Calculations of the Packet Loss Ratio in Finite-Buffer Queues

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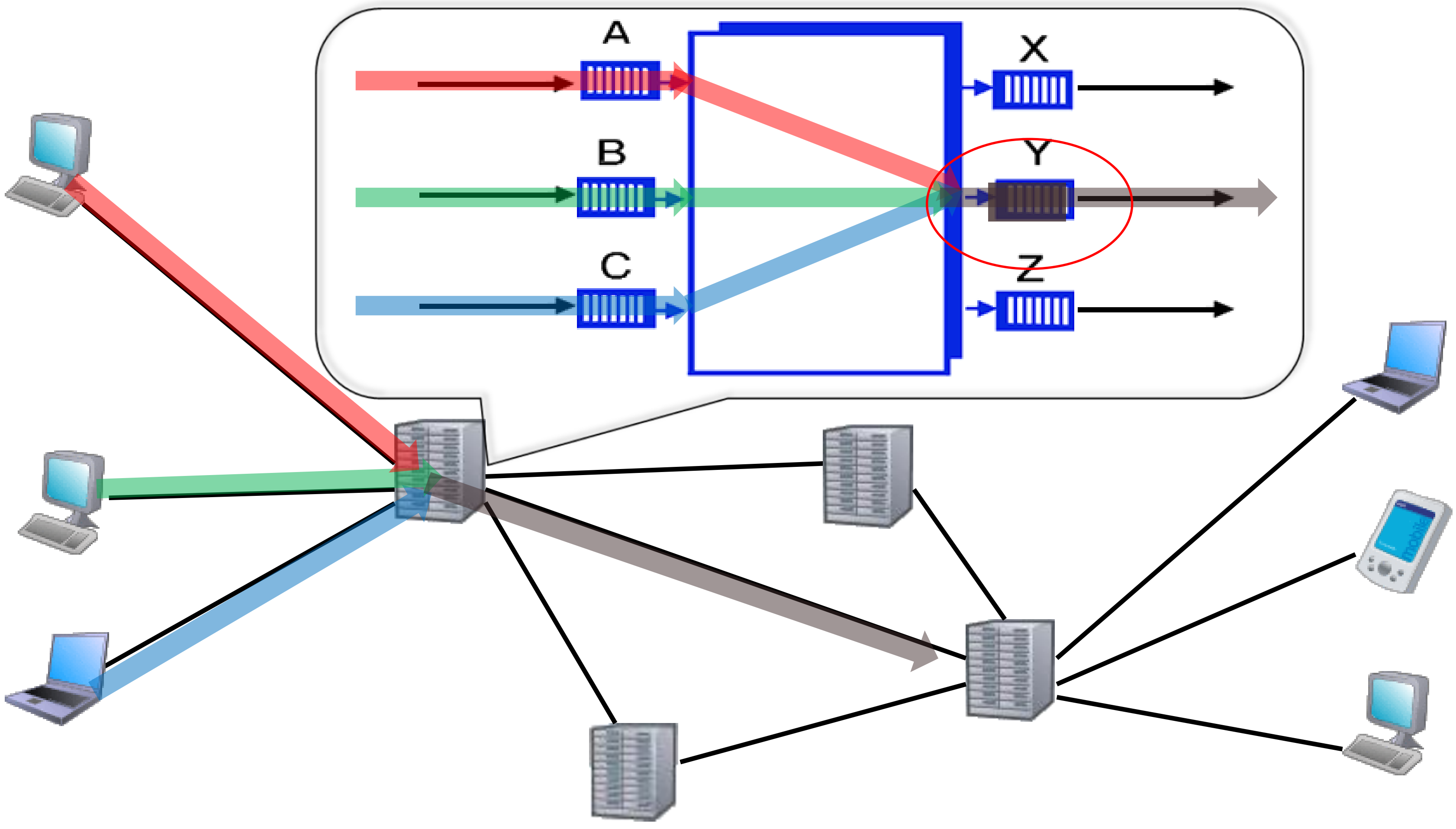
Outline

- Packet losses in the network layer
- Loss ratio
- Methods of computation
- The method based on the empty queue probability
- Results
- Conclusions

Packet losses in the network layer

- overflows at output ports of routers



Loss ratio – the main loss characteristic

- ***L***: the long run fraction of lost packets

L= #lost_packets / #all_packets

- can be measured globally (in the whole network), between two end systems, at a router's output interface, etc.
- we are interested in the latter

Methods of computation of L

- Direct – using the Laplace transform of the number of losses:

$$L = \lim_{t\to\infty} L(t) = \lim_{t\to\infty} \frac{M(t)}{\lambda t}$$

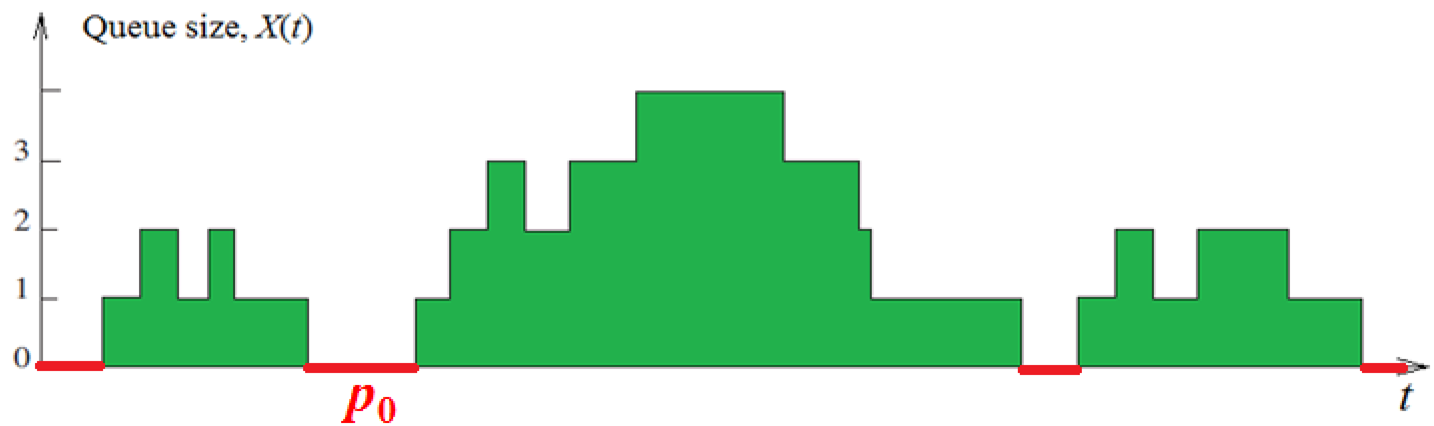
  - universal, but very demanding analytically and computationally
- Indirect – using special properties of the arrival process
  - e.g. when the arrivals are Poisson, using the PASTA property we have $L=P_B$= probability that the buffer is full
  - nice and easy, but works only in the very special case of the arrival process
- Indirect – using the empty queue probability:

$$L = 1 - \frac{1-p_0}{\rho}$$

  - universal, very underappreciated!

Dependence between L and p_0

- Theorem: $$L = 1 - \frac{1-p_0}{\rho}$$
- $\rho = \lambda/\mu$ is the offered load
- Proof. In a long time interval of length T, the output link is busy for $(1-p_0)T$ time



- # of packets sent in a long interval of length T is: $(1-p_0)T\mu$
- on the other hand, there are λT new new packets arriving in this interval
- Thus: $$L = 1 - \frac{(1-p_0)T\mu}{\lambda T} = 1 - \frac{1-p_0}{\rho}$$

The method based on the empty queue probability cont.

- Is valid for an arbitrary (can be very complex) arrival process, and arbitrary service time distribution (packet size distribution)
  - including MMPP, BMAPs, ARIMA etc.
- Is valid for an arbitrary loss mechanism.
  - Including AQMs, e.g. RED
- The empty queue probability is a part of the queue size distribution. It is known for most popular arrival process models, e.g. Poisson, batch Poisson, MMPP, BMAP etc.

Results in the paper

- **Several ready-to use formulas for the loss ratio in queueing models with Poisson, batch Poisson, MMPP, BMAP arrivals:**

$$L = \frac{1}{\rho}\left[\rho - 1 + \frac{1}{\rho\sum_{k=0}^{N-1}\xi_k + 1}\right],$$

$$L = \frac{1}{\rho}\left[\rho - 1 + \frac{\pi_0^\infty}{\pi_0^\infty + \sum_{j=0}^{N-1}\pi_j^\infty \rho/\overline{b}}\right]$$

$$L = \frac{1}{\rho}\left[\rho - 1 + \lim_{s\to 0+} sM_N^{-1}(s)l_N(s)\right]$$

$$L = \frac{1}{\rho}\left[\rho - 1 + \lim_{s\to 0+} sH_N^{-1}(s)m_N(s)\right]$$

Results in the paper cont.

- **Numerical examples of calculation of the loss ratio for two most complex arrival processes, i.e. MMPP and BMAP.**

$$D_0 = \begin{bmatrix} -45.5935855 & 1.95261616 & 0.19526161 \\ 0.01952616 & -4.55935855 & 0.19526161 \\ 0.00195261 & 0.01952616 & -0.45593586 \end{bmatrix},$$

$$D_2 = \begin{bmatrix} 0.06508720 & 0.52069762 & 5.20697622 \\ 0.52069762 & 0.00065087 & 0.05792761 \\ 0.05076801 & 0.00650872 & 0.00065087 \end{bmatrix},$$

$$D_4 = \begin{bmatrix} 0.06508720 & 0.52069762 & 5.20697622 \\ 0.52069762 & 0.00065087 & 0.05792761 \\ 0.05076801 & 0.00650872 & 0.00065087 \end{bmatrix},$$

$$D_8 = \begin{bmatrix} 0.35797962 & 2.86383692 & 28.6383692 \\ 2.86383692 & 0.00357979 & 0.31860186 \\ 0.27922410 & 0.03579796 & 0.00357979 \end{bmatrix}.$$

Firstly, using (32) we can calculate the stationary vector for matrix $D = D_0 + D_2 + D_4 + D_8$. We get:

$$\pi = (0.010598, 0.028056, 0.961345).$$

Then, using (31), we can compute the arrival rate:

$$\lambda = 6.666666.$$

Therefore, from (33) it follows that:

$$\rho = 0.666666.$$

Using (34) with $s = 10^{-9}$, we can obtain the empty queue probability:

$$p_0 = 0.377399.$$

Finally, from (38) we obtain the loss ratio:

$$L = 0.066099.$$

Conclusions

- In the paper the method of calculating the packet loss probability based on the empty queue probability is presented and discussed
- The method is easy to use
- Valid for an arbitrary type of the arrival process
- Valid for an arbitrary loss mechanism
- Ready-to-use formulas given for popular types of the arrival process
- Numerical examples given

Thank you.