Multi-objective Optimization of Dynamic Networks

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Parimala Thulasiraman is a Professor with the department of Computer Science at the University of Manitoba. She received her B.Eng. (Honours) and M.A.Sc. degrees in Computer Engineering from Concordia University in Montreal, QC, Canada and obtained her Ph.D. from the University of Delaware in Newark, DE, USA after finishing most of her formalities at McGill University, Department of Computer Science in Montreal, QC, Canada. Parimala's research interests are in the intersection of high performance parallel/distributing computing and graph analytics for real world applications in network science. Her laboratory, Inter-Disciplinary Evolutionary Algorithmic Sciences (IDEAS), studies innovative, adaptive, self-learning approaches for solving modeling, simulation, and optimization problems. She explores novel algorithmic optimization techniques to efficiently map, design, and develop scalable algorithms for distributed and many-core architectures. She has published over 200 research articles in conferences, journals, a book, and several book chapters. She has received best paper awards in leading high performance computing conferences. Her research is supported through the national grant from the Natural Sciences and Engineering Research Council of Canada, Research Manitoba, MITACS ACCELERATE, MITACS Globalink, as well as other local and industry grants.

Ying Ying Liu received her Ph.D. in February 2023 from the University of Manitoba, Canada, under the supervision of Dr. Parimala Thulasiraman. She completed her M.Sc. in 2016 and her B.C.S. (First Class Honors) in 2013, both from the Department of Computer Science, University of Manitoba. Her current research interest is Distributed Machine Learning on dynamic, evolving and spatial/temporal varying graph networks. Dr. Liu published 11 articles in reputable and top conferences and journals during her research activities at the Inter-Disciplinary Evolutionary Algorithmic Sciences (IDEAS) laboratory. Dr. Liu won multiple awards including the University Outstanding Student Award, the Canada Graduate Scholarship from the Natural Sciences and Engineering Research Council of Canada (NSERC), and the Department of Computer Science Award. Dr. Liu has over a decade of industry experience. As the Lead Data Scientist at Manitoba Hydro, she leads multiple Artificial Intelligence and Machine Learning initiatives and full-life-cycle projects at the Canadian public utility corporation.

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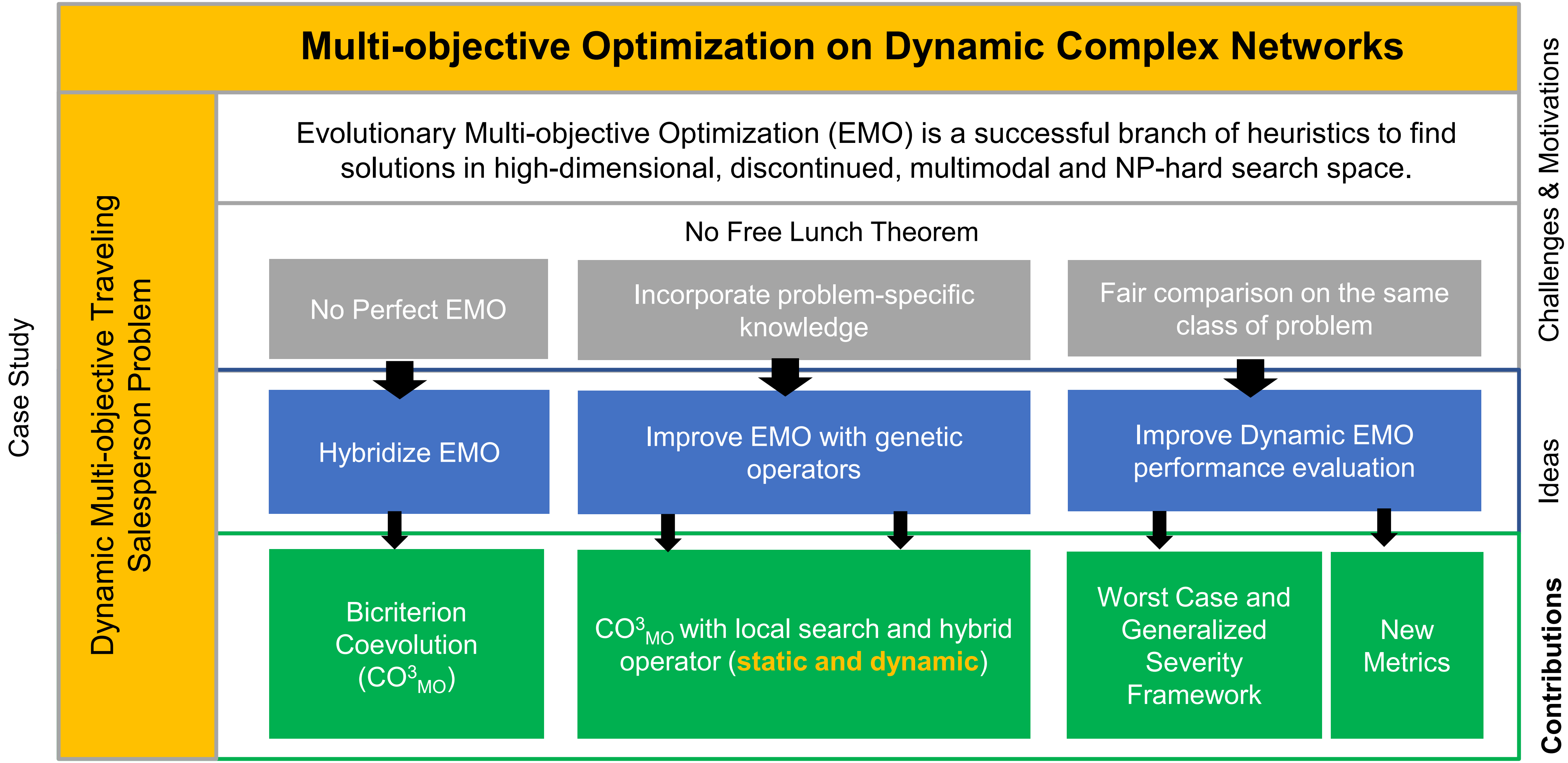
Contributions



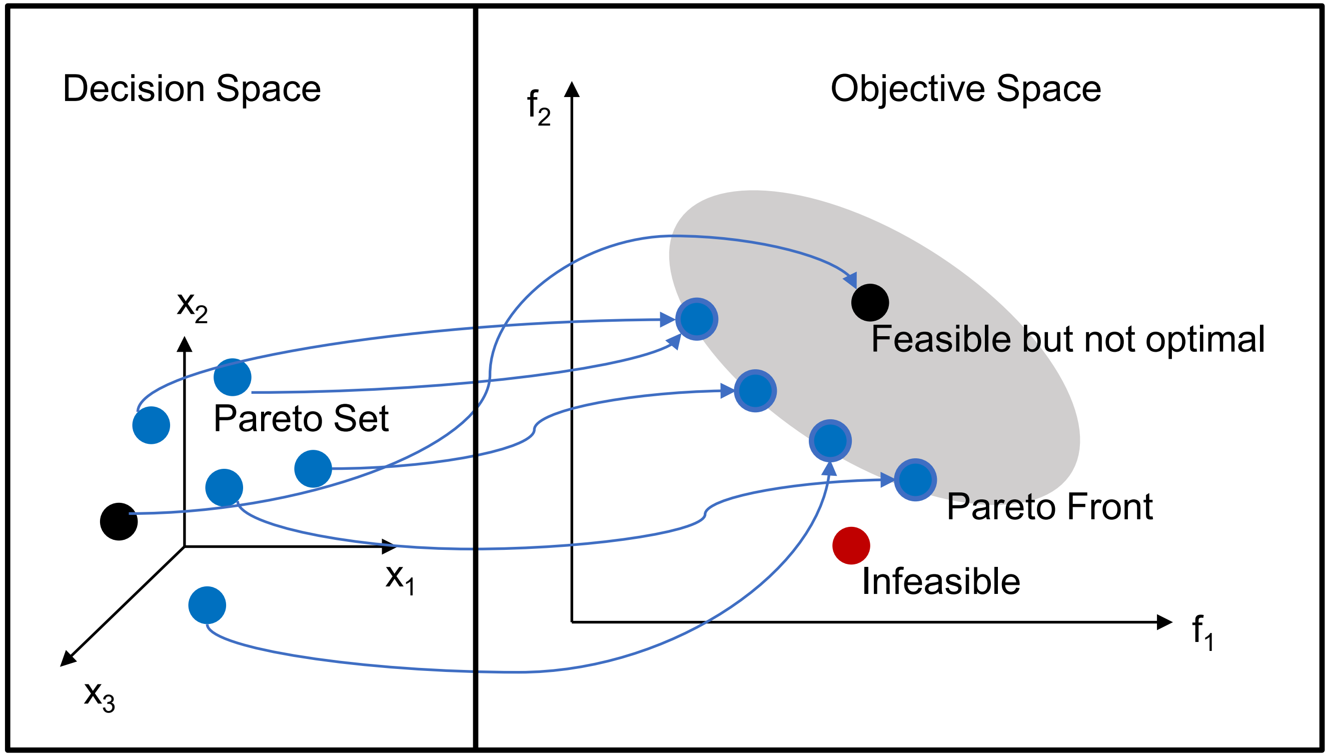
Overview

1. **Introduction/Background**
2. Proposed Algorithm CO^3_{MO}
   - Bicriterion Coevolution for Static and Dynamic MOP
   - Experiment Results
3. Proposed Evaluation Framework and Metrics
   - Worst and Generalized Severity Evaluation for Dynamic EMO
   - Case Study
4. Conclusion and Future Work

MOP Fundamentals: Pareto Optimality

Optimization: make the "best" decisions, given minimization or maximization objectives and constraints

- **Multi-objective Optimization Problem (MOP)** requires many solutions

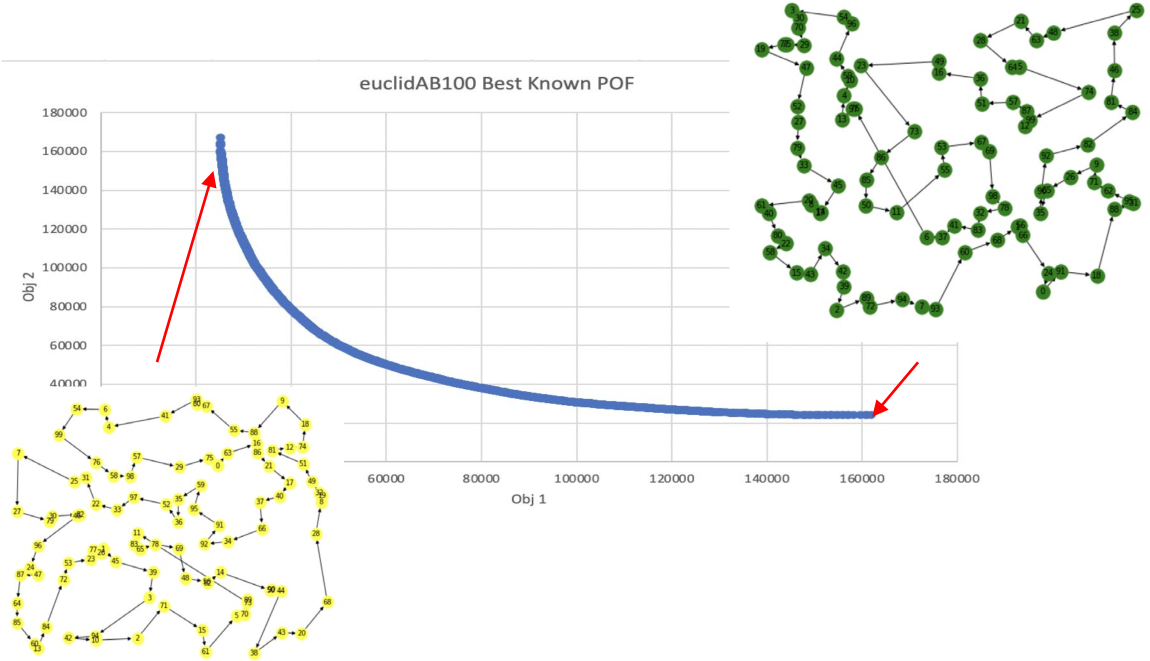


Map from Decision Space to Objective Space

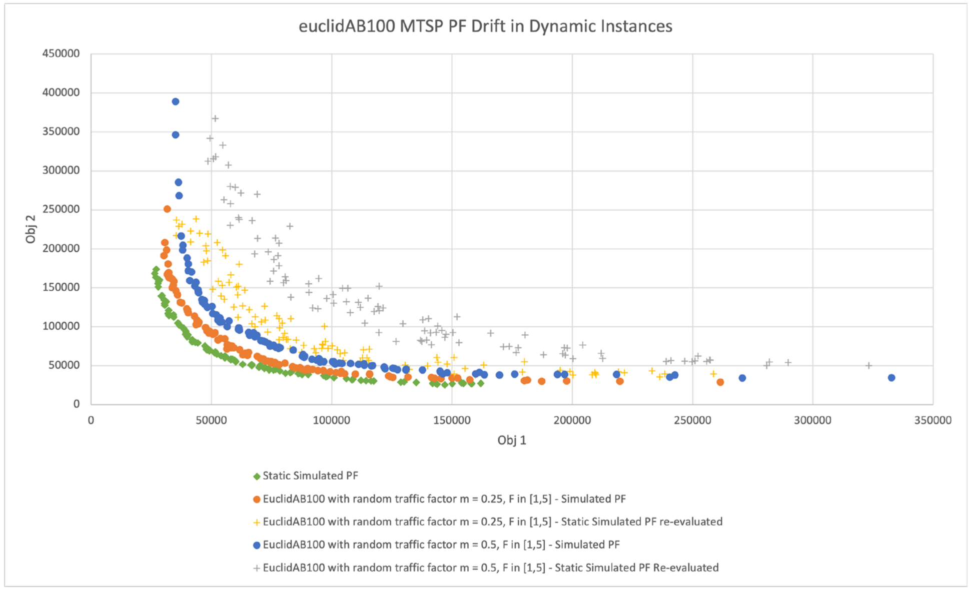
Discrete MOPs on Complex Networks

Example: two-objective traveling salesperson problem with 100 cities

Static MOP: 1719 points on the Best-Known Pareto Front



Dynamic MOP: Pareto Front drifts when the environment changes



Challenges

- Challenges for Static MOP
  - Complex search space
  - Traditional global search methods become ineffective

- Challenges for Dynamic MOP
  - Adapt quickly to the changing environment

- Challenges for MOPs on Complex Networks:
  - Combinatorial and often NP-hard problems even for SOP
  - High-dimensional, discontinued, multimodal search space

- Evolutionary Multi-objective Optimization (EMO)
  - Popular method for solving both static and dynamic MOPs with no assumption of the convexity of the search space
  - Strengths: evolve an entire population of solutions at the same time, allowing simulation of PF iteratively

EMO Convergence and Diversity

- What is Convergence?
  - The algorithm has reached a stable state and the solutions are close to the optimal values.
- Why is Convergence Important?
  - Fast convergence and optimality of the solutions are ideal for the algorithm.

- What is Diversity?
  - The solutions have good spread and evenness in the objective space.
- Why is Diversity important?
  - In the absence of any preference information, the Diversity of the solution set is important for the decision-maker.

EMO Literature Review

- EMO for Static MOP – Gap: convergency and diversity trade-off
  - A priori (before) preference
  - Progressive (during) preference
  - A posteriori (generating) preference
    - Pareto Criterion (**PC**) selection: **NSGA-II** [Deb et al., 2002]
    - Non-Pareto Criterion (**NPC**) selection: **MOEA/D** [Zhang and Li, 2007]
    - Hybrid: NSGA-III [Deb and Jain, 2013], Divide-and-Conquer Cooperative Coevolution [Potter and Jong, 2000], Bicriterion Evolution [Li et al., 2015]

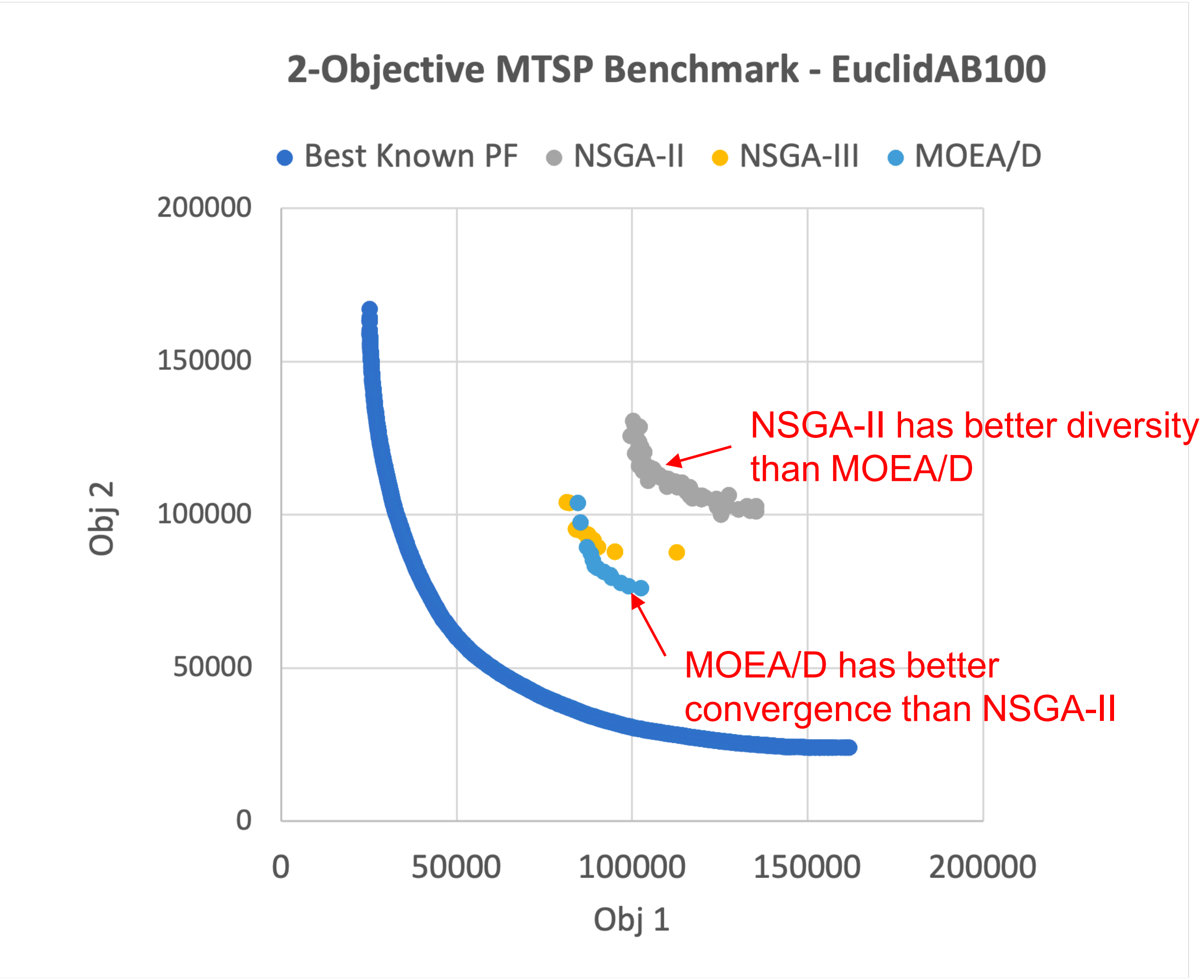
- EMO for Dynamic MOP – Gap: effective algorithms and evaluation methods
  - Diversity Increase/ Maintenance [Deb et al., 2007] [Ma et al., 2020]
  - Multi-population [Goh and Tan, 2009][Xu et al., 2017]
  - Prediction [Hatzakis and Wallace, 2006] [Zou et al., 2021]
  - Memory [Koo et al., 2010] [Chen et al., 2019]
  - Local search [Mavrovouniotis et al., 2016]
  - etc

- EMO for Complex Networks – Gap: limited studies
  - Some work in multi-objective versions of
    - Community detection [Shi et al., 2011] [Zou et al., 2017]
    - Vulnerability analysis [Rocco et al., 2010]
    - Capacitated arc routing problem [Mei et al., 2011]
    - Traveling salesperson problem [Lust and Teghem, 2010] [Cai et al., 2019]

Background: PC Evolution vs NPC Evolution

	Pareto Criterion (PC) Evolution	Non-Pareto Criterion (NPC) Evolution
Main Idea	Search for the entire PF iteratively	Solve all scalar subproblems at the same time
Pros	• **Diversity maintenance** • Adaptive to nonconvex and discontinued PF	• **Fast convergence** • Easy to combine with local search • Scalable to many-objective optimization
Cons	• **Slow convergence** • Curse of dimensionality in the objective space	• Sensitive to the shape of PF • **Loss of diversity**
Representative Algorithm	**NSGA-II**	**MOEA/D**

Challenge: The Convergence/ Diversity Tradeoff of PC and NPC Evolutions



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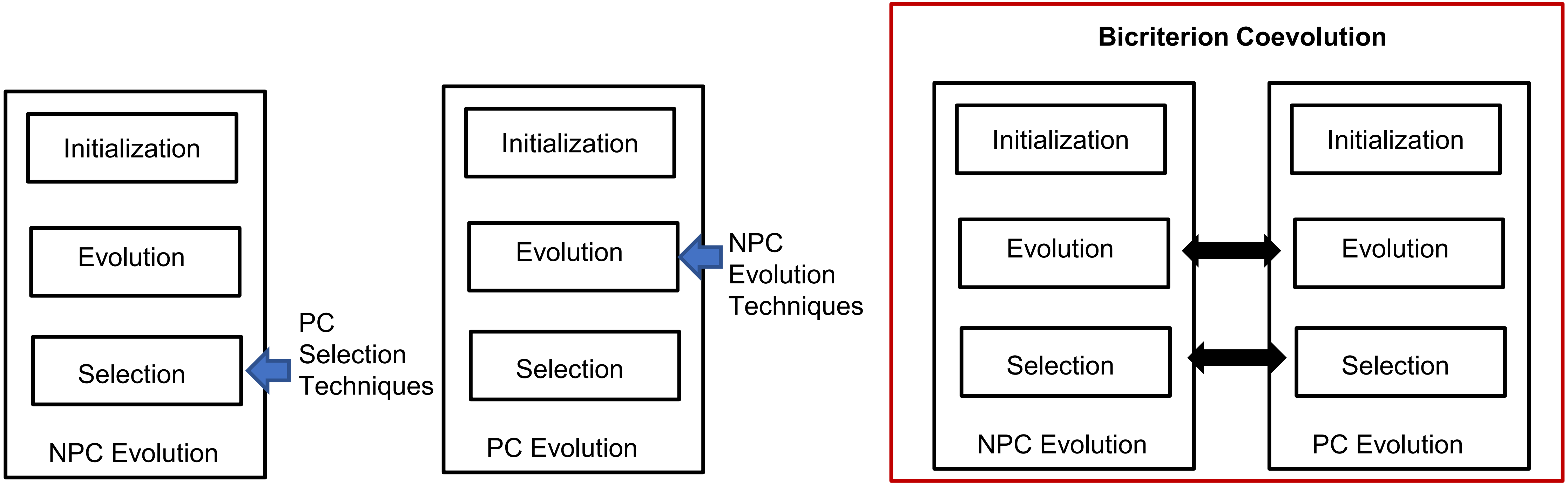
Ideas to Hybridize PC and NPC



Key idea:

A general framework to coevolve NPC and PC selections

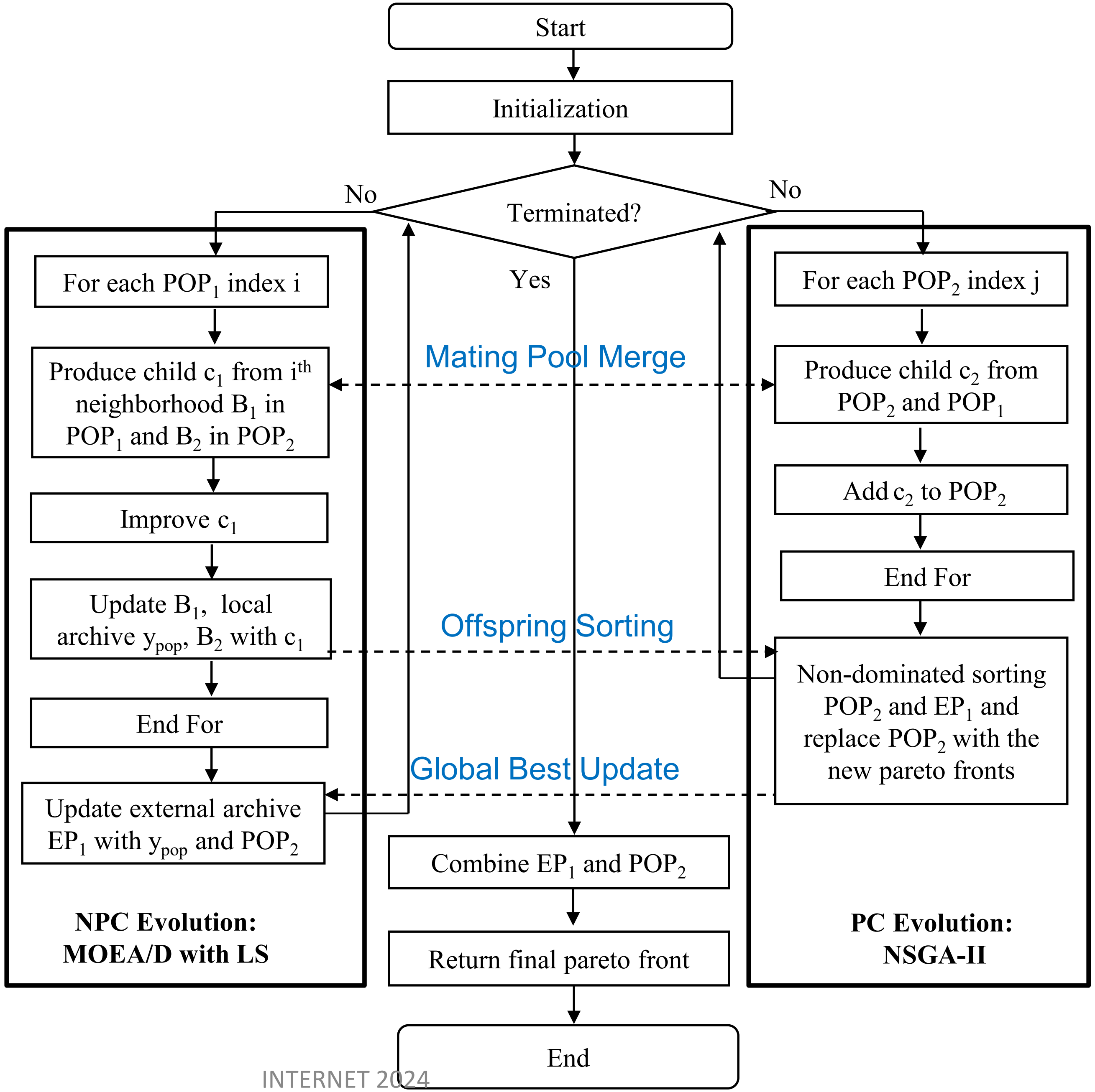
Li, Miqing, Shengxiang Yang, and Xiaohui Liu. "Pareto or non-Pareto: Bi-criterion evolution in multiobjective optimization." *IEEE Transactions on Evolutionary Computation* 20.5 (2015): 645-665.

Proposed Bicriterion Coevolution Algorithm: Co^3_{MO}

- **Co**operative: PC and NPC evolutions interchange information
- **Co**ncurrent: PC and NPC populations evolve in parallel
- **Co**evolutionary: PC and NPC form mutualism symbiosis
- The effect of **local search** and **hybrid operator** can benefit the convergence for both PC and NPC
- Achieve **Convergence** and **Diversity** at the same time

- PC: NSGA-II
- NPC: MOEA/D + LS

$Co^3{}_{MO}$ for Static MOP



Multi-objective Traveling Salesperson Problem (MTSP)

- Static MTSP
  - Minimization of all M objective functions with permutations of n cities
  - k = 1,…,M
  - c is the cost vector
  - π is a permutation of n cities

$$f_k(\pi) = \sum_{j=1}^{n-1} c^k_{\pi(j),\pi(j+1)} + c^k_{\pi(n),\pi(1)}$$

- Dynamic MTSP
  - Minimization of $f_k(\pi, t)$
  - t is a timestamp

Single-objective TSP and MTSP are both NP-hard

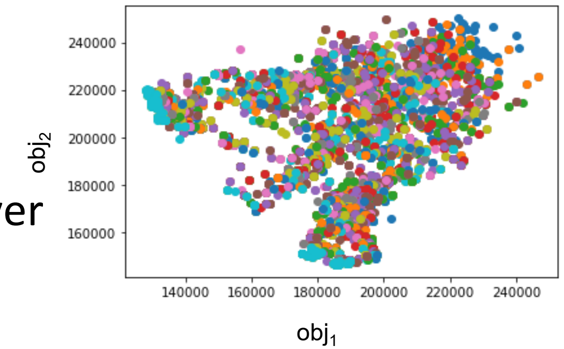
MTSP Literature Review

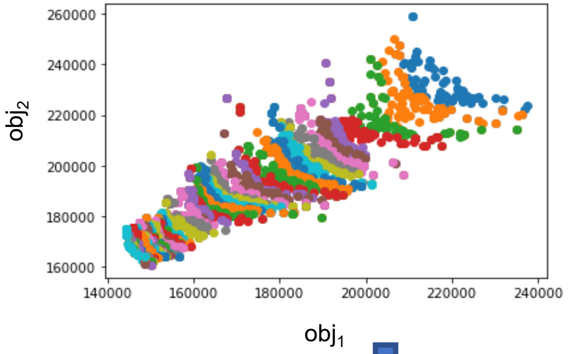
DMOP / MOP	None	Diversity Maintenance	Local search	Memory	Multi-population or coevolution	Prediction
NPC	[Wei et al., 2009]: MOEA/D with local search for Static MTSP [Lust and Teghem, 2010]: Decomposition based 2-phase pareto local search for Static MTSP				[Cai et al., 2019]: coevolution for decomposition local search and for Static MTSP	
PC	[Andrzej, 2002]: pareto ranking is not well suited for hybridization with local search for MOCO	[Ming et al., 2008]: ensemble of EA algorithms with different operations for DMTSP	[Michalak, 2021]: NSGA-II with local search and random immigrants for DMTSP			
Hybrid						[Gupta and Nanda, 2021]: NSGA-III with SVR-RBF kernel predictor for type I DMTSP of 16 nodes

Solving Static MTSP with Co^3_{MO}

100-Generation Evolution for MOEA/D and NSGA-II on 2-objective TSP benchmark: randomAB100

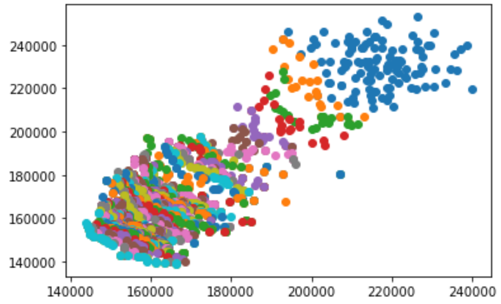
MOEA/D alone:
- Exploration at extreme points
- Evolution stress over certain areas

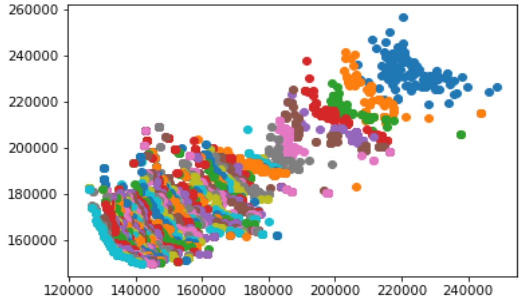




NSGA-II alone:
- Even distribution
- Reducing width

MOEA/D component in Co^3_{MO}:
- Better distribution
- 31% less IGD than MOEA/D alone

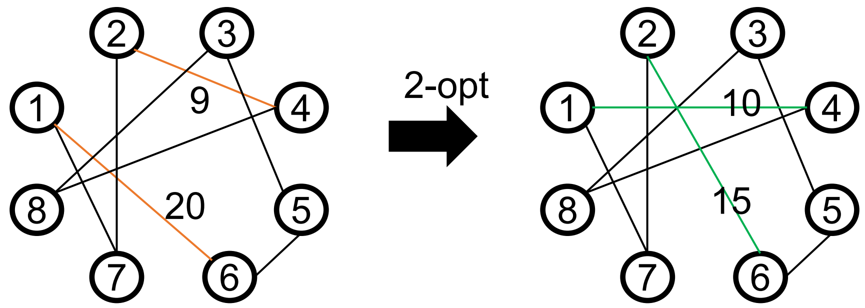




NSGA-II component in Co^3_{MO} :
- Better diversity
- 21% less IGD than NSGA-II alone

Improve Convergence with Local Search and Hybrid Operator

- Local Search



- Hybrid Operator

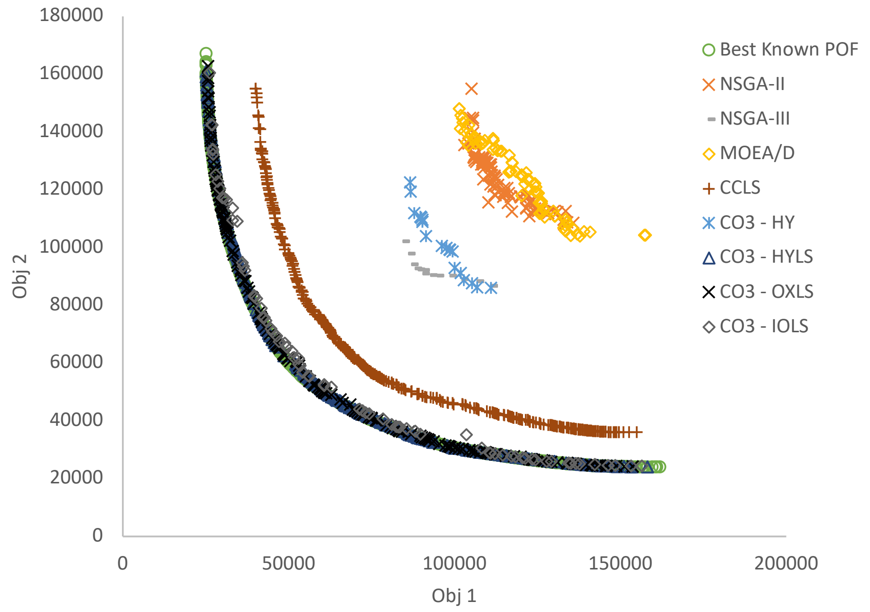
Order Crossover

parent1 = (8,3,2,6,5,1,4,7)
parent2 = (4,6,3,1,8,7,5,2)
pos1 = 3, pos2 = 6
child = (3,8,7,6,5,1,4,2)

Inver-over

Let n=8, δ = 50%, s'=(8,3,2,6,5,1,4,7), c =3:
Begin loop:
if rand()> δ: ← iteration 1
select another random tour s_k = (4,6,3,1,8,7,5,2)
c' (next city to c in s_k) = 1
c'' (next city to c in s') = 2
s' = (8,3,1,5,6,2,4,7), c = 1
if rand()<= δ: ← iteration 2
c' (next city to c in s') = 2
if c' != c'': continue ← iteration 1
Exit loop ← iteration 2

Result Summary for Static MTSP

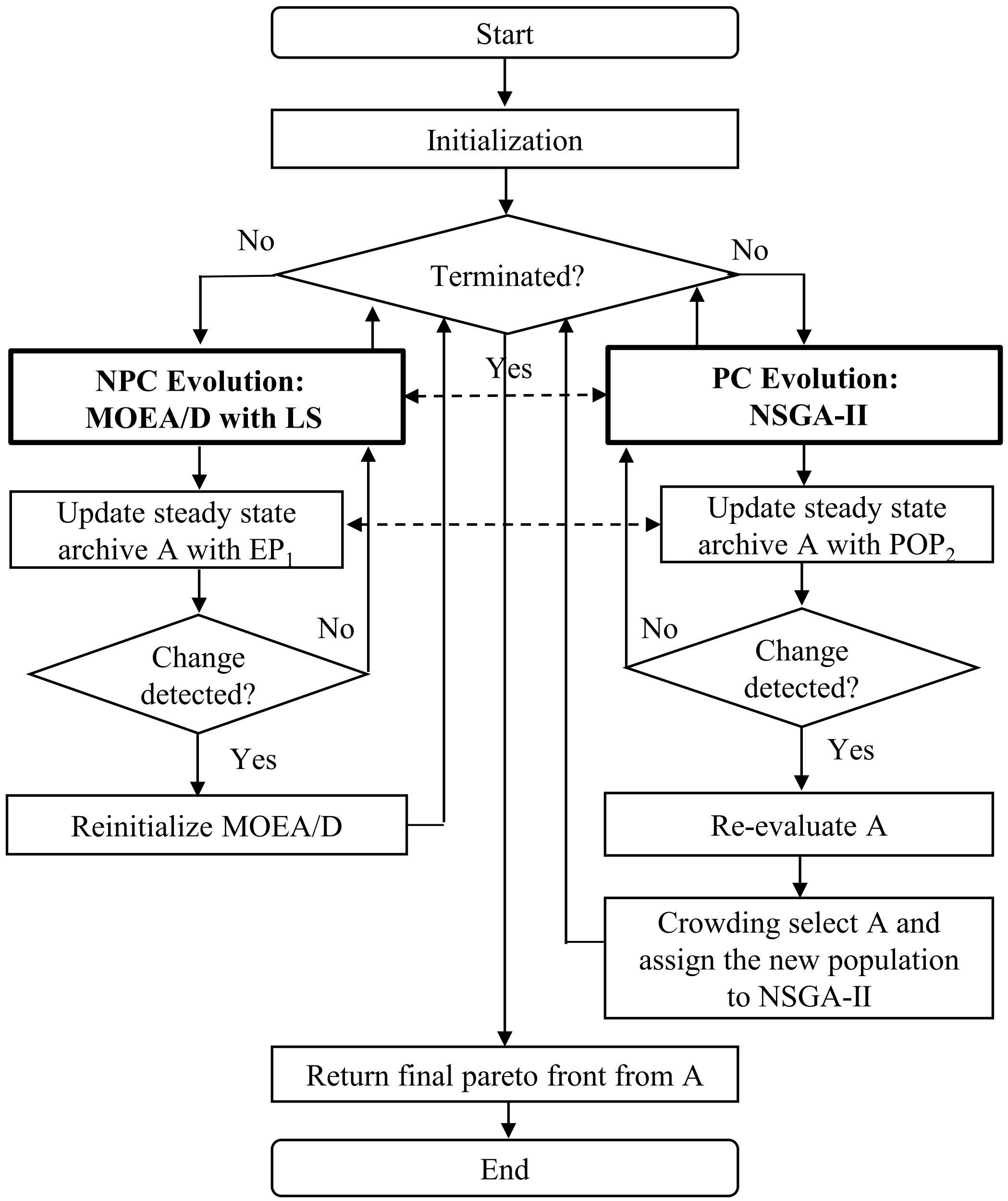


Pareto Front Visualization of Static euclidAB100

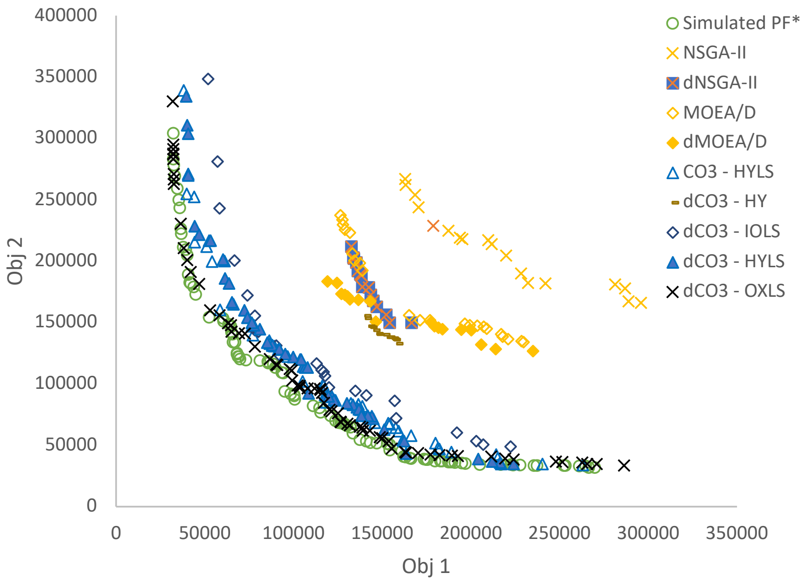
Friedman Test Ranking	Inverted Generational Distance (IGD)	Hypervolume (HV)	Spacing Metric (SM)
NSGA-II	54	36	20
NSGA-III	39	49	51
MOEA/D	63	50	21
CO3-HY	42	63	37
CO3-HYLS	10	12	24
CO3-OXLS	17	15	38
CO3-IOLS	27	27	61

- The three Co^3_{MO} - LS configurations outperform compared standard EMOs and state-of-the-art literature result for MTSP
- Co^3_{MO} with hybrid operator and local search performs the best

dCo$^3{}_{MO}$ for Dynamic MOP



Result Summary for Dynamic MTSP



Pareto Front Visualization of Dynamic euclidAB100

Friedman Test Ranking on Dynamic euclidAB100	Mean Inverted Generational Distance (MIGD)	Mean Hypervolume (MHV)	Mean Spacing Metric (MSM)
NSGA-II	89	58	61
MOEA/D	70	85	38
CO3-HYLS	29	32	71
dNSGA-II	81	65	25
dMOEA/D	55	82	42
dCO3-HY	55	60	14
dCO3-HYLS	19	22	59
dCO3-OXLS	12	10	54
dCO3-IOLS	40	36	86

- For DMTSP, dCo^3_{MO} with order crossover and local search outperforms dCo^3_{MO} with hybrid operator and local search
- The evenness of dCo^3_{MO} - LS configurations is affected by the weighted sum decomposition

Dynamic vs Static Algorithms

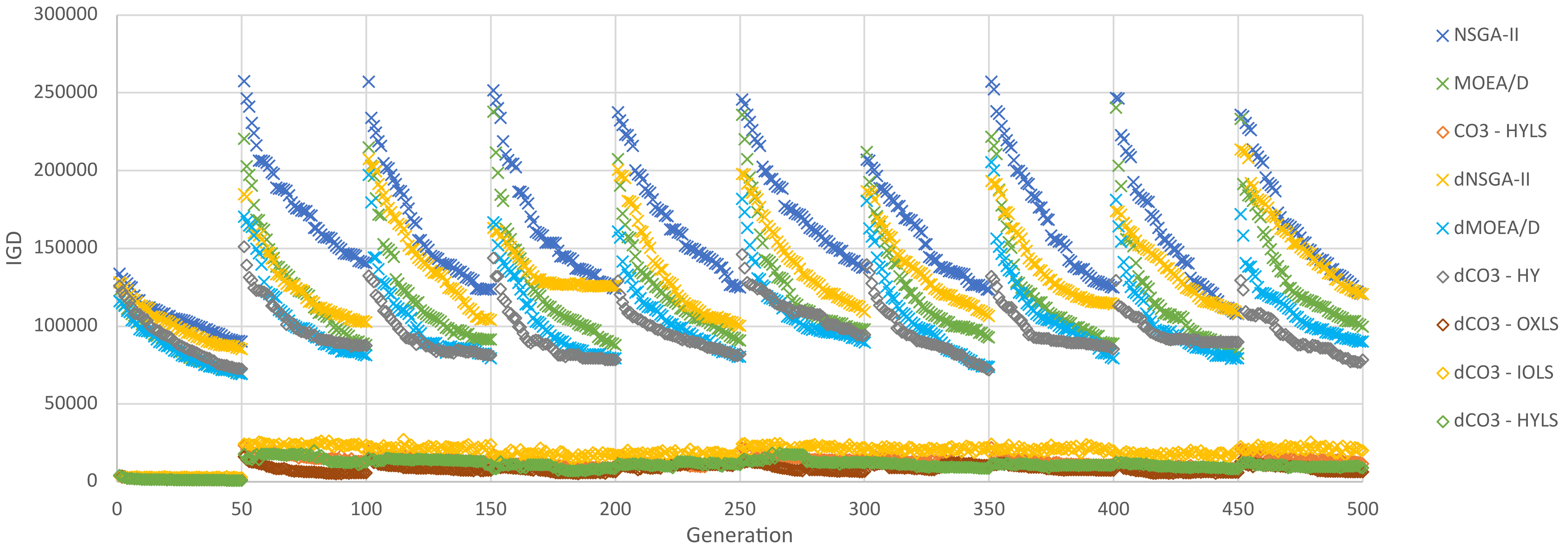


Figure 5.9: Evolution on Dynamic euclidAB100 with Random Traffic Every 50 Generations.

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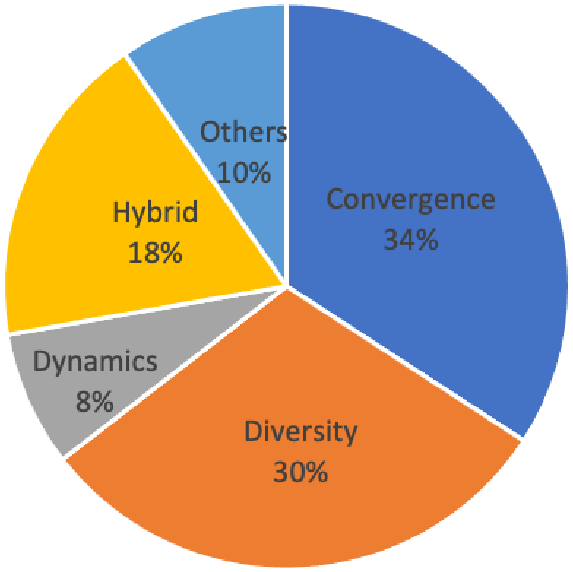
DEMO Evaluation Literature Review

Evaluation Goal	Common Metrics
Convergence	Mean Inverted Generational Distance (MIGD) [Zhou et al., 2007], Variational Distance (VD) [Goh and Tan, 2009], Convergence Measure [Farina et al., 2003]
Diversity	Mean Maximum Spread (MMS) [Goh and Tan, 2009], Mean Spacing Metric (MSP) [Jiang and Yang, 2016], Average Density (AD) [Zhang, 2008], Front Diversity (H_N) [Azevedo and Ara´ujo, 2011]
Hybrid (Measure both Convergence and Diversity)	Mean Hypervolume [Zhou et al., 2007], Hypervolume Ratio (HVR) [Camara, 2010], Mean Hypervolume Difference (MHVD) [Zhou et al., 2014]
Dynamics	Robustness [Jiang and Yang, 2016], Stability [Sola, 2010], Reaction Time [Sola, 2010]
Others (Problem Specific)	Optimal objectives (minimum carbon emission, vehicle waiting time and the number of vehicles) for DMOP vehicle routing [Guo et al, 2017]

DEMO Evaluation Literature Gap

1. The use of more advanced dynamics metrics for DEMO evaluation is still at an early stage.



DEMO Metrics Usage Sampled from 81 Papers

2. Lack of Worst-case Evaluation
   - There are few explicit definitions of worst-case scenarios, especially in the application papers.
   - Instead of being considered side by side with average-case evaluation, the worst-case scenarios are generally treated as “just another test case”.
   - Beyond worst-case, we also need to generalize the test cases to represent a DMOP with all severity levels.

Proposed 3-Layer Generalized Severity Framework

Layer 1: Severity Level

Layer 2: Change scope at decision space

Layer 3: Change scope at objective space

Severity	Change Scope	Both objectives change at the same rate	One objective changes	Both objectives change at different rates
Mild	Global	Scenario 1	Scenario 2	Scenario 3
	Local	Scenario 4	Scenario 5	Scenario 6
Medium	Global	Scenario 1	Scenario 2	Scenario 3
	Local	Scenario 4	Scenario 5	Scenario 6
Worst	Global	Scenario 1	Scenario 2	Scenario 3
	Local	Scenario 4	Scenario 5	Scenario 6

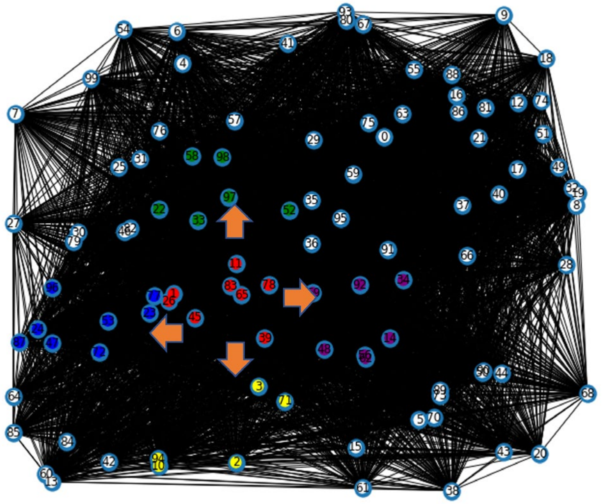
Proposed New Metrics for Evaluation

Name	Formula	Purpose
Stress Response SR	**SR(σ)** = $\frac{1}{T_s}\sum_{t=1}^{T_s} Stb(\sigma, t)$	Stability of the DEMO under maximum severity of change
Generalized Severity Response GSR	**GSR** = $Avg_{\varphi \in \Phi}(\frac{1}{T_s}\sum_{t=1}^{T_s} Stb(\varphi, t))$	Average stability of the DEMO under varying severity changes
Generalized Severity Average Performance Ranking (GSAPR)	Avg (rankings of generalized severity metrics)	Composite overall metric can be used as a single metric to compare different dynamic EMOs

Note:

- Stb(t) = max{0, acc(t-1) – acc(t)} (0 means good stability) [Camara et al., 2007]
- acc(t) = HVR(t) = $\frac{HV(t)}{HV^{max}(t)}$ = $\frac{HV(t,POF^*)}{HV(t,POF)}$ for minimization problems
- σ is the timeseries input sequence that results in **maximum severity of change**
- Φ is a set of timeseries inputs with varying severity changes, ideally with mild, medium and worst severities. Severity for benchmark datasets is usually configurable. Applications can use assumed range of changes to define severities.
- In this thesis, we use the rankings of GSMIGD, GSMHV, GSMSM, GSR to calculate GSAPR. However, GSAPR can be calculated from any selected metrics

Case Study: Experiment on Dynamic Multi-objective Traveling Salesperson Problem (DMTSP)



Layer 1:
- Mild Traffic
- Medium Traffic
- Worst Traffic

Layer 2:
- Global traffic change
- Zone based local traffic propagation

Layer 3:
- Both objectives change with the same rate
- One objective changes
- Both objectives change with different rates

Metric	Convergence	Diversity	Dynamics
Generalized Severity Mean Inverted Generational Distance (GSMIGD)	√	√	
Generalized Severity Mean Hypervolume (GSMHV)	√	√	
Generalized Severity Mean Spacing Metric (GSMSM)		√	
NEW: Stress Response (SR)	√	√	√
NEW: Generalized Severity Response (GSR)	√	√	√
NEW: Generalized Severity Average Performance Ranking (GSAPR)	√	√	√

DMTSP Evaluation Result

Compared to common metrics such as MIGD, MHV, MSM, the proposed three-layer framework and new metrics offer a more comprehensive and granular understanding of the evaluated algorithms.

GSR Ranking

Level	Measure	NSGA-II	MOEA/D	$Co^3_{MO}-HYLS$	dNSGA-II	dMOEA/D	$dCo^3_{MO}-HY$	$dCo^3_{MO}-HYLS$	$dCo^3_{MO}-OXLS$	$dCo^3_{MO}-IOLS$
All	Avg	7	9	3	8	6	5	2	1	4
Mild	Avg	7	9	3	8	6	5	2	1	4
	Scenario 1	5	9	1	8	7	6	3	2	4
	Scenario 2	7	9	3	8	6	5	2	1	4
	Scenario 3	6	9	2	8	7	5	3	1	4
	Scenario 4	7	9	3	8	6	5	2	1	4
	Scenario 5	8	9	3	6	7	5	2	1	4
	Scenario 6	8	9	3	7	6	5	2	1	4
Mediu	Avg	7	9	3	8	6	5	2	1	4
	Scenario 1	5	9	3	8	7	6	2	1	4
	Scenario 2	6	9	3	8	7	4	2	1	5
	Scenario 3	7	8	3	9	6	5	2	1	4
	Scenario 4	8	9	2	6	7	5	3	1	4
	Scenario 5	8	9	3	7	6	5	2	1	4
	Scenario 6	8	9	3	6	7	5	2	1	4
Worst	Avg	7	9	3	8	6	5	2	1	4
	Scenario 1	6	8	3	9	7	5	2	1	4
	Scenario 2	5	9	3	6	7	4	2	1	8
	Scenario 3	6	8	3	9	7	5	2	1	4
	Scenario 4	7	9	3	8	5	6	2	1	4
	Scenario 5	8	9	3	7	5	6	2	1	4
	Scenario 6	8	9	3	7	6	5	2	1	4

SR Ranking

GSAPR Ranking

Level	Measure	NSGA-II	MOEA/D	$Co^3_{MO}-HYLS$	dNSGA-II	dMOEA/D	$dCo^3_{MO}-HY$	$dCo^3_{MO}-HYLS$	$dCo^3_{MO}-OXLS$	$dCo^3_{MO}-IOLS$
All	Avg	6.75	7.0	4.0	6.25	5.25	4.75	3.0	2.75	5.25
Mild	Avg	6.75	7.25	4.0	6.5	5.25	4.25	3.25	2.5	5.25
	Scenario 1	6.0	7.25	3.25	6.5	5.5	4.75	3.5	3.0	5.25
	Scenario 2	6.5	8.0	4.25	6.5	5.5	4.5	3.0	1.5	5.25
	Scenario 3	6.75	7.0	3.25	6.25	5.75	4.75	3.5	2.5	5.25
	Scenario 4	7.5	8.0	3.25	6.25	6.25	4.25	2.25	2.0	5.25
	Scenario 5	7.75	8.25	3.25	5.5	6.5	5.0	2.0	1.5	5.25
	Scenario 6	7.75	8.25	3.5	5.75	6.25	4.25	2.0	2.0	5.25
Mediu	Avg	6.75	7.25	4.25	6.5	5.25	4.25	3.0	2.5	5.25
	Scenario 1	6.0	7.25	3.5	6.5	5.5	4.75	3.5	2.75	5.25
	Scenario 2	6.25	7.5	4.0	6.25	7.0	4.25	2.5	1.5	5.75
	Scenario 3	6.5	6.75	3.5	7.0	5.0	4.75	3.5	2.75	5.25
	Scenario 4	7.75	8.0	3.5	5.75	6.25	4.25	2.75	1.5	5.25
	Scenario 5	7.75	7.75	4.0	6.0	6.0	4.25	2.25	1.75	5.25
	Scenario 6	7.75	8.0	3.5	5.75	6.5	4.25	2.25	1.75	5.25
Worst	Avg	6.75	7.0	4.0	6.25	5.25	4.75	3.0	2.75	5.25
	Scenario 1	6.25	7.0	3.75	6.5	5.5	4.75	3.25	2.75	5.25
	Scenario 2	5.0	7.25	4.25	5.75	6.75	4.0	2.0	2.5	7.5
	Scenario 3	6.25	6.5	3.5	6.75	5.5	5.0	3.5	3.0	5.0
	Scenario 4	7.0	7.5	4.25	6.25	5.25	4.75	3.0	1.75	5.25
	Scenario 5	7.75	7.75	4.0	6.0	5.75	4.75	2.25	1.5	5.25
	Scenario 6	7.5	7.25	4.25	6.0	5.5	4.25	2.75	2.25	5.25

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1. Introduction/Background
2. Proposed Algorithm
   - Bicriterion Coevolution for Static and Dynamic MOP
   - Experiment Results
3. Proposed Evaluation Framework and Metrics
   - Worst and Generalized Severity Evaluation for Dynamic EMO
   - Case Study
4. **Conclusion and Future Work**

Conclusion

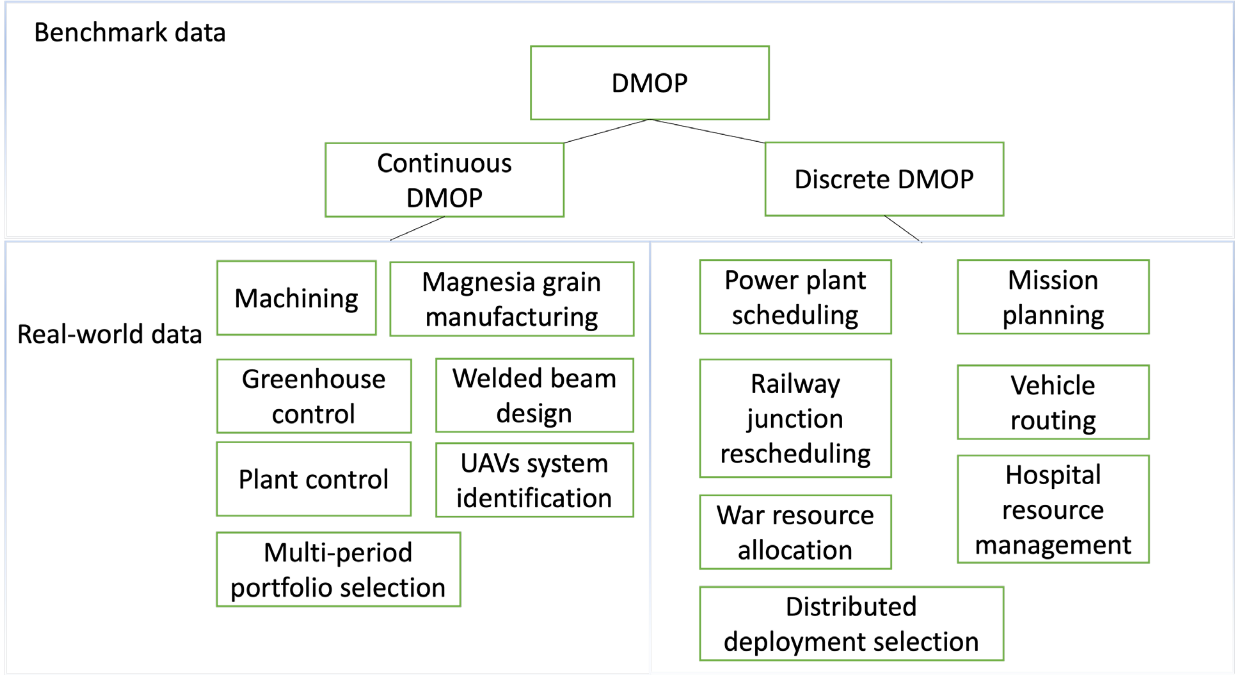
- In this research, we have proposed **CO^3_{MO}**, a **Co**operative, **Co**ncurrent, **Co**evolutionary technique to solve multi-objective optimization in complex networks.
- We also propose a **worst and generalized severity framework and new metrics** to improve the evaluation of convergence, diversity, and dynamics for DEMOs.

Future Work

1. Improve the distribution evenness of CO^3_{MO} by adopting more advanced decomposition techniques
2. Use Genetic Programming to optimize the ”plug-and-play” components for
   - the configuration of operators of CO^3_{MO}
   - the configuration of severity change level or metrics to include in GSAPR
3. Extend the experiments to more real-world DMOP applications
4. Enhance the integration of complex network features with CO^3_{MO}, for example, extend current multi-population strategy with predictions using Temporal Graph Forecasting

The Proposed Three-Layer Evaluation Framework is Extensible to General Purpose

- Different DMOP problems can have their own definitions of mild, medium and worst severity changes
- The proposed GSAPR metric can include any combination of metrics
- The different configurations and metrics fit a ”plug-and-play” paradigm and can be automated using Genetic Programming



Related Publications

- Liu, Ying Ying, Parimala Thulasiraman, and Nelishia Pillay. “Generalized Severity Evaluation for Evolutionary Dynamic Multi-objective Optimization.” In 2022 IEEE Symposium Series on Computational Intelligence (SSCI), December 4, 2022 to December 7, 2022 - Singapore.
- Liu, Ying Ying, Parimala Thulasiraman, and Nelishia Pillay. “Bicriterion Coevolution for the Multi-objective Travelling Salesperson Problem.” In 2022 IEEE Congress on Evolutionary Computation (IEEE CEC 2022), July 18, 2022 to July 23, 2022 – Padua, Italy.
- Liu, Ying Ying, Parimala Thulasiraman. “Integrating traffic network clustering to multi-objective route planning: a heuristic approach.” In The Thirteenth International Conference on Evolving Internet (INTERNET 2021), July 18, 2021 to July 22, 2021 - Nice, France.

Thank you!