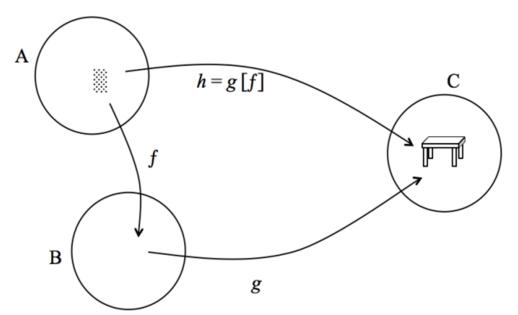
There is no need to define consciousness; we only observe some of its properties, namely geometric and topological properties of the visual consciousness, and show that these properties cannot be based on physics only. Now, if a part of consciousness cannot be grounded on physics only, it is the same for consciousness as a whole and we speak of the irreducibility of consciousness to physical data. We do not consider philosophical questions or issues; in a simple physical and mathematical frame, we give a logical proof of this irreducibility.

We consider the *visual space*, the space we see when looking at something. We study its *continuity* and *unity*. Although the visual space depends on the brain of the observer, these properties of continuity and unity of the visual space can be seen by everybody: their study is therefore perfectly objective.

Let A be the space of 'physical reality' as known by physics, it is the 'real' space of matter with what is included in it: moving atoms, particles and waves; let B be the observer's brain regarded as a space, with its physiological and neural activity; and let C be the observer's visual space. Clearly C differs from A and B.



Qualities (*qualias*) of a colour cannot be given physically, but they do correspond to physical data: a green object reflects the light waves with a frequency that we see as 'green'. We say that this property of being green can be *reduced* to physical properties.

The question then arises whether there are properties of the visual space that cannot be reduced to physical ones. If so, we speak of *irreducible* properties.

Continuity

In the visual space, there are no gaps or moving separate points; e.g., an ordinary white sheet of paper appears uniformly, permanently white and still, for at least a while. In contrast, physical reality at the atomic level is essentially discrete, non uniform, never motionless, and full of collisions.

How can a discrete moving atomic reality be represented in a continuous way (both in space and time)?

Perceptions are transmitted and received (if we remain in a purely physical world) by discrete processes of particles and moving waves, particularly photons, and charges. How does this produce a continuous image, and where does this image appear? It appears in the space C which, because of this property of continuity, does not belong to the physical or brain space, and is a specific space of non-material, non-physical nature.

If everything were created, transmitted and received by physical spaces, it would remain permanently moving and discrete. As we say in French, "the most beautiful girl in the world cannot give more than what she has": physics gives no more than physics.

But what about the physical notion of a *field*? The only physical approach to continuity is given by the notion of field. There is a brain electromagnetic field, but we do not see this field, nor the gravitational one, and the discrete aspect remains. The very notion of continuity, before the complicated mathematical one, comes from our visual perception of lines and surfaces of different objects (e.g., water), which physically are far from being continuous. If, indeed, we understand consciousness as a field, this field is of different – non physical – nature.

Unity

One of the most remarkable properties of consciousness space — and, moreover, difficult to understand — is its unity, that is the capacity that consciousness has to gather perceptions as a whole; from a multiplicity of independent nervous impulses and neuronal processes, consciousness produces a unified whole. We do not have consciousness of separated elements, but always of a coherent whole, even when looking at an isolated object.

This unity principle is the following: given separate elements x_1 , ..., x_n , it is the actual capacity to conceive their totality at once as a whole, i.e. the set $\{x_1, ..., x_n\}$. But this is an axiom of Set Theory. It is not physically explicable. The argument of simultaneity of the appearance of all particles and waves on the retina and in the brain does not work, because perception of simultaneity needs already consciousness, i.e., an observer.

But, even if the notion of simultaneity is given, the probability that all the possible visible 'dots' of our visual neurology (e.g., retina) – their number can be estimated of the order of 10^7 – are grasped together in a coherent unity, this probability is of order of $2^{\Lambda}(10^7)$ (2 to the power of 10 to the power 7), which is well beyond any physical meaning even at the level of light-wave length. Unity cannot emerge 'by chance'; moreover, it is permanent, continuous in time. The probability for this continuous unity is physically without meaning.

Finally, we have the capacity to extract and see particular forms, objects or parts from the unity of what we see. This corresponds to the *Comprehension Axiom* of Set Theory. It has no physical evidence or grounds.

Conclusion

That consciousness space is relatively independent from external physical reality is a classical statement. It is worth quoting Berkeley, "The proper objects of sight not without the mind; nor the images of anything without the mind" and also "Images in the eye are not pictures of external objects" and quoting Max Planck, "I regard consciousness as fundamental. I regard matter as derivative from consciousness. We cannot get behind consciousness. Everything that we talk about, everything that we regard as existing, postulates consciousness". For intelligent systems, this is a challenge, it reveals a limit to Al and shows the necessity of deeper rationale in this domain.