On the Proportional-Integral-Derivative Based Trading Algorithm under the Condition of the log-Normal Distribution of Stock Market Data

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Outline

CONTROL THEORETICAL FOUNDATIONS OF THE OPTIMAL TRADING

Main Ideas and Concepts

the formal discrete-time PID type profit - investment relation

$$\delta I(t+1) = K_P(t)\Delta g(t) + K_D(t)\dot{\Delta} g(t) + K_I(t)\int_{t-T}^{t} h(\tau)\Delta g(\tau)d\tau, \quad \Delta I(t+1) = \chi(\delta I(t+1)), \text{ for } t = 1, \dots, \; I(1) = I_1 \tag{1}$$

where $K(\cdot) := \{K_P(\cdot),\ K_D(\cdot),\ K_I(\cdot)\}$ are dynamic gains and

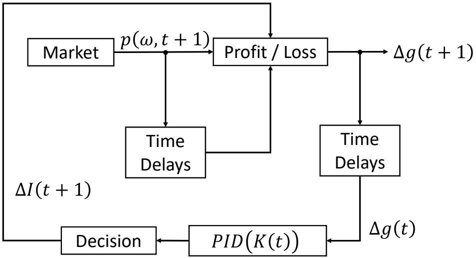
$$\chi(\delta I) := \begin{cases} \delta I, & \text{if } \delta I^{\min} \leq |\delta I| \leq \delta I^{\max}; \\ \pm\delta I^{\max}, & \text{if } |\delta I| > \delta I^{\max}; \\ 0, & \text{if } |\delta I| < \delta I^{\min}. \end{cases}$$

Main Ideas and Concepts

the current profit calculation

$$\Delta g(t+1) = \frac{(p(\cdot, t+1) - p(\cdot, t))}{p(\cdot, t)} \Delta I(t+1) \tag{2}$$

the model-free PID trading strategy



Statistical Analysis of the Stock Data

the log-normal pdf $\rho(\theta)$ of the price/volume ratio

$$\theta(\omega, t+1) := \frac{p(\omega, t+1)}{v(t+1)}$$

where the $t+1$ investment volume is next determined as:

$$v(t+1) := \frac{\Delta I(t+1)}{p(\cdot, t)} \tag{3}$$

in the praxis the above investment volume is restricted and

$$\rho(\theta) = \frac{a}{\sqrt{2\pi}\sigma(\theta - s)} \exp -(0.5\sigma^2)(\ln(\theta - s) - \mu)^2, \tag{4}$$

is the the (stationary) statistical law for $\theta(\omega, t+1)$

Decision Making Related to the Investment Volume

the daily market prices and investment volumes data

$$\{p(1),...,p(T)\}, \ \{v(1),...,v(T)\}$$

the log-normal pdf for $p(\omega, t+1)/p(\cdot, t)$ implies that

$$\ln\Big(\frac{v(t+1)}{v(t)}\Big) = \ln\Big(\frac{p(\omega, t+1)}{p(\cdot, t)}\Big) + \ln\theta(\cdot, t) - \ln\theta(\omega, t+1)$$

and finally we get

$$v(t+1) = \exp\Big[\ln\Big(\frac{p(\omega, t+1)}{p(\cdot, t)}\Big) + \ln\theta(\cdot, t) - \ln\theta(\omega, t+1) + \ln v(t)\Big]$$

Optimal Calibration of the PID Gains

the main optimization problem for PID gains calibration

$$\sum_{j=1}^{M} \left(\chi(\delta I(t+1)) - v^j(t+1)p(\cdot,t)\right)^2 \rightarrow \min$$
$$\delta I(t+1) = K_P(t)\Delta g(t) + K_D(t)\dot{\Delta} g(t) +$$
$$K_I(t)\int_{t-T}^{t} h(\tau)\Delta g(\tau)d\tau, \tag{5}$$

it leads to $K^{opt}(\cdot) := \{K_P^{opt}(\cdot),\ K_D^{opt}(\cdot),\ K_I^{opt}(\cdot)\}$ and finally to the optimal investment volumes

$$v^{opt}(t+1) := \frac{\Delta I^{opt}(t+1)}{p(\cdot,t)}.$$

A Computational Example

the Binance Bitcoin / USD futures

Figure: Binance BTC / USD one day price index

A Computational Example

the corresponding profit dynamics

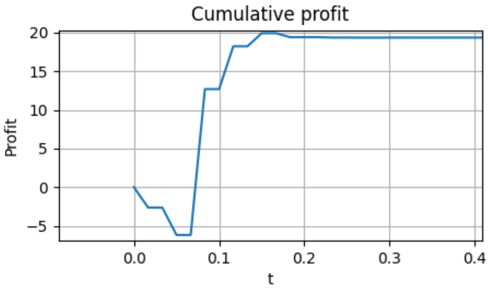


Figure: Binance BTC / USD one day price index

Conclusion

- development of a combined PID based trading algorithm
- a priori given statistical characteristics of the stock market data
- the PID calibration is based on the backtesting procedure
- a specific application of the data driven optimization

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THANKS!