A Hypergraph Approach for Logic-based Abduction

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About us

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I am a PhD student at LISN, Université Paris-Saclay. My research interests are extremal graph theory and its applications, including knowledge representation and reasoning.

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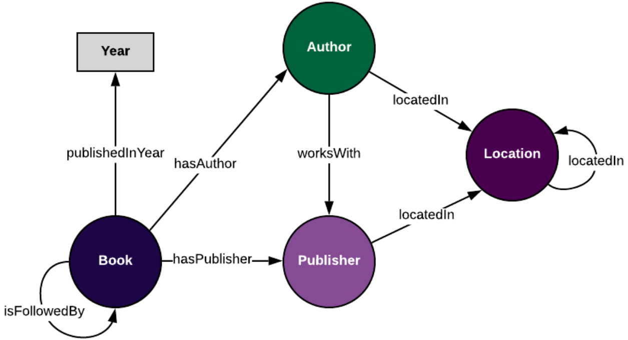
I am an associate professor at Université Paris-Saclay, France. My research interests are situated within the domain of knowledge representation and its applications, concerned three aspects: Knowledge Representation (the logic approach for Artificial Intelligence), Semantic Web (the next generation of the web), and Semantic methods for heterogenous data processing.

What is an ontology?

Ontologies consist of axioms that state the relationship of different concepts and relationships over a specific domain.



A simple $\mathcal{EL}$-ontology

Concept names: $\{A, B, \cdots\}$

Definition

A normalized *$\mathcal{EL}$-ontology* $\mathcal{O}$ is a finite set of axioms of the form:

$$A \sqsubseteq B_1 \sqcap \cdots \sqcap B_n, \qquad B_1 \sqcap \cdots \sqcap B_n \sqsubseteq A.$$

$$peopleWithDiploma \sqsubseteq doctor$$
$$peopleHasPaper \sqsubseteq researcher$$
$$doctor \sqcap employeeWithUniversityChair \sqsubseteq professor$$

Abduction problem

Classical abduction problem consists of three parts:

- a given background knowledge (i.e., an existing ontology $\mathcal{O}$);
- a set of hypotheses (i.e., a set of axioms $\mathcal{H}$);
- a given conclusion (i.e., a single axiom α),

satisfying:

$$\mathcal{O} \not\models \alpha, \mathcal{O} \cup \mathcal{H} \models \alpha.$$

We also take into account a user's interests represented by a set of concepts Σ.

Our abduction problem

Abduction problem

An abduction problem is a tuple $\langle \mathcal{O},\ \Sigma,\ A_1 \sqcap \cdots \sqcap A_n \sqsubseteq B \rangle$, where $\Sigma = \{A', B', \cdots\}$ is a set of concept names. A solution of this problem is a minimal ontology

$$\mathcal{H} = \{A'_1 \sqcap \cdots \sqcap A'_n \sqsubseteq B' \mid A'_i, B' \in \Sigma, n \geq 0\}$$

such that $\mathcal{O} \cup \mathcal{H} \models A_1 \sqcap \cdots \sqcap A_n \sqsubseteq B$. A solution $\mathcal{H}$ is called a hypothesis with respect to Σ.

Our abduction problem

Example

$$peopleWithDiploma \sqsubseteq doctor$$
$$peopleHasPaper \sqsubseteq researcher$$
$$doctor \sqcap employeeWithUniversityChair \sqsubseteq professor$$

- $\mathcal{O}_0$ can not derive α_0:

$$doctor \sqcap employeeWithUniversityChair \sqsubseteq researcher.$$

- Consider $\Sigma_0 = \{professor,\ peopleHasPaper\}$ and a hypothesis $\mathcal{H}_0 = \{professor \sqsubseteq peopleHasPaper\}$.
- Then, $\mathcal{O}_0 \cup \mathcal{H}_0 \models \alpha_0$.
- Therefore, $\mathcal{H}_0$ is a solution of the abduction problem $\mathcal{A}_0 = \langle \mathcal{O}_0,\ \Sigma_0,\ \alpha_0 \rangle$.

Related work

- Complexity of abduction over $\mathcal{EL}$ and their application to repairing ontologies;
- Abduction over $\mathcal{EL}$ by translation to first-order logic;
- Forgetting-based or signature-based abduction over expressive ontologies.

In this presentation, we propose a new solution of abduction over a special $\mathcal{EL}$-ontology (free of role restrictions), based on a hypergraph representation of ontologies.

Hypergraph

Hypergraph

A (directed) hypergraph $\mathcal{H} = \{\mathcal{V}, \mathcal{E}\}$ consists of a node set $\mathcal{V} = \{v_1, v_2, \cdots, v_n\}$ and a hyperedge set $\mathcal{E} = \{e_1, e_2 \cdots, e_m\}$, where $e_i = \langle T(e_i), f(e_i)\rangle$ with $T(e_i) \subseteq \mathcal{V}$ being a subset and $f(e_i) \in \mathcal{V}$ being a node.

Definition

Given an ontology $\mathcal{O}$, we define a hypergraph $H_{\mathcal{O}} = (\mathcal{N}_h, \mathcal{E}_h)$, where $\mathcal{N}_h := \{N_A \mid A \in \mathsf{N_C}\}$ and

$$\mathcal{E}_h := \{\{N_{A_1}, \cdots, N_{A_n}\} \to N_A \mid A_1 \sqcap \cdots \sqcap A_n \sqsubseteq A \in \mathcal{O}\}$$

N_1: peopleWithDiploma

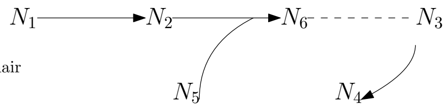
N_2: doctor

N_3: peopleHasPaper

N_4: researcher

N_5: employeeWithUniversityChair

N_6: professor



Hyperpath

Hyperpath

Given a hypergraph $\mathcal{H} = \{\mathcal{V}, \mathcal{E}\}$, assume $S \subseteq \mathcal{V}$ and $v \in \mathcal{V}$. A hyperpath from S to v is a sequence $h = [e_1, e_2, \cdots, e_n]$ of hyperedges such that

- $f(e_n) = \{v\}$;
- for $i = 1, \cdots, n$, $T(e_i) \subseteq S \cup \{f(e_1), \cdots, f(e_{i-1})\}$;
- for $i = 1, \cdots, n$, $f(e_i) \in \bigcup_{i<j\leq n} T(e_j)$.

N_1: peopleWithDiploma

N_2: doctor

N_3: peopleHasPaper

N_4: researcher

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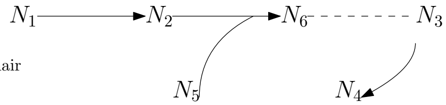

- A hyperpath h from $\{N_2,\ N_5\}$ to N_4:
  $h = [\{N_2, N_5\} \to \{N_6\},\ \{N_6\} \to N_3,\ \{N_3\} \to N_4]$

Our result

Theorem

Given an ontology $\mathcal{O}$ and its associated hypergraph $H_{\mathcal{O}}$, an ontology $\mathcal{H}$ is a (minimal) solution to the abduction problem $\langle \mathcal{O}, \Sigma, A_1 \sqcap \cdots \sqcap A_n \sqsubseteq B \rangle$ iff $H_{\mathcal{H}}$ is a (minimal) hypergraph s.t.

- There is a hyperpath from $N_{A_1}, \cdots, N_{A_n}$ to N_B in $H_{\mathcal{O}} \cup H_{\mathcal{H}}$;
- And all nodes in $H_{\mathcal{H}}$ are of the form N_A, $A \in \Sigma$.

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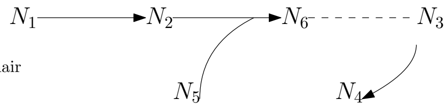
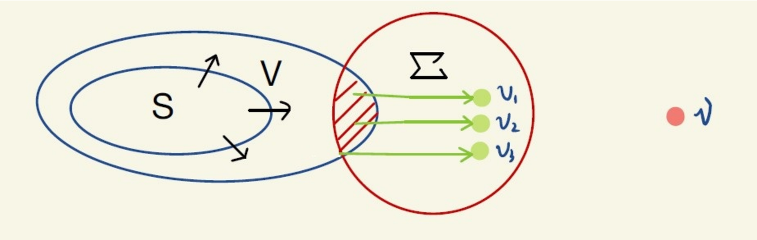
N_3: peopleHasPaper
N_4: researcher
N_5: employeeWithUniversityChair
N_6: professor

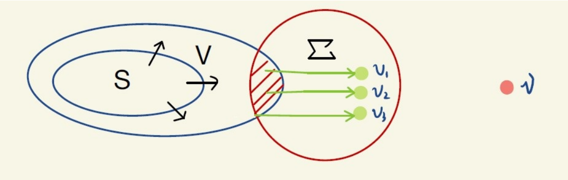


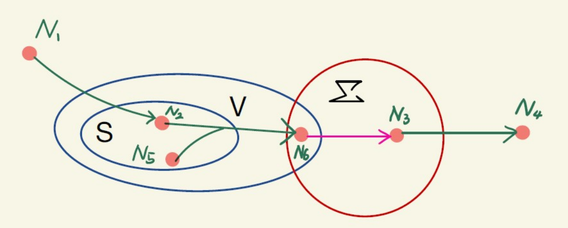
To find an $H_{\mathcal{H}}$ such that there exists a hyperpath from $\{N_2, N_5\}$ to N_4 in $H_{\mathcal{O}_0} \cup H_{\mathcal{H}}$. The hypergraph $H_{\mathcal{H}}$ consists of a single edge $\{N_6\} \to N_3$ satisfying the requirement.

GOAL: To find a hyperpath from S to v in $H_{\mathcal{O}_0} \cup H_{\mathcal{H}}$.



- $\Sigma \setminus V = \{v_1, v_2, v_3\}$
- Choose any 3 hyper-edges $\mathcal{E}' = \{e_1, e_2, e_3\}$ where $T(e_i) \subset \Sigma \cap V$ and $f(e_i) = v_i$ for $i = 1, 2, 3$.
- if $v \notin Span(\mathcal{V}, \mathcal{E} \cup \mathcal{E}', V \cup \Sigma)$, then non-existence.
- Minimize $\mathcal{E}'$ (check if there exists $e \in \mathcal{E}'$ such that $\mathcal{E}' - e$ satisfies until we get a minimal size).


S
V
Σ
v_1
v_2
v_3
v


N_1
S
V
N_2
N_5
N_6
Σ
N_3
N_4

Thanks for your attention!