A Refined ERR-based Method for Nonlinear System Identification. Application to Epilepsy.

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Summary

I – Context

II – Connectivity

III – Method

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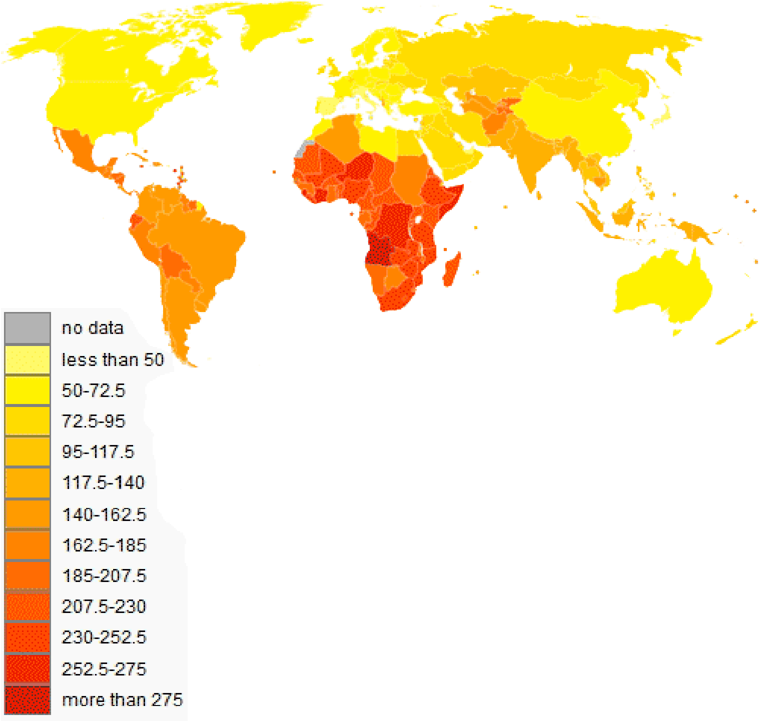
Epilepsy



Clinical context – Epilepsy

- A chronic **neurological** disease that affects about 0.6% to 0.7% of the world population
- Seizures are marked by sudden and recurrent electro-chemical discharges in groups of neurons
- An epileptic seizure is usually divided into three phases: **Pre-ictal**, **Ictal** and **Post-ictal** phases

[1] globometer.com/maladies-epilepsie.php
[2] who.int/news-room/fact-sheets/detail/epilepsy

Epilepsy – Symptoms

Tremors

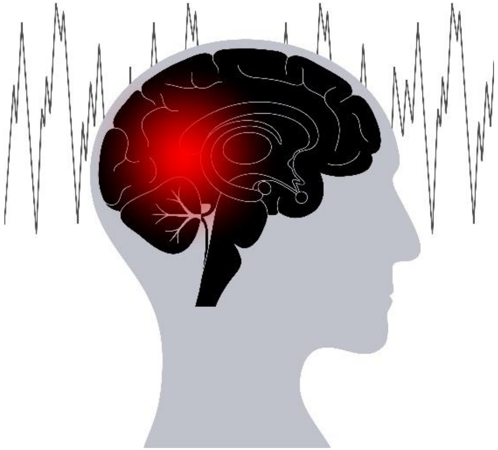
Muscle twitches

Hearing impairments

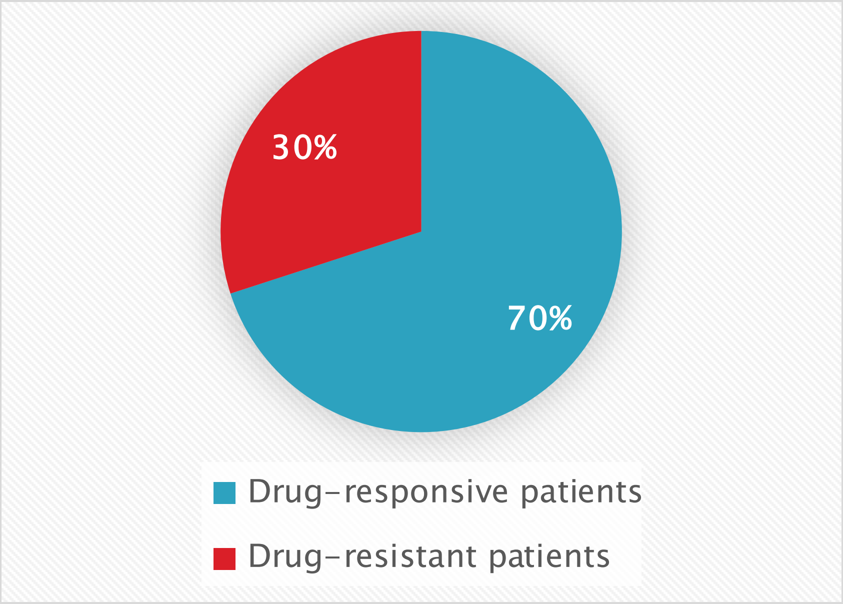
Language disorders

Visual hallucinations

Epilepsy – Drug Responsiveness



Extracted from [3]

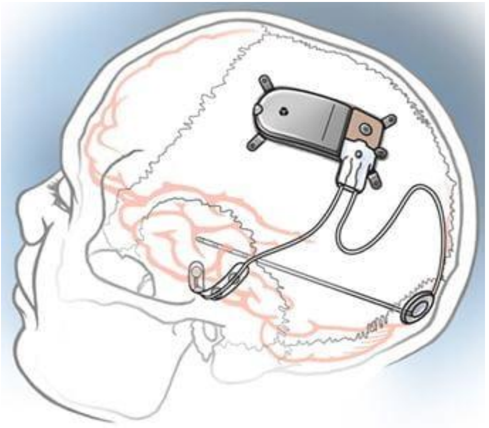


[3] https://www.buzzfeed.com/drumoorhouse/17-things-people-with-epilepsy-want-you-to-know

Epilepsy – Therapy

Drug-resistant patients

Ketogenic diet



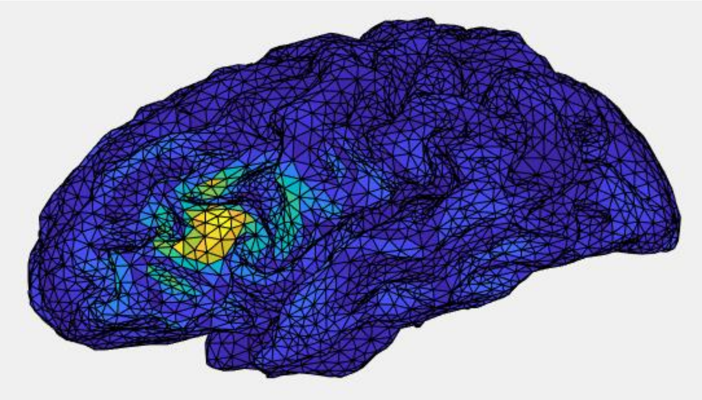
Electric stimulation

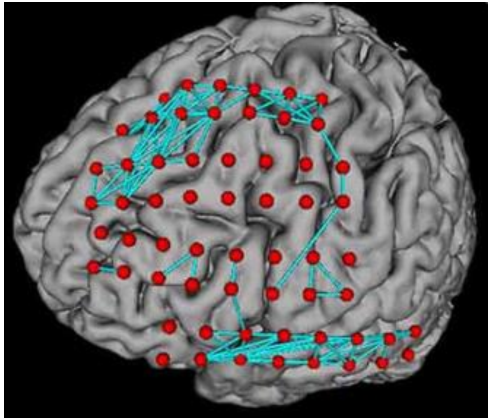
Surgery

[4] https://medicine.iu.edu/news/2015/06/salanova-epilepsy-brain-stimulation-1
[5] https://www.epilepsy.com/treatment/dietary-therapies/ketogenic-diet
[6] https://www.epilepsy.com/treatment/surgery/types

Motivation

Identify and characterize regions responsible for the seizure onset





Regions responsible for seizures

Brain connectivity

[7] https://www.frontiersin.org/articles/10.3389/fninf.2016.00015/full

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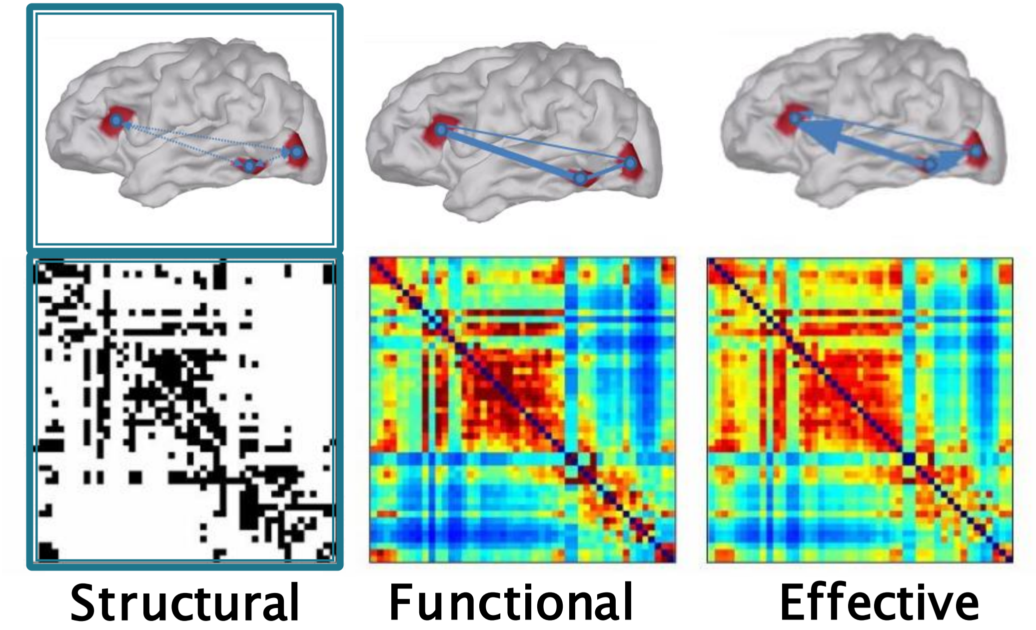
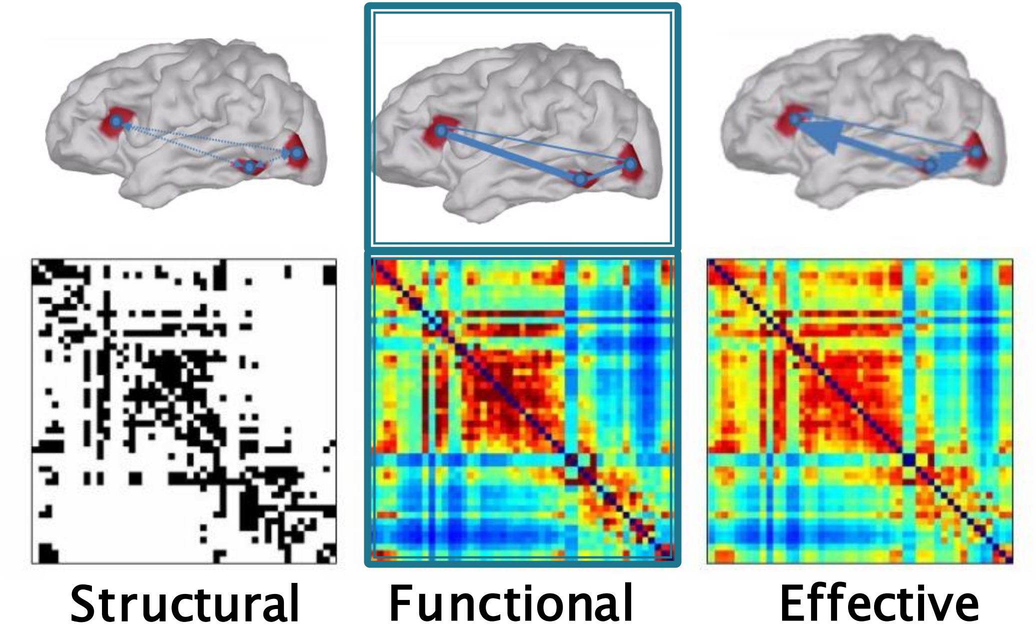
Brain Connectivity (1/3)



- Structural connectivity also called "**anatomical connectivity**"
- A network of **physical** or **structural links** (**synaptic connections**) between pairs of brain regions

[8] *O. Sporns, Brain Connectivity, Scholarpedia, 2(10), 2007.*

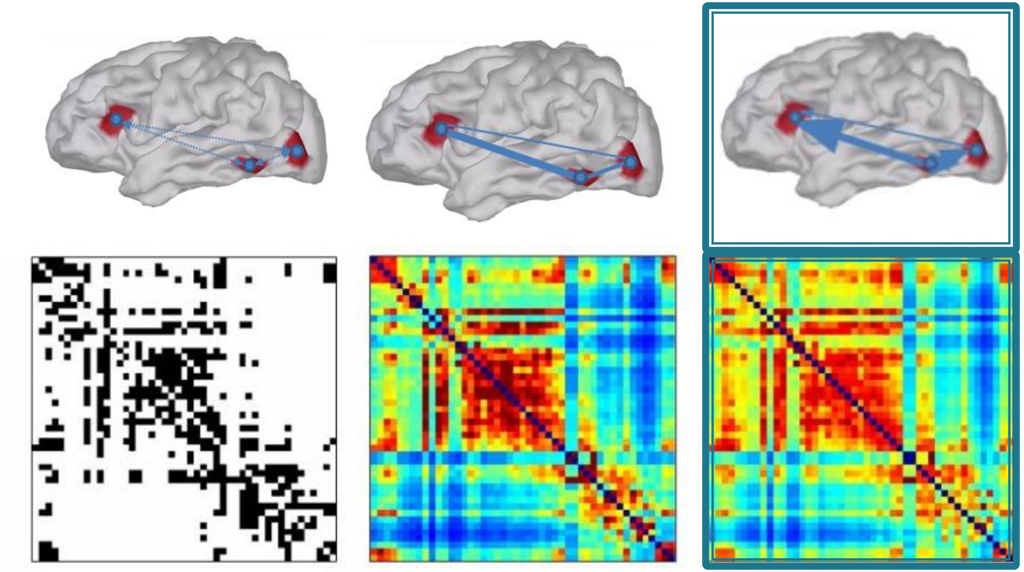
Brain Connectivity (2/3)



- **Statistical dependencies** among **remote** neurophysiological events
- **Temporal correlation** among the activities of neural assemblies

[8] *O. Sporns, Brain Connectivity, Scholarpedia, 2(10), 2007.*

Brain Connectivity (3/3)



Structural Functional Effective

- Completes the notions of structural and functional connectivities
- **Causal influences** between different neurons or neuronal populations

[8] *O. Sporns, Brain Connectivity, Scholarpedia, 2(10), 2007.*

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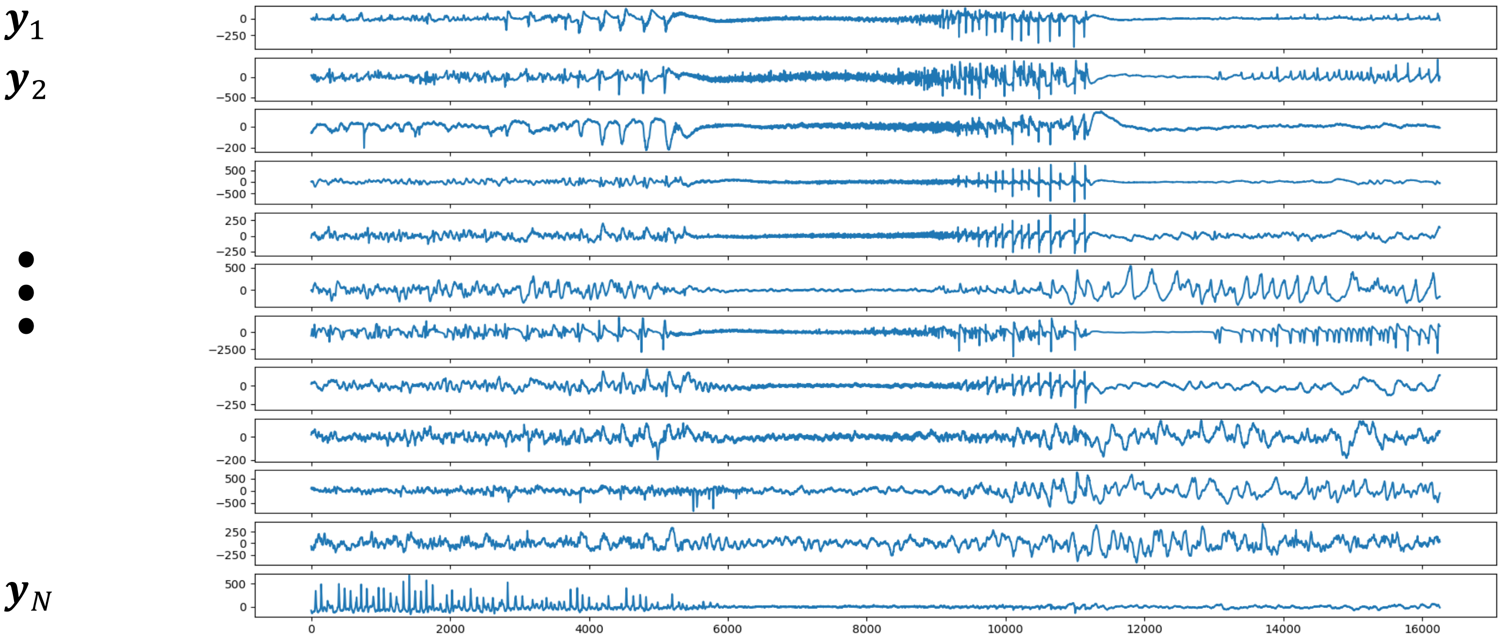
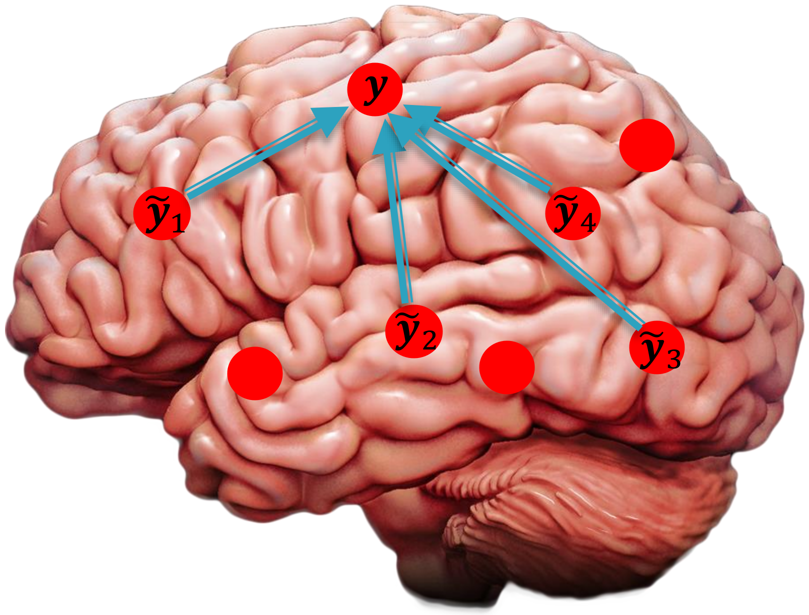
iEEG Channels



N: total number of channels
iEEG: intracranial ElectroEncephaloGraphy

iEEG signals



$$\boldsymbol{y} = \sum_{i=1}^{N_m} \boldsymbol{\alpha}_i \tilde{\boldsymbol{y}}_i + \boldsymbol{w}$$

N_m: number of signals influencing $\boldsymbol{y}$

α_i: i–th decomposition coefficient

$\boldsymbol{\alpha} = [\alpha_1, \alpha_2, \dots, \alpha_{N_m}]$

$\tilde{\boldsymbol{y}}_i$: i–th signal influencing $\boldsymbol{y}$

$\boldsymbol{w}$: model residual related to $\boldsymbol{y}$

rERR-based method: Principle

1. Select candidates from an initial dictionary using the error reduction ratio (ERR)-based method

2. Refine the ERR solution based on the assumption of a sparse representation of the model coefficient vector

[9] S. A. Billings, M. J. Korenberg, and S. Chen, "Identification of nonlinear output-affine systems using an orthogonal least squares algorithm," Tech. Rep. 1-11, Sheffield University, 1987.

Candidates selection (1/3)

Create an initial dictionary $\boldsymbol{D}$

$$\boldsymbol{D} : \quad [\boldsymbol{d}^{(1)},\ \boldsymbol{d}^{(2)}, \dots, \boldsymbol{d}^{(N)}]$$

$\boldsymbol{D} \in \mathbb{R}^{T \times N}$

N: total number of candidates

Candidates selection (1/3)

Create an initial dictionary $\boldsymbol{D}$

$$\boldsymbol{D} : \quad \left[\boldsymbol{d}^{(1)},\ \boldsymbol{d}^{(2)}, \dots, \boldsymbol{d}^{(N)}\right]$$

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N: total number of candidates

$$\begin{aligned} \boldsymbol{y}_m &= \boldsymbol{D}_m \boldsymbol{\alpha}_m + \boldsymbol{w}_m \\ &= \boldsymbol{D}\boldsymbol{\Pi}\boldsymbol{\Pi}^{-1}\boldsymbol{\theta}_{\boldsymbol{m}} + \boldsymbol{w}_m \end{aligned}$$

$\boldsymbol{\Pi}$: selection matrix

Candidates selection (1/3)

Create an initial dictionary $\boldsymbol{D}$ → $\boldsymbol{D}$: $[\boldsymbol{d}^{(1)},\ \boldsymbol{d}^{(2)}, \ldots, \boldsymbol{d}^{(N)}]$

$\boldsymbol{D} \in \mathbb{R}^{T \times N}$

N: total number of candidates

$$\boldsymbol{y}_m = \boldsymbol{D}_m \boldsymbol{\alpha}_m + \boldsymbol{w}_m$$
$$= \boldsymbol{D}\boldsymbol{\Pi}\boldsymbol{\Pi}^{-1}\boldsymbol{\theta}_m + \boldsymbol{w}_m$$

$\boldsymbol{\Pi}$: selection matrix

Decompose the signal $\boldsymbol{y}_m$ → $\boldsymbol{D}$ is decomposed into $\boldsymbol{D} = \boldsymbol{U}\boldsymbol{W}$ where $\boldsymbol{U} \in \mathbb{R}^{T \times N}, \boldsymbol{W} \in \mathbb{R}^{N \times N}$

→ $\boldsymbol{y} = \boldsymbol{U}\boldsymbol{W}\boldsymbol{\theta} + \boldsymbol{w} = \boldsymbol{U}\tilde{\boldsymbol{\theta}} + \boldsymbol{w}$

$\boldsymbol{U}$: orthogonal upper matrix

$\boldsymbol{W}$: upper triangular matrix

$\tilde{\boldsymbol{\theta}}$: coefficient vector

Candidates selection (2/3)

Most relevant column vectors of $\boldsymbol{U}$ encoded in $\widetilde{\boldsymbol{U}}$

$$\widetilde{\boldsymbol{U}}_{k_i} = \boldsymbol{D}^{-(k_i-1)} - \boldsymbol{H}_{k_i}\widetilde{\boldsymbol{U}}_{k_i-1}$$

$\boldsymbol{H}_{k_i}$: diagonal matrix related to the k_i–th selected candidate, obtained by solving the following optimization problem:

$$\boldsymbol{H}^{*}_{k_i} = argmin_{\boldsymbol{H}_{k_i}} \left\| \boldsymbol{D}^{-(k_i-1)} - \widetilde{\boldsymbol{U}}_{k_{i-1}} \boldsymbol{H}_{k_i} \right\|_F^2$$

$$s.t.\ \boldsymbol{H}_{k_i,i,j} = 0, \forall i \neq j$$

where $\boldsymbol{H}_{k_i,i,j}$ is the (i,j)–th entry of $\boldsymbol{H}_{k_i}$

Candidates selection (2/3)

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Related coefficient vector $\widetilde{\boldsymbol{\theta}}$

$$\widetilde{\boldsymbol{\theta}}_{k_i}^{*} = argmin_{\widetilde{\boldsymbol{\theta}}_{k_i}} \left\| \boldsymbol{y} - \widetilde{\boldsymbol{U}}_{k_i}\widetilde{\boldsymbol{\theta}}_{k_i} \right\|_2^2$$

Candidates selection (3/3)

ERR vector $\boldsymbol{e}$

$$\boldsymbol{e}_{k_i} = \boldsymbol{\Lambda}\boldsymbol{\Psi}\widetilde{\boldsymbol{\theta}}_{k_i}^{\odot 2}$$

$$\boldsymbol{\Lambda} = \mathrm{diag}(\left\|u_{k_i}^1\right\|_2^2, \dots, \left\|u_{N-k_i+1}^1\right\|_2^2)$$

$\widetilde{\boldsymbol{\theta}}_{k_i}^{\odot 2} = \widetilde{\boldsymbol{\theta}} \odot \widetilde{\boldsymbol{\theta}}$, $\odot$: Hadamard product

$$\boldsymbol{\Psi} = \frac{1}{\|\boldsymbol{y}\|_2^2}\boldsymbol{I}_{N-k_i+1}$$

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Repeat until a certain threshold is reached

$$1 - \sum_{i=1}^{N_m} \boldsymbol{e}_{max}^{(i)} < \varepsilon$$

where N_m is the number of retained candidates and ε is a predefined threshold

Candidates selection (3/3)

ERR vector $\boldsymbol{e}$

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where N_m is the number of retained candidates and ε is a predefined threshold

New dictionary $\boldsymbol{D}_1$ containing the most relevant candidates of $\boldsymbol{D}$

$$\boldsymbol{D}_1 : [\boldsymbol{d}_1{}^{(1)}, \boldsymbol{d}_1{}^{(2)}, \dots, \boldsymbol{d}_1{}^{(N_m)}]$$

Refined ERR selection (1/2)

Hypothesis: sparse representation of the coefficient vector $\boldsymbol{\theta}$

$$\boldsymbol{\theta}^* = argmin_{\boldsymbol{\theta}} \frac{\lambda}{2} \|\boldsymbol{y} - \boldsymbol{x}\|_2^2 + \|\boldsymbol{z}\|_1$$

$$s.t. \ \boldsymbol{x} = \boldsymbol{D}_1 \boldsymbol{\theta} \ and \ \boldsymbol{z} = \boldsymbol{\theta}$$

$$\boldsymbol{\theta}^* = argmin_{\boldsymbol{\theta}} L(\boldsymbol{x}, \boldsymbol{z}, \boldsymbol{\theta}, \boldsymbol{v}, \boldsymbol{g}, \lambda)$$

$$\boldsymbol{\theta}^* = argmin_{\boldsymbol{\theta}} \frac{\lambda}{2} \|\boldsymbol{y} - \boldsymbol{x}\|_2^2 + \|\boldsymbol{z}\|_1 + \frac{\rho_1}{2} \|\boldsymbol{\theta} - \boldsymbol{z}\|_2^2 + \boldsymbol{v}^T(\boldsymbol{\theta} - \boldsymbol{z}) + \frac{\rho_2}{2} \|\boldsymbol{D}_1 \boldsymbol{\theta} - \boldsymbol{x}\|_2^2 + \boldsymbol{g}^T(\boldsymbol{D}_1 \boldsymbol{\theta} - \boldsymbol{x})$$

L: augmented Lagrangian
$\boldsymbol{x}, \boldsymbol{z}$: dual variables
$\boldsymbol{v}, \boldsymbol{g}$: Lagrangian multipliers
λ, ρ_1, ρ_2: regularization parameters

Refined ERR selection (2/2)

Resolution using the PALM method

Update rules:

$$\boldsymbol{\theta} = (\rho_1 \boldsymbol{I}_N + \rho_2 \boldsymbol{D}_1^T \boldsymbol{D}_1)^{-1} \, (\boldsymbol{v} + \rho_1 \boldsymbol{z} + \boldsymbol{D}_1^T (\rho_2 \boldsymbol{x} - \boldsymbol{g}))$$

$$\boldsymbol{x} = \frac{\lambda \boldsymbol{y} + \boldsymbol{g} + \rho_2 \boldsymbol{D}_1 \boldsymbol{\theta}}{\lambda + \rho_2}$$

$$\Delta \boldsymbol{v} = \rho_1 (\boldsymbol{\theta} - \boldsymbol{z}), \qquad \Delta \boldsymbol{g} = \rho_2 (\boldsymbol{D}_1 \boldsymbol{\theta} - \boldsymbol{x})$$

$$\boldsymbol{z} = prox_{\phi, \lambda c_{\boldsymbol{z}}} (\boldsymbol{z} - \frac{1}{c_{\boldsymbol{z}}} \nabla_{\boldsymbol{z}} L(\boldsymbol{x}, \boldsymbol{z}, \boldsymbol{\theta}, \boldsymbol{v}, \boldsymbol{g}, \lambda))$$

$prox$: shrinkage operator

$\phi = \|.\|_1$

[10] J. Bolte, S. Sabach, and M. Teboulle, “Proximal alternating linearized minimization for nonconvex and nonsmooth problems,” Mathematical Programming, vol. 146, pp. 459–494, 2014.

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Steepest descent rule

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[10] J. Bolte, S. Sabach, and M. Teboulle, "Proximal alternating linearized minimization for nonconvex and nonsmooth problems," Mathematical Programming, vol. 146, pp. 459-494, 2014.

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step-size

$prox$: shrinkage operator

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[10] J. Bolte, S. Sabach, and M. Teboulle, "Proximal alternating linearized minimization for nonconvex and nonsmooth problems," Mathematical Programming, vol. 146, pp. 459-494, 2014.

Parameters optimization (1/2)

Specifying $c_{\boldsymbol{z}}$

$$\boldsymbol{z} = prox_{\phi,\lambda c_{\boldsymbol{z}}}(\boldsymbol{z} - \frac{1}{c_{\boldsymbol{z}}} \nabla_{\boldsymbol{z}} L(\boldsymbol{x}, \boldsymbol{z}, \boldsymbol{\theta}, \boldsymbol{v}, \boldsymbol{g}, \lambda))$$

$c_z \geq \gamma_z L_z$ $(\gamma_z > 1)$, $c_{\boldsymbol{z}} \in \mathbb{R}$

L_z: Lipschitz modulus

A good behavior of PALM is guaranteed when $c_z = \gamma_z L_z$

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L_z is a Lipschitz modulus $\Rightarrow L_{\boldsymbol{z}} \geq \rho_1 \Rightarrow$ $c_z \geq \gamma_z \rho_1$

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L_z is a Lipschitz modulus $\Rightarrow L_{\boldsymbol{z}} \geq \rho_1 \Rightarrow c_z \geq \gamma_z \rho_1$

The equal part: $c_z = \gamma_z \rho_1$ is taken into account in this study

[10] J. Bolte, S. Sabach, and M. Teboulle, "Proximal alternating linearized minimization for nonconvex and nonsmooth problems," Mathematical Programming, vol. 146, pp. 459–494, 2014.

Parameters optimization (2/2)

Optimal computation of λ

Discrepancy principle: λ is laying in the set $\{\boldsymbol{x}: \|\boldsymbol{x}-\boldsymbol{y}\|_2^2 \leq c\}$ $(c \in \mathbb{R})$
where c is a coefficient related to the noise variance

$$\boldsymbol{x} = \frac{\lambda \boldsymbol{y} + \boldsymbol{g} + \rho_2 \boldsymbol{D}_1 \boldsymbol{\theta}}{\lambda + \rho_2} \Rightarrow \lambda = \frac{\|\rho_2(\boldsymbol{y} - \boldsymbol{D}_1\boldsymbol{\theta}) - \boldsymbol{g}\|_2}{\sqrt{c}} - \rho_2$$

- Stopping criterion:
  - $\frac{\|\boldsymbol{\theta}_{it}-\boldsymbol{\theta}_{it-1}\|_F}{\|\boldsymbol{\theta}_{it}\|_F} < predefined\ threshold$
  - Max_{it} is reached

[11] K. El Houari et al., "Investigating transmembrane current source formulation for solving the ecg inverse problem," in 2018 IEEE 10th Sensor Array and Multichannel Signal Processing Workshop (SAM), pp. 371–375, 2018.

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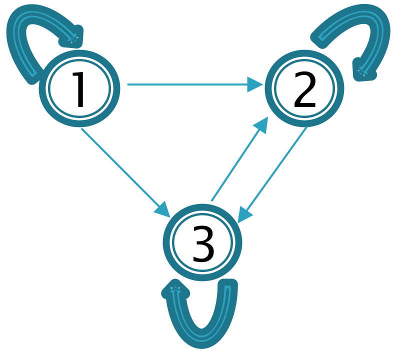
Experimental Data

Simulated data



$$y_1(k) = 3{,}4y_1(k-1)\left(1 - y_1^2(k-1)\right)e^{-y_1^2(k-1)} + w_1(k)$$

$$y_2(k) = 3{,}4y_2(k-1)\left(1 - y_2^2(k-1)\right)e^{-y_2^2(k-1)} - 0{,}5y_1^2(k-1) + 0{,}25\sqrt{2}y_2(k-1) - 0{,}5y_3(k-3) + w_2(k)$$

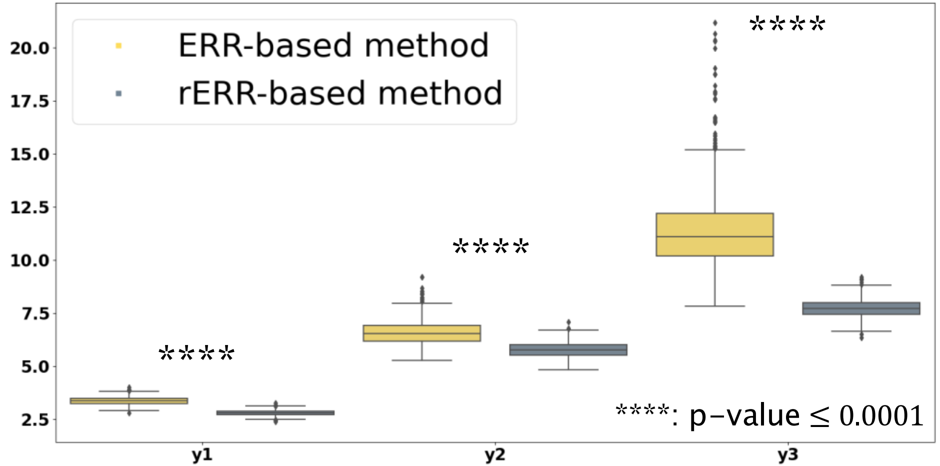
$$y_3(k) = 3{,}4y_3(k-1)\left(1 - y_3^2(k-1)\right)e^{-y_3^2(k-1)} - 0{,}5y_1^2(k-2) - 0{,}5y_2(k-2) - 0{,}25\sqrt{2}y_3(k-2) + w_3(k)$$

where $w_m \sim N(0{,}1), 1 \leq m \leq 3$

Simulated Data – Results

$MSE \pm SD$

	ERR-based method	rERR-based method
y_1	3.39 ± 0.22	2.84 ± 0.18
y_2	6.51 ± 0.72	5.81 ± 0.37
y_3	11.50 ± 2.34	7.70 ± 0.54



$$MSE^{(m)} = \frac{1}{K}\sum_{k=1}^{K}\left\| \boldsymbol{y}^{(m)} - \hat{\boldsymbol{y}}_k{}^{(m)} \right\|_2^2, \forall m \in \{1, \dots, M\}$$

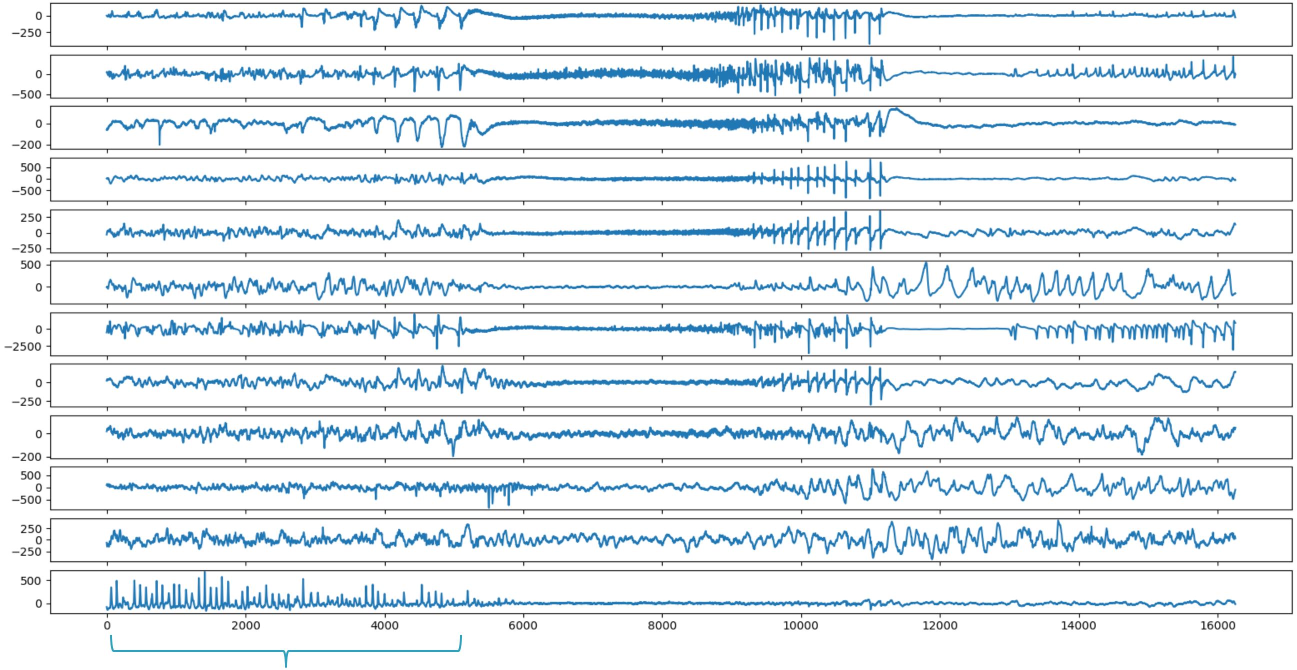
M: total number of channels
$K = 1000$ Monte-Carlo
$\hat{\boldsymbol{y}}_k{}^{(m)}$ is the estimate of $\boldsymbol{y}^{(m)}$

- ✓ Better reconstruction using the rERR-based method
- ✓ Better stability in terms of prediction

Real signals (1/2)

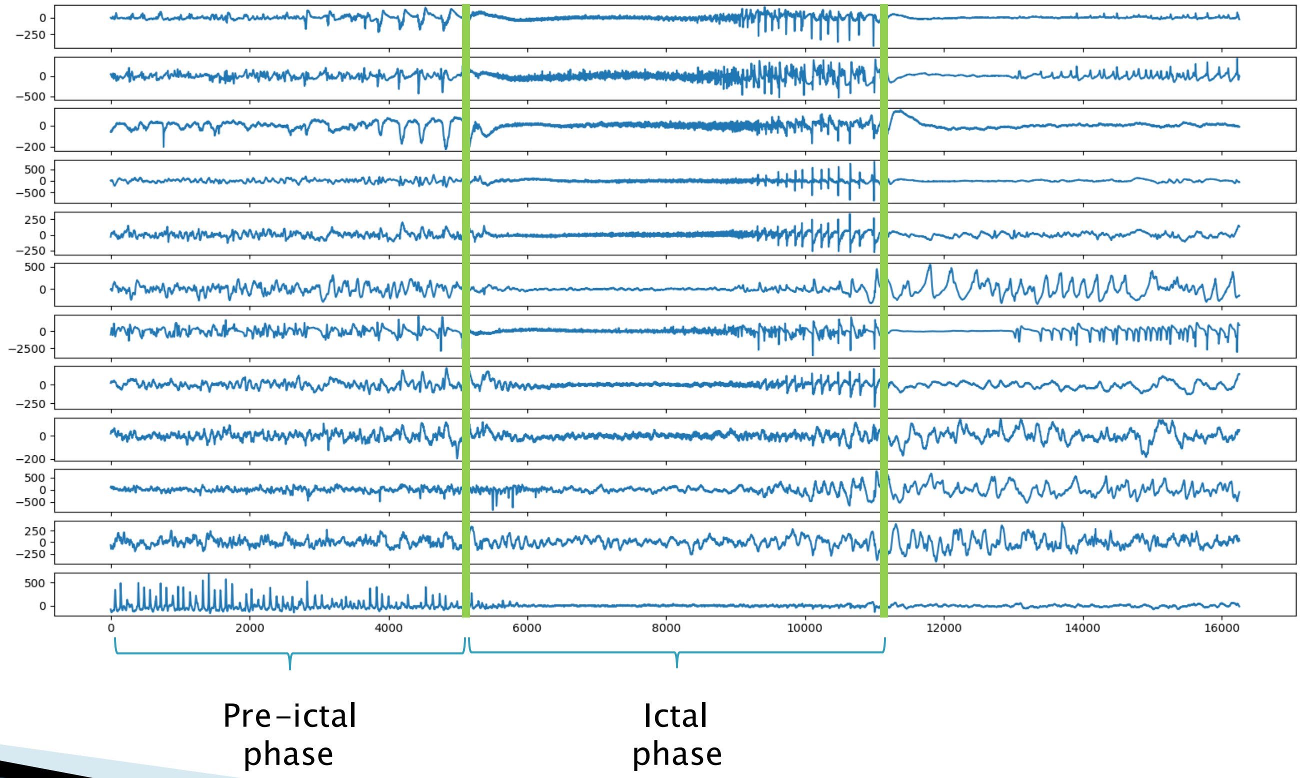
- Female patient, aged 35 and suffering from temporal lobe epilepsy
- 64-second length iEEG signals using invasive electrodes equipped with 128 channels in the cerebral cortex of the patient
- 256-Hz sampling frequency
- 12 bipolar channels selected according to the clinical expert
- Channels are classified into 3 major groups :
    - Onset 'O' group
    - Propagation Internal 'P_I' group
    - Propagation Sink 'P_s' group

Real signals (2/2)



Pre-ictal phase

Real signals (2/2)



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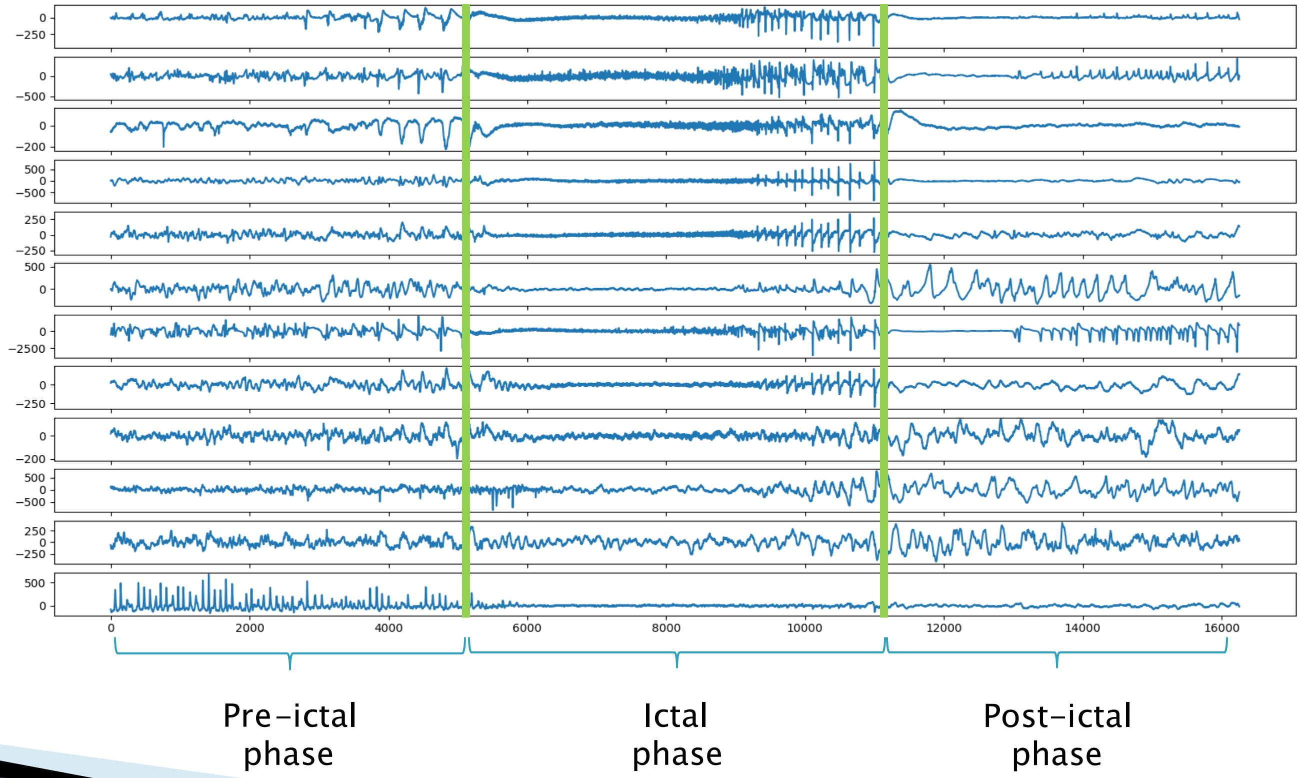
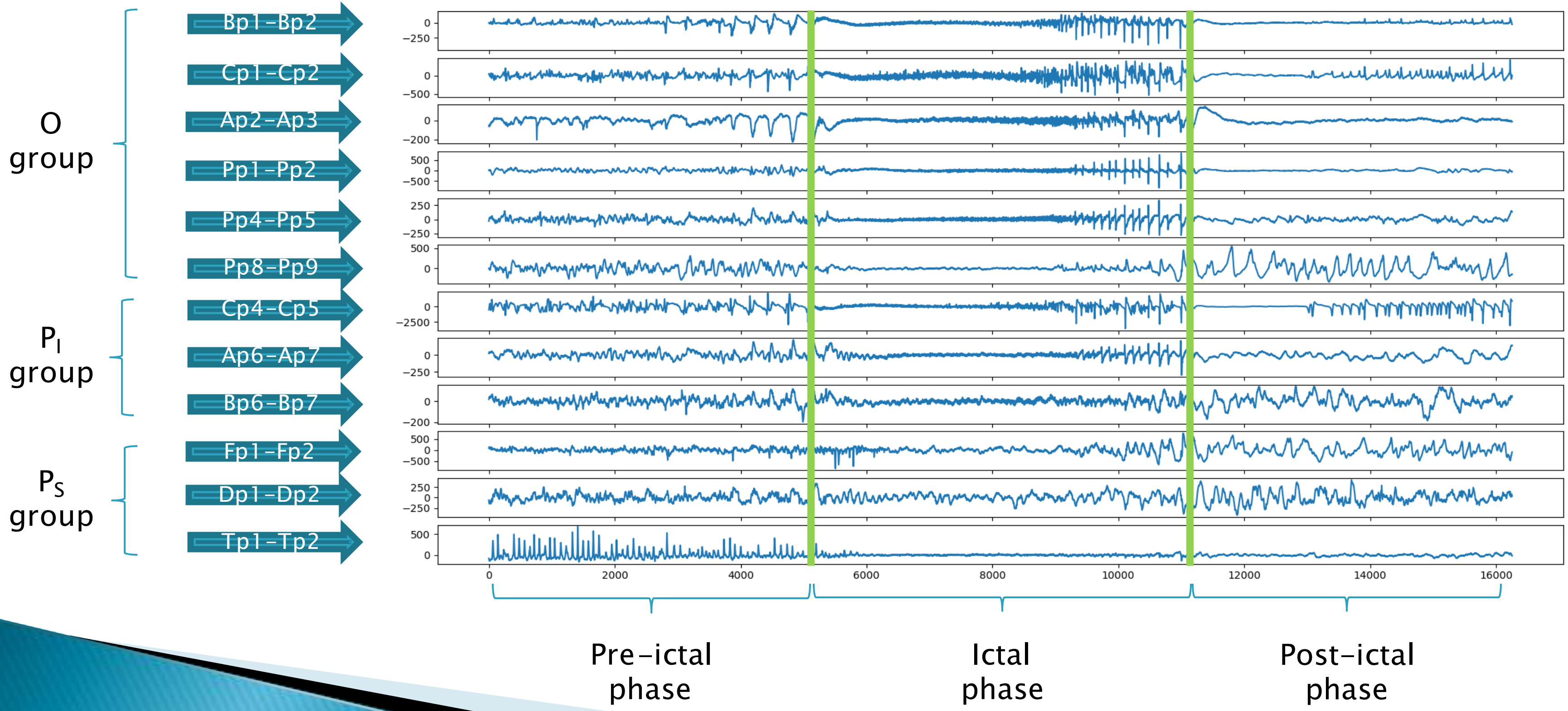
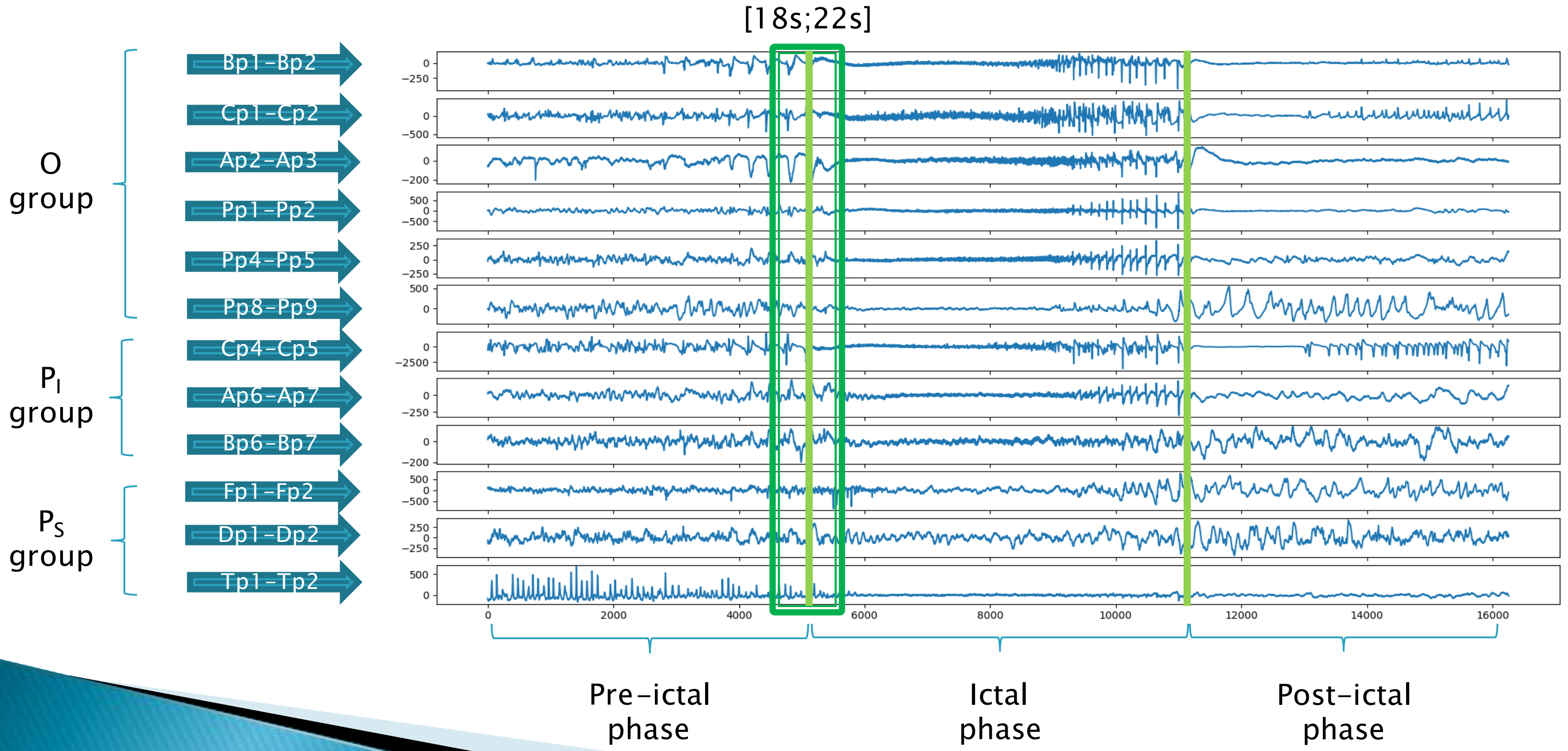


Real signals (2/2)



Real signals (2/2)



Classification rule

Threshold: $\phi_{th} = \frac{1}{4M}\sum_{m=1}^{M}|\phi_m|$ M: total number of channels

For every channel: $\phi_m = \frac{OD_m - ID_m}{OD_m + ID_m}$ OD: Outward Degree
ID: Inward Degree

$$OD_m = \sum_{i=1}^{M}\boldsymbol{\Theta}_{m,i}\,, \qquad ID_m = \sum_{i=1}^{M}\boldsymbol{\Theta}_{i,m} \qquad \boldsymbol{\Theta} = [\boldsymbol{\theta}_1, \ldots, \boldsymbol{\theta}_M] \in \mathbb{R}^{M\times M}$$

Classification rule for $\boldsymbol{y}_m$: $\boldsymbol{y}_m \in \begin{cases} O, & if\, \phi_m \geq \phi_{th} \\ P_I, & if - \phi_{th} \leq \phi_m \leq \phi_{th} \\ P_S, & if\, \phi_m \leq -\phi_{th} \end{cases}$

Expert's classification

Expert	Classification	Expert	Classification
Bp1–Bp2	O	Cp4-Cp5	P_I
Cp1–Cp2	O	Ap6-Ap7	P_I
Ap2–Ap3	O	Bp6-Bp7	P_I
Pp1–Pp2	O	Fp1-Fp2	P_S
Pp4–Pp5	O	Dp1-Dp2	P_S
Pp8–Pp9	O	Tp1-Tp2	P_S

Real signals – Results

Expert	Classification	Expert	Classification
Bp1–Bp2	O	Cp4-Cp5	P_I
Cp1–Cp2	O	Ap6-Ap7	P_I
Ap2–Ap3	O	Bp6-Bp7	P_I
Pp1–Pp2	O	Fp1-Fp2	P_S
Pp4–Pp5	O	Dp1-Dp2	P_S
Pp8–Pp9	O	Tp1-Tp2	P_S

ERR-based method	Classification	ERR-based method	Classification
Bp1–Bp2	P_S	Cp4-Cp5	P_S
Cp1–Cp2	P_I	Ap6-Ap7	O
Ap2–Ap3	O	Bp6-Bp7	O
Pp1–Pp2	P_I	Fp1-Fp2	P_S
Pp4–Pp5	O	Dp1-Dp2	P_S
Pp8–Pp9	P_I	Tp1-Tp2	P_S

rERR-based method	Classification	rERR-based method	Classification
Bp1–Bp2	P_S	Cp4-Cp5	P_S
Cp1–Cp2	O	Ap6-Ap7	O
Ap2–Ap3	O	Bp6-Bp7	O
Pp1–Pp2	O	Fp1-Fp2	P_S
Pp4–Pp5	O	Dp1-Dp2	P_S
Pp8–Pp9	P_S	Tp1-Tp2	P_S

Real signals – Results

Properly classified

Fairly classified

Misclassified

Expert	Classification	Expert	Classification
Bp1–Bp2	O	Cp4-Cp5	P_I
Cp1–Cp2	O	Ap6-Ap7	P_I
Ap2–Ap3	O	Bp6-Bp7	P_I
Pp1–Pp2	O	Fp1-Fp2	P_S
Pp4–Pp5	O	Dp1-Dp2	P_S
Pp8–Pp9	O	Tp1-Tp2	P_S

ERR-based method	Classification	ERR-based method	Classification
Bp1–Bp2	P_S	Cp4-Cp5	P_S
Cp1–Cp2	P_I	Ap6-Ap7	O
Ap2–Ap3	O	Bp6-Bp7	O
Pp1–Pp2	P_I	Fp1-Fp2	P_S
Pp4–Pp5	O	Dp1-Dp2	P_S
Pp8–Pp9	P_I	Tp1-Tp2	P_S

rERR-based method	Classification	rERR-based method	Classification
Bp1–Bp2	P_S	Cp4-Cp5	P_S
Cp1–Cp2	O	Ap6-Ap7	O
Ap2–Ap3	O	Bp6-Bp7	O
Pp1–Pp2	O	Fp1-Fp2	P_S
Pp4–Pp5	O	Dp1-Dp2	P_S
Pp8–Pp9	P_S	Tp1-Tp2	P_S

Real signals – Results

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Fairly classified

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Bp1–Bp2	O	Cp4-Cp5	P_I
Cp1–Cp2	O	Ap6-Ap7	P_I
Ap2–Ap3	O	Bp6-Bp7	P_I
Pp1–Pp2	O	Fp1-Fp2	P_S
Pp4–Pp5	O	Dp1-Dp2	P_S
Pp8–Pp9	O	Tp1-Tp2	P_S

In other seizures, and for the same patient:

- Ap6-Ap7 was classified in the O group
- Pp8–Pp9 was classified in the P_I/ P_S groups

ERR-based method	Classification	ERR-based method	Classification
Bp1–Bp2	P_S	Cp4-Cp5	P_S
Cp1–Cp2	P_I	Ap6-Ap7	O
Ap2–Ap3	O	Bp6-Bp7	O
Pp1–Pp2	P_I	Fp1-Fp2	P_S
Pp4–Pp5	O	Dp1-Dp2	P_S
Pp8–Pp9	P_I	Tp1-Tp2	P_S

rERR-based method	Classification	rERR-based method	Classification
Bp1–Bp2	P_S	Cp4-Cp5	P_S
Cp1–Cp2	O	Ap6-Ap7	O
Ap2–Ap3	O	Bp6-Bp7	O
Pp1–Pp2	O	Fp1-Fp2	P_S
Pp4–Pp5	O	Dp1-Dp2	P_S
Pp8–Pp9	P_S	Tp1-Tp2	P_S

Real signals – Results

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Fairly classified

Misclassified

Expert	Classification	Expert	Classification
Bp1–Bp2	O	Cp4-Cp5	P_I
Cp1–Cp2	O	Ap6-Ap7	P_I
Ap2–Ap3	O	Bp6-Bp7	P_I
Pp1–Pp2	O	Fp1-Fp2	P_S
Pp4–Pp5	O	Dp1-Dp2	P_S
Pp8–Pp9	O	Tp1-Tp2	P_S

ERR-based method	Classification	ERR-based method	Classification
Bp1–Bp2	P_S	Cp4-Cp5	P_S
Cp1–Cp2	P_I	Ap6-Ap7	O
Ap2–Ap3	O	Bp6-Bp7	O
Pp1–Pp2	P_I	Fp1-Fp2	P_S
Pp4–Pp5	O	Dp1-Dp2	P_S
Pp8–Pp9	P_I	Tp1-Tp2	P_S

rERR-based method	Classification	rERR-based method	Classification
Bp1–Bp2	P_S	Cp4-Cp5	P_S
Cp1–Cp2	O	Ap6-Ap7	O
Ap2–Ap3	O	Bp6-Bp7	O
Pp1–Pp2	O	Fp1-Fp2	P_S
Pp4–Pp5	O	Dp1-Dp2	P_S
Pp8–Pp9	P_S	Tp1-Tp2	P_S

Real signals – Results

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Bp1–Bp2	O	Cp4-Cp5	P_I
Cp1–Cp2	O	Ap6-Ap7	P_I
Ap2–Ap3	O	Bp6-Bp7	P_I
Pp1–Pp2	O	Fp1-Fp2	P_S
Pp4–Pp5	O	Dp1-Dp2	P_S
Pp8–Pp9	O	Tp1-Tp2	P_S

ERR-based method	Classification	ERR-based method	Classification
Bp1–Bp2	P_S	Cp4-Cp5	P_S
Cp1–Cp2	P_I	Ap6-Ap7	O
Ap2–Ap3	O	Bp6-Bp7	O
Pp1–Pp2	P_I	Fp1-Fp2	P_S
Pp4–Pp5	O	Dp1-Dp2	P_S
Pp8–Pp9	P_I	Tp1-Tp2	P_S

rERR-based method	Classification	rERR-based method	Classification
Bp1–Bp2	P_S	Cp4-Cp5	P_S
Cp1–Cp2	O	Ap6-Ap7	O
Ap2–Ap3	O	Bp6-Bp7	O
Pp1–Pp2	O	Fp1-Fp2	P_S
Pp4–Pp5	O	Dp1-Dp2	P_S
Pp8–Pp9	P_S	Tp1-Tp2	P_S

Summary

I - Context

II - EEG and connectivity

III - Method

IV - Results

V – Conclusion

Conclusion

- Proposal of a refined ERR-based approach based on a sparsity representation of the coefficient vector for nonlinear system identification in the context of epilepsy

- Performance validated on simulated and real iEEG signals

- Ongoing work: more robust and optimized dictionary-based nonlinear identification system

Thank you