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On the Regularization of a Low-Complexity Recursive Least-Squares Adaptive Algorithm

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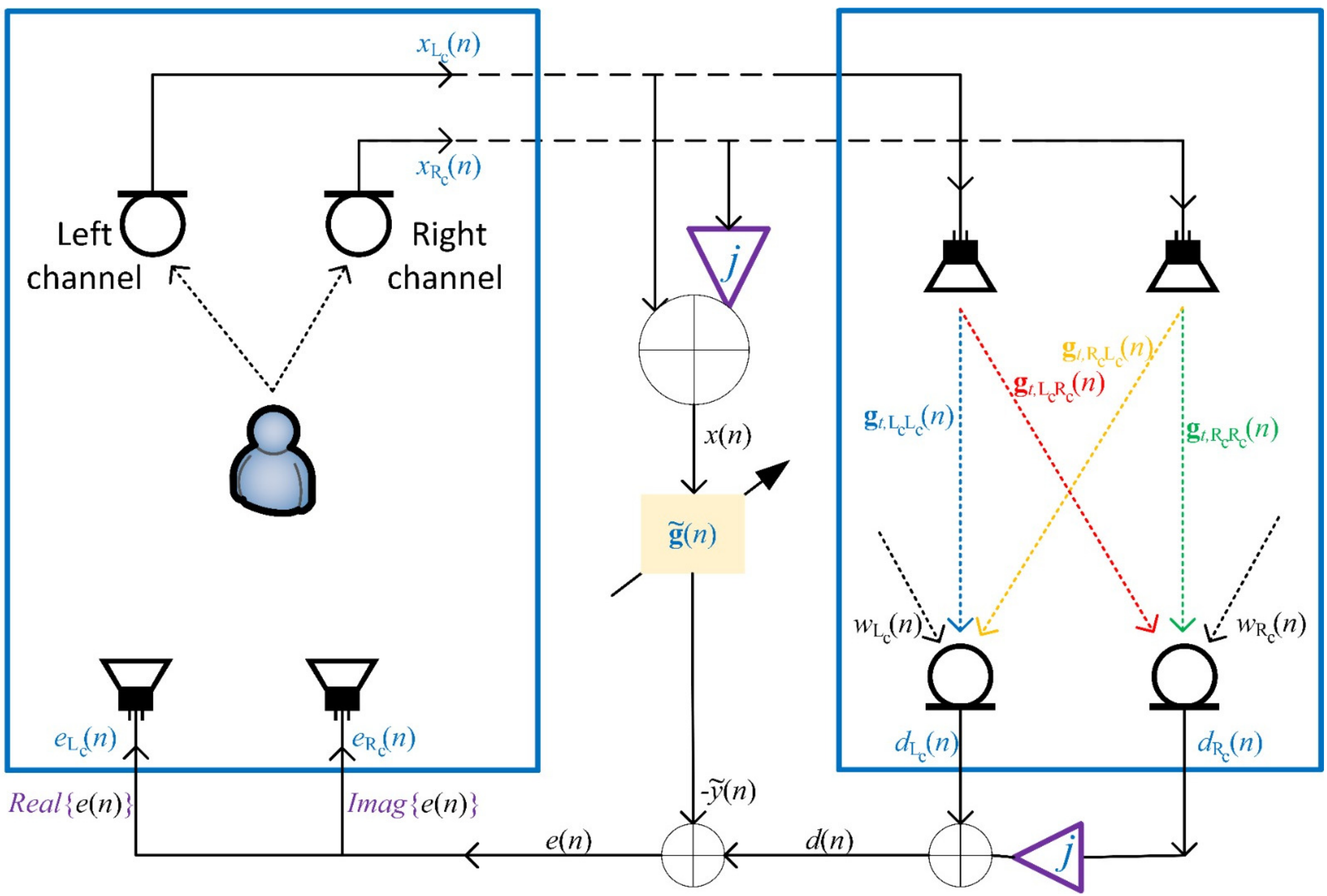
Summary

- Introduction
- System Model – WL Framework
- WL-RLS-LSM
- VR-WL-RLS-LSM
- Simulation Results
- Conclusions

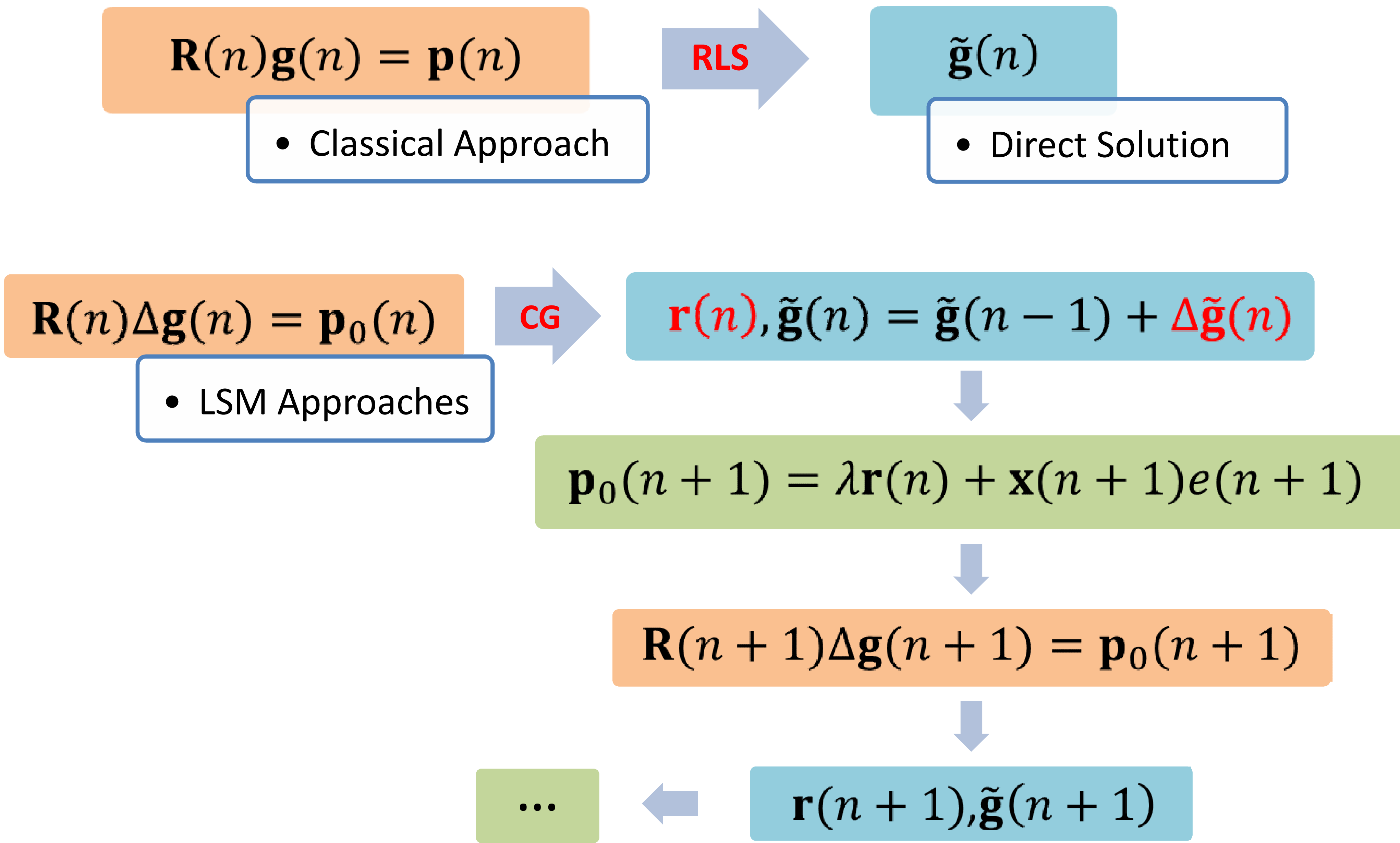
Introduction

- Stereophonic Acoustic Echo Cancellation (**SAEC**):
  - Estimation of 4 echo paths => the loudspeaker-to-microphone pairs
- The **LMS** method (standard approach):
  - 4 individual systems => low arithmetic complexity and limited performance for highly correlated signals due to gradient noise
- The **WL-RLS-LSM** algorithms:
  - Complex-valued line search methods
  - Solve an auxiliary system of equations
  - Have a key parameter – performance vs complexity
- The **VR-WL-RLS-LSM** algorithms:
  - Improved performance for small echo-to-noise (ENR) values
  - Improved performance when double-talk (DT) situations occur

System Model – WL



WL-RLS-LSM



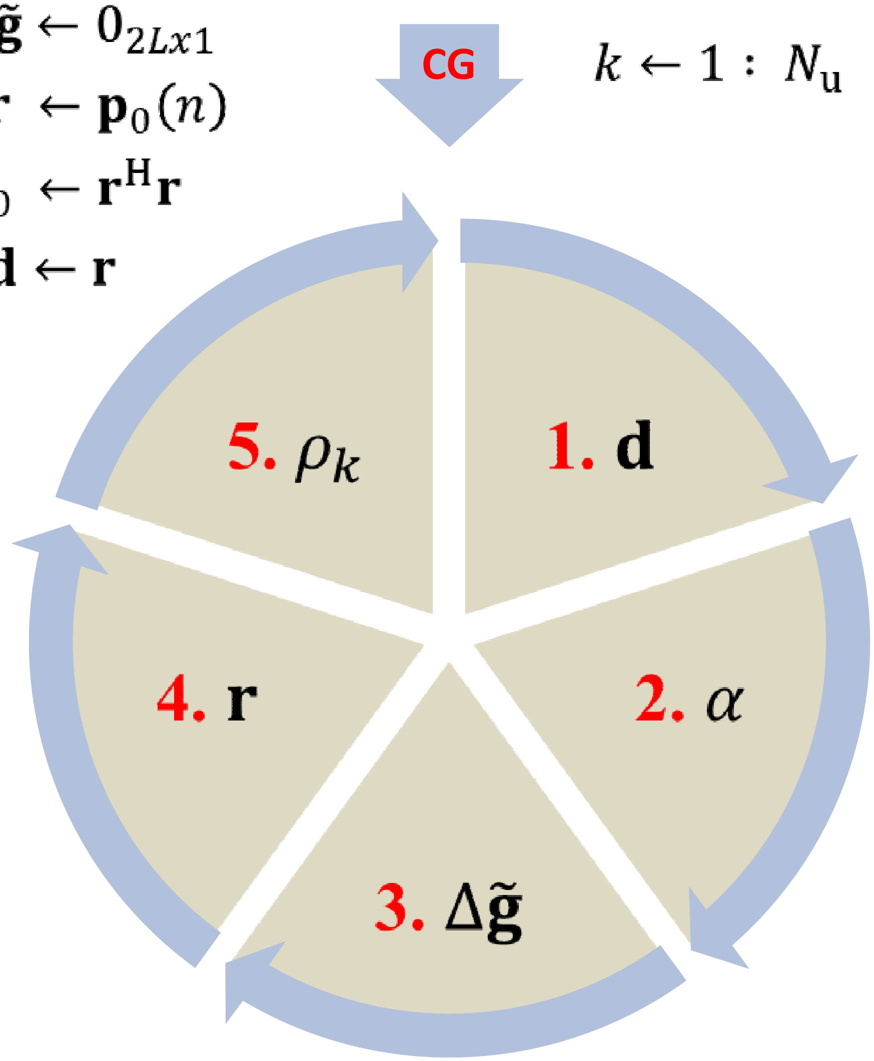
$\mathbf{R}(n)\mathbf{g}(n) = \mathbf{p}(n)$
• Classical Approach
RLS
$\tilde{\mathbf{g}}(n)$
• Direct Solution
$\mathbf{R}(n)\Delta\mathbf{g}(n) = \mathbf{p}_0(n)$
• LSM Approaches
CG
$\mathbf{r}(n), \tilde{\mathbf{g}}(n) = \tilde{\mathbf{g}}(n-1) + \Delta\tilde{\mathbf{g}}(n)$
$\mathbf{p}_0(n+1) = \lambda\mathbf{r}(n) + \mathbf{x}(n+1)e(n+1)$
$\mathbf{R}(n+1)\Delta\mathbf{g}(n+1) = \mathbf{p}_0(n+1)$
$\mathbf{r}(n+1), \tilde{\mathbf{g}}(n+1)$
…

Conjugate Gradient

$$\mathbf{R}(n)\Delta\mathbf{g}(n) = \mathbf{p}_0(n)$$

$$\Delta\tilde{\mathbf{g}} \leftarrow 0_{2Lx1}$$
$$\mathbf{r} \leftarrow \mathbf{p}_0(n)$$
$$\rho_0 \leftarrow \mathbf{r}^{\mathrm{H}}\mathbf{r}$$
$$\mathbf{d} \leftarrow \mathbf{r}$$

$$k \leftarrow 1 : N_{\mathrm{u}}$$



Conjugate Gradient

$$\mathbf{R}(n)\Delta\mathbf{g}(n) = \mathbf{p}_0(n)$$

$$\Delta\tilde{\mathbf{g}} \leftarrow 0_{2Lx1}$$
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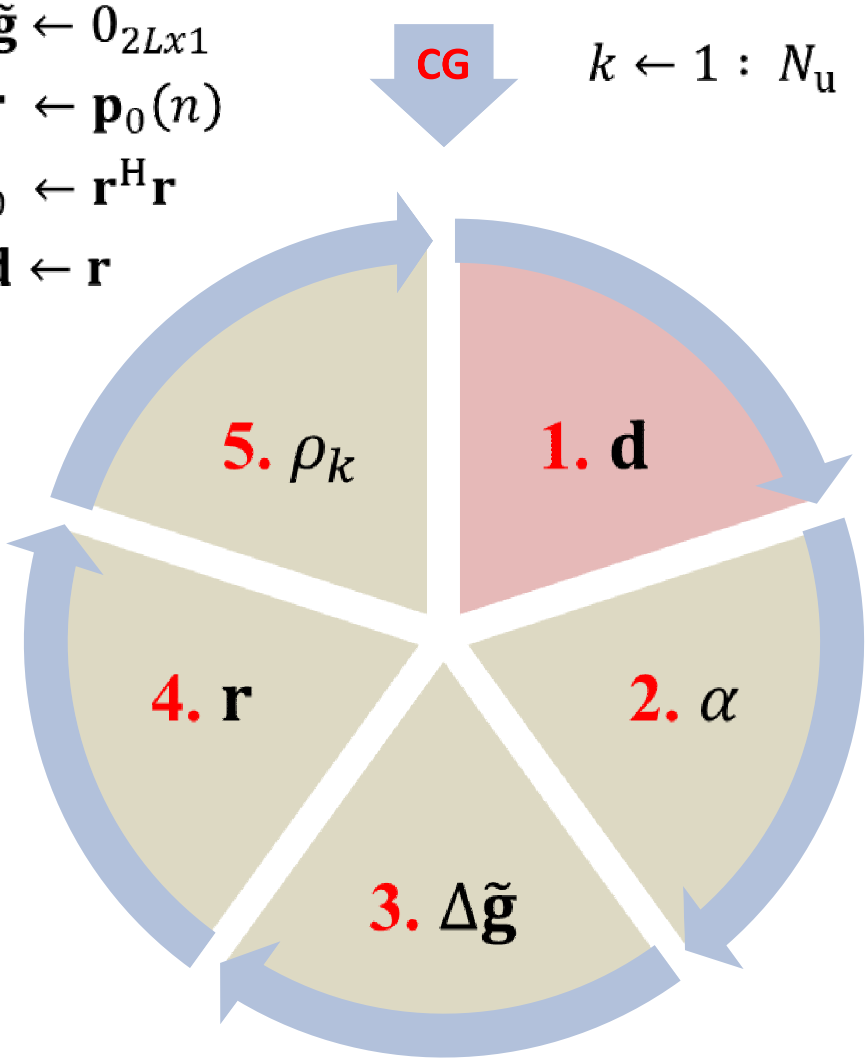

$$k > 1 \,?\; \mathbf{d} \leftarrow \mathbf{r} + \frac{\rho_{k-1}}{\rho_{k-2}}\mathbf{d}$$

Conjugate Gradient

$$\mathbf{R}(n)\Delta\mathbf{g}(n) = \mathbf{p}_0(n)$$

$\Delta\tilde{\mathbf{g}} \leftarrow 0_{2Lx1}$

$\mathbf{r} \leftarrow \mathbf{p}_0(n)$

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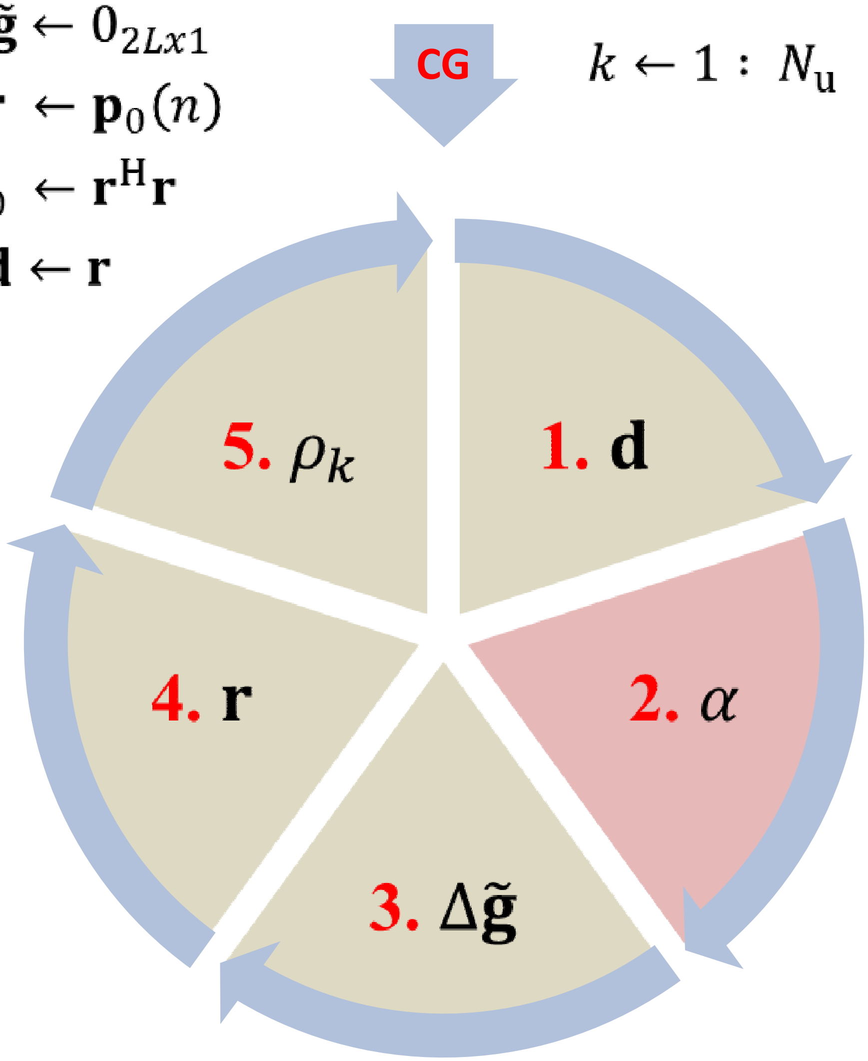


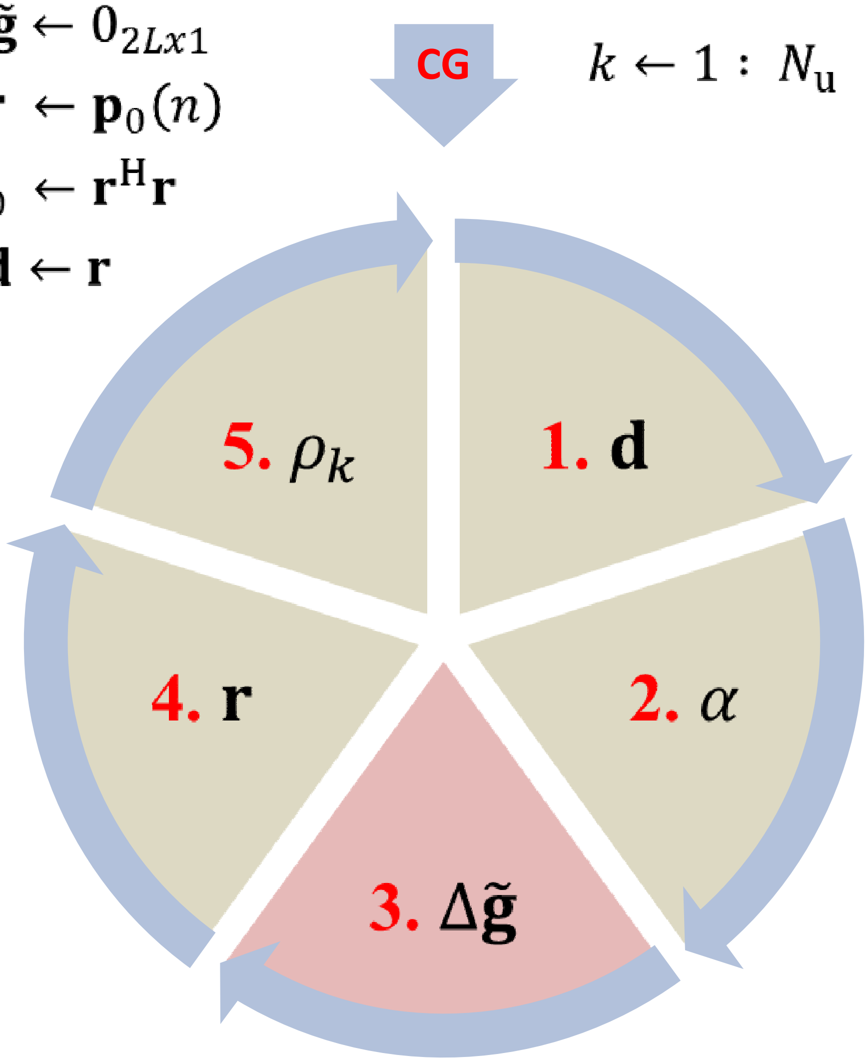
$$\alpha \leftarrow \frac{\rho_{k-1}}{\mathbf{d}^{\mathrm{H}}\mathbf{R}_{\tilde{\mathbf{x}}}(n)\mathbf{d}}$$

Conjugate Gradient

$$\mathbf{R}(n)\Delta\mathbf{g}(n) = \mathbf{p}_0(n)$$

$\Delta\tilde{\mathbf{g}} \leftarrow 0_{2Lx1}$
$\mathbf{r} \leftarrow \mathbf{p}_0(n)$
$\rho_0 \leftarrow \mathbf{r}^{\mathrm{H}}\mathbf{r}$
$\mathbf{d} \leftarrow \mathbf{r}$

$k \leftarrow 1 : N_{\mathrm{u}}$

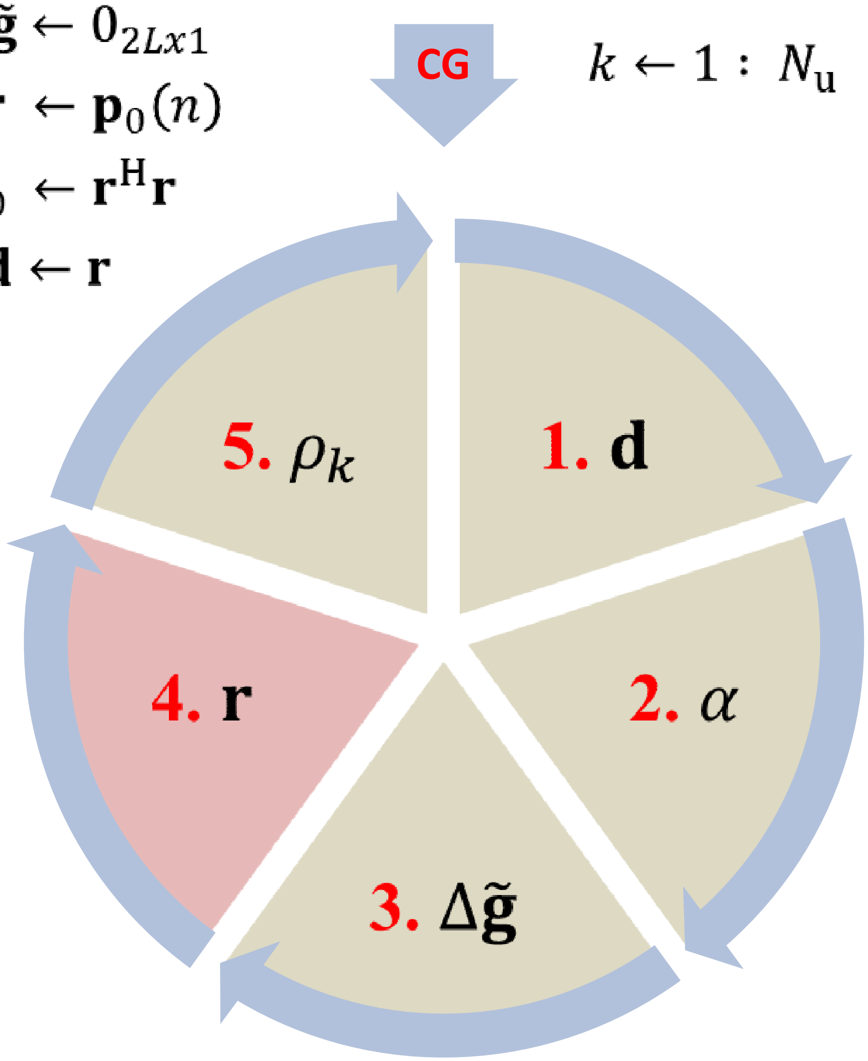


$$\Delta\tilde{\mathbf{g}} \leftarrow \Delta\tilde{\mathbf{g}} + \alpha\mathbf{d}$$

Conjugate Gradient

$$\mathbf{R}(n)\Delta\mathbf{g}(n) = \mathbf{p}_0(n)$$

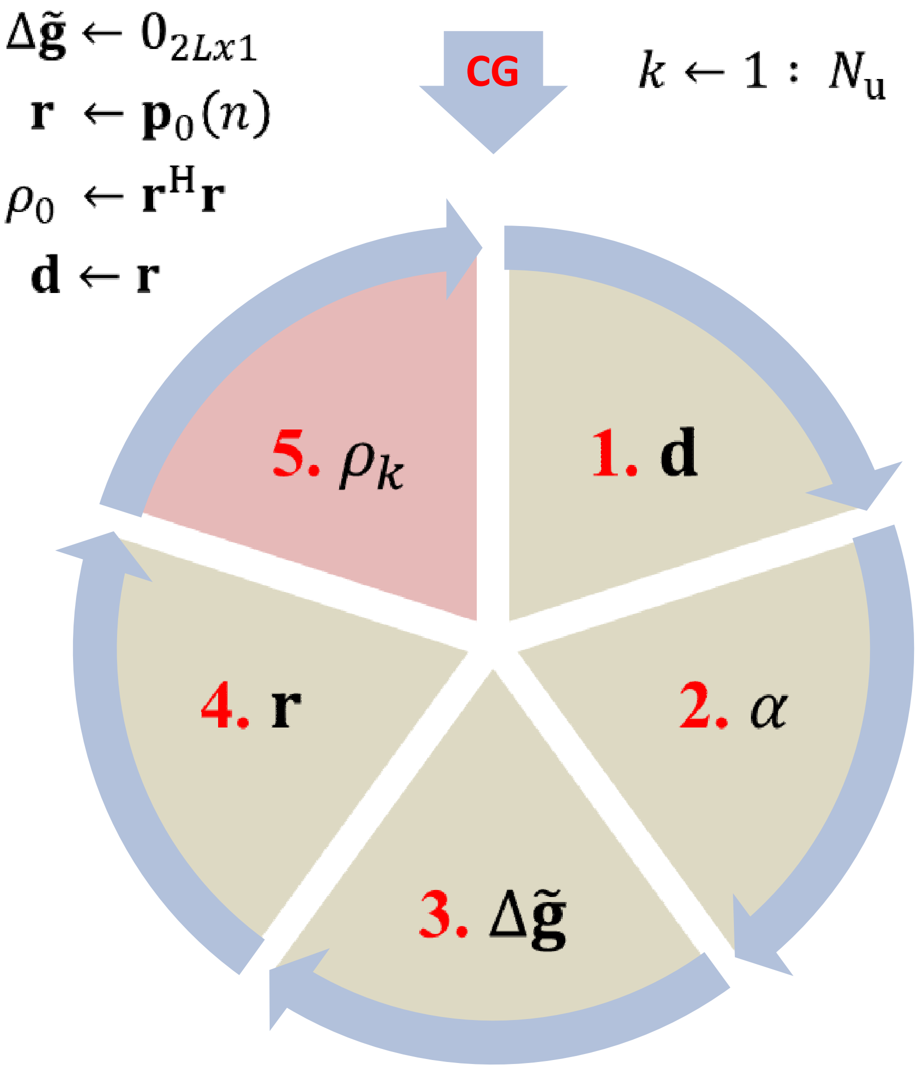
$$\Delta\tilde{\mathbf{g}} \leftarrow 0_{2Lx1}$$
$$\mathbf{r} \leftarrow \mathbf{p}_0(n)$$
$$\rho_0 \leftarrow \mathbf{r}^{\mathrm{H}}\mathbf{r}$$
$$\mathbf{d} \leftarrow \mathbf{r}$$



$$\mathbf{r} \leftarrow \mathbf{r} - \alpha\mathbf{R}_{\tilde{\mathbf{x}}}(n)\mathbf{d}$$

Conjugate Gradient

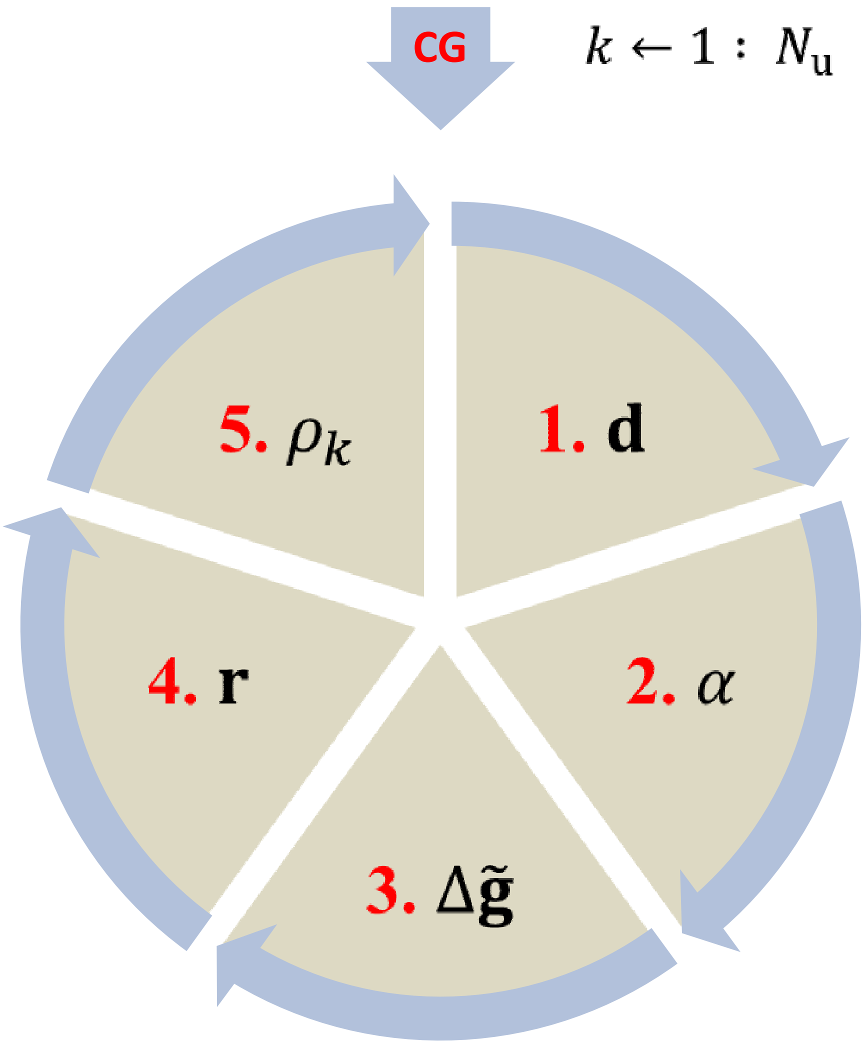
$$\mathbf{R}(n)\Delta\mathbf{g}(n) = \mathbf{p}_0(n)$$



$$\rho_k \leftarrow \mathbf{r}^{\mathrm{H}}\mathbf{r}$$

Conjugate Gradient

$$\mathbf{R}(n)\Delta\mathbf{g}(n) = \mathbf{p}_0(n)$$



Complexity: $O(8L^2)$ multiplications/it.

$$\tilde{\mathbf{g}}(n+1) = \tilde{\mathbf{g}}(n) + \Delta\tilde{\mathbf{g}}$$

Dichotomous Coordinate Descent

$$\mathbf{R}(n)\Delta\mathbf{g}(n) = \mathbf{p}_0(n)$$

Complexity: **no** multiplications/it.

$\Delta\tilde{\mathbf{g}} \leftarrow 0_{2Lx1}$
$\mathbf{r} \leftarrow \mathbf{p}_0(n)$
$\alpha \leftarrow H$
$m \leftarrow 0$

DCD

$k \leftarrow 1 : N_u$

Step	Action
1	$\{val_1; pos_1\} \leftarrow \max_{i=1...2L}\{\lvert Re\{r_i\}\rvert\}$ $\{val_2; pos_2\} \leftarrow \max_{i=1...2L}\{\lvert Im\{r_i\}\rvert\}$ $(p;\eta) \leftarrow (val_1 > val_2)\ ?\ (pos_1; 1) : (pos_2; j)$ **Jump to step 4**
2	$\alpha \leftarrow \alpha/2\,; \quad m \leftarrow m+1$
3	**If** $(m > M_b)$ **RETURN**
4	$r_{temp} \leftarrow (\eta == 1)\ ?\ Re\{r_p\} : Im\{r_p\}$
5	**If** $(\lvert r_{temp}\rvert \leq \frac{\alpha}{2} R_{p,p})$ **Jump to step 2**
6	$\Delta\tilde{g}_p \leftarrow \Delta\tilde{g}_p + \text{sign}\{r_{temp}\}\eta\alpha$
7	$\mathbf{r} \leftarrow \mathbf{r} - \text{sign}\{r_{temp}\}\eta\alpha\mathbf{R}^{(p)}(n)$

$$\tilde{\mathbf{g}}(n+1) = \tilde{\mathbf{g}}(n) + \Delta\tilde{\mathbf{g}}$$

Variable Regularization

$$\mathbf{R}(n)\Delta\mathbf{g}(n) = \mathbf{p}_0(n)$$

VR

$$[\mathbf{R}(n) + \phi(n)\mathbf{I}_{2L}]\Delta\mathbf{g}(n) = \mathbf{p}_0(n)$$

$$\Phi(n) = 2L\widetilde{\sigma}_x^2(n)(1 + \sqrt{1 + \widetilde{\text{ENR}}})/(\widetilde{\text{ENR}})$$

$$\tilde{\mathbf{g}}(n+1) = \tilde{\mathbf{g}}(n) + \Delta\tilde{\mathbf{g}}$$

Simulation Results

- **Tracking & Double Talk:**
  - WL-RLS-CG vs WL-RLS-DCD vs VR-WL-RLS-CG vs VR-WL-RLS-DCD

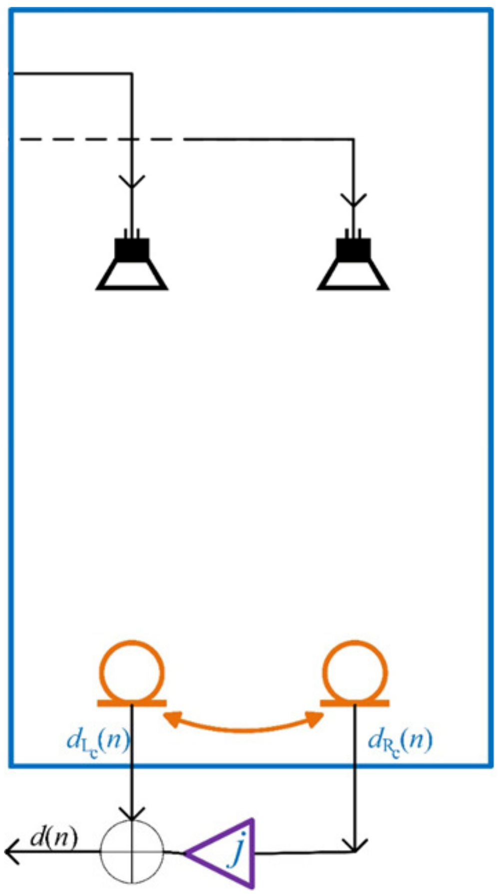
- **Input signal:**
  - High quality speech sequence

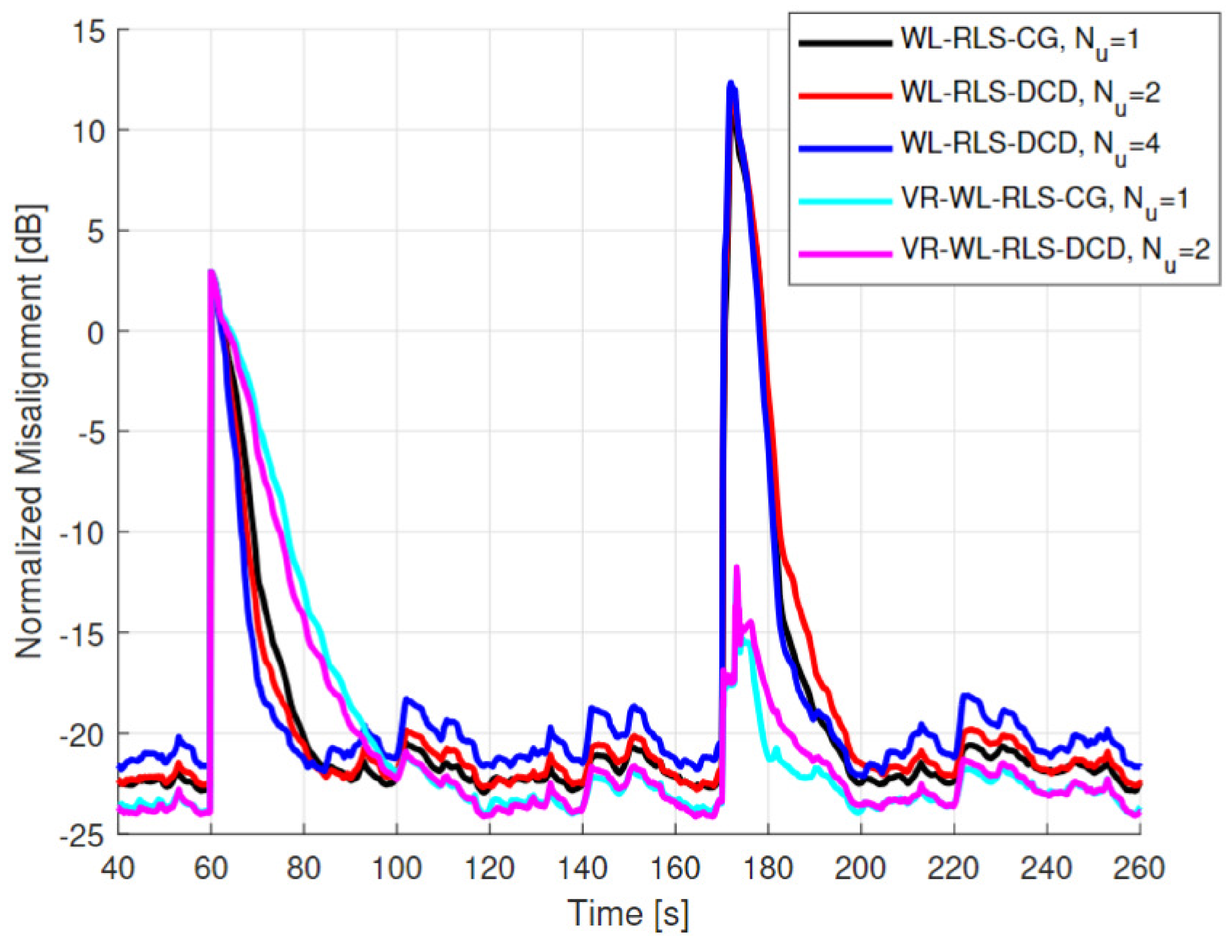
- **Unknown systems** (*left* and *right*):
  - Real measured impulse responses (decimated)
  - L = 256
  - ENR = 25 dB & 10 dB

- **Performance measure:**

$$\mathrm{Mis}(n) = 20\log_{10}\left[\frac{\|\tilde{\mathbf{g}}_{\mathrm{t}} - \tilde{\mathbf{g}}\,(n)\|_2}{\|\tilde{\mathbf{g}}_{\mathrm{t}}\|_2}\right] (\mathrm{dB})$$

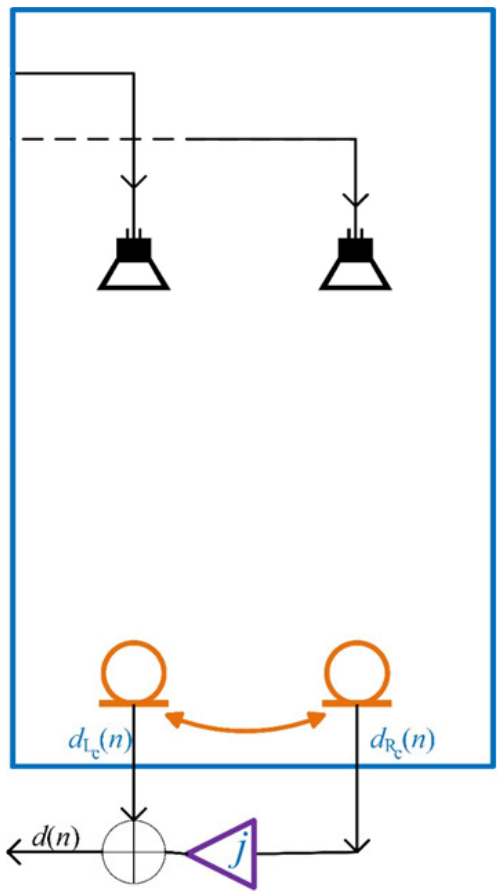
Simulation Results

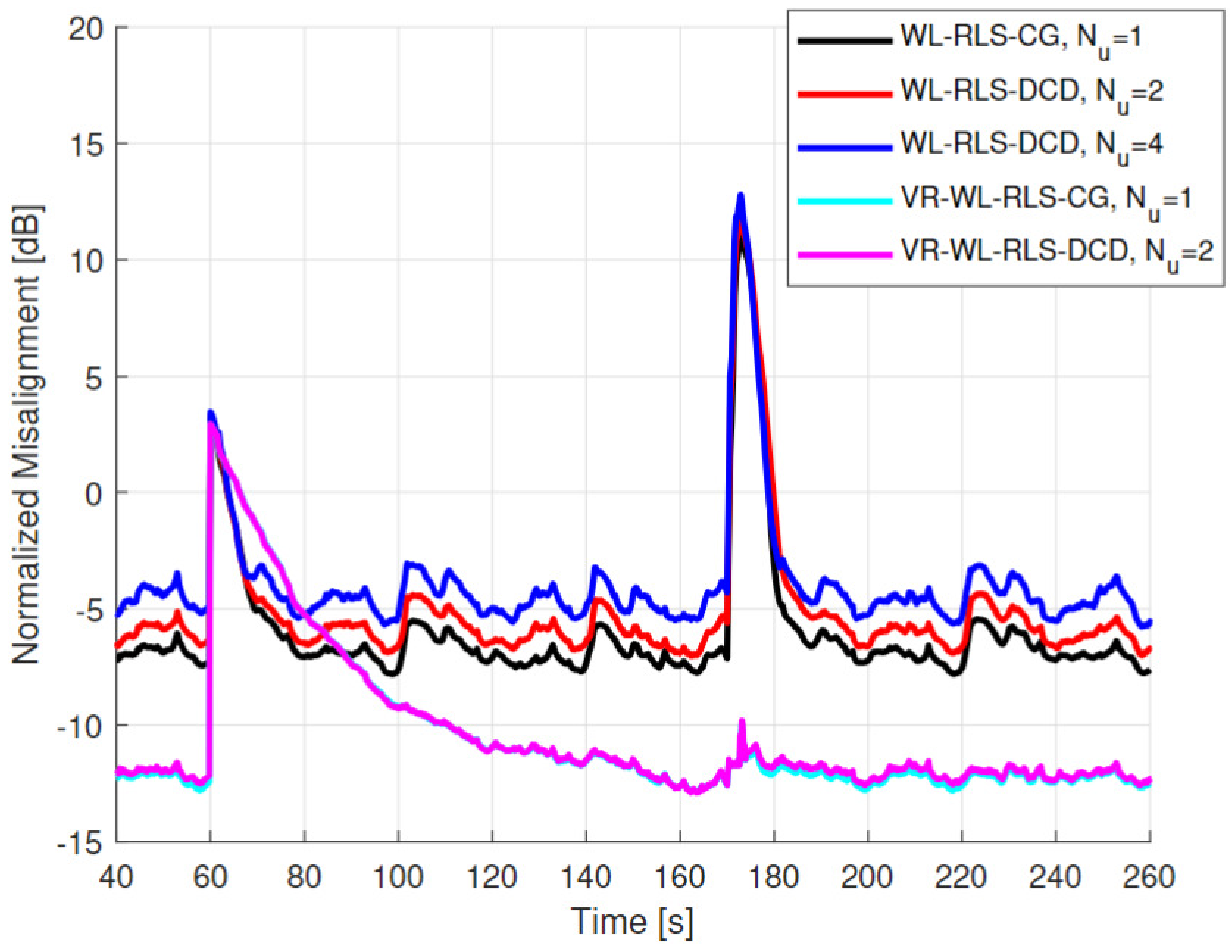




Performance of the LSM algorithms for L=256 and ENR = 25dB. The unknown system **changes at t_0 = 60s due to the interchanging of the microphone positions**. **Double-talk** situation in the **time interval [170; 174]s**.

Simulation Results





Performance of the LSM algorithms for L=256 and ENR = 10dB. The unknown system **changes at t_0 = 60s due to the interchanging of the microphone positions**. **Double-talk** situation in the **time interval [170; 174]s**.

Conclusions

- **VR-WL-RLS-LSM:**
  - Improves the performance of WL-RLS-LSM working in the SAEC setup with small ENR values and/or when double-talk situations occur
  - Extra arithmetical effort
  - Tracking speed loses

Thank you!

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