Identification of Multilinear Forms Using Combinations of Adaptive Algorithms

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Presenter's Biography

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- **Past:**
  - **Bachelor's degree** (Valedictorian)
    - @ Telecommunications Technologies and Systems (TST), University Politehnica of Bucharest (2014 - 2018)
    - **Diploma thesis:** Convolutional Neural Networks for Object Segmentation and Tracking in Video Sequences
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    - @ Advanced Digital Imaging Techniques (TAID), University Politehnica of Bucharest (2018 - 2020)
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Outline

- Introduction
- Identification of multilinear forms
- Combinations of adaptive algorithms
- Experiment
- Conclusions

Introduction

- Adaptive filters → frequently used in system identification problems
- Targets → fast convergence rate, accurate estimation, and low computational complexity
- MISO systems → large parameter space → linearly separable systems (array beamforming, nonlinear acoustic echo cancellation, channel equalization, and source separation)
- Efficient solutions → tensor-based adaptive filters → decomposition of rank-1 tensors → the global solution = a combination of shorter adaptive filters
- In this work → different adaptation modes usage for the individual filters → RLS + NLMS

Identification of multilinear forms

- **Framework = a real-valued MISO system** ➜ **the output signal:**

$$y(n) = \sum_{l_1=1}^{L_1}\sum_{l_2=1}^{L_2}\dots\sum_{l_N=1}^{L_N} x_{l_1 l_2 \dots l_N}(n) h_{1,l_1} h_{2,l_2} \dots h_{N,l_N},$$

where $\mathbf{h}_i = \left[h_{i,1}\ \ h_{i,2}\ \ \dots\ \ h_{1,L_i}\right]^T$ are N individual channels of length L_i

- **The input signals** ➜ **Tensorial form:** $\boldsymbol{\chi}(n) \in \mathbb{R}^{L_1 \times L_2 \times \cdots \times L_N}$, with $(\boldsymbol{\chi})_{l_1 l_2 \dots l_N}(n) = x_{l_1 l_2 \dots l_N}(n)$

- **The output signal becomes:** $y(n) = \boldsymbol{\chi}(n) \times_1 \mathbf{h}_1^T \times_2 \mathbf{h}_2^T \times_3 \dots \times_N \mathbf{h}_N^T,$ (mode-N product)

- $y(n)$ has a linear function of each h_i, when the other $N-1$ components are fixed

- $y(n)$ **is s a multilinear form**

Identification of multilinear forms

- Let us consider the **tensor** $\mathcal{H} \in \mathbb{R}^{L_1 \times L_2 \times \cdots \times L_N}$ with the **elements** $(\mathcal{H})_{l_1,l_2,\ldots,l_N} = h_{1,l_1} h_{2,l_2} \ldots h_{N,l_N}$ such that $\mathcal{H} = \mathbf{h}_1 \circ \mathbf{h}_2 \circ \cdots \circ \mathbf{h}_N$ (outer product)

- Also, $\mathrm{vec}(\mathcal{H}) = \mathbf{h}_N \otimes \mathbf{h}_{N-1} \otimes \cdots \otimes \mathbf{h}_1$ (vectorization; Kronecker product) ⇒ $y(n) = \mathrm{vec}^T(\mathcal{H})\, \mathrm{vec}[\chi(n)]$

- Additionally, $\mathrm{x}(n) = \mathrm{vec}[\chi(n)]$ (input vector of length $L_1 L_2 \ldots L_N$) and $\mathbf{g} = \mathrm{vec}(\mathcal{H})$ (global impulse response of length $L_1 L_2 \ldots L_N$)

- **The output signal becomes:** $y(n) = \mathbf{g}^T \mathrm{x}(n)$

- **The reference signal usually results as:** $d(n) = \mathbf{g}^T \mathrm{x}(n) + w(n)$ (measurement noise)

- The main **goal** is the identification of the global system $\mathbf{g}$ with its individual components $\mathbf{h}_i$, with $i = 1,2,\ldots,N$

Combinations of adaptive algorithms

The NLMS-T algorithm:

$$e_{\hat{\mathbf{h}}_i}(n) = d(n) - \hat{\mathbf{h}}_i^T(n-1)\mathbf{x}_{\hat{\mathbf{h}}_i}(n), \qquad i = \overline{1, N}$$

Identity matrices of sizes $L_i \times L_i$

$$\mathbf{x}_{\hat{\mathbf{h}}_i}(n) = \left[\hat{\mathbf{h}}_N(n-1) \otimes \hat{\mathbf{h}}_{N-2}(n-1) \otimes \cdots \otimes \hat{\mathbf{h}}_{i+1}(n-1) \otimes \mathbf{I}_{L_i} \otimes_{i-1} (n-1) \otimes \cdots \otimes \hat{\mathbf{h}}_2(n-1) \otimes \hat{\mathbf{h}}_1(n-1)\right]^T \mathbf{x}(n)$$

Step-size parameters

$$e_{\hat{\mathbf{h}}_1}(n) = \cdots = e_{\hat{\mathbf{h}}_N}(n)$$

Multilinear optimization strategy → MSE criterion

$$\hat{\mathbf{h}}_i(n) = \hat{\mathbf{h}}_i(n-1) + \mu_i(n)\,\mathbf{x}_{\hat{\mathbf{h}}_i}(n)\, e_{\hat{\mathbf{h}}_i}(n)$$

$$\hat{\mathbf{g}}(n) = \hat{\mathbf{h}}_N(n) \otimes \hat{\mathbf{h}}_{N-1}(n) \otimes \cdots \otimes \hat{\mathbf{h}}_1(n)$$

Normalized step-sizes — Regularization constants

NLMS-T

$$\mu_i(n) = \frac{\alpha_i}{\delta_i + \mathbf{x}_{\hat{\mathbf{h}}_i}^T(n)\mathbf{x}_{\hat{\mathbf{h}}_i}(n)}, \qquad i = \overline{1, N}, \qquad 0 < \alpha_i < 1, \qquad \delta_i > 0$$

Combinations of adaptive algorithms

The RLS-T algorithm:

$$e_{\hat{\mathbf{h}}_i}(n) = d(n) - \hat{\mathbf{h}}_i^T(n-1)\mathrm{x}_{\hat{\mathbf{h}}_i}(n)$$

LS error criterion + Minimization of cost functions →

Kalman gain vectors

$$\hat{\mathbf{h}}_i(n) = \hat{\mathbf{h}}_i(n-1) + \mathbf{k}_i(n)\, e_{\hat{\mathbf{h}}_i}(n)$$

$$\mathrm{x}_{\hat{\mathbf{h}}_i}(n) = \left[\hat{\mathbf{h}}_N(n-1) \otimes \hat{\mathbf{h}}_{N-2}(n-1) \otimes \cdots \otimes \hat{\mathbf{h}}_{i+1}(n-1) \otimes \mathbf{I}_{L_i} \otimes \hat{\mathbf{h}}_{i-1}(n-1) \otimes \cdots \otimes \hat{\mathbf{h}}_2(n-1) \otimes \hat{\mathbf{h}}_1(n-1)\right]^T \mathrm{x}(n)$$

$$\mathbf{k}_i(n) = \frac{\mathbf{R}_i^{-1}(n-1)\,\mathrm{x}_{\hat{\mathbf{h}}_i}(n)}{\lambda_i + \mathrm{x}_{\hat{\mathbf{h}}_i}^T(n)\mathbf{R}_i^{-1}(n-1)\,\mathrm{x}_{\hat{\mathbf{h}}_i}(n)}$$

Matrix inversion lemma →

$$\mathbf{R}_i^{-1}(n) = \frac{1}{\lambda_i}\left[\mathbf{I}_{L_i} - \mathbf{k}_i(n)\,\mathrm{x}_{\hat{\mathbf{h}}_i}^T(n)\right]\mathbf{R}_i^{-1}(n-1)$$

Individual forgetting factors

The RLS-NLMS-T algorithm: a **combination** of adaptive filters that uses the **RLS-T** algorithm for the **longest filter**; the **rest of them** are updated with the **NLMS-T** algorithm.

Experiment

- **Conditions:**

  - Input signals – AR(1) processes; each one is generated by filtering a white Gaussian noise through a first-order system with the pole 0.99
  - Additive noise $w(n)$ – WGN with the variance 0.01
  - The order of the system: $N = 4$
  - Individual impulse response $\mathbf{h}_i$ generated using: $L_1 = 32,\ L_2 = 8,\ L_3 = L_4 = 4$
  - $\alpha_i = 0.25, \quad \text{for } i = 1, 2, \dots, N$
  - $\lambda_i = 1 - 1/(50L_i), \quad \text{for } i = 1, 2, \dots, N$
  - Measure of performance:

$$\text{NM}[\mathbf{g}, \hat{\mathbf{g}}(n)] = \left[\frac{\|\mathbf{g} - \hat{\mathbf{g}}(n)\|_2}{\|\mathbf{g}\|_2}\right]^2 \ [\text{dB}]$$

Experiment

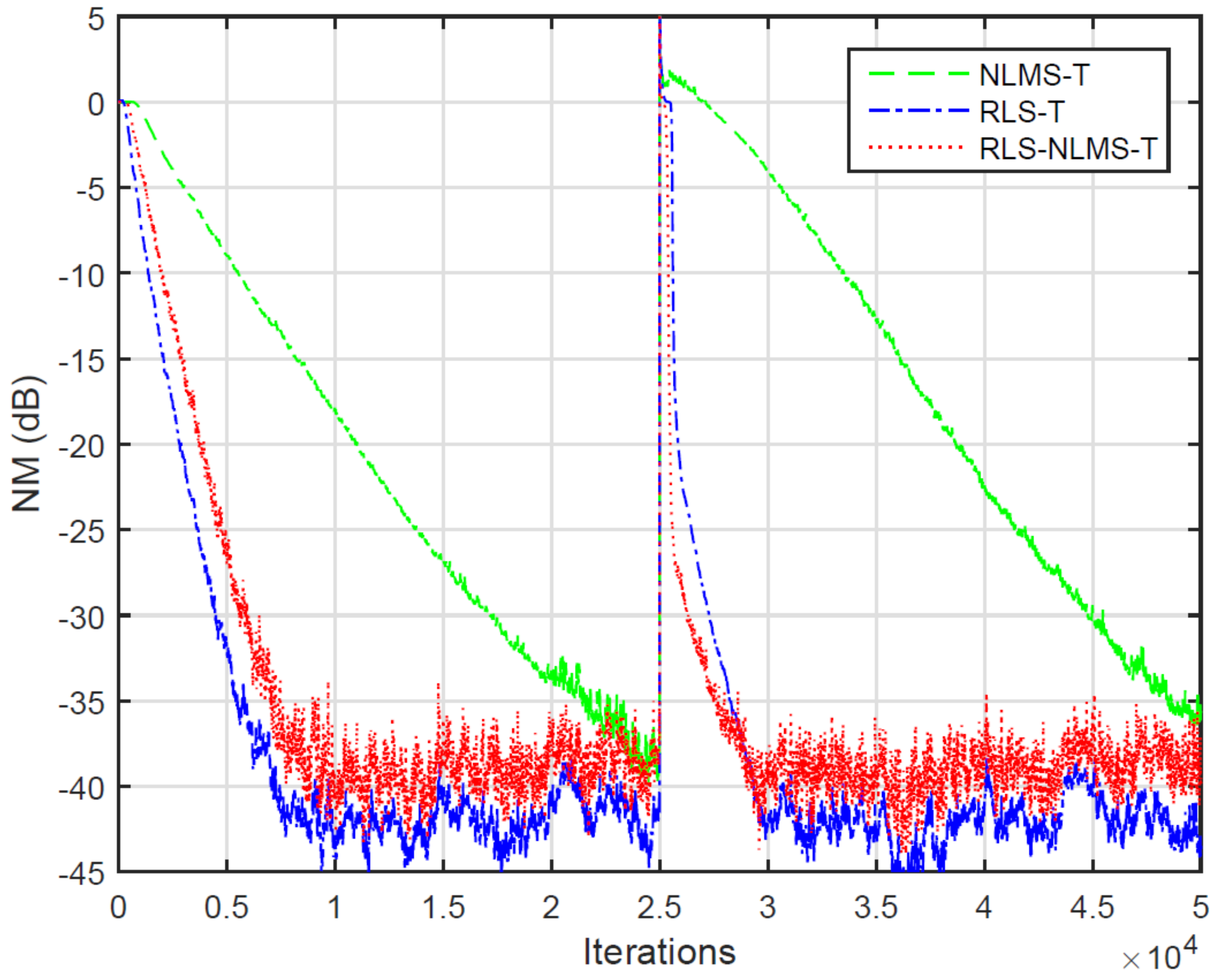

Figure 1. NM of the NLMS-T, RLS-T, and RLS-NLMS-T algorithms.

Conclusions

- In this work, we have explored the idea of using a combination of adaptive filters for multilinear forms.
- The proposed RLS-NLMS-T algorithm achieves a fast convergence rate, while having a lower computational complexity as compared to the RLS-T algorithm.
- Future works will investigate computationally efficient versions of the RLS-NLMS-T algorithm, which could be based on the coordinate descent iterations.

Thank you for your attention!

This work was supported by a grant of the Romanian Ministry of Education and Research, CNCS-UEFISCDI, project number PN-III-P1-1.1-PD-2019-0340 and project number PN-III-P1-1.1-TE-2019-0529, within PNCDI III.