and internal forces (actuators, e.g.)

→ include **DYNAMICS!!!**



Spikes (kinematic constraint)

$$\dot{x}_i \geq 0, \quad \forall i \in 0, \dots, n$$



$$\dot{x}_0 - \dot{S}_i \geq 0 \Leftrightarrow \dot{x}_0 \geq \dot{S}_i \quad \forall i$$



$$\dot{x}_0 \geq V_0 := \max \{ \dot{S}_i | i \in \{0, \dots, n\} \}$$

$$\Rightarrow \dot{x}_0 = V_0 + w, \quad w \geq 0$$

$$\text{and } \dot{x}_i = V_0 - \dot{S}_i + w$$

actual links **lengths** $l_j := x_{j-1} - x_j$

distances of mass points from head

$$S_i := x_0 - x_i = \sum_{j=1}^i l_j$$



$$\dot{x}_i = \dot{x}_0 - \dot{S}_i, \quad \forall i$$



- w unknown
- ‚rigid‘ forward velocity part
- drive must guarantee $w = 0$ (no skidding forward)

Simple kinematic theory

1. Prescribe: $l_j(\cdot) \in D^2(\mathbb{R}) : t \mapsto l_j(t) > 0, \quad j = 1 \dots n.$

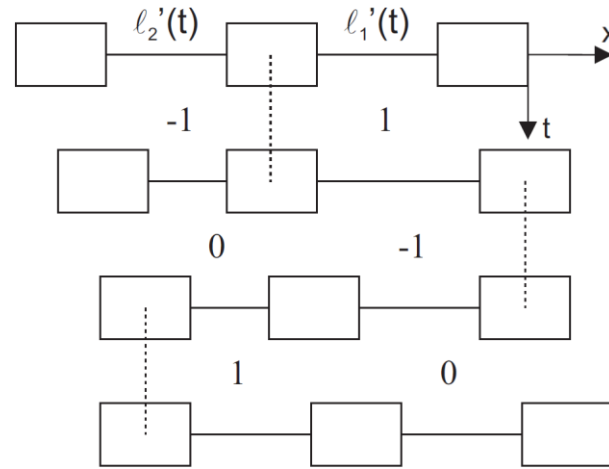
2. Determine: $S_i := \sum_{j=1}^i l_j, \quad V_0 := \max \{ \dot{S}_i | i \in \{0 \dots, n\} \} \in D^1(\mathbb{R}).$

3. Result: $x_0(t) = \int_0^t V_0(s) ds, \quad x_j(t) = x_0(t) - S_j(t), \quad j = 1, \dots, n.$

 derive appropriate gaits, how?

Example: $n = 2$
pure kinematics

mode = $\{1, 0, 2\}$

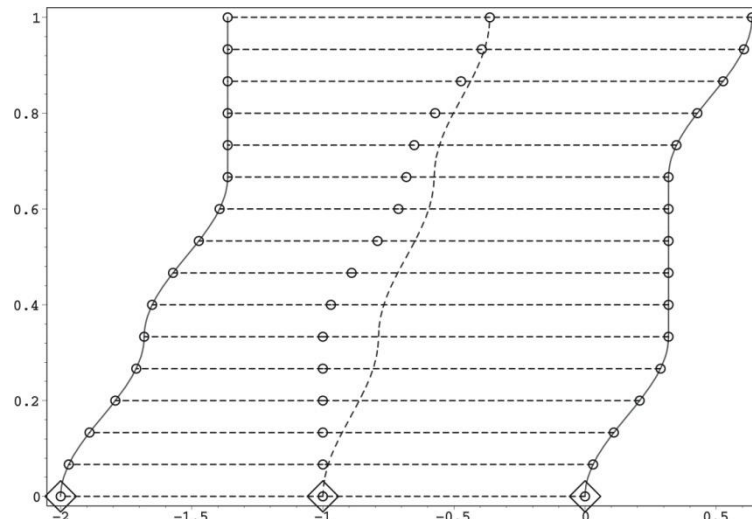
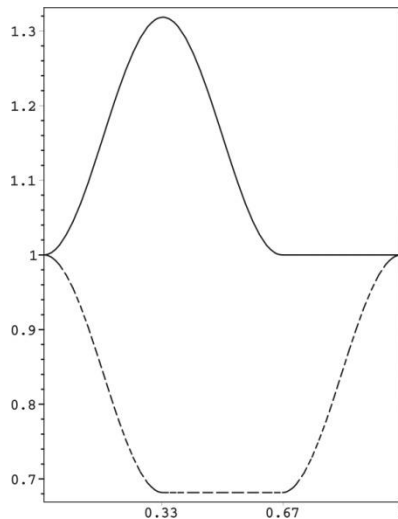


cyclically repeated
consecutive spikes active

$l_2'(t)$	$l_1'(t)$
-1	1
0	-1
1	0

$\varepsilon = 0.3, l^0 = 1,$
 $\omega = 2\pi, T = 1$

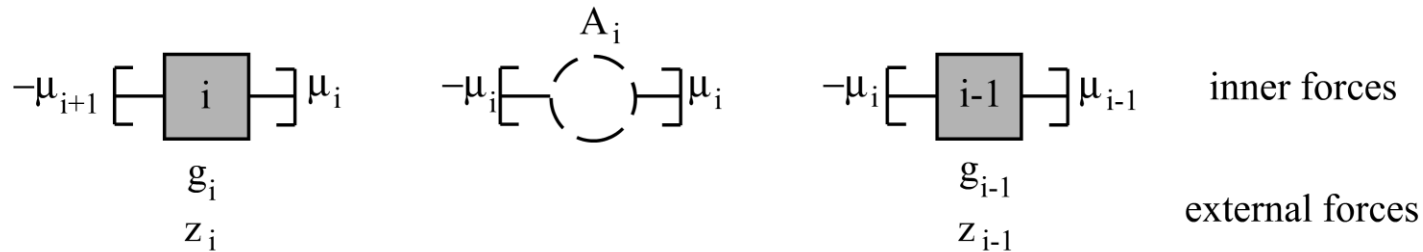
$$\dot{l}_j(t) = \varepsilon l^0 \frac{N}{\pi} \omega \sin^2 \left(N \frac{\omega}{2} t \right)$$



Which gait
for what kind
of motion?



DYNAMICS



- g_i external impressed force, e.g., resultant of weight Γ_i , visc. friction $-k_0 \dot{x}_i$
- μ_i stress resultant, cut force
- z_i external reaction force caused by the kinematic constraint (spikes)



complementary slackness condition $\dot{x}_i \geq 0, \quad z_i \geq 0, \quad \dot{x}_i z_i = 0 \quad \forall i$

satisfied by $z_i(f_i, \dot{x}_i) = -\frac{1}{2} (1 - \text{sign}(\dot{x}_i)) (1 - \text{sign}(f_i)) f_i$

(f_i resultant of all further forces acting on masspoint i)



Newton's 2nd law: $m \ddot{x}_i = \underbrace{g_i + \mu_i - \mu_{i+1} + z_i}_{=: f_i}, \quad \forall i$

summing up all the Newton's 2nd laws for the mass points, using $g_i := -k_0 \dot{x}_i - \Gamma_i$

$$m \dot{v}^* + k_0 v^* + \Gamma^* = z^*$$

$$\text{using } \Gamma^* := \frac{1}{n+1} \sum_i \Gamma_i, \quad z^* := \frac{1}{n+1} \sum_i z_i$$

$$\begin{aligned} \text{from kinematics: } \dot{x}_i = V_0 - \dot{S}_i + w &\Rightarrow \dot{x}_i^* = \frac{1}{n+1} \sum_i \dot{x}_i \\ &\Rightarrow v^* = V_0 - \underbrace{\frac{1}{n+1} \sum_i \dot{S}_i}_{=:\dot{S}} + w \\ &\quad \underbrace{\hspace{10em}}_{=:W_0 \text{ (defined via gait!!)}} \end{aligned}$$

$$\text{This yields: } m \dot{w} + k_0 w + \underbrace{m \dot{W}_0 + k_0 W_0}_{=:\sigma} + \Gamma^* = z^*$$

$$\Rightarrow m \dot{w} + k_0 w + \sigma = z^*$$

$$\text{with } w \geq 0, z^* \geq 0, w z^* = 0$$

central equation

with **slackness condition**

Remind of: $m \dot{w} + k_0 w + \sigma = z^*$ with $w \geq 0$, $z^* \geq 0$, $w z^* = 0$

Observations:

1. No skidding forward $w = 0 \Leftrightarrow z^* > 0$ (at least one active spike, obvious)
2. $z^* = 0 \Rightarrow w > 0$, i.e., $m \dot{w} + k_0 w = -\sigma \Rightarrow \sigma < 0$
spikes forces zero, no active spike, i.e., skidding forward

Finally: 1.) Prescribe gait and surrounding (friction, landscape)

\Rightarrow we have $l_j, \dot{l}_j, S_i, \dot{S}_i, V_0, W_0, \dot{W}_0, k_0, \Gamma \Rightarrow \sigma = \sigma(t)$

2.) Gait, such that

- a) no skidding forward $\sigma(t) \geq 0 \Rightarrow m \varepsilon l^0 \omega^2 \frac{N(N-a)}{2\pi} \leq \Gamma^*$
- b) finite strength of spikes $m \varepsilon l^0 \omega^2 \frac{N(N-a)}{2\pi} \leq \frac{a}{N} \hat{z} - \Gamma^*$

Gait construction considering dynamics!!

For details see:



Contents lists available at ScienceDirect

Robotics and Autonomous Systems

journal homepage: www.elsevier.com/locate/robot



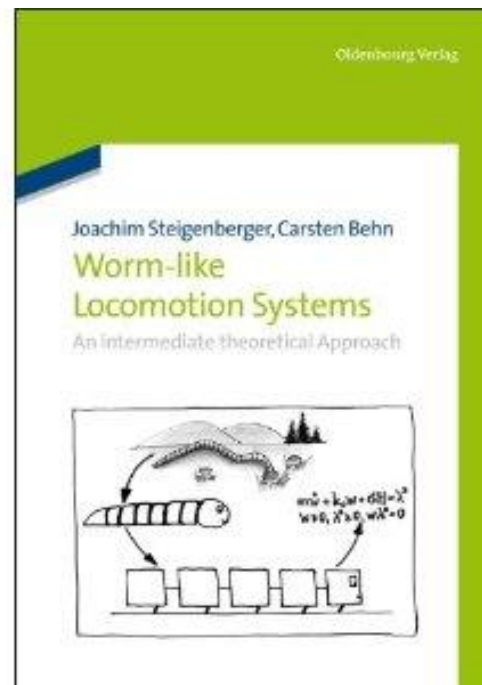
Gait generation considering dynamics for artificial segmented worms

Joachim Steigenberger^a, Carsten Behn^{b,*}

^a Institute of Mathematics, Ilmenau University of Technology, Weimarer Straße 25, 98693 Ilmenau, Germany

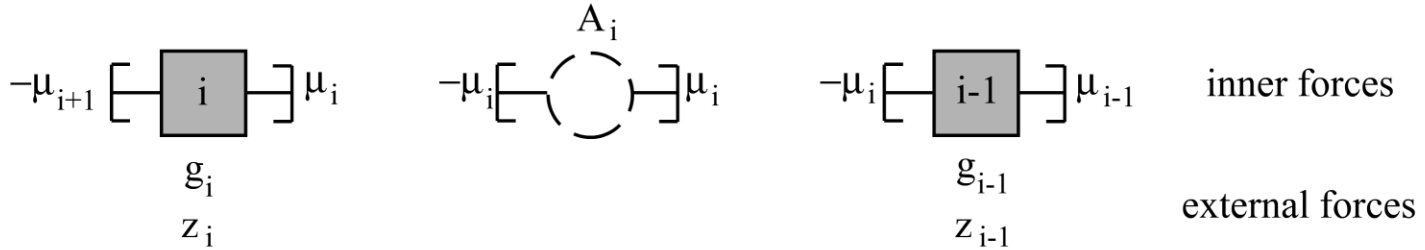
^b Department of Technical Mechanics, Ilmenau University of Technology, Max-Planck-Ring 12, Building F, 98693 Ilmenau, Germany

For more details see:



But: How to realize a gait???

Cut forces? Each link carries an actuator:

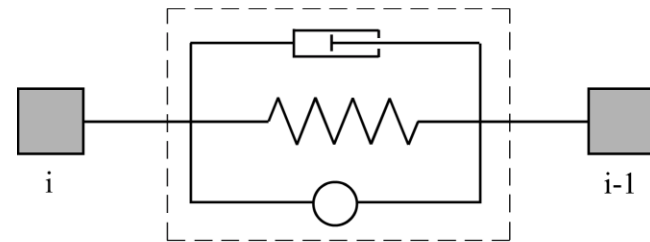


Black Box

output $l_i(t)$
(via stepping motor, piezo, ...)

White Box

one possibility: output force $\mu_i(t)$ (impressed)



$$\mu_i(t, x, \dot{x}) := c(x_{i-1} - x_i - l^0) + k_{00}(\dot{x}_{i-1} - \dot{x}_i) + u_i(t)$$

$u_i(t)$ – control force



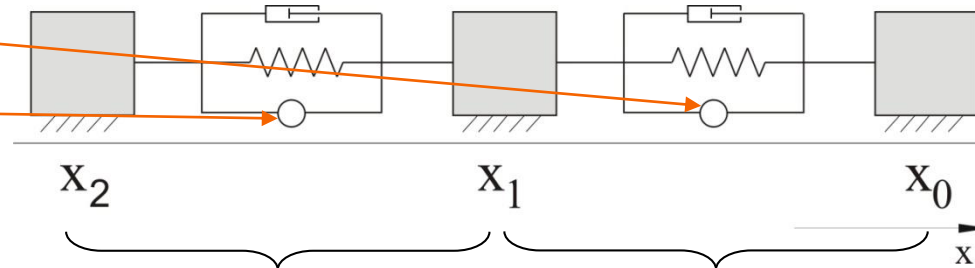
Kinematical Theory



Dynamical (Control) Theory

Control inputs

$$u(t) = \begin{pmatrix} u_1(t) \\ u_2(t) \end{pmatrix}$$



Outputs

$$y(t) = \begin{pmatrix} x_0(t) - x_1(t) \\ x_1(t) - x_2(t) \end{pmatrix}$$

distances

desired distances: **References**

$$y_{\text{ref}}(t) = \begin{pmatrix} l_1(t) \\ l_2(t) \end{pmatrix}$$

prescribed gait from kinematics

problem:

- lack of precise knowledge of actuator data
- moreover, worm system parameter not exactly known as well
- ➡ uncertain systems

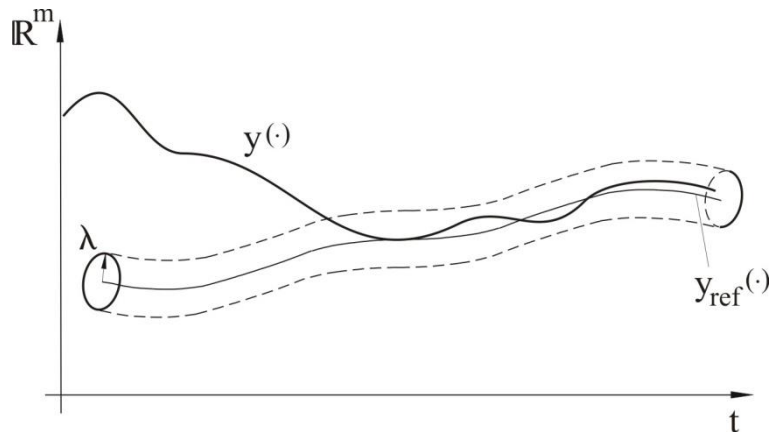
task:

- design a controller which generates the necessary output forces on its own
- no identification, simply controlling of the WLLS in order to track a given reference trajectory (kinematic gait)
- ➡ movement of the system

solution:

- adaptive tracking controller (learning controller)
- has to achieve λ -tracking (not exact tracking):

- every solution of the closed-loop system is defined and bounded on $\mathbb{R}_{\geq 0}$,
- the output $y(\cdot)$ tracks the given reference signal with asymptotic accuracy λ .



λ -tracking \Rightarrow simple controller:

$$\left. \begin{aligned} e(t) &:= y(t) - y_{\text{ref}}(t) \\ u(t) &= k(t)e(t) + \frac{d}{dt}(k(t)e(t)) \\ \dot{k}(t) &= \gamma (\max\{0, \|e(t)\| - \lambda\})^2 \end{aligned} \right\}$$

$$\lambda > 0, \gamma \gg 1, k(0) \in \mathbb{R}$$

Remark: Spikes (complementary slackness) force for dynamical control system

$$z_i(f_i, \dot{x}_i) = -\frac{1}{2} (1 - \text{sign}(\dot{x}_i)) (1 - \text{sign}(f_i)) f_i$$

General
System
Class:

$$\left. \begin{aligned} \begin{pmatrix} \dot{y}(t) \\ \dot{z}(t) \end{pmatrix} &= \begin{bmatrix} 0 & I_m & 0 \\ 0 & A_2 & 0 \\ 0 & A_0 & A_5 \end{bmatrix} \begin{pmatrix} y(t) \\ \dot{y}(t) \\ z(t) \end{pmatrix} + \begin{bmatrix} 0 \\ G \\ 0 \end{bmatrix} u(t) + \begin{bmatrix} 0 \\ g_1(s_1(t), y(t), z(t)) \\ g_2(s_2(t), y(t)) \end{bmatrix}, \\ y(0) &= y_0, \quad \dot{y}(0) = y_1, \quad z(0) = z_0, \end{aligned} \right\}$$

- $y(t), \dot{y}(t), u(t) \in \mathbb{R}^m, z(t) \in \mathbb{R}^{n-2m}$
- real matrices $A_2, G \in \mathbb{R}^{m \times m}, A_0 \in \mathbb{R}^{(n-2m) \times m}, A_5 \in \mathbb{R}^{(n-2m) \times (n-2m)}$
- $n \geq 2m$.

Properties:

- quadratic, nonlinearly perturbed multi-input $u(\cdot)$, multi-output (MIMO) $y(\cdot)$ control system with strict relative degree two;
- $\sigma(G) \subset \mathbb{C}_+$, i.e. the spectrum of the ‘high-frequency gain’ G lies in the open right-half complex plane;
- unperturbed system is minimum phase (stable zero dynamics): $\sigma(A_5) \subset \mathbb{C}_-$;
- A_0 is coupling term, necessary for underactuated systems;
- functions g_1 and g_2 are continuous and linearly affine bounded;
- $s_1(\cdot)$ and $s_2(\cdot)$ are (bounded) disturbance terms, where we have to claim the following dependence (all theorems and results remain valid):

$$s_1(t) = \psi_1(t, y(t), \dot{y}(t), z(t), u(t)) \quad \text{and} \quad s_2(t) = \psi_2(t, y(t), \dot{y}(t), z(t), u(t));$$

Theorem:

Let $\lambda > 0$, $\mathbf{y}_{\text{ref}}(\cdot) \in \mathcal{R}$, $s_1(\cdot) \in \mathcal{L}^\infty(\mathbb{R}_{\geq 0}; \mathbb{R}^{q_1})$ and $s_2(\cdot) \in \mathcal{L}^\infty(\mathbb{R}_{\geq 0}; \mathbb{R}^{q_2})$. Then the presented adaptive λ -tracker applied to every system of the general system class yields for any initial data $(\mathbf{y}_0, \mathbf{y}_1, \mathbf{z}_0, \mathbf{k}_0) \in \mathbb{R}^m \times \mathbb{R}^m \times \mathbb{R}^{n-2m} \times \mathbb{R}$

$$\left. \begin{aligned} \dot{\mathbf{y}}(t) &= \zeta(t), & \mathbf{y}(0) &= \mathbf{y}_0, \\ \dot{\zeta}(t) &= \mathbf{A}_2 \zeta(t) + f_1(s_1(t), \mathbf{y}(t), \mathbf{z}(t)) \\ &\quad - \mathbf{G} \left[k(t) (\mathbf{y}(t) - \mathbf{y}_{\text{ref}}(t)) + k(t) (\zeta(t) - \dot{\mathbf{y}}_{\text{ref}}(t)) \right. \\ &\quad \left. + \max \left\{ 0, \|\mathbf{y}(t) - \mathbf{y}_{\text{ref}}(t)\| - \lambda \right\}^2 (\mathbf{y}(t) - \mathbf{y}_{\text{ref}}(t)) \right], & \zeta(0) &= \mathbf{y}_1, \\ \dot{\mathbf{z}}(t) &= \mathbf{A}_5 \mathbf{z}(t) + \mathbf{A}_0 \zeta(t) + f_2(s_2(t), \mathbf{y}(t)), & \mathbf{z}(0) &= \mathbf{z}_0, \\ \dot{\mathbf{k}}(t) &= \max \left\{ 0, \|\mathbf{y}(t) - \mathbf{y}_{\text{ref}}(t)\| - \lambda \right\}^2, & \mathbf{k}(0) &= \mathbf{k}_0, \end{aligned} \right\}$$

which has a maximal solution $(\mathbf{y}, \zeta, \mathbf{z}, \mathbf{k}) : [0, t') \rightarrow \mathbb{R}^m \times \mathbb{R}^m \times \mathbb{R}^{n-2m} \times \mathbb{R}$ with:

- (i) $t' = \infty$, i.e. there does not exist a finite escape time;
- (ii) $\lim_{t \rightarrow \infty} k(t)$ exists and is finite;
- (iii) the solution, $\dot{\zeta}(\cdot)$, $\dot{\mathbf{z}}(\cdot)$ and $\mathbf{u}(\cdot)$ are bounded;
- (iv) $\limsup_{t \rightarrow \infty} \|\mathbf{y}(t) - \mathbf{y}_{\text{ref}}(t)\| \leq \lambda$.

The presented controller
achieves lambda-tracking!
(PhD thesis C. Behn)

Idea:

- Prescribe an (optimal) gait from the kinematical theory.
- This gait serves as a reference signal to be tracked by the controller.
- Controller adjusted distances between the mass point
- Results in a locomotion of the whole system (while in contact to an environment)

Remarks:

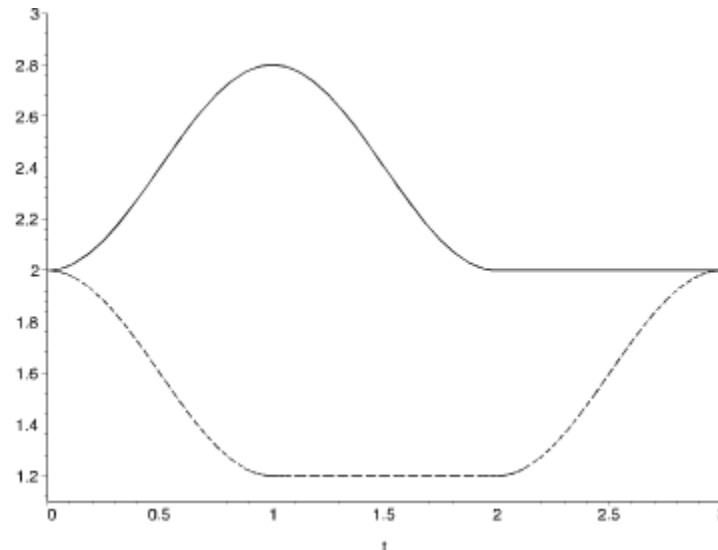
- The adaptive nature of the controller is expressed by **arbitrary choice of the system parameters**
- Obviously, for numerical simulation, these parameters / system data is fixed and known
- But, the controllers adjust their gain parameters to each set of system data.

Data:

Worm system $m_0 = m_1 = m_2 = 1$, $c = 10$, $k_{00} = 5$

Environment $k_0 = 0$, $\Gamma_{1,2,3} = 2.7$
(ensures kinematical theory to be dynamically feasible)

Reference Gait



Gait from Kinematics
(Slide 022)

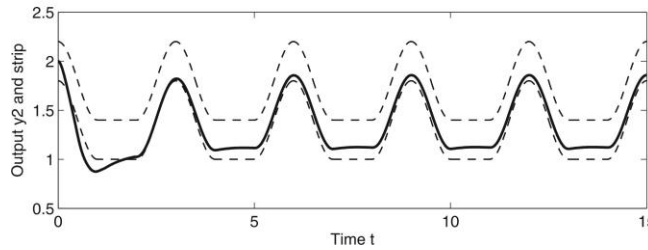
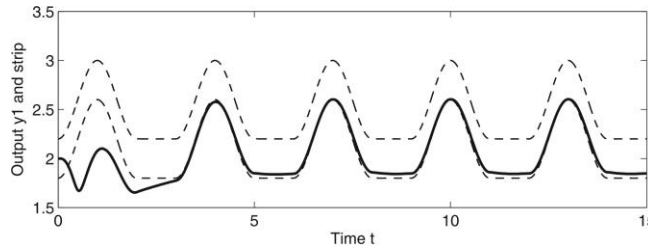
upper curve: $l_1(t)$
lower curve: $l_2(t)$

(one resting point mass at
any time: fast in-plane gait)

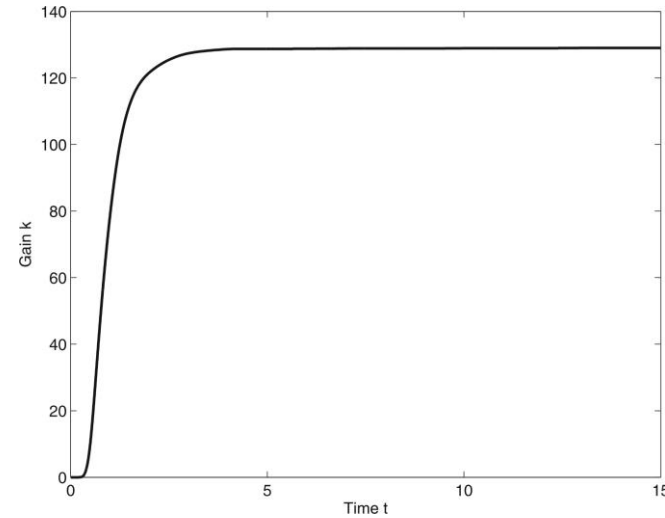
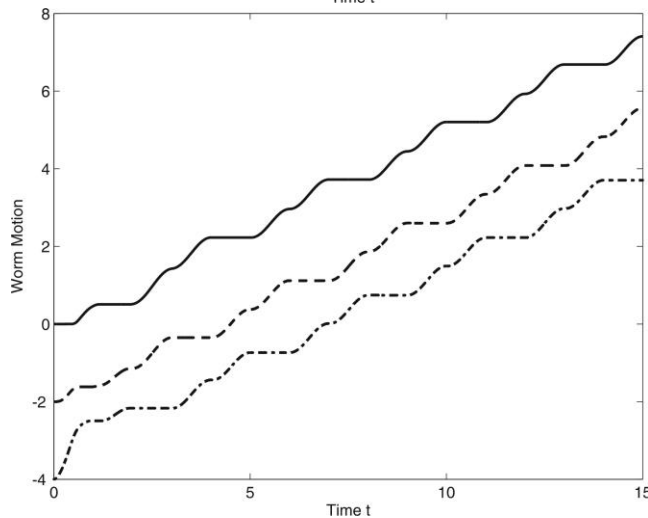
Controller $k(0) = 0$, $\lambda = 0.2$, $\gamma = 100$

Simulation 1: worm with spikes and adaptive control

Outputs
and tubes



Worm
motion



Gain
parameter

- good tracking behavior after transient phase
- *average speed = 0.4938*
- monotonic increase and convergence of gain to a finite limit

Remark

- high gain values, k still stays constant although control objective is achieved
- high feedback values
- caused by monotonic increasing of the gain parameter (also literature)

3 – Stages – Adaptation – Law:

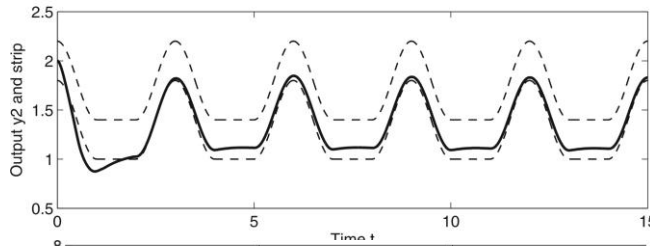
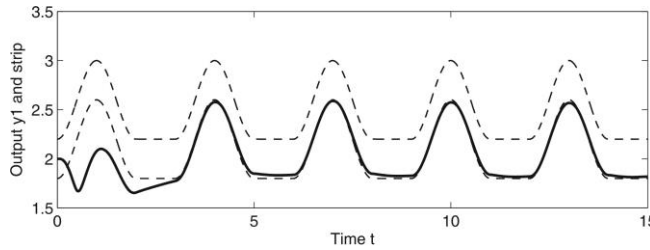
1. increasing $k(\cdot)$ while e is outside the tube,
2. constant $k(\cdot)$ after e entered the tube - no longer than a pre-specified duration of stay,
3. decreasing $k(\cdot)$ after this duration has been exceeded:

$$\dot{k}(t) = \begin{cases} \gamma (\|e(t)\| - \lambda)^2, & \|e(t)\| \geq \lambda \\ 0, & (\|e(t)\| < \lambda) \wedge (t - t_E < t_d) \\ -\sigma k(t), & (\|e(t)\| < \lambda) \wedge (t - t_E \geq t_d) \end{cases}$$

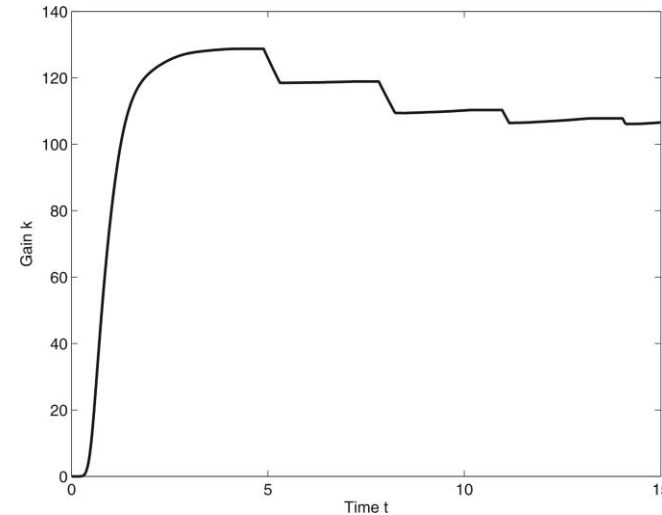
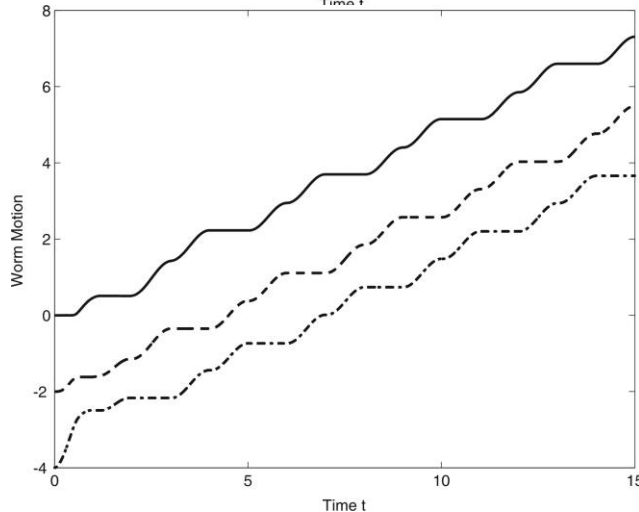
$$\sigma = 0.2 > 0, \quad t_d = 1$$

Simulation 2: worm with spikes and new adapter

Outputs and tubes



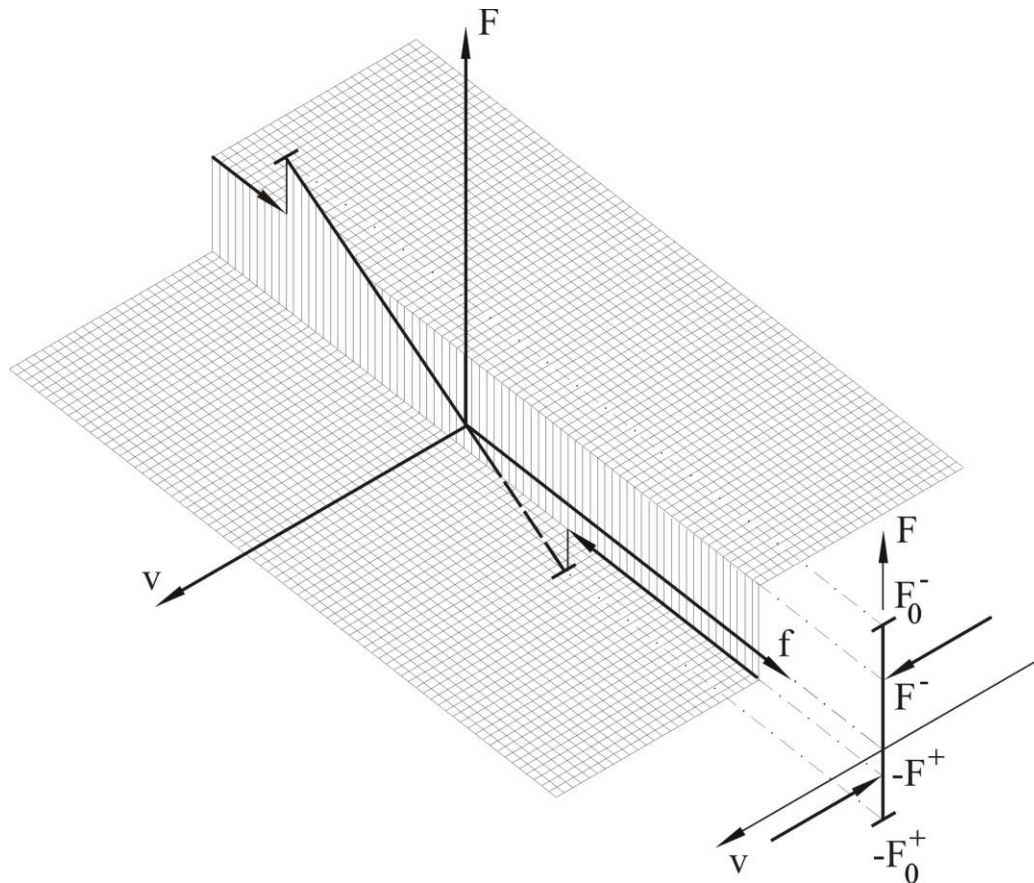
Worm motion



Gain parameter

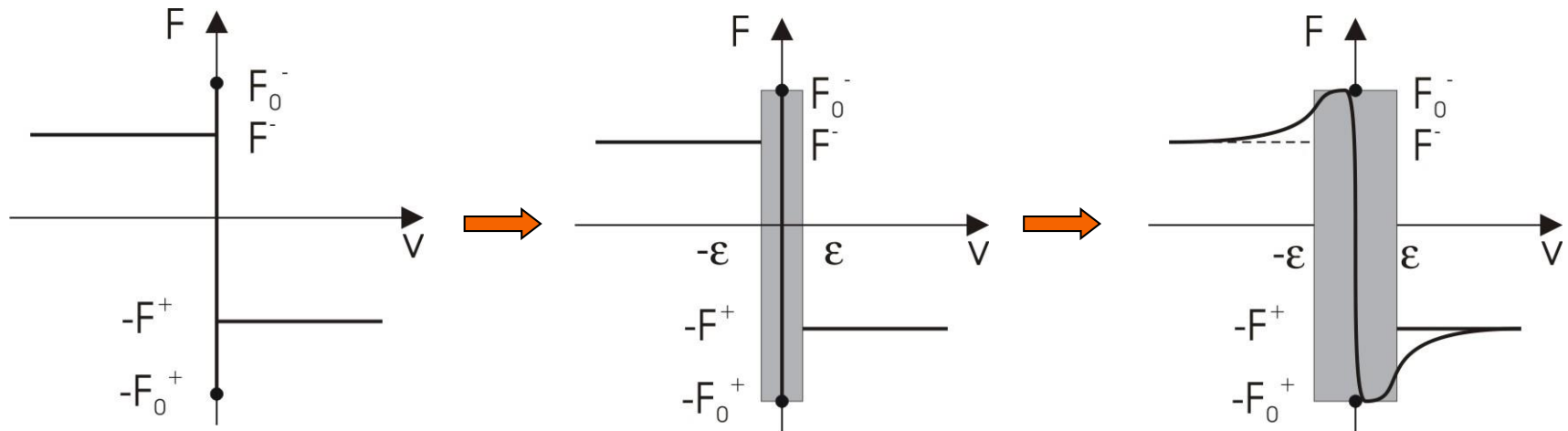
- good tracking behavior
- decreasing of the gain
- minimum high-gain $k^*=100$
- *average speed = 0.4859*

- practical failing of ideal spikes (finite strength) requires friction
- new **approach of interaction**: Coulomb law - stiction combined with dry sliding friction
- **but**: modeling makes friction a function of **two** arguments



- projection to the (v, F) -plane shows the typical F vs. v diagram that
- this is often preferred in literature
- but there F appears as a set-valued function, here well-defined
- tackled in dynamics by means of fitting theories
- now: relaxation ...

- Relaxation:
 ε blow-up interval (Karnopp), ε replaces computer accuracy



- friction law with stiction in the literature $F_R = F_R(v)$
- grasp $v=0$ with PC?
- really stiction?
- discontinuities (jumps)

- status $v=0$ is blown up to an interval for simulating stick-slip
- Karnopp model
- discontinuity still present

- adequate, smooth approximation, but...

$$(v, f_a) \mapsto F_R(v, f_a) = \begin{cases} -F^+ & , v > 0 \quad \vee \quad (v = 0 \wedge f_a > F_0^+) \\ -f_a & , v = 0 \wedge f_a \in [-F_0^-, F_0^+] \\ F^- & , v < 0 \quad \vee \quad (v = 0 \wedge f_a < -F_0^-) \end{cases}$$

- **new model**: theoretically transparent, handy in computing, simpler than various sophisticated ones in literature and captures stick-slip

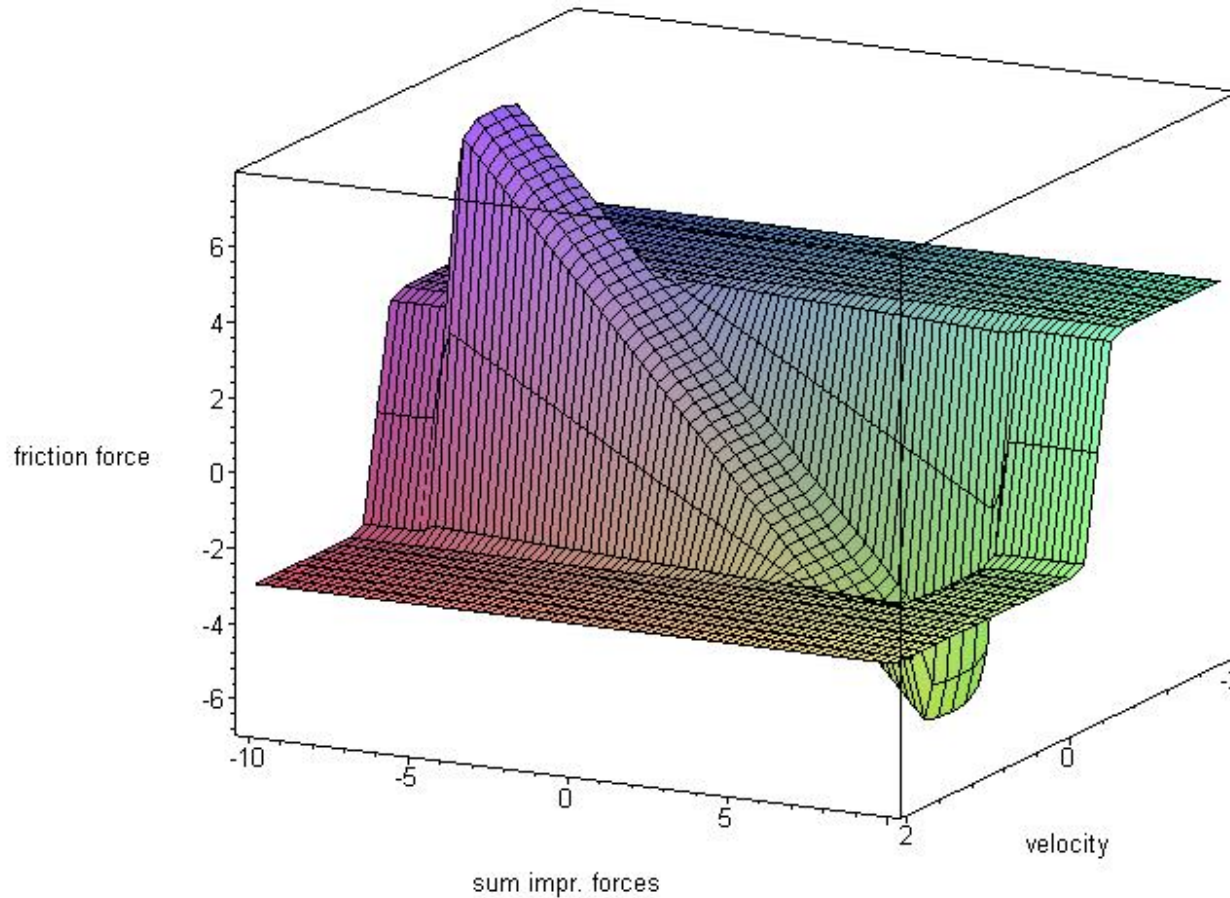
$$(v, f_a) \mapsto F_R(v, f_a) = \begin{cases} -F^+ & , v > \varepsilon \quad \vee \quad (v \in [0, \varepsilon] \wedge f_a > F_0^+) \\ -f_a & , |v| \leq \varepsilon \wedge f_a \in [-F_0^-, F_0^+] \\ F^- & , v < -\varepsilon \quad \vee \quad (v \in [-\varepsilon, 0] \wedge f_a < -F_0^-) \end{cases}$$

- mathematical model in closed analytical form
(jump approximation with a smooth Heaviside function):

$$H(a, b, x) := \frac{1}{2} \left\{ \tanh(A(x - a)) + \tanh(A(b - x)) \right\}$$

➡ no theory of differential inclusions is necessary!

Graph of the friction function: with $A=100$, $\varepsilon=0.5$ (only for visualization of smoothness).

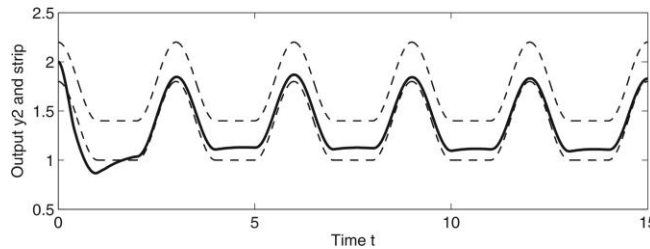
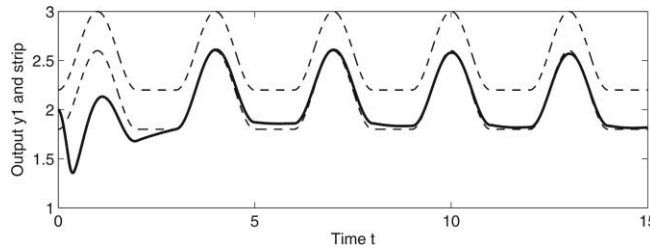


each
space curve
 $\{v, f_a, F_R\}$
has to lie
in
this graph

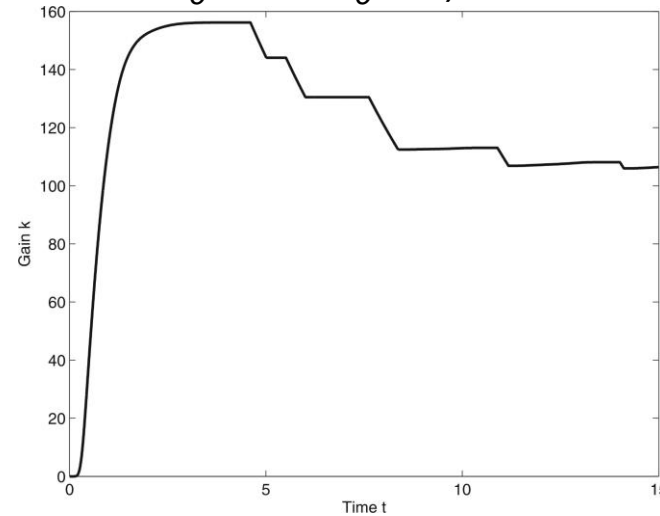
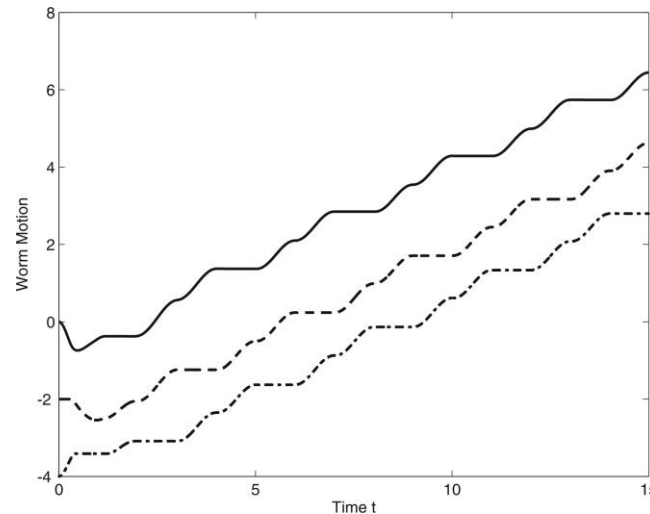
- **but:** randomly changing coefficients again requires adaptive control

Simulation 3: worm with new friction (only stiction $F_0^- = 16$, $F_0^+ = 3$) and new adapter

Outputs and tubes



Worm motion

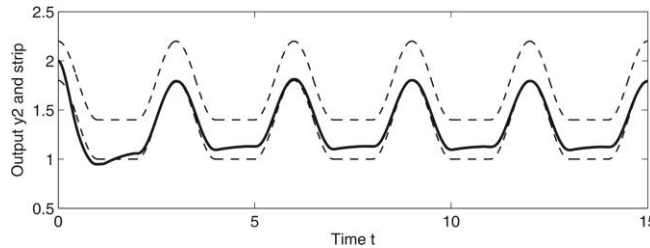
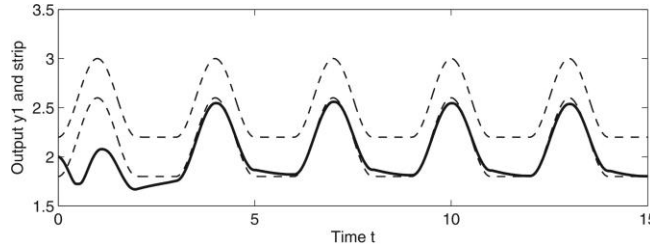


Gain parameter

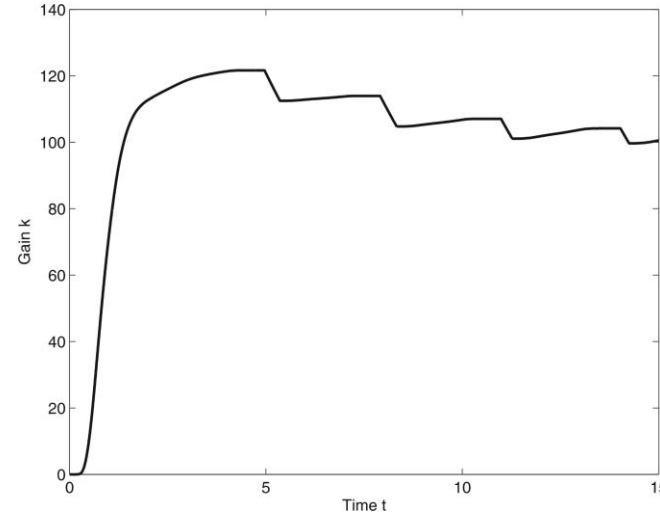
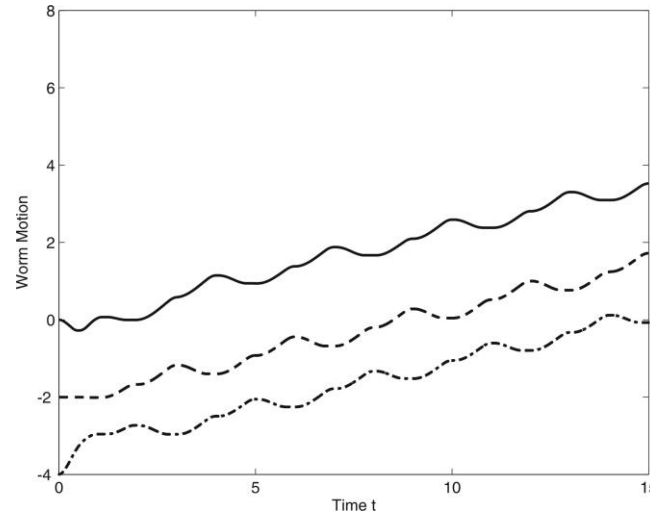
- stiction values guided by spikes theory
- short backward motions
- afterwards coincidence to previous motions
- *average speed = 0.4857*

Simulation 4: worm with new friction (only sliding friction $F=16$, $F^+=3$) and new adapter

Outputs and tubes



Worm motion

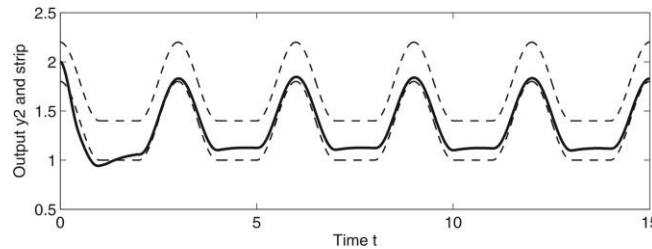
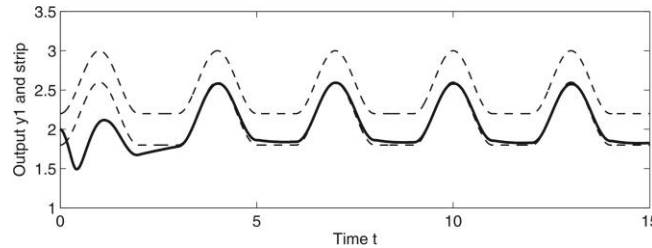


Gain parameter

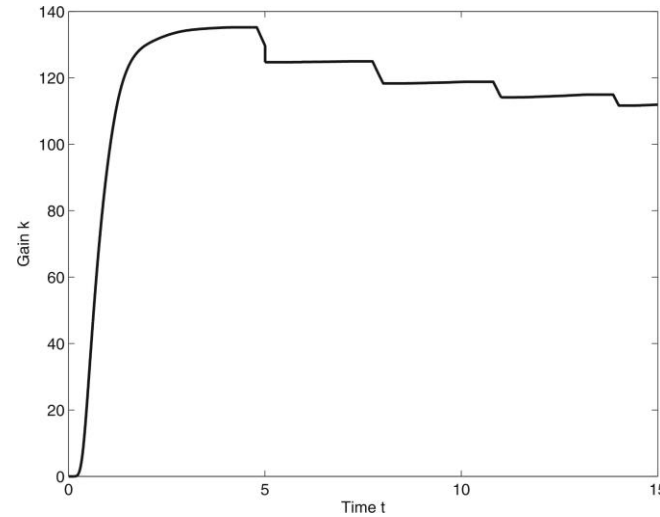
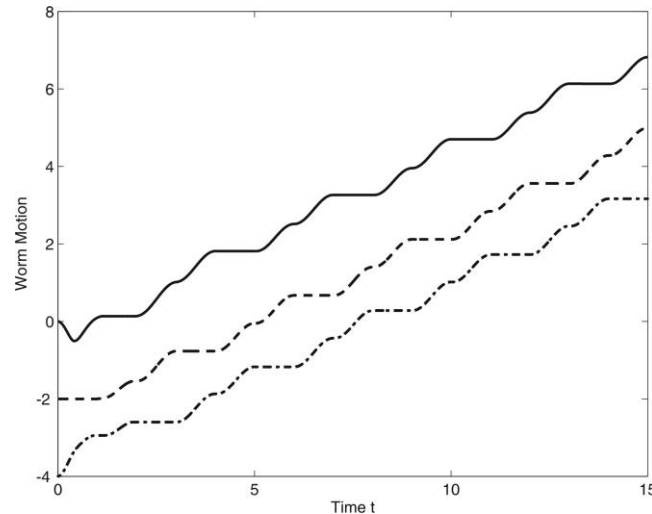
- replacing stiction by sliding friction values
- good tracking
- but observe an unsatisfactory external behavior (negative velocities)
- owing to stiction cancelling
- *average speed = 0.2399*

Simulation 5: worm with new friction (stiction & sliding friction) and new adapter

Outputs and tubes



Worm motion



Gain parameter

- $F_0^- = 18, F_0^+ = 3, F = 8, F^+ = 1$, two sliding mass points to be compensated by stiction
- good behavior as before
- bit smaller *average speed* = 0.4793
- if $F_0^- = 16$ worm runs backwards
- we need $F_0^- = 16 + 2$, two sliding mass points at every moment

Conclusions:

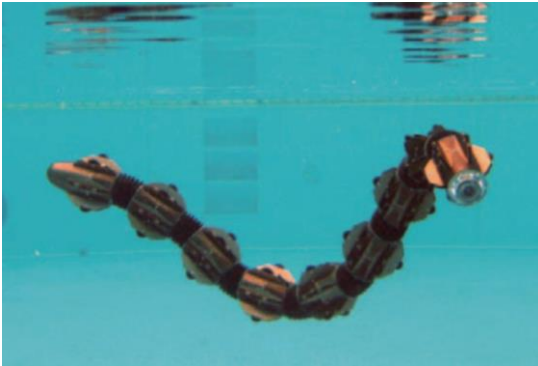
- (adaptive) control has been directed to ensure a prescribed gait
- obviously that a changing environment or changing type of interaction influences the global movement and the driving forces
- simulations have shown that adaptive control is promising in application to WLLS
- points out: stiction is the essential part of Coulomb interaction with the ground
- **warm warning of** a careless reducing of the interaction to pure sliding friction
- improved adaptive controllers are useful and should be developed further
- summarizing: adaptive control with minimal knowledge of system parameters (self-adjusting, robust and universal)

**NOW:**

- transfer results to Snake-like Locomotion Systems
- Snakes: „definition“ – 2-dimensional movement (no longer rectilinear motion)

Snake-Like Locomotion Systems (SLLS)

12. State of the Art
13. Kinematics – Model 1
14. Kinematics – Model 2
15. Dynamics & Adaptive Control
16. Conclusions Part 2
17. Future Work



[Active Cord Mechanism R5]



[Omni Thread OT-4]



[Rigid-Type Robot]



[SoftWorm]



[GMD-Snake2]



[Kulko]



[SlimSlime]

Model, Proto- type	Dimension	Active joints	Peristaltic	Undulation	Locomotion
Worm (Bio)	1D (3D)	allusively (variated linear motion)	yes	yes	rectilinear
Snake (Bio)	2D, 3D (3D)	yes (vertebrate)	no (not by definition)	yes	rectilinear, serpentine, concertina, crotaline
masspoint-model according to [13]	2D	no	no	yes	rectilinear, serpentine(?)
ACM-III, -R2	1D (with lateral movement)	yes	no	yes	“swimming”
ACM-R3, -R4	3D	yes	no	yes	“swimming”, serpentine, concertina
ACM-R5	3D	yes	no	yes	serpentine, concertina, swimming
ACM-R7	3D	yes	no	yes	concertina, “winding”
ACM-S1	3D	allusively (like worms)	no	yes	rectilinear(?), concertina
Aiko	3D	yes	no	yes	concertina
AmphiBot	3D	yes	no	yes	“swimming”, swimming

Name	Dimension	Active joints	Peristaltic	Undulation	Locomotion
Genbu	2D	no	no	no	driving
GMD-Snake	3D	yes	yes ¹	yes	rectilinear, concertina
GMD-Snake 2	3D	yes	no	yes	rectilinear, concertina
Kairo II	3D	yes	no	no	serpentine/ driving
Kohga	2D/3D	yes	no	no	driving
Kulko	3D	yes	no	yes	serpentine, concertina, swimming
OmniTread	3D	yes	no	yes	concertina, driving
Perambulator II	3D	yes	no	yes	“swimming”, serpentine, concertina
Polychaete-Roboter	2D	yes	no	yes	“swimming”
Rigid-Type Robot	2D	yes	yes	yes	rectilinear, concertina
Screw-Drive Mechanism Robot	2D/3D	yes	no	yes	driving, concertina
Slim Slime Robot	3D	yes	no	yes	concertina
SoftWorm (MeshWorm)	1D	no	yes	yes	rectilinear
WormBot	2D	yes	no	yes	“swimming”

Interim Conclusion:

- 18 prototypes use active joints for their movement
- 18 prototypes use an undulatory locomotion pattern
- 6 prototypes perform a “swimming” movement
- 3 prototypes use wheels or something similar for a straight driving locomotion

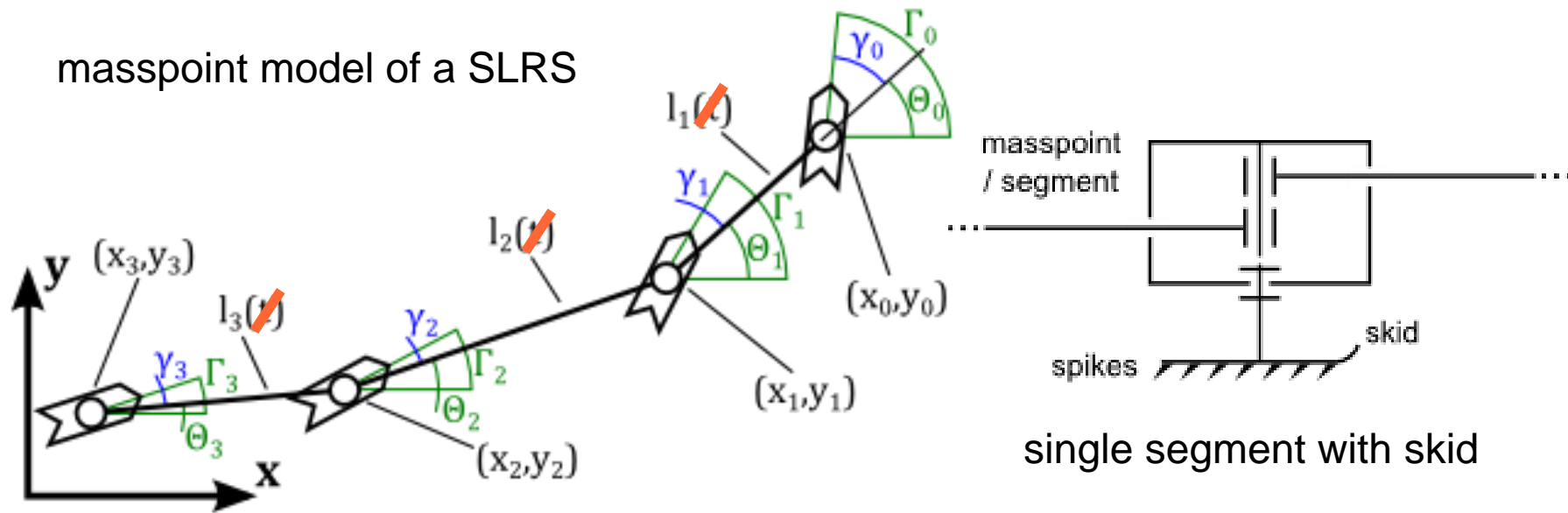
Focus now:

- still using undulatory locomotion
- introducing peristalsis of WLLS to SLLS
- using passive joints

Goal:

- easy description of a multi-segmented system in kinematics and dynamics
- development of steering mechanisms to change movement direction and to avoid obstacles

masspoint model of a SLRS



model with **constant** link lengths: $l_j(t) = l = const$, $\Gamma_0(t)$, $v_0(t)$

$$\Gamma_0 = \Theta_1 + \gamma_0$$

$$\Gamma_i = \Theta_i + \gamma_i$$

$$v_i = v_{i-1} \frac{\cos(\gamma_{i-1} + \Theta_{i-1} - \Theta_i)}{\cos(\gamma_i)}$$

$$\dot{x}_0 = v_0 \cos(\Gamma_0)$$

$$\dot{y}_0 = v_0 \sin(\Gamma_0)$$

$$\dot{\Theta}_\nu = \frac{v_{i-1} \sin(\gamma_{i-1} + \Theta_{i-1} - \Theta_i) - v_i \sin(\gamma_i)}{l}$$

$$\{0, \dots, N-1\} = \{0, \dots, n\} \quad , \quad j = i \neq 0$$

Skid Control Mechanisms:

- classic tractrix
- directional stable control
- obstacle avoidance backwards
- obstacle avoidance forwards

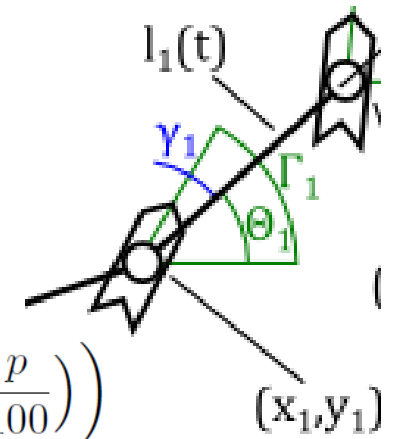
$$\Gamma_i = \Theta_i$$

$$\Gamma_i = \Gamma_i(t) = \Gamma_0 \left(t - \frac{il}{v_i} \right)$$

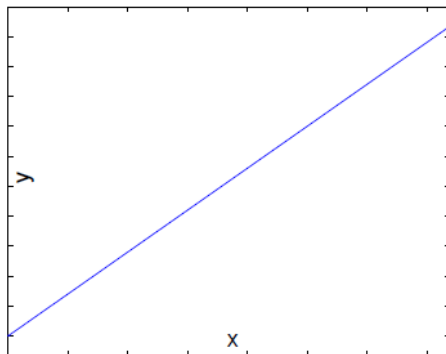
$$\Gamma_i = \Gamma_i(t) = \Gamma_0 \left(t - \frac{il}{v_i} \left(1 - \frac{p}{100} \right) \right)$$

$$\Gamma_i = \Gamma_i(t) = \Gamma_0 \left(t - \frac{il}{v_i} \left(1 + \frac{p}{100} \right) \right)$$

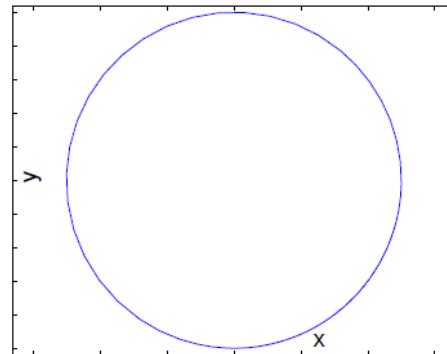
$$i = 0, 1, 2, \dots, n \quad (n = N - 1)$$



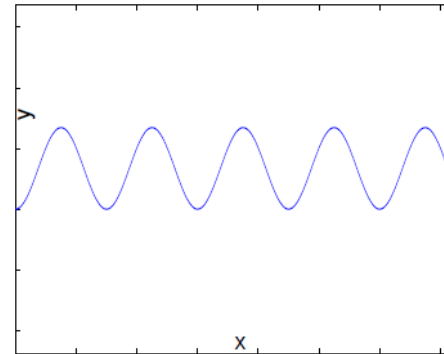
Test paths from literature:



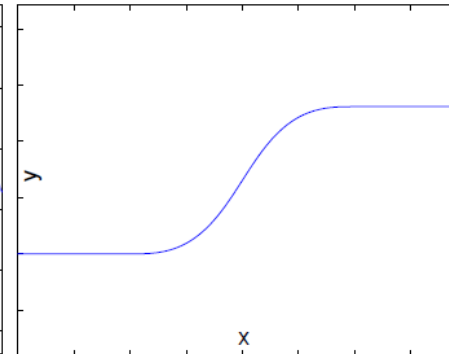
straight line



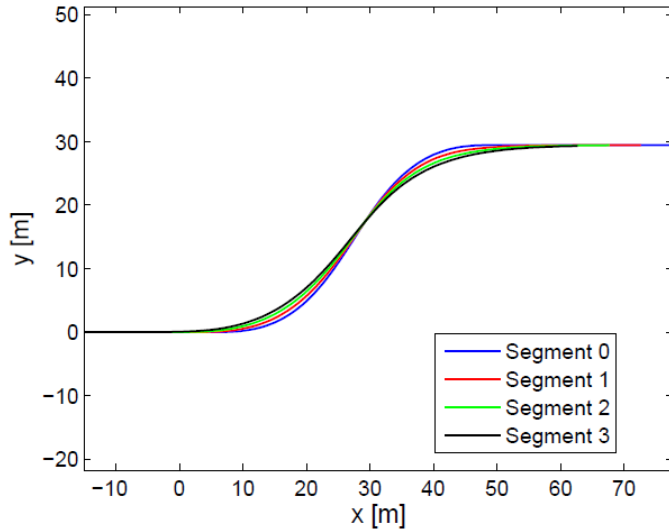
circular path



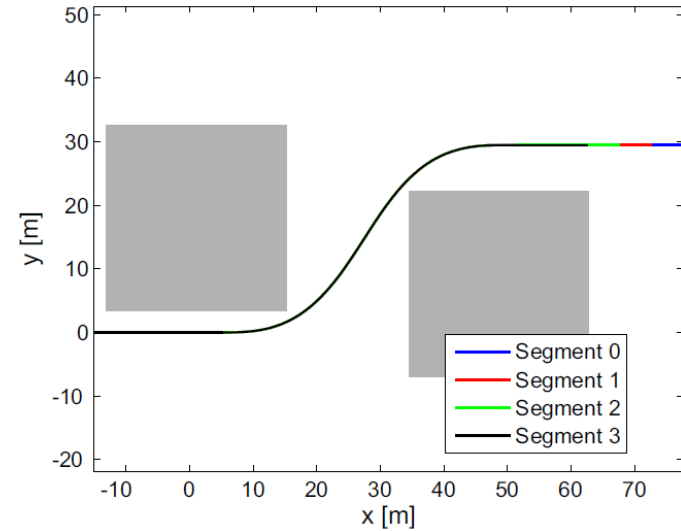
sinus-shaped path



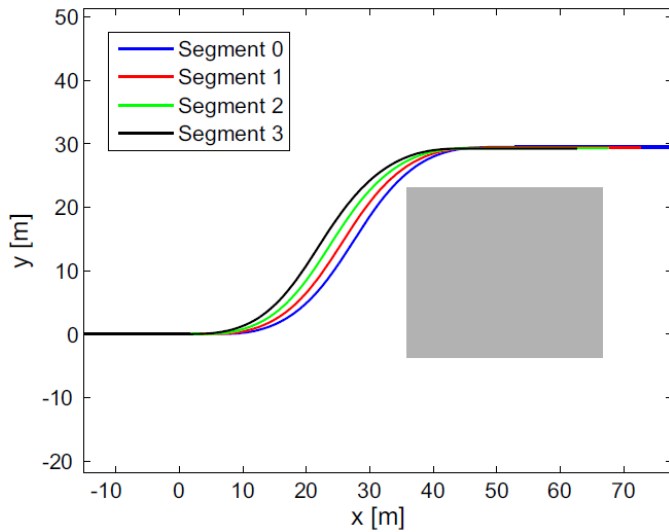
sinus-lane change



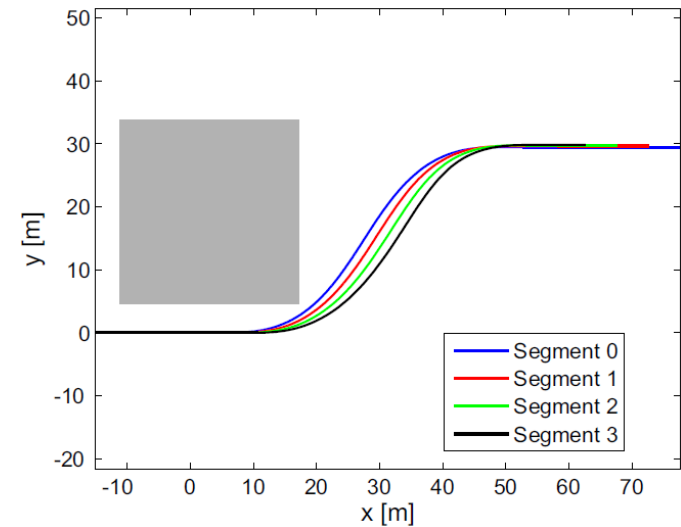
classic tractrix



directional stable control

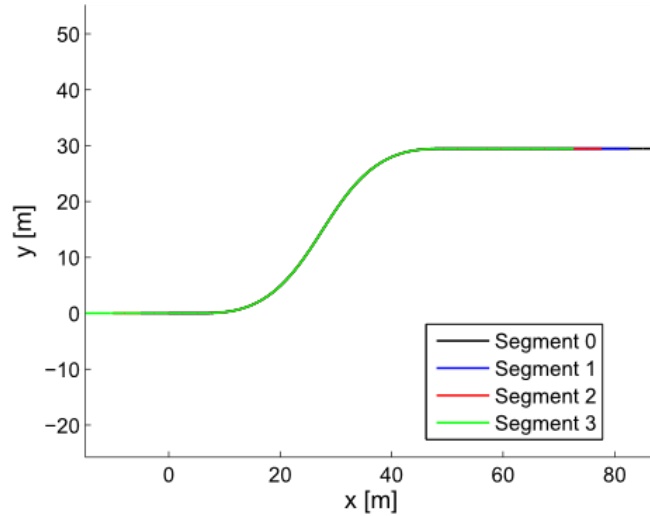


obstacle avoidance backwards

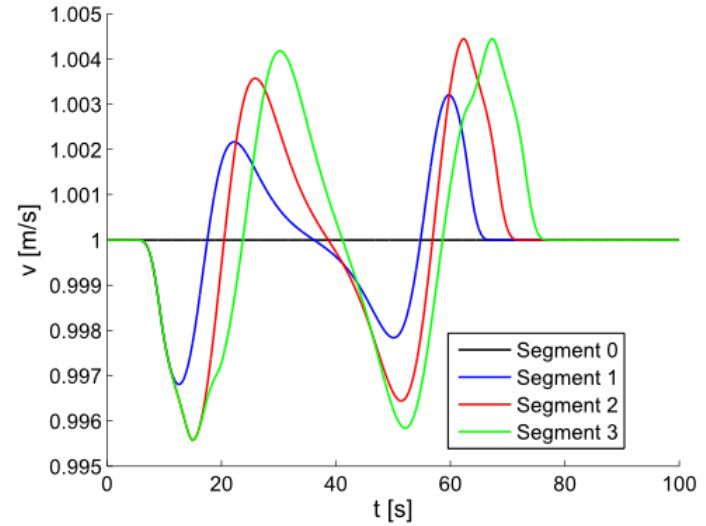


obstacle avoidance forwards

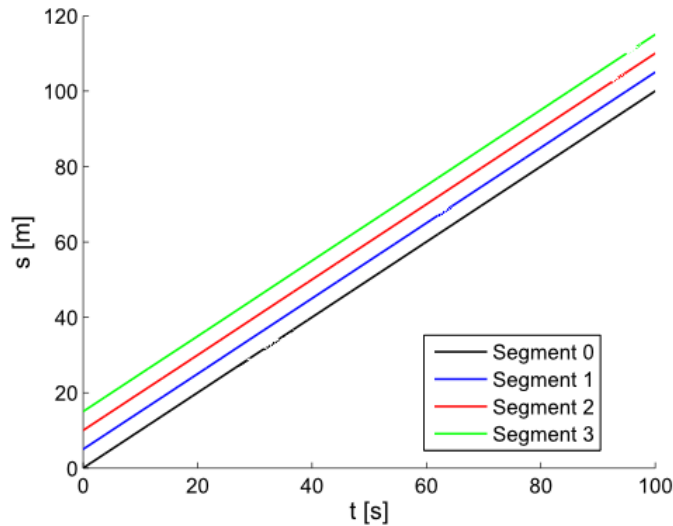
Example: Simulation of a stable directional control (Kinematics)



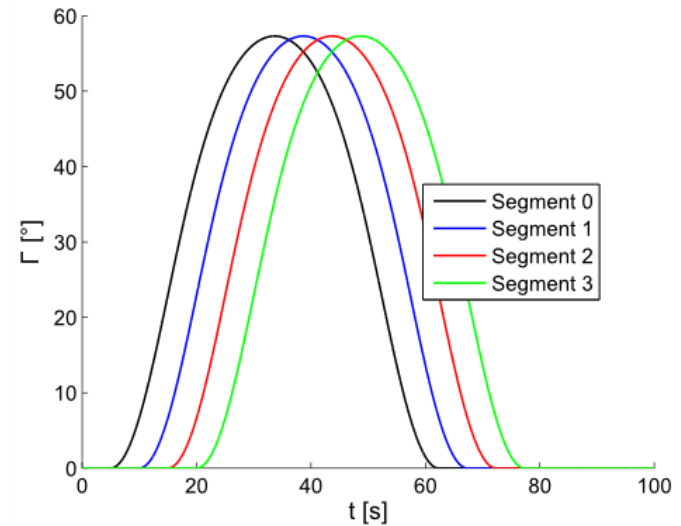
segment paths



segment velocities



travel distance



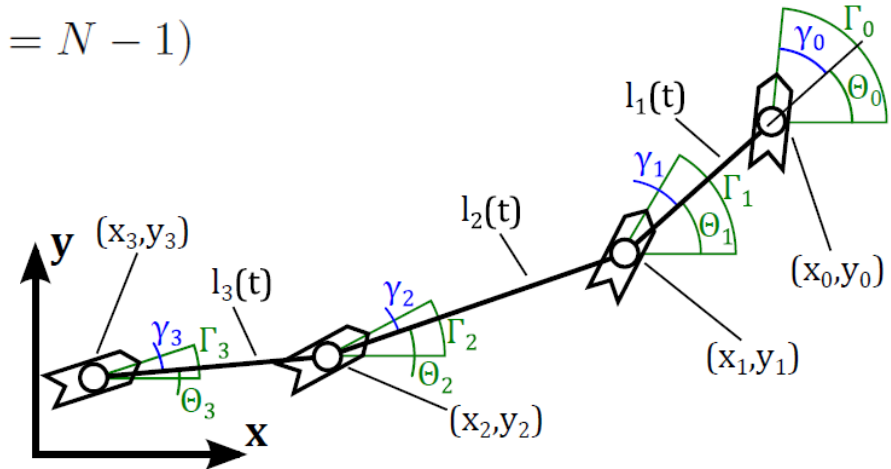
skid angles

Masspoint model with **time-dependent** link length

$$l_j \rightarrow t \mapsto l_j(t) \quad , \quad j = 1, \dots, n \quad (n = N - 1)$$

Step 1:

- no backward motion due to spikes!!
- realization in kinematics?
- identify reference segment q supporting the motion

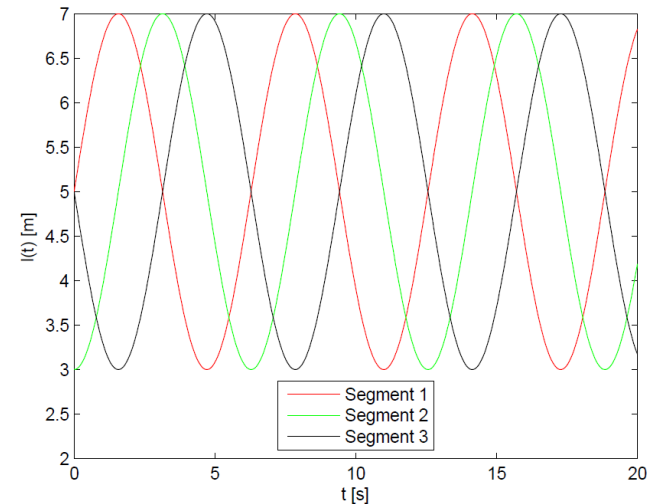


Step 2:

- prescribed link lengths in time (arbitrarily chosen function)

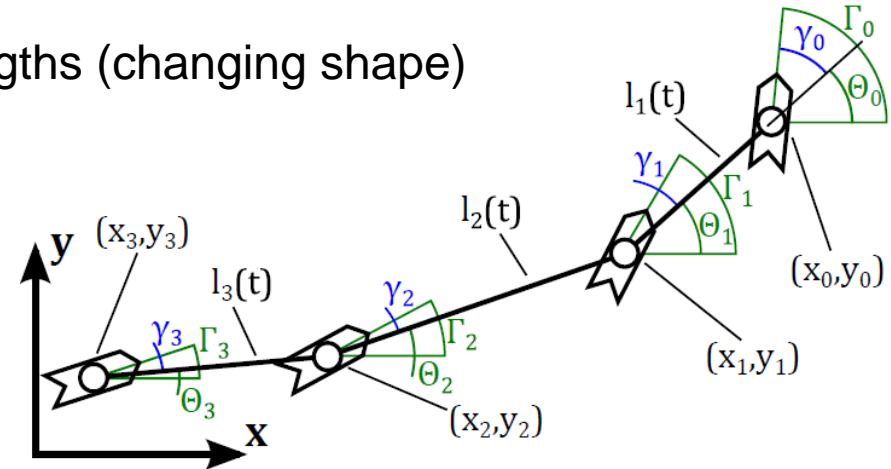
$$l_j(t) = l_0 + A \sin \left(t f_0 - 2\pi f_0 \frac{j-1}{N} \right)$$

$$\dot{l}_j(t) = A \cos \left(t f_0 - 2\pi f_0 \frac{j-1}{N} \right) f_0$$



Step 3:

- model equations
- pumping mechanism changing link lengths (changing shape)
- skid angles can be actively steered



$$\begin{aligned} \Gamma_i &= \Theta_i + \gamma_i \\ \Gamma_0 &= \Theta_1 + \gamma_0 \\ \dot{x}_i &= v_i \cos(\Gamma_i) \\ \dot{y}_i &= v_i \sin(\Gamma_i) \end{aligned}$$

posterior: $v_i = \frac{v_{i-1} \cos(\Gamma_{i-1} - \Theta_i) - \dot{l}_i}{\cos(\gamma_i)} \quad (i = q + 1..n)$

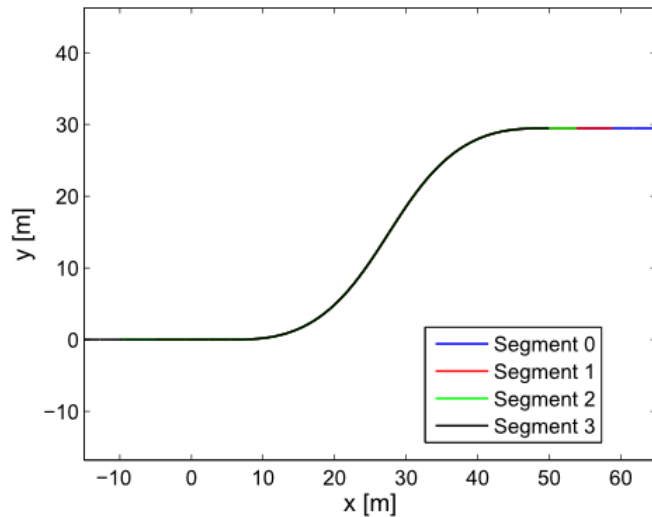
anterior: $v_i = \frac{v_{i+1} \cos(\gamma_{i+1}) + \dot{l}_{i+1}}{\cos(\Gamma_i - \Theta_{i+1})} \quad (i = q..0)$

$$\forall i \in \{0, \dots, n\} \quad , \quad j = i \neq 0$$

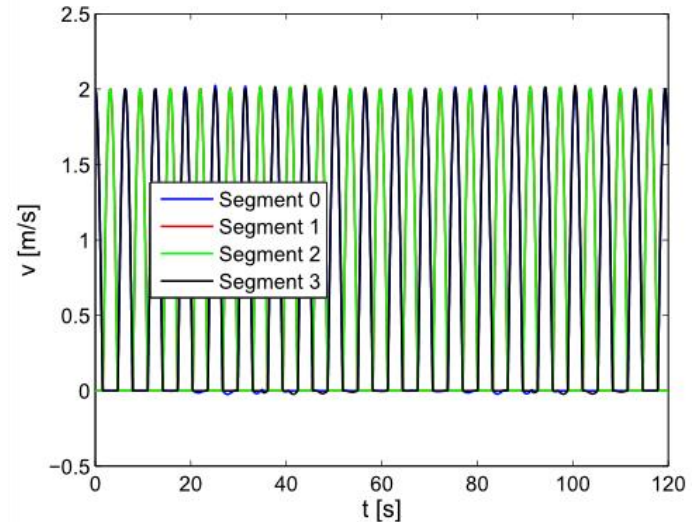
generally: $\dot{\Theta}_i = \frac{v_{i-1} \sin(\Gamma_{i-1} - \Theta_i) - v_i \sin(\gamma_i)}{l_i}$

\bigcirc - inputs (i.e., skid control algorithm and link lengths)

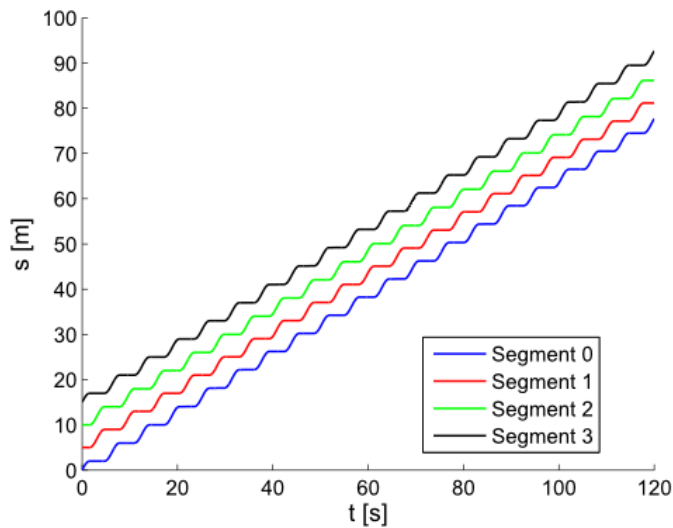
Example: Simulation of a stable directional control (Kinematics)



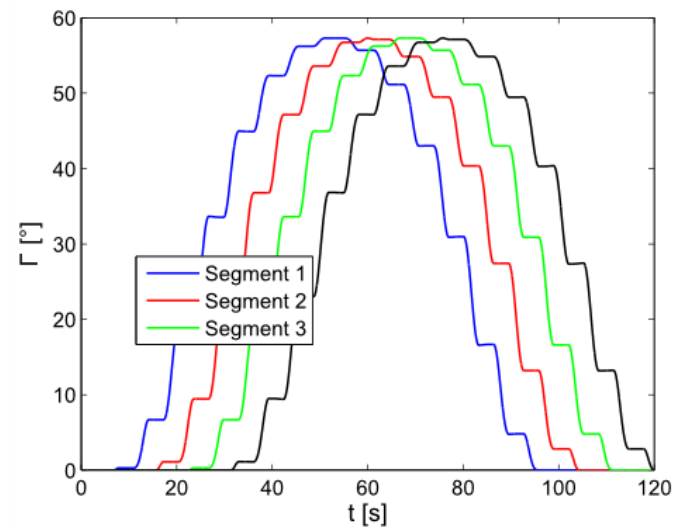
segment paths



segment velocities



travel distance

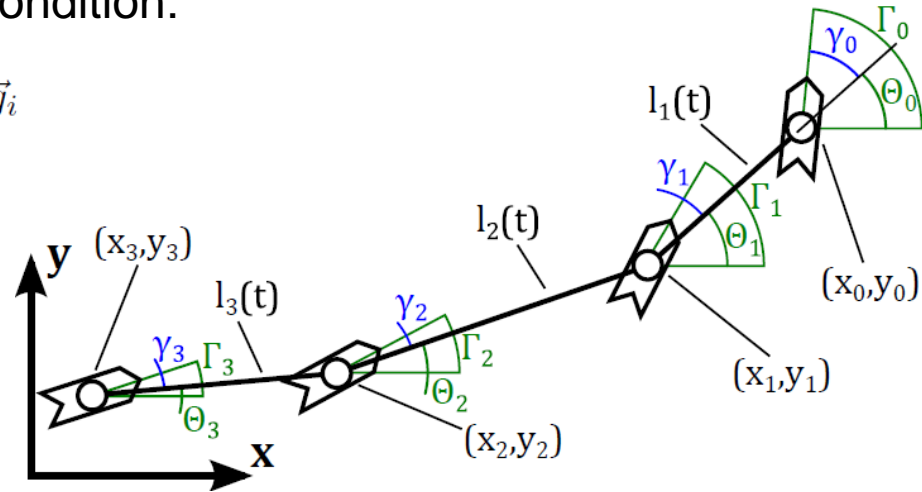


skid angles

Equations of motion

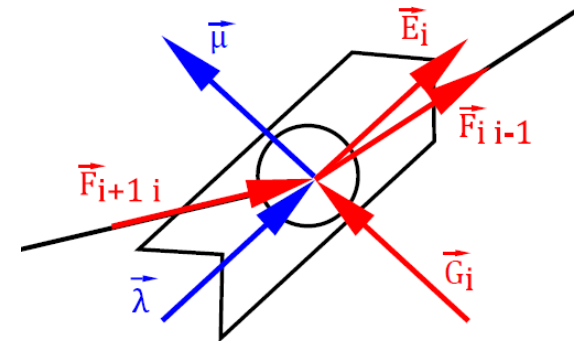
Forces and complementary slackness condition:

$$\begin{aligned}\vec{F}_i &= F_{i,i-1}\vec{e}_{i,i-1} - F_{i+1,i}\vec{e}_{i+1,i} + E_i\vec{e}_i + G_i\vec{g}_i \\ \vec{R}_i &= \lambda_i\vec{e}_i + \mu_i\vec{g}_i \\ m_i\ddot{\vec{x}}_i &= \vec{F}_i + \vec{R}_i \\ v_i &\geq 0, \quad \lambda_i \geq 0, \quad v_i\lambda_i = 0\end{aligned}$$



Model equations:

$$\begin{aligned}\dot{x}_i &= v_i \cos(\Gamma_i) \\ \dot{y}_i &= v_i \sin(\Gamma_i) \\ \dot{l}_i &= v_{i-1} \cos(\gamma_{i-1} + \Theta_{i-1} - \Theta_i) - v_i \cos(\gamma_i) \\ \dot{\Theta}_i &= \frac{v_{i-1} \sin(\gamma_{i-1} + \Theta_{i-1} - \Theta_i) - v_i \sin(\gamma_i)}{l_i} \\ f_i &= F_{Ai} v_{i-1} \cos(\gamma_i) - F_{Ai+1,i} \cos(\Gamma_i - \Theta_{i+1}) - k_{St} v_i \\ v_i &= \frac{f_i + \lambda_i}{m_i} \\ \lambda_i &= -\frac{1}{2}(1 - \text{sign}(v_i))(1 - \text{sign}(f_i))f_i\end{aligned}$$



$$\forall i \in \{0, \dots, n\}$$

- realization of prescribed link lengths in time in dynamics?
- actuator force control to track masspoint distances
- due to supposed uncertainty of parameters: adaptive λ -tracking controller

- given:

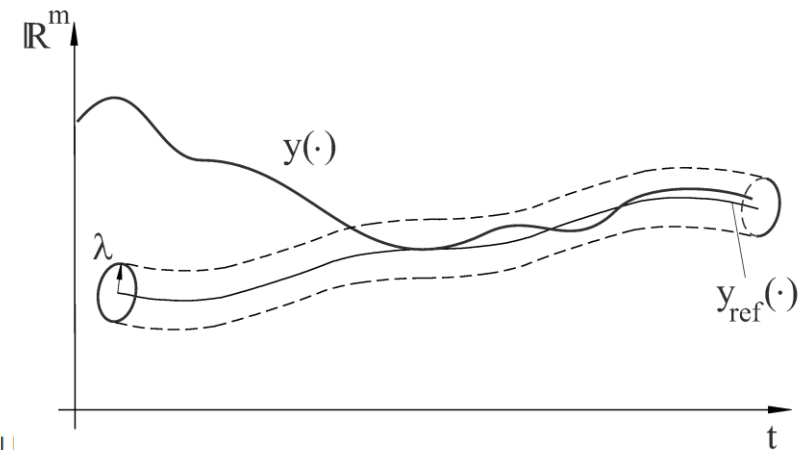
$$y_{ref}(t) = l(t) = (l_1(t), \dots, l_n(t))^T$$

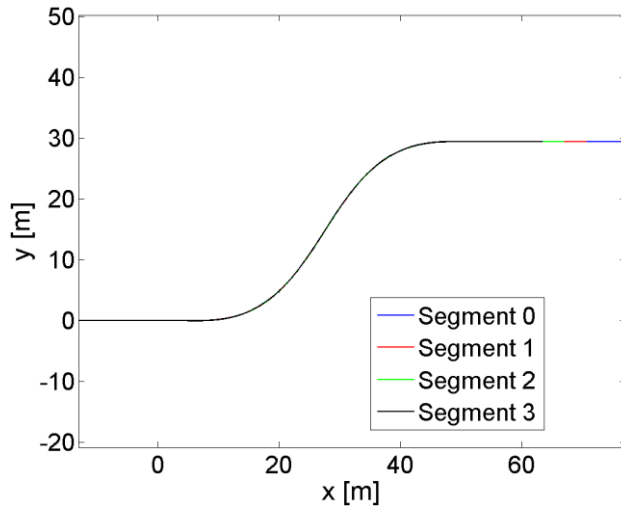
- controller:

$$\begin{aligned} e(t) &:= y(t) - y_{ref}(t) \\ F_A(t) &= -k(t)e(t) - \kappa k(t)\dot{e}(t) \end{aligned}$$

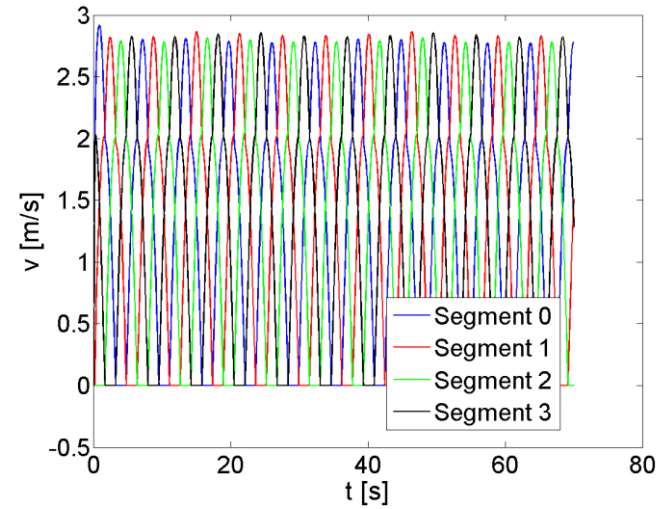
$$\dot{k}(t) = \begin{cases} \gamma(\|e(t)\| - \lambda)^2 & , \quad \lambda + 1 \leq \|e(t)\| \\ \gamma(\|e(t)\| - \lambda)^{0.5} & , \quad \lambda \leq \|e(t)\| < \lambda + 1 \\ 0 & , \quad \|e(t)\| < \lambda \wedge t - t_e < t_d \\ -\sigma k(t) & , \quad \|e(t)\| < \lambda \wedge t - t_e \geq t_d \end{cases}$$

$$k(t_0) = k_0$$

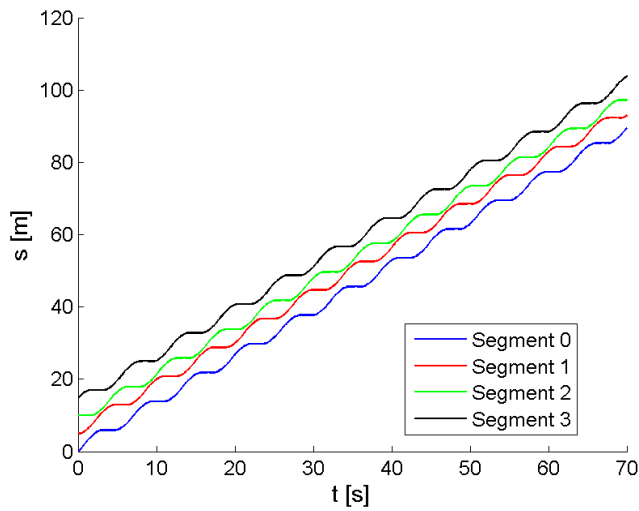




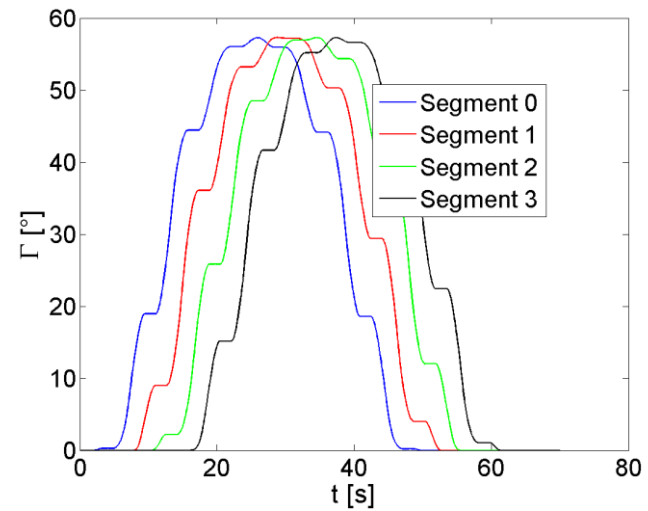
segment paths



segment velocities



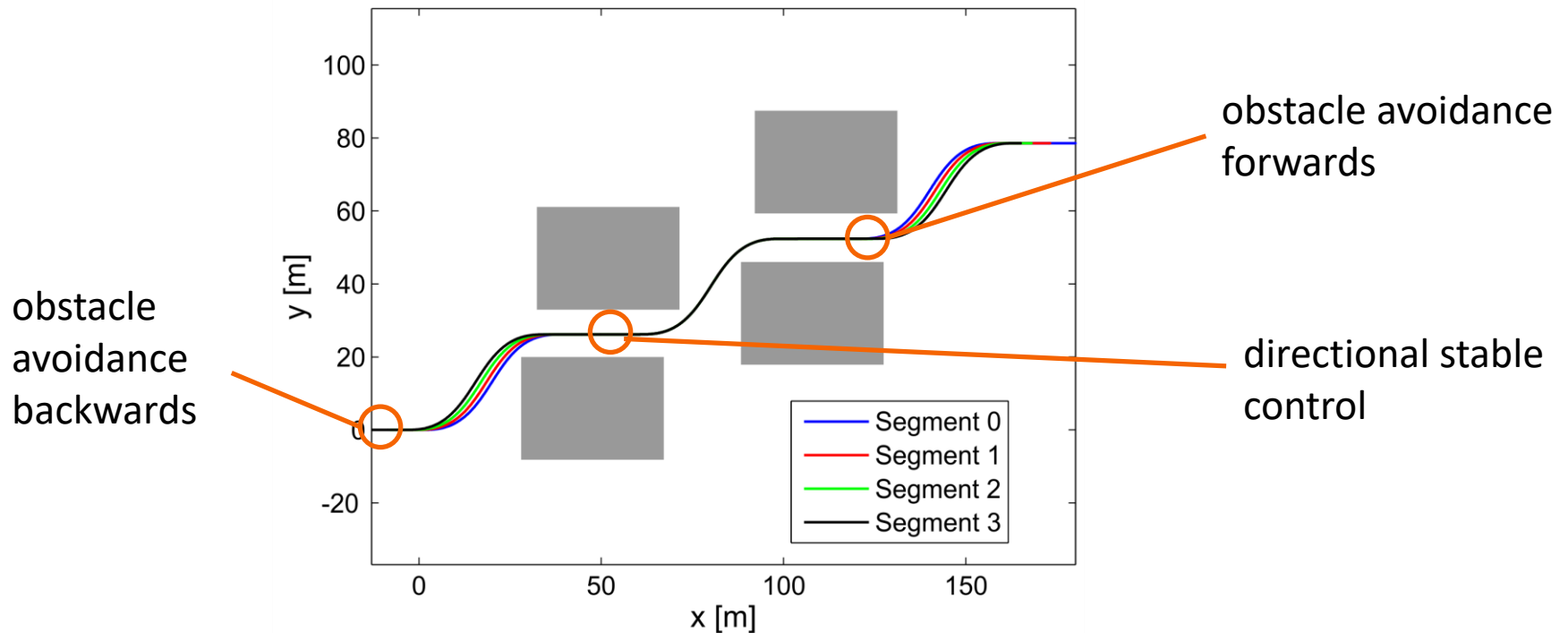
travel distance



skid angles

- development of SLRS with passive joints, peristalsis, skids and spikes:
 - active joints substituted by passive ones, change of shape via controlled, time-dependent link lengths (peristalsis)
 - ground contact via spiked skids
- description of a multi-segmented model:
 - masspoint model (kinematics) with constant link length
 - masspoint model (kinematics) with time-dependent link length
 - masspoint model (dynamics and adaptive control) with time-dependent link lengths
- development of skid control mechanisms, introducing a simple gait-function and an adaptive actuator force controller
- validation via simulations of several test paths

- parameter studies: increase number of mass points, analyse corresponding gaits, identification of optimal gaits
- switching of gaits (gear shift) in dependence of actuator loads, spike loads ...
- switching of skid control mechanisms to navigate in uncertain terrain:



Application: traveling through a labyrinth with switching skid control mechanisms

Thanks!

Take care!

Stay healthy!