

Performance of Linear Programming in Optimizing the Energy Schedule of a Grid-connected Hybrid System Compared to Particle Swarm Optimization

Hoda ELAOUNI^{1,2}

Hussein Obeid¹, Stéphane Le Masson², Olivier Foucault² et Hamid Gualous¹

1. LUSAC Laboratory, University of Caen Normandy,
2. Orange Innovation, Lannion France



Context

Objectives :

Integrate **renewable energies** in the energy production of the Telecoms sites.

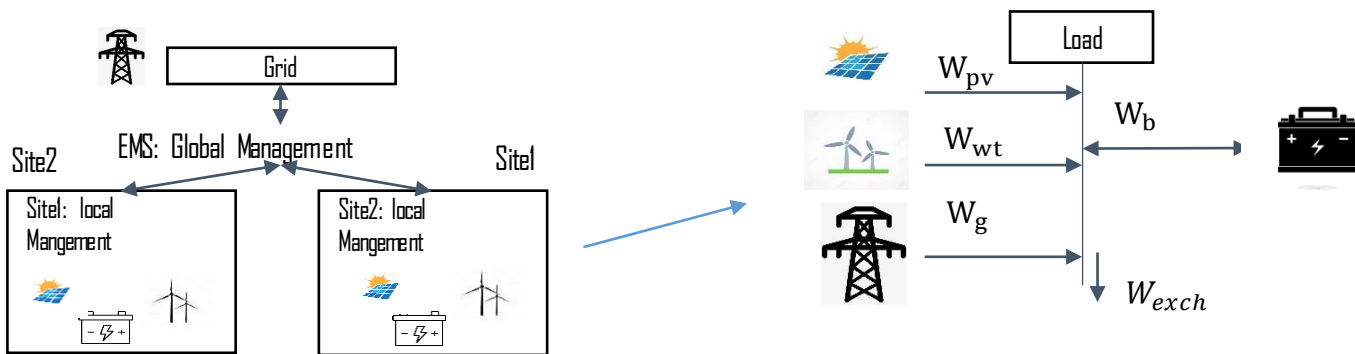
Minimize the **energy cost of the Telecoms sites**.

Interconnect the Telecom sites.

Optimize the **energy exchanged** between two Telecoms sites.

Problematic:

How to optimize the energy between two Microgrids (sites) **globally** and **locally**?

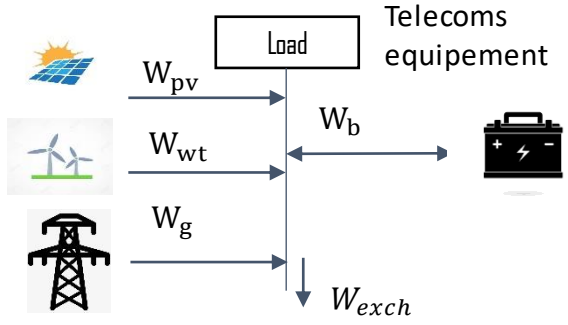


Architecture of the site 1 (Microgrid)

A. SYSTEM DESCRIPTION

Assumptions:

- Load demand is constant.
- No sale of energy to the Grid.
- Charging battery should be only by the remaining energy.
- W_{exch} refers to the energy lost.



- | | | | | |
|---|---|--|--|--|
| <ul style="list-style-type: none"> • Meteorological data (Sunshine, Wind speed, Load demand). • Purchased energy price. | <ul style="list-style-type: none"> • Calculate the renewable energy production (i.e., PV, and Wind Turbine) • Define the battery state of charge. | <ul style="list-style-type: none"> • Energy balance • Maintain the values of $(W_g, W_b, \text{ and } W_{exch})$ within their lower and upper bounds. | <ul style="list-style-type: none"> • Minimize the operating cost of the architecture studied. | <ul style="list-style-type: none"> • Determine the optimal value of $(W_g, W_b \text{ and } W_{exch})$ |
|---|---|--|--|--|



B. OPTIMIZATION ALGORITHM

❑ Linear Programming (LP)

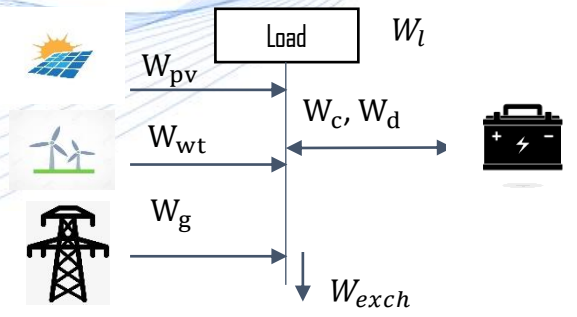
1. An exact optimization method.
2. A mathematical modeling technique.
3. The objective function and constraints must be linear.

❑ Particle Swarm Optimization (PSO)

1. A bio-inspired algorithm (behavior of birds).
2. An heuristic optimization method.
3. The optimization performance stands into the choice of the PSO parameters.

B. OPTIMIZATION ALGORITHM

1. Linear Programming Model



- **Decision variables:**
 $x = [W_g \ W_c \ W_d \ SOC \ W_{exch}]$
- **Input:**
 - W_{pv}, W_{wt}, W_l
- **Objective:**
 Minimize the cost of energy purchased from the grid in the deficit case.
- **Assumptions:**
 - The $K(t)$ refers to the charging state of the battery.
 - The purchased energy price is considered constant.

- **Objective function:**

$$\min C = \sum_{t=1}^T W_g(t) C_g k(t)$$

- **Penalty function:**

$$k(t) = \begin{cases} 1, & \text{if } d(t) \geq 0 \\ 0, & \text{Otherwise} \end{cases}$$

$$d(t) = W_l(t) - W_{pv}(t) - W_{wt}(t)$$

- **Constraints:**

- **Energy balance:**

$$d(t) = k(t)(W_g(t) - W_d(t)) + (k(t) - 1)(W_c(t) - W_{exch}(t))$$

- **Battery:**

$$SOC(t+1) = SOC(t) + \frac{(1 - k(t))W_c(t) + k(t)W_d(t)}{E_c}$$

$$SOC_{min} \leq SOC(t) \leq SOC_{max}$$

$$0 \leq P_c(t) \leq P_{cmax}, P_{dmax} \leq P_d(t) \leq 0$$

$$|SOC(T) - SOC(1)| \leq \epsilon$$

- **Grid:**

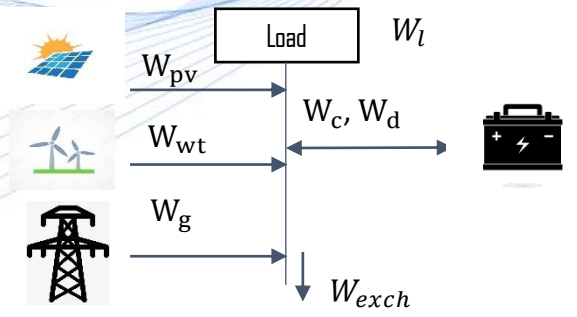
$$0 \leq W_g(t) \leq W_{gmax}$$

- **Exchange:**

$$W_{exch} \geq 0$$

B. OPTIMIZATION ALGORITHM

1. Linear Programming Model



- **Decision variables:**
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- **Grid:**

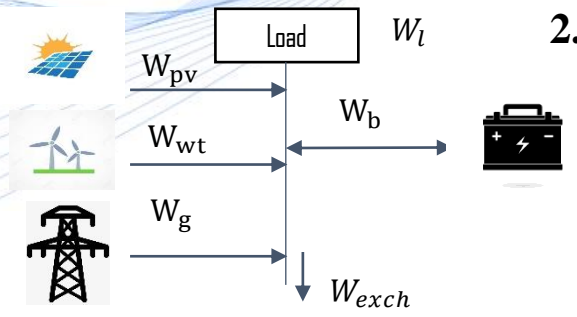
$$0 \leq W_g(t) \leq W_{gmax}$$

- **Exchange:**

$$W_{exch} \geq 0$$

B. OPTIMIZATION ALGORITHM

2. Particle Swarm Optimization Model



- Decision variables:**

$$x = [W_b]$$

- Input:**

- W_{pv}, W_{wt}, W_l

- Objective:**

- Minimize the cost of energy purchased from the Grid in the deficit case.

- Hypothesis:**

- Penalty is applied to avoid the following scenarios:
 1. Charging and discharging the battery in the steady state
 2. Discharging the battery if there is an excess of energy
 3. Energy left is not sufficient to charge the battery

- Objective function:**

$$\min C = P + \sum_{t=1}^T W_g(t) C_g$$

- Penalty function:**

$$W_g(t) = \begin{cases} W_l(t) - d(t) + W_b(t), & \text{if } d(t) \geq 0 \\ 0, & \text{Otherwise} \end{cases}$$

$$W_{exch}(t) = \begin{cases} -d(t) - W_b(t), & \text{if } d(t) \leq 0 \\ 0, & \text{Otherwise} \end{cases}$$

$$d(t) = W_l(t) - W_{pv}(t) - W_{wt}(t)$$

- Constraints:**

- Battery:**

$$SOC(t+1) = SOC(t) + \frac{(1 - k(t))W_c(t) + k(t)W_d(t)}{E_c}$$

$$SOC_{min} \leq SOC(t) \leq SOC_{max}$$

$$0 \leq P_c(t) \leq P_{cmax}, P_{dmax} \leq P_d(t) \leq 0$$

$$|SOC(T) - SOC(1)| \leq \epsilon$$

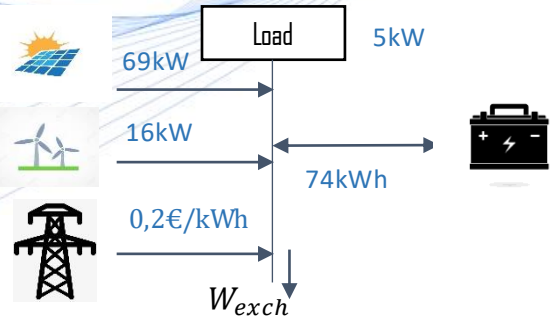
- Grid:**

$$0 \leq W_g(t) \leq W_{gmax}$$

- Exchange:**

$$W_{exch} \geq 0$$

A. COMPARATIVE STUDY



Conclusion

- LP suggests the best schedule of the considered system compared to the PSO algorithm.
- PSO becomes avoidable when the numbers of decision variables increase.

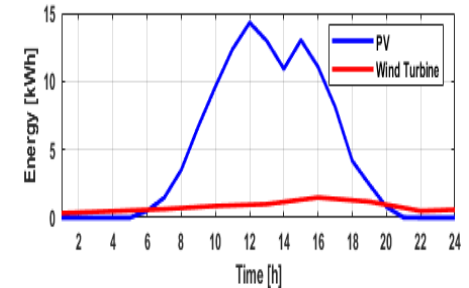
- Operating cost (C) and exchanged energy (W_{exch}) proposed by the both approaches:

Scénario	PSO		LP	
	C (€)	W_{exch} (kWh)	C (€)	W_{exch} (kWh)
Cas 1: (July)	1,85	32	0,68	24
Cas 2: (May)	2,61	27	1,18	18
Cas 3: (October)	2,38	18,5	1,2	10

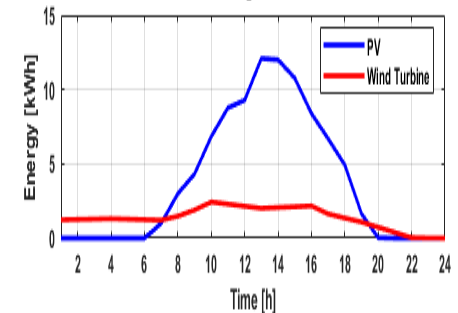
- Comparaison between the characteristics of the two algorithms:

	LP	PSO
Computational time	< 1min	> 1h
Robustness	Confirmed	Confirmed

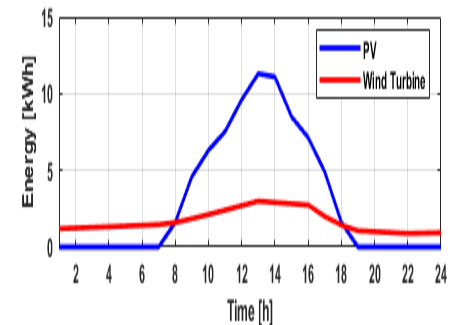
Case 1: 1st of July in Lannion



Case2: 1st of May in Lannion



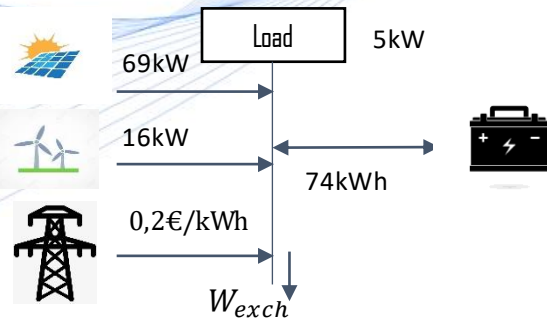
Case3 : 1st of October in Lannion



A. COMPARATIVE STUDY

The energy scheduling on the 1st of July in Lannion

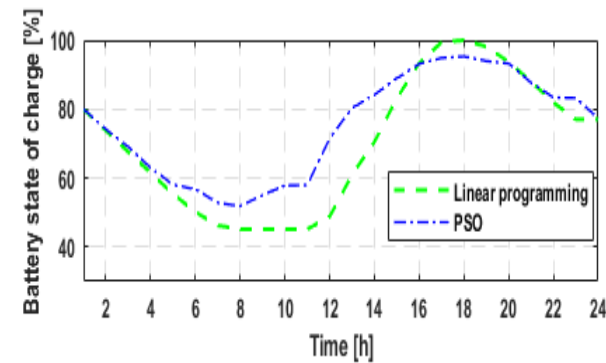
Case 1: 1st of July in Lannion



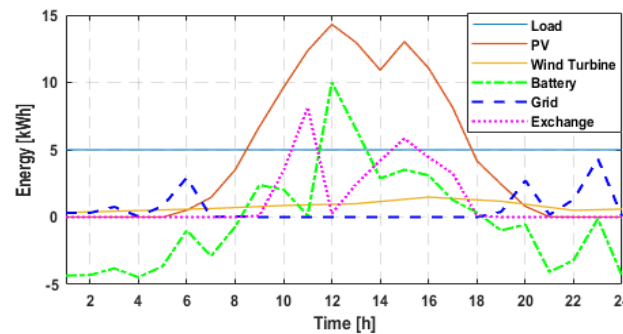
Conclusion

- The energy dispatch proposed by the both approaches is globally similar.
- The difference occurs in the variation of the battery state of charge.

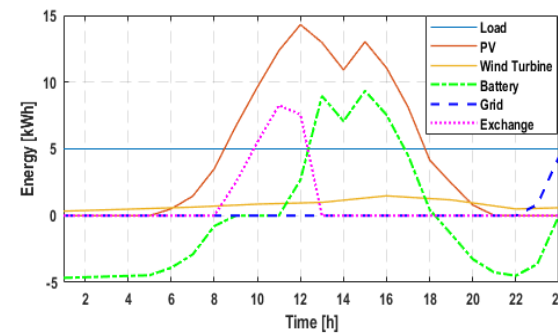
Battery state of charge

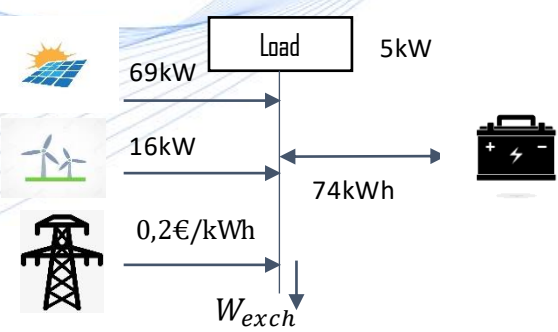


Energy scheduling proposed by PSO



Energy scheduling proposed by LP



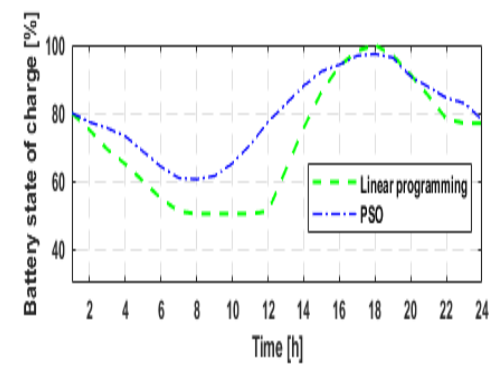
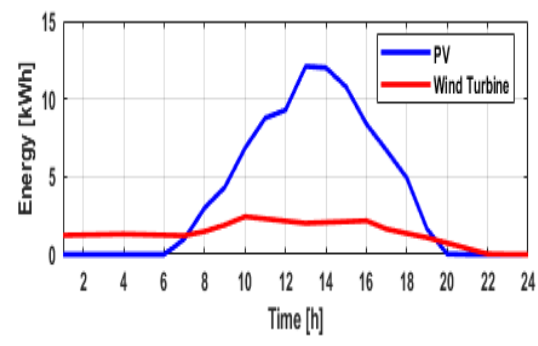


A. COMPARATIVE STUDY

The energy scheduling on the 1st of May in Lannion

Case 2: 1st of May in Lannion

Battery state of charge

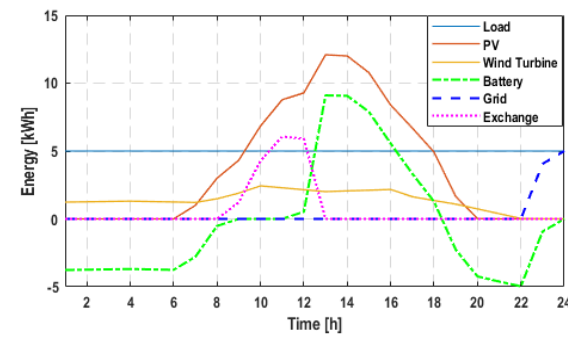
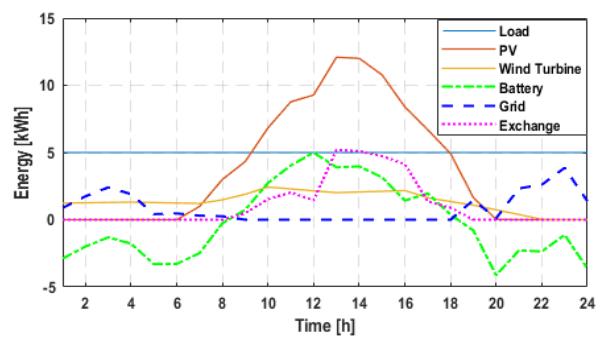


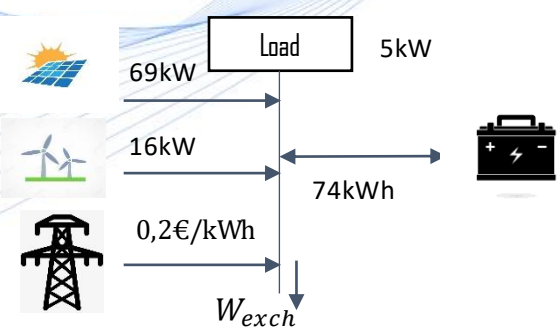
Conclusion

- The energy dispatch proposed by the both approaches is globally similar.
- The difference occurs in the variation of the battery state of charge.

Energy scheduling proposed by PSO

Energy scheduling proposed by LP

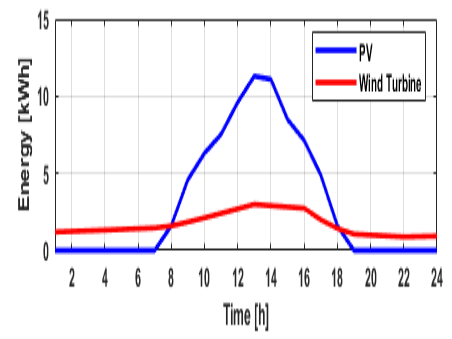




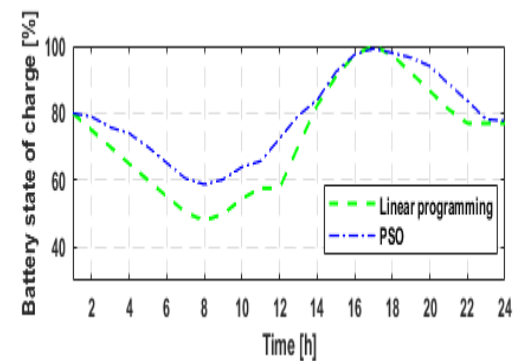
A. COMPARATIVE STUDY

The energy scheduling on the 1st of October in Lannion

Case 3: 1st of October in Lannion



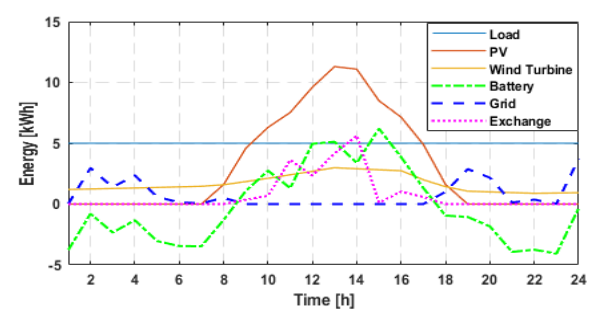
Battery state of charge



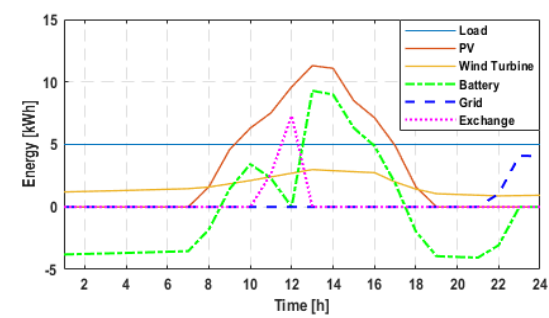
Conclusion

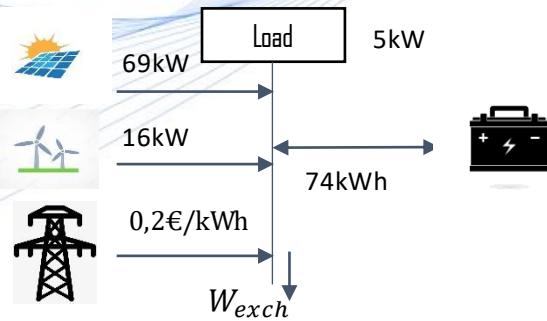
- The energy dispatch proposed by the both approaches is globally similar.
- The difference occurs in the variation of the battery state of charge.

Energy scheduling proposed by PSO



Energy scheduling proposed by LP





B. SENSITIVITY ANALYSIS

The energy scheduling on 1st of October in Lannion

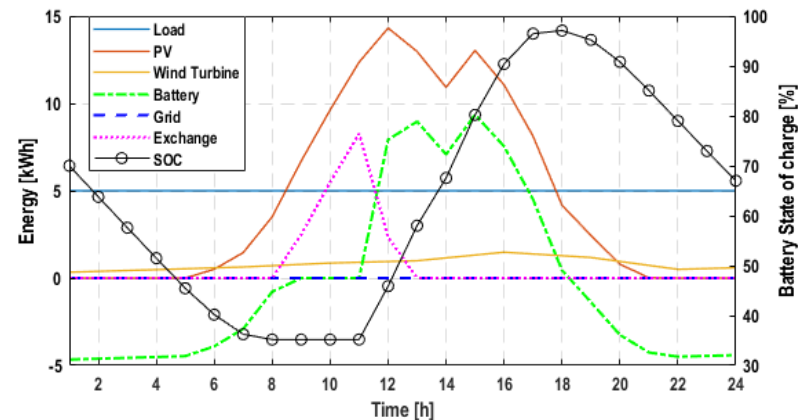
- Operating cost (C) and exchanged energy (W_{exch}) for different values of the initial battery state of charge using LP.

SOC(t0)	100%	90%	80%	70%	60%	50%	40%
C (€)	2,16	1,64	0,68	0	0,46	1,42	2,39
W_{exch} (kWh)	39	31	24	19	22	30	37

Conclusion

- The economic scenario to adopt is: $SOC(t_0) = 70\%$

- Energy scheduling of the system studied on the 1st of July in Lannion





Linear Programming	Particle Swarm Optimization
<ul style="list-style-type: none">• Positive aspect:<ol style="list-style-type: none">1. Effectiveness (operating cost).2. Rapidity (Computational time).3. Robust• Negative aspect:<ol style="list-style-type: none">1. Problem formulation	<ul style="list-style-type: none">• Positive aspect:<ol style="list-style-type: none">1. Effectiveness (operating cost).2. Robust• Negative aspect<ol style="list-style-type: none">1. Rapidity(Computational time).2. Convergence requires some parameters to be tuned (i.e., Population size).

- The comparative study confirms the effectiveness and rapidity of the LP in front of the PSO in terms of computational time and operational cost.
- The sensitivity analysis shows the impact of the initial battery state of charge on the energy scheduling.



Thank you for your attention !

A decorative graphic of blue wavy lines, resembling a stylized ribbon or wave, that curves from the top left towards the center of the slide.

Annex 1: Simulation data

Component	Values
Load demand	Constant rated power: 5kW
Photovoltaic	Installed peak power: 69kW
Wind Turbine	Rated power: 16 kW
Battery	Rated energy: 74 kWh
Grid	Purchased electricity price: 0,2€/kWh

Battery PARAMETERS:

SOCmin=30%, SOCmax=100%, SOC(1)=80%, ϵ =3%

PSO PARAMETERS:

Numbers of variables: 24

Numbers of iterations: 300

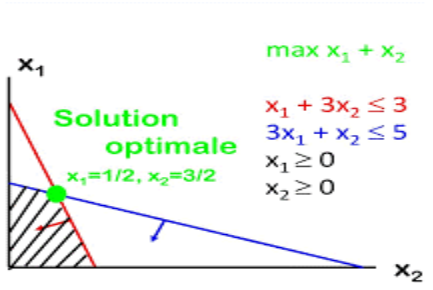
Population size: 1000

OPTIMIZATION ALGORITHM

LP

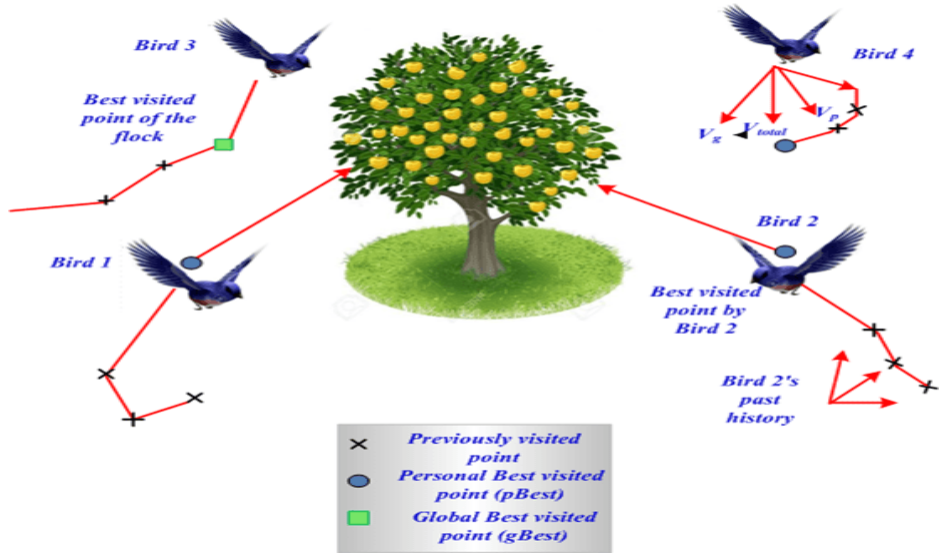
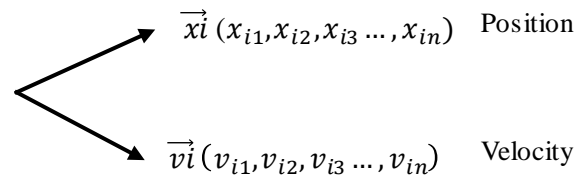
$$\min_x f^T x \text{ such that } \begin{cases} A \cdot x \leq b \\ A_{eq} \cdot x = b_{eq} \end{cases}$$

Example



PSO

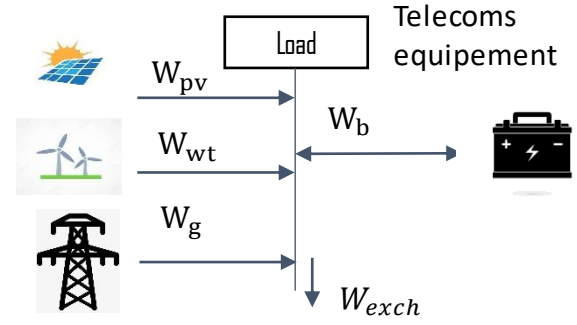
Particle i :



A. SYSTEM DESCRIPTION

Hypothesis:

- Load demand is constant.
- No sale of energy to the Grid.
- Charging battery should be only by the remaining energy.
- W_{exch} refers to the energy lost.



Photovoltaic model:

(Lan and al.,2015)

$$P_{pv}(t) = P_p f_c G(t) \left(\frac{1 + \beta(T_c(t) - T_{ref})}{G_r} \right)$$

$$T_c(t) = T_a(t) + G(t) \left(\frac{NOCT - 20}{800} \right)$$

Wind Turbine model:

(Diaf and al.,2007)

$$P_{wt}(t) = \begin{cases} 0, & \text{if } V_i \leq V(t) \leq V_n \\ P_r & \text{if } V_n \leq V(t) \leq V_o \\ 0, & \text{Otherwise} \end{cases}$$

$$v(t) = V_0(t) \left(\frac{H}{H_0} \right)^\alpha$$

Battery model:

- Charging mode:
- Discharging mode:

$$W_b(t) = W_b(t-1) + (W_{pv}(t) + W_{wt}(t) - W_l(t))$$

$$W_b(t) = W_b(t-1) - (W_{pv}(t) + W_{wt}(t) - W_l(t))$$

(Hossain and al.,2019)