Near-Optimal Coordination of Vehicles at an Intersection Plaza Using Bézier Curves

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- **Elham Ahmadi** is a Ph.D. student in the Postgraduate Program in Automation and Systems Engineering at the Federal University of Santa Catarina, Brazil. She received her master’s and bachelor’s degree from Shiraz University of Technology, Iran, in 2017 and 2015, respectively.

- Her main areas of expertise and research are control of positive systems, distributed convex optimization and its application to transportation systems, Model Predictive Control (MPC), modeling and control of urban traffic networks and intelligent transportation systems.

Main objectives

- Propose a new model for vehicle coordination at intersections
  - Break long-lasting paradigm of road use to maximize the traffic efficiency
  - Better use of the available intersection infrastructure
- Generate optimal trajectories without predefined paths to minimize some criteria while strictly avoiding vehicle collision
- Compare the obtained results with an approach based on Finite Fourier Series (FFS)

Connected Vehicles under Automated Driving (CVAD)

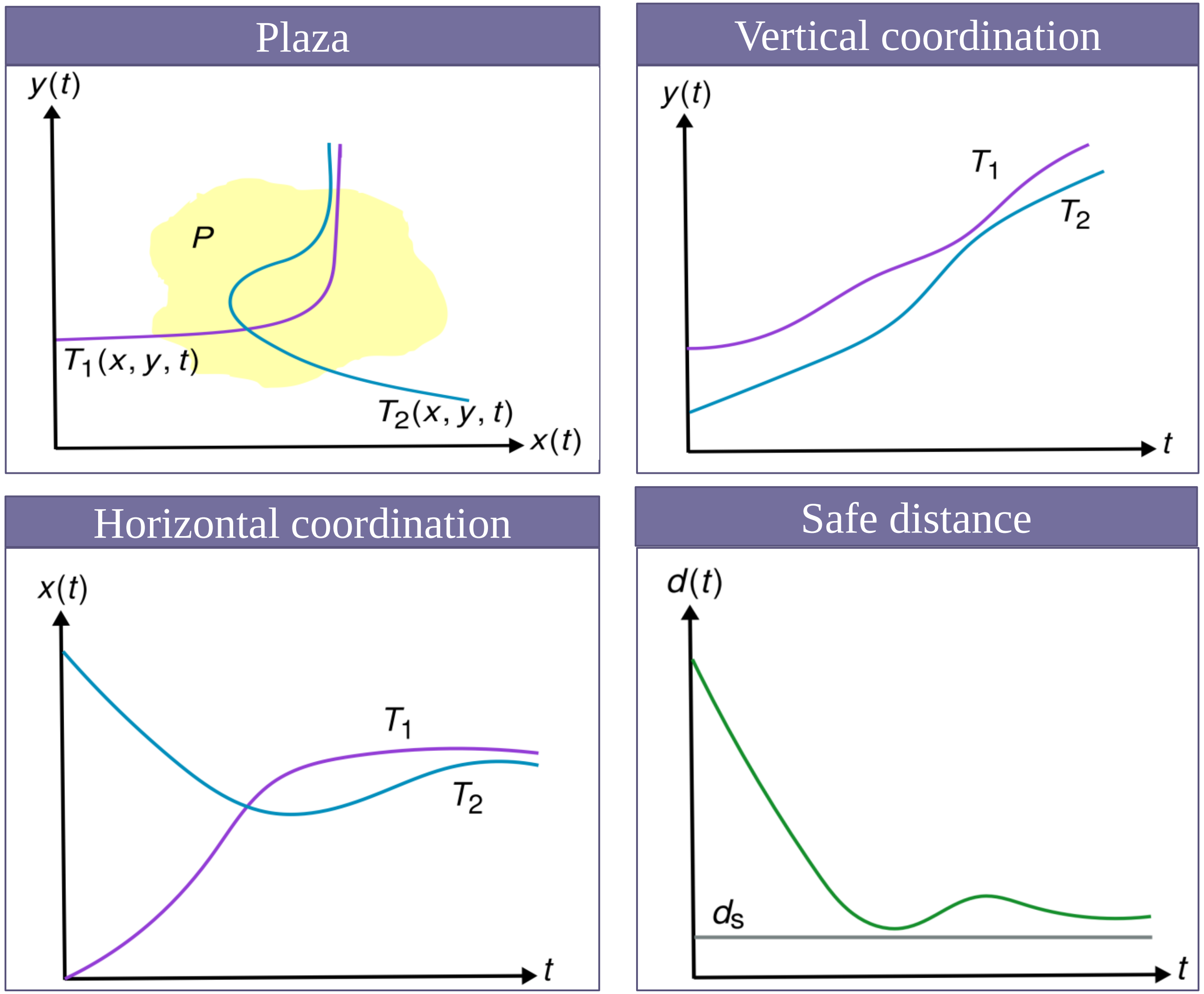
- **Automated Driving:**
  - An automated system instead of human driver
  - Six levels of automation, e.g. ACC to full automation

- **Connected Vehicles:**
  - Capable of sharing information
  - Vehicle to vehicle (V2V)
  - Vehicle to infrastructure (V2I)

Waymo: Google's fully automated driving vehicle

Intersection plaza

- A space with free-movement traffic
  - No horizontal road markings
  - Signal-free
  - Various layouts
  - More driving flexibility
  - Maximum infrastructure utilization



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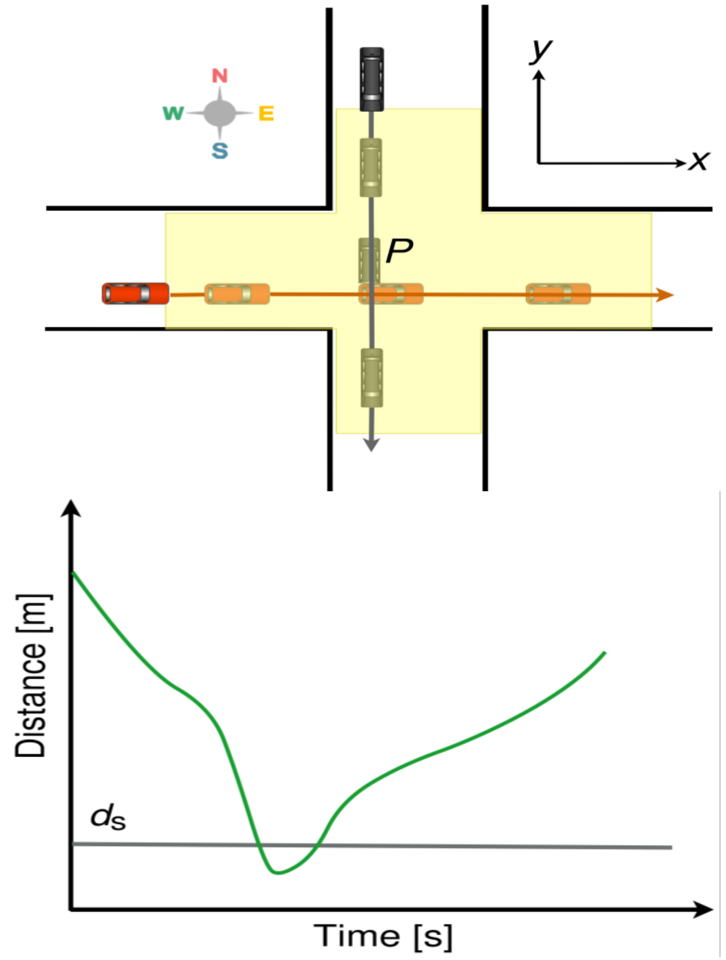
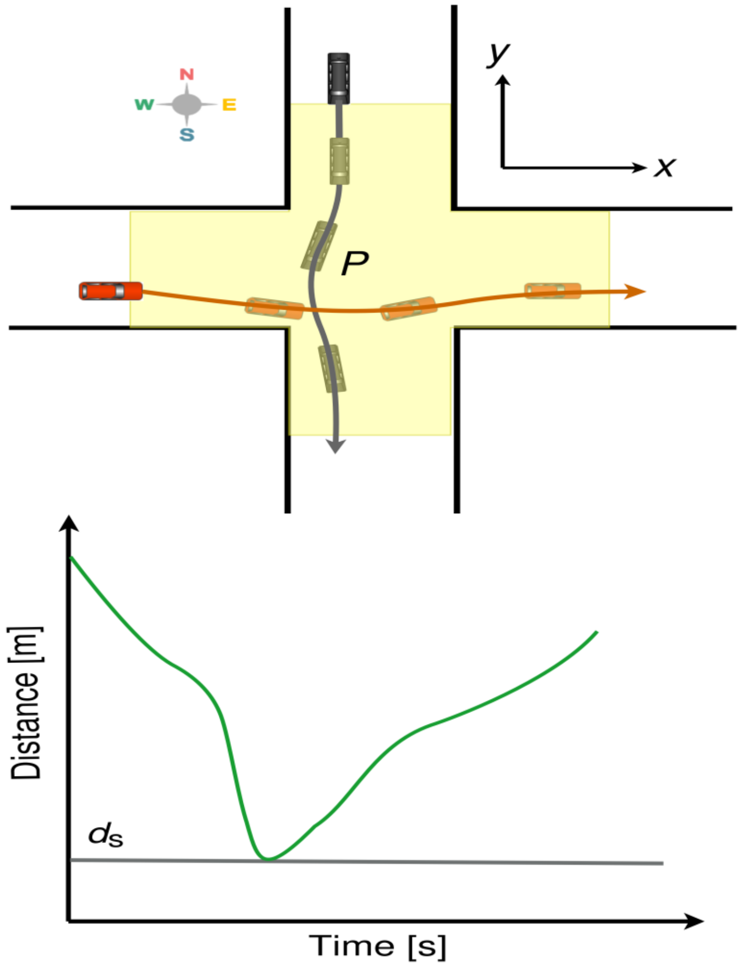
Intersection plaza

Collision scenario



Collision-free scenario



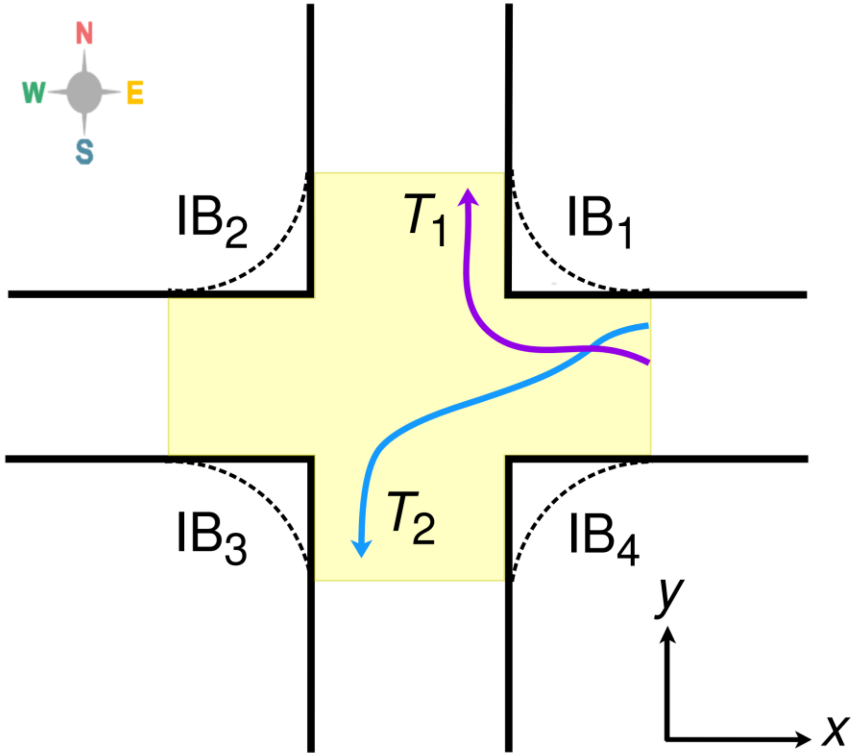
Plaza modeling

- A plaza is modeled by four Intersection Boundaries (IB)

- Each $\mathrm{IB}_h, h = 1, \cdots, 4$ is modeled by an exponential function:

$$y_h = f_h(x(t))$$

$$f_h\big(x(t)\big) = r_{0,h} + r_{1,h} \cdot e^{r_{2,h} \cdot (x(t) + r_{3,h})}$$



Vehicle's equation of motion

$$\begin{cases} \ddot{x}_j(t) = a_{x_j}(t) \\ \ddot{y}_j(t) = a_{y_j}(t) \end{cases} \xrightarrow{j = 1, \cdots, k}$$

- Total absolute acceleration:

$$a_j(t) = \sqrt{a_{x_j}^2(t) + a_{y_j}^2(t)}$$

- Total speed increment:

$$\Delta v(t) = \sum_{j=1}^{k} \Delta v_j(t)$$

$$\Delta v_j(t) = \int_0^T a_j(t)\, d(t)$$

Problem constraints

- Vehicle's physical constraints

$$a_j(t) \leq a_{\max}$$

$$0 \leq v_j(t) \leq v_{\max}$$

- V2V collision avoidance

$$d_{i,j}(t) \geq d_s$$

$$i = 1, \cdots, k \quad j = 1, \cdots, k \quad i \neq j$$

- Plaza boundaries

$$\begin{cases} y_j(t) \leq f_h\left(x_j(t)\right), \text{if } h = 1, 2 \\ y_j(t) \geq f_h\left(x_j(t)\right), \text{if } h = 3, 4 \end{cases}$$

Intersection Trajectory Optimal Control Problem (ITOP)

minimize $\mathcal{J} = w_1 \cdot \Delta v + w_2 \cdot T$

s.t.

- Vehicle's equations of motion
- Vehicle's physical constraints
- V2V collision avoidance constraints
- Plaza boundaries constraints

- **States:**
  - Position
  - Speed
- **Control variables:**
  - Completion time (T)
  - Acceleration

Bézier curves approximation

- Approximate the position state variable (x_i and y_i) :

Number of Bézier terms

Bézier coefficients

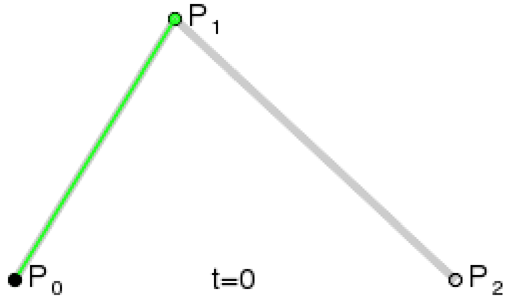
$$\boldsymbol{z}(\tau) = \sum_{l=0}^{n_{\boldsymbol{z}}} B_{\boldsymbol{z},l}(\tau)\, P_{\boldsymbol{z},l}$$

Scaled time:

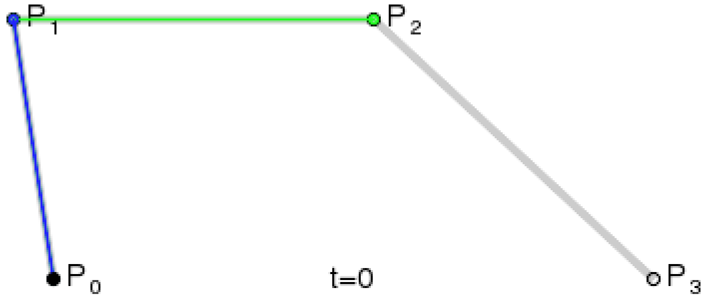
$$\tau = \frac{t}{T}, \quad 0 \leq \tau \leq 1$$

Bernstein basis polynomials:

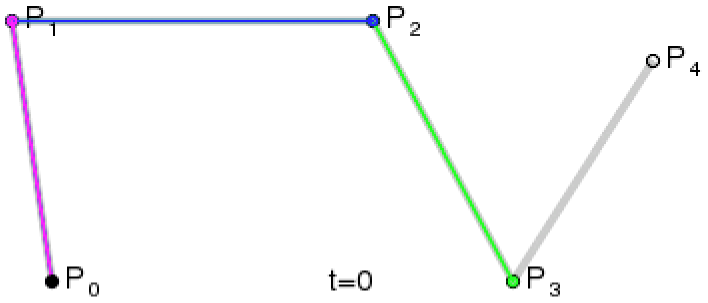
$$B_{\boldsymbol{z},\boldsymbol{l}}(\tau) = \binom{n_z}{l} \tau^l (1 - \tau)^{n_z - 1}, \; l \in \{0,1,\ldots,n_{\boldsymbol{z}}\}$$



Quadratic Bézier curve (Wikipedia)



Cubic Bézier curve (Wikipedia)



Quartic Bézier curve (Wikipedia)

Boundary conditions (BCs)

$$\boldsymbol{z}(\tau = 0) = \boldsymbol{z}_\mathrm{I} \qquad \boldsymbol{z}(\tau = 1) = \boldsymbol{z}_\mathrm{f}$$

$$\boldsymbol{z}'(\tau = 0) = T\dot{\boldsymbol{z}}_\mathrm{I} \qquad \boldsymbol{z}'(\tau = 1) = T\dot{\boldsymbol{z}}_\mathrm{f}$$

Discretization points (DPs)

$$\tau_1 = 0 < \tau_2 < \cdots < \tau_{m-1} < \tau_m = 1$$

Using BCs to express four coefficients

$$P_{\boldsymbol{z},0} = \boldsymbol{z}_I$$

$$P_{\boldsymbol{z},1} = \boldsymbol{z}_I + \frac{T\dot{\boldsymbol{z}}_I}{n_{\boldsymbol{z}}}$$

$$P_{\boldsymbol{z},\boldsymbol{n_z}-\boldsymbol{1}} = \boldsymbol{z}_f - \frac{T\dot{\boldsymbol{z}}_f}{n_{\boldsymbol{z}}}$$

$$P_{\boldsymbol{z},n_{\boldsymbol{z}}} = \boldsymbol{z}_f$$

Compact matrix form

$$[\boldsymbol{z}]_{m\times 1} = [B_{\boldsymbol{z}}]_{m\times(n_{\boldsymbol{z}}-3)}\boxed{[X_{\boldsymbol{z}}]}_{(n_{\boldsymbol{z}}-3)\times 1} + [F_{\boldsymbol{z}}]_{m\times 1}$$

$$[\boldsymbol{z}']_{m\times 1} = [B_{\boldsymbol{z}'}]_{m\times(n_{\boldsymbol{z}}-3)}\boxed{[X_{\boldsymbol{z}}]}_{(n_{\boldsymbol{z}}-3)\times 1} + [F'_{\boldsymbol{z}}]_{m\times 1}$$

$$[\boldsymbol{z}'']_{m\times 1} = [B_{\boldsymbol{z}''}]_{m\times(n_{\boldsymbol{z}}-3)}\boxed{[X_{\boldsymbol{z}}]}_{(n_{\boldsymbol{z}}-3)\times 1} + [F''_{\boldsymbol{z}}]_{m\times 1}$$

Coefficient matrices

Unknown Bézier coefficients

Vectors obtained from BCs

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NonLinear Programming (NLP) formulation

minimize $\mathcal{J} = w_1 \cdot \Delta v + w_2 \cdot T$

s.t.

$$\left[a_j(t)\right] \leq [u_{\text{max}}]$$

$$0 \leq \left[v_j(t)\right] \leq [v_{\text{max}}]$$

$$\left[d_{i,j}(t)\right] \geq [d_{\text{s}}]$$

$$[y_j(t)] \leq [f_h\left(x_j(t)\right)], \text{if } h = 1, 2$$

$$[y_j(t)] \geq [f_h\left(x_j(t)\right)], \text{if } h = 3,4$$

- Decision variables:
  - Completion time (T)
  - Unknown Bézier coefficients ($[X_{\boldsymbol{z}}]$)

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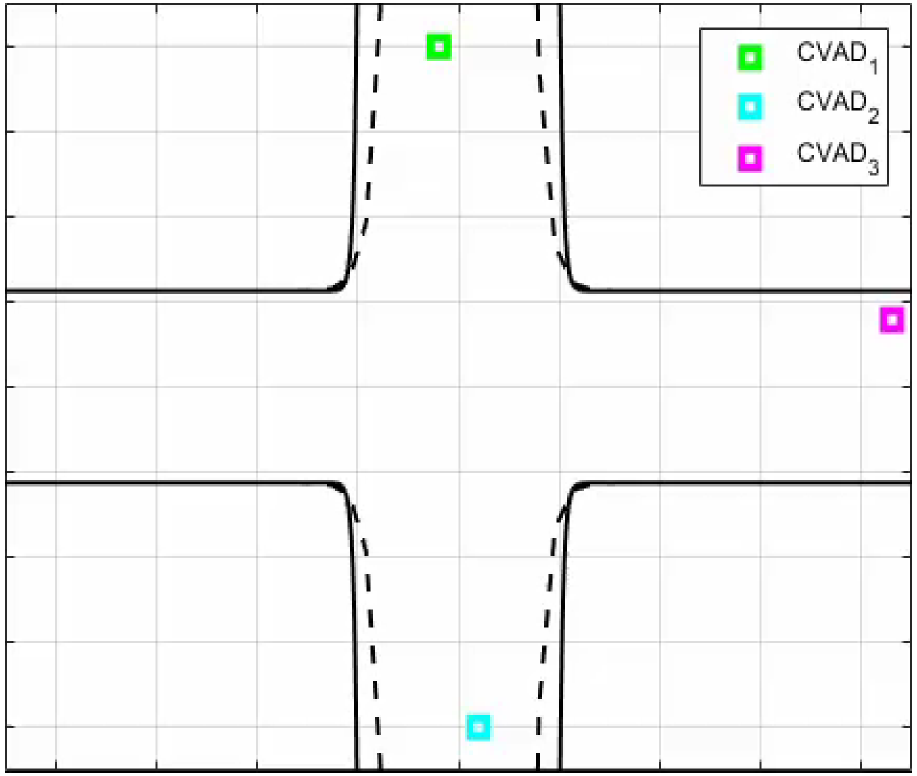
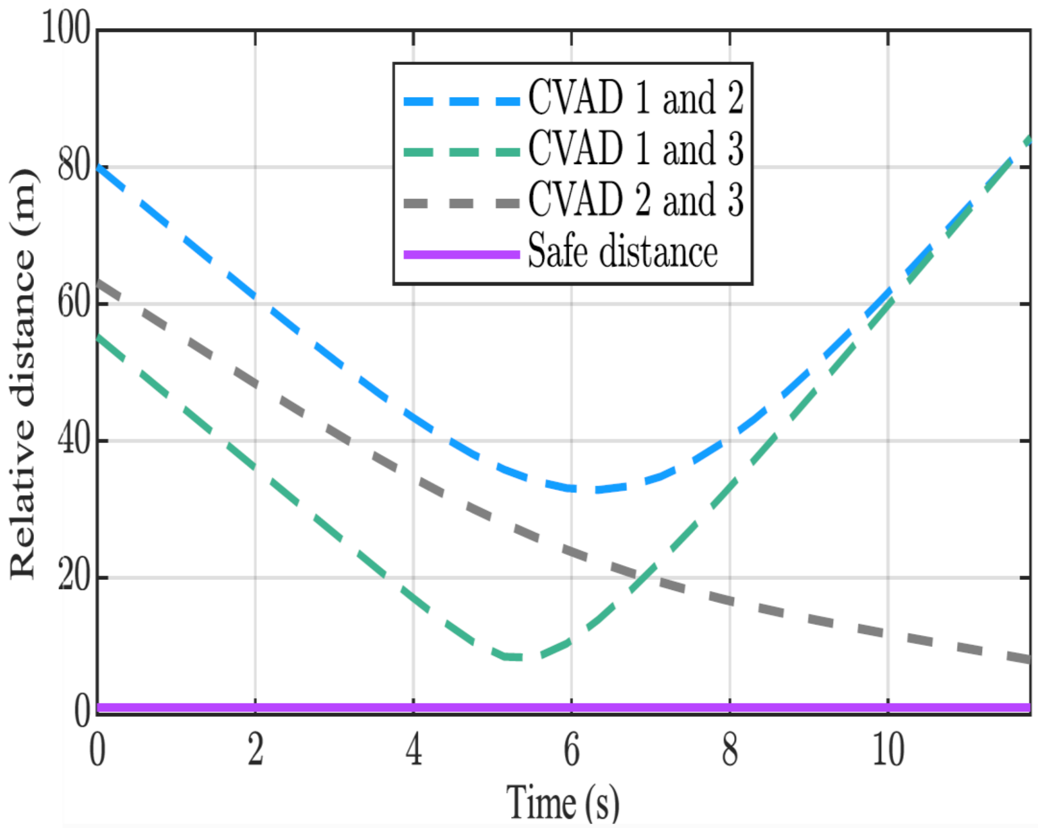
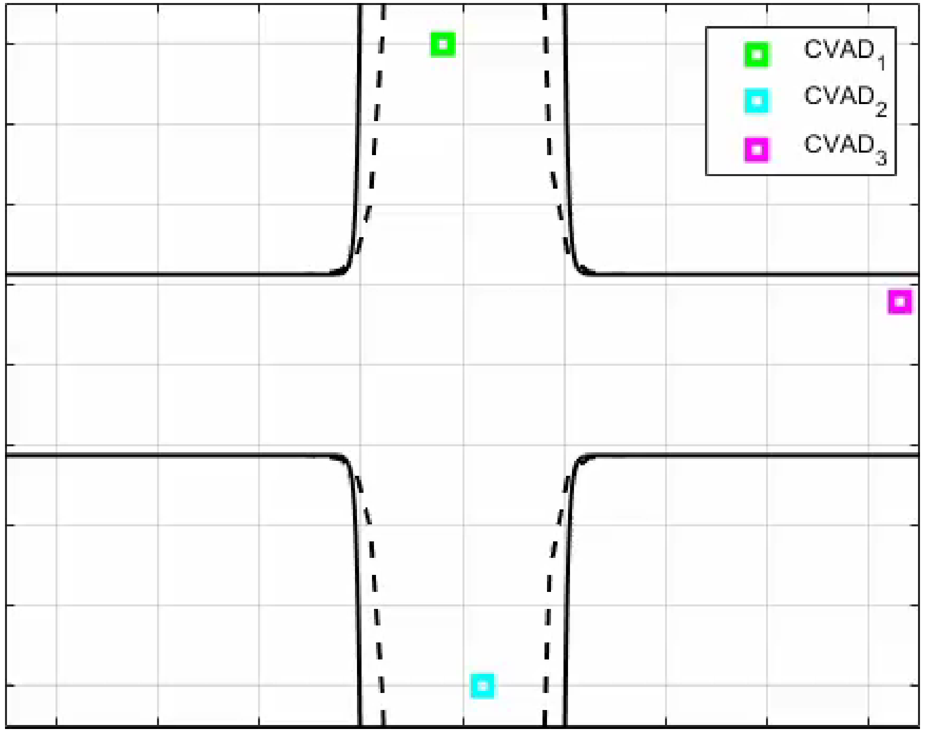
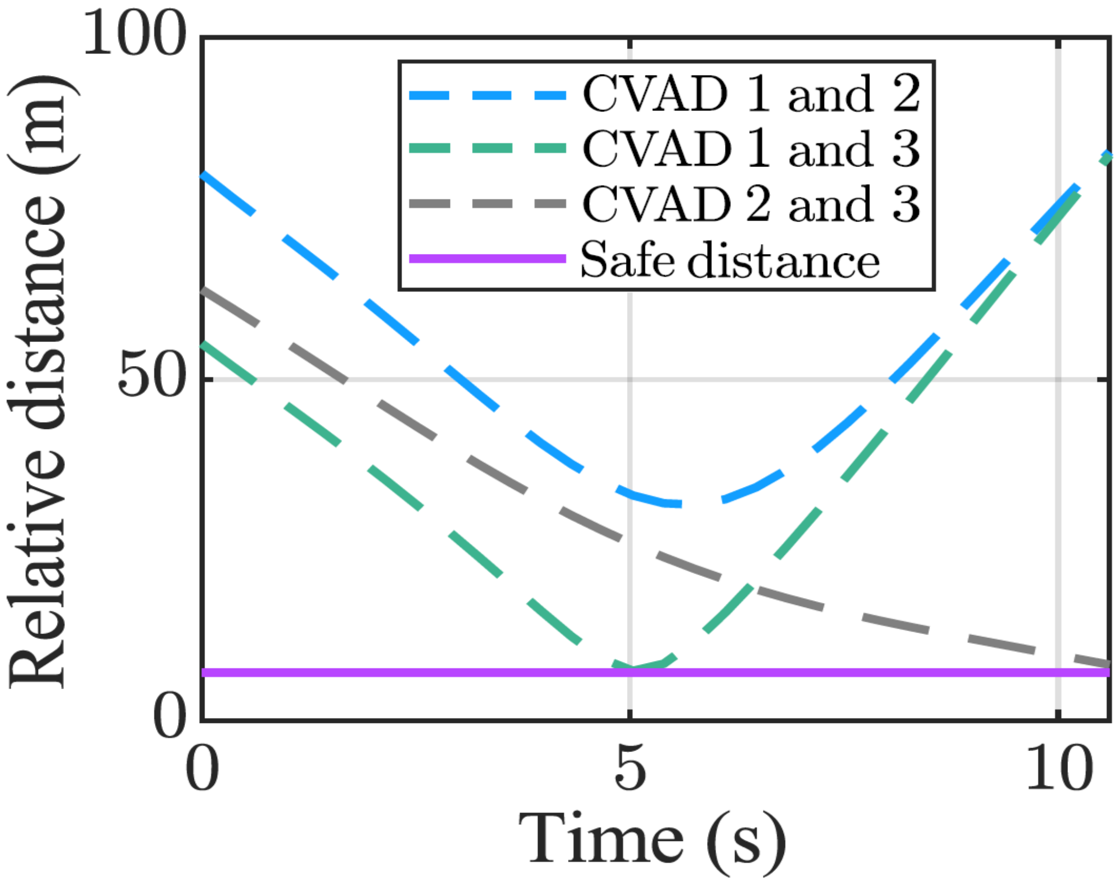
Scenario I: 3 CVAD and $d_s = 1$



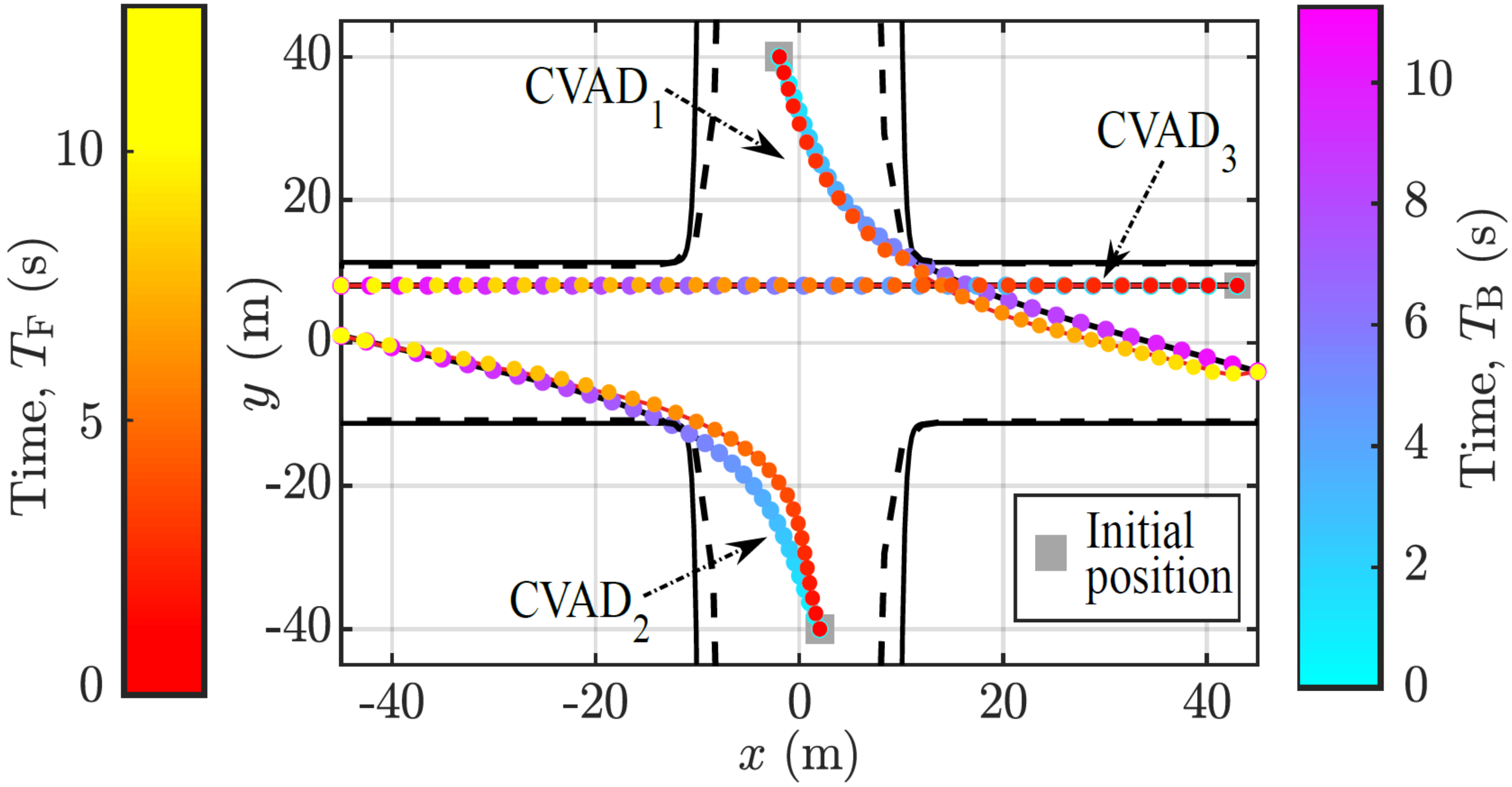


Scenario II: 3 CVAD and $d_s = 7$

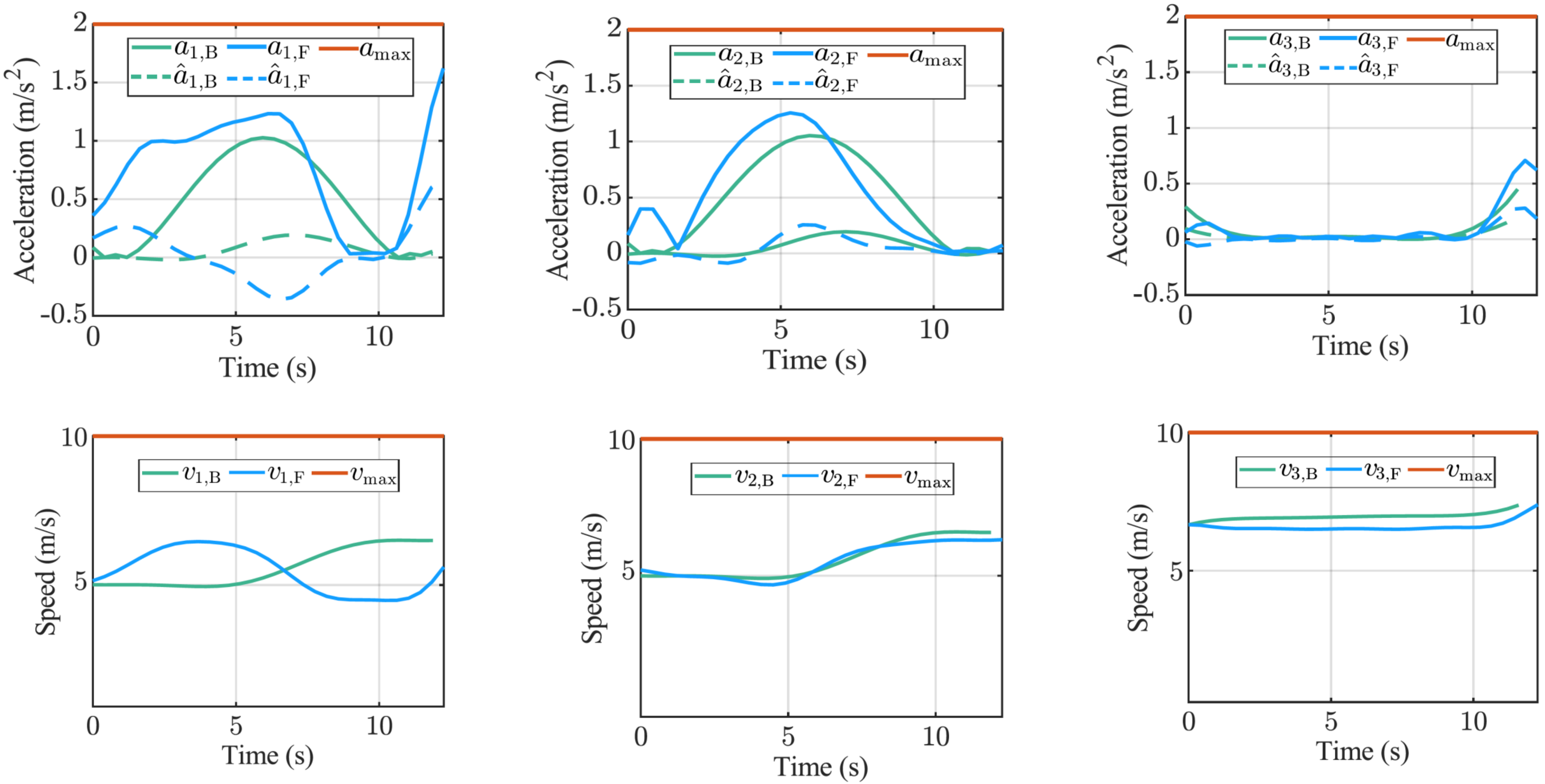




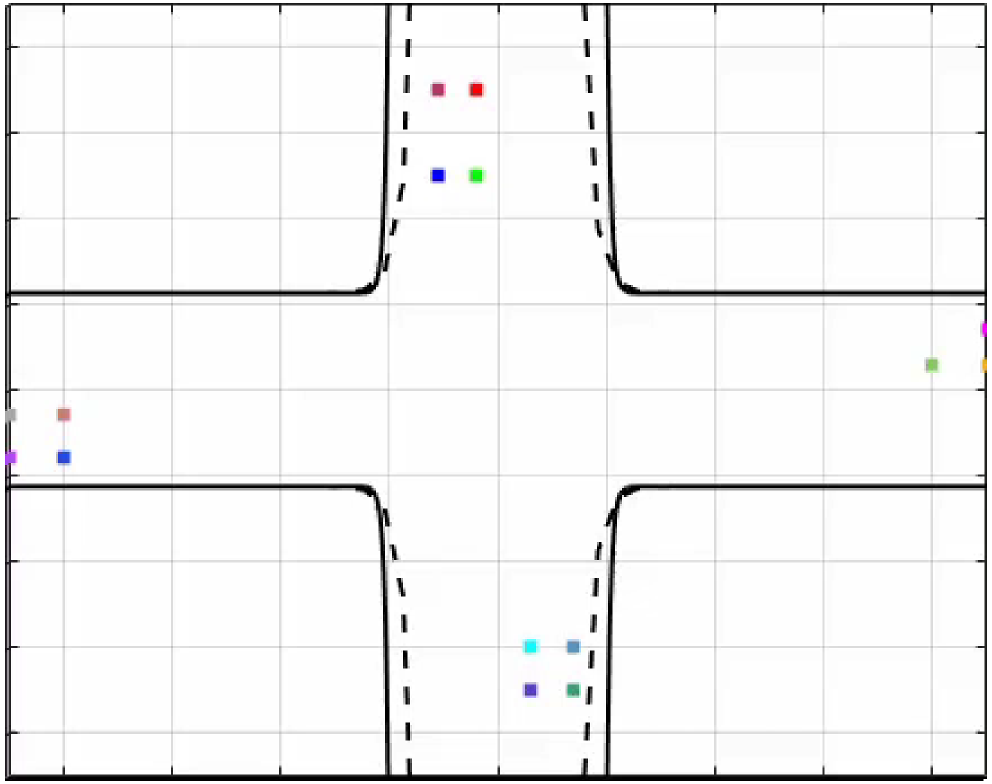
Scenario III: compare Bézier and FFS methods (3 CVAD and $d_s = 1$)

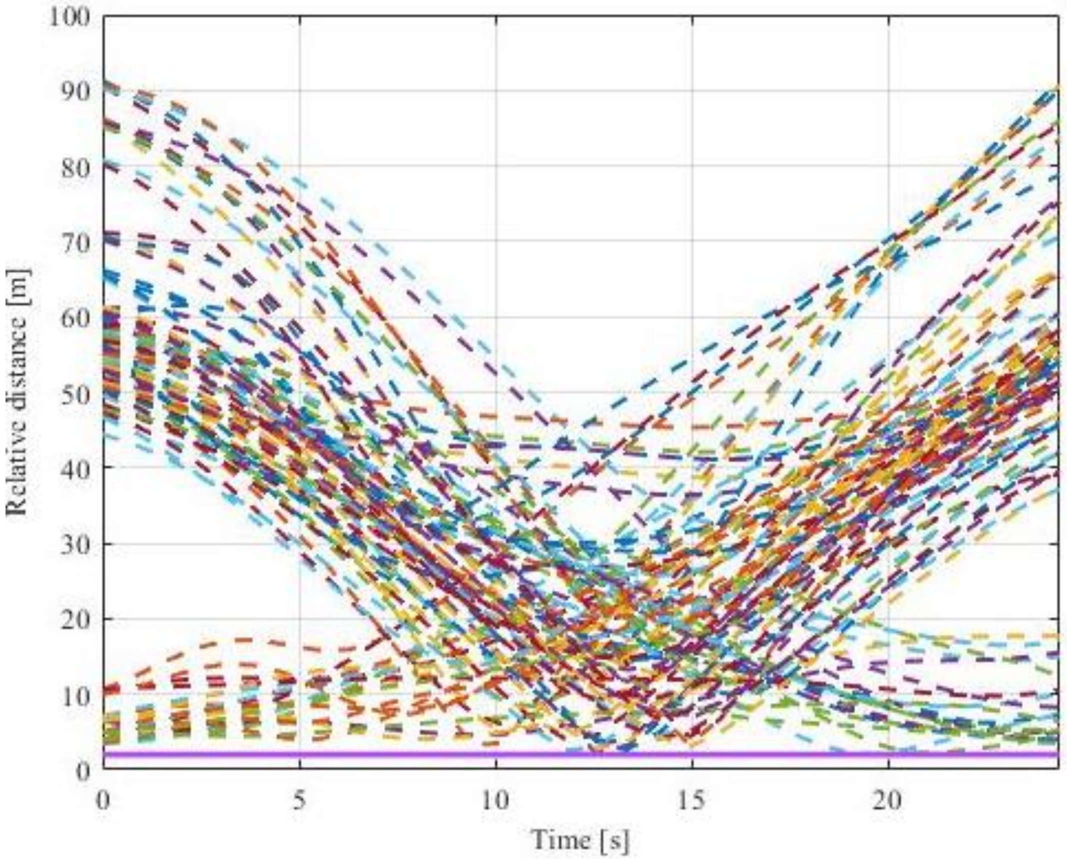


Scenario III: acceleration and speed profiles



Scenario IV: 15 CVAD and $d_s = 2$





Conclusions

- The proposed Bézier method
  - generates feasible trajectories
  - generates near-optimal collision-free trajectories
- The computational time of the Bézier method depends on
  - the number of Bézier terms
  - the number of discretization points
  - the number of vehicles
- The Bezier method shows slightly better results compared to the FFS method
- The generated trajectories by both Bézier and FFS methods can be used as an initial guess for other optimal control methods

THANK YOU !

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QUESTIONS