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Robust Representations in Deep Learning

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Robust deep neural networks

We introduce two robust deep neural networks, the robust deep feedforward neural network and the robust long short-term memory neural network.

- Correntropy loss

$$\mathcal{L}(y_i, \hat{y}_i) = \sigma^2 \left(1 - \exp \left(-\frac{(y_i - \hat{y}_i)^2}{\sigma^2} \right) \right)$$

- Robust Deep Forward Neural Network

We implement the robust deep forward neural network (RFNN) by minimizing the mean correntropy loss

$$\min_f \frac{1}{n} \sum_{i=1}^n \mathcal{L}(y_i, \hat{y}_i)$$

Robust deep neural networks

- Robust Long Short-Term Memory Neural Network

Given a sequence of time series x_1, x_2, \dots, x_T and corresponding response series y_1, y_2, \dots, y_T , the robust LSTM will minimize the mean correntropy loss

$$\frac{1}{T} \sum_{i=1}^T \mathcal{L}(y_t, \hat{y}_t)$$

over the historical period to estimate the network parameters.

Two Stage Algorithms

We extract the features and run the linear regression with either least square (LS) approach or the robust regression (RR) with correntropy loss. This leads to four two-stage algorithms:

- FNN+LS
- FNN+RR
- RFNN+LS
- RFNN+RR

where the FNN+LS uses the features extracted from FNN and least square regression to predict the response variable, FNN+RR uses the features extracted from FNN and robust regression, RFNN+LS uses the features extracted from RFNN and least square regression, and RFNN+LS uses the features extracted from RFNN and robust regression.

Applications

We apply our algorithms to real-world applications and illustrate their effectiveness. There are four data sets are used in two stage RFNN and one data set is used in two stage RLSTM.

TABLE I
MAE ON AIRFOIL AND BOSTON HOUSING DATA

Method	Airfoil	Boston Housing	Agroecosystem
FNN	0.2366 (0.0023)	0.2761 (0.0045)	0.1877 (0.0026)
FNN+LS	0.2235 (0.0016)	0.2719 (0.0040)	0.1720 (0.0007)
FNN+RR	0.2223 (0.0016)	0.2702 (0.0040)	0.1719 (0.0007)
RFNN	0.2279 (0.0018)	0.2706 (0.0035)	0.1779 (0.0014)
RFNN+LS	0.2173 (0.0014)	0.2681 (0.0035)	0.1714 (0.0009)
RFNN+RR	0.2161 (0.0014)	0.2669 (0.0035)	0.1713 (0.0008)

TABLE II
MAE ON AIU AND CSI300 DATA

Method	CSI300
LSTM	0.2221 (0.0007)
LSTM+LS	0.2133 (0.0007)
LSTM+RR	0.2016 (0.0006)
RLSTM	0.2197 (0.0008)
RLSTM+LS	0.2116 (0.0007)
RLSTM+RR	0.2012 (0.0007)

Conclusions and future works

Conclusions

- We proposed to implement robust deep neural networks by using the correntropy loss and four two-stage algorithms.
- Simulation studies on four real data applications show that the robust deep neural networks are more efficient to handle data with outliers or skewed.
- The robust deep neural networks are able to efficiently extract more informative features, indicating the entropy loss plays more roles in robust representation of the data.
- The superiority of two-stage algorithms is a serendipity. The simulations surprisingly show that all two-stage algorithms are consistently better than their one-stage counterparts, regardless the loss function used.

Future Works

- We omitted the study of robust Convolutional Neural Network in this paper. But the idea of two-stage training is promising and it would be interesting to develop two-stage CNN algorithms with appropriate classification loss functions, such as the cross entropy loss.

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