A Novel Approach for Sparse System Identification – Boosting the Performance of Adaptive Algorithms

Laura-Maria Dogariu and Camelia Elisei-Iliescu

Department of Telecommunications
Faculty of Electronics, Telecommunications, and Information Technology
University Politehnica of Bucharest, Romania
Email: ldogariu@comm.pub.ro
Web: www.comm.pub.ro/ldogariu
www.comm.pub.ro/camelia_elisei

The Fifteenth International Conference on Sensor Technologies and Applications
SENSORCOMM 2021
14 -18 November 2021, Athens, Greece

Outline

- Introduction
- Sparse System Identification
- Decomposition-Based Approach
- Recursive Least-Squares (RLS) Algorithm
- Simulation Results
- Conclusions and Perspectives

Laura-Maria Dogariu received a Bachelor degree in telecommunications systems from the Faculty of Electronics and Telecommunications (ETTI), University Politehnica of Bucharest (UPB), Romania, in 2014, and a double Master degree in wireless communications systems from UPB and Centrale Supélec, Université Paris-Saclay (with "*Distinction*" mention), in 2016. She received a PhD degree with "*Excellent*" mention (*SUMMA CUM LAUDE*) in 2019 from UPB and is currently a postdoctoral researcher and lecturer at the same university. Her research interests include adaptive filtering algorithms and signal processing.

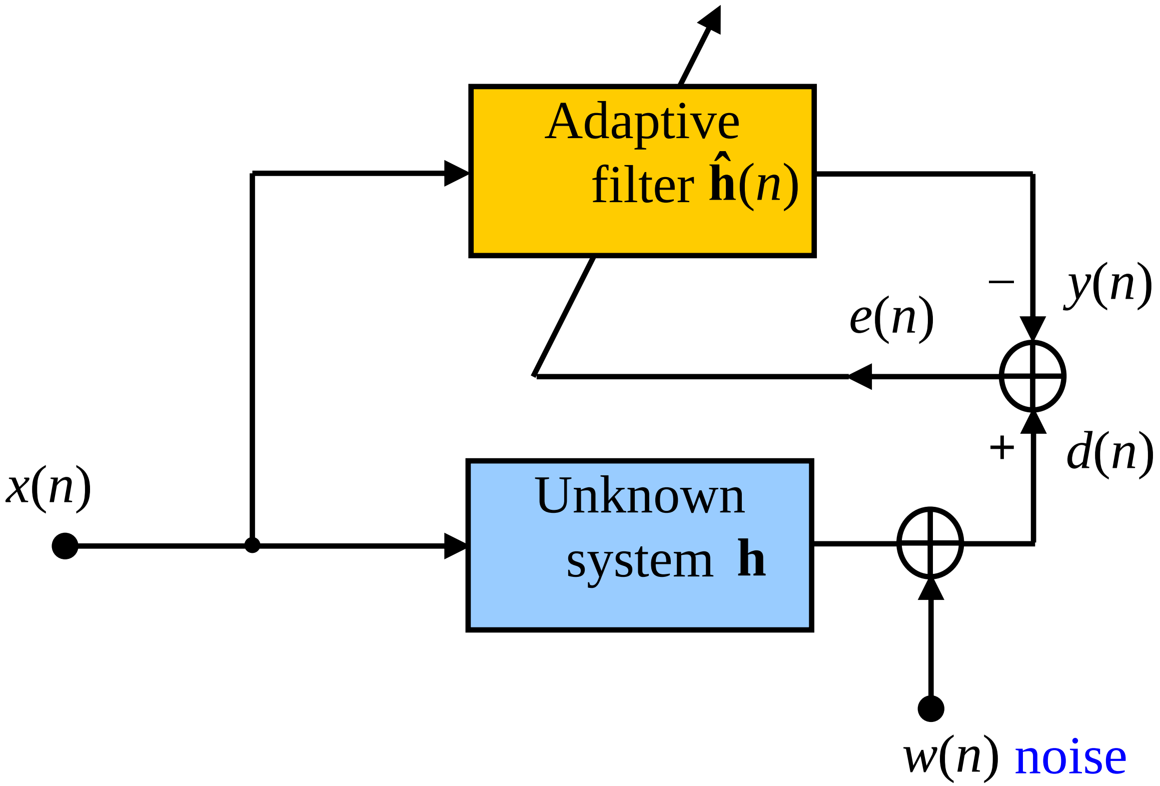
She acts as a reviewer for several important journals and conferences, such as *IEEE Transactions on Signal Processing, Signal Processing, IEEE International Symposium on Signals, Circuits and Systems (ISSCS).* She was the recipient of several prizes and scholarships, among which the Paris-Saclay scholarship, the excellence scholarship offered by Orange Romania, and an excellence scholarship from UPB. Laura Dogariu is also the winner of the competition for a postdoctoral research grant on adaptive algorithms for multilinear system identification using tensor modelling, financed by the Romanian Government, starting in 2021 (first place, with the maximum score).

Camelia Elisei-Iliescu was born in Romania in 1991. In 2014, she received the B.Sc. in telecommunications systems from Faculty of Electronics, Telecommunications, and Information Technology, University Politehnica of Bucharest (UPB), Romania. She also received the Master degree in integrated circuits and systems for communications in 2016, and a Ph.D. degree (*SUMMA CUM LAUDE*) in adaptive signal processing in 2019 coordinated by prof. Constantin Paleologu, both from the same institution. Currently, she is involved as a postdoctoral researcher (with UPB) in a research grant on adaptive algorithms for sparse system identification. Her research interests also include acoustic signal processing and DSP/FPGA implementation.

Introduction

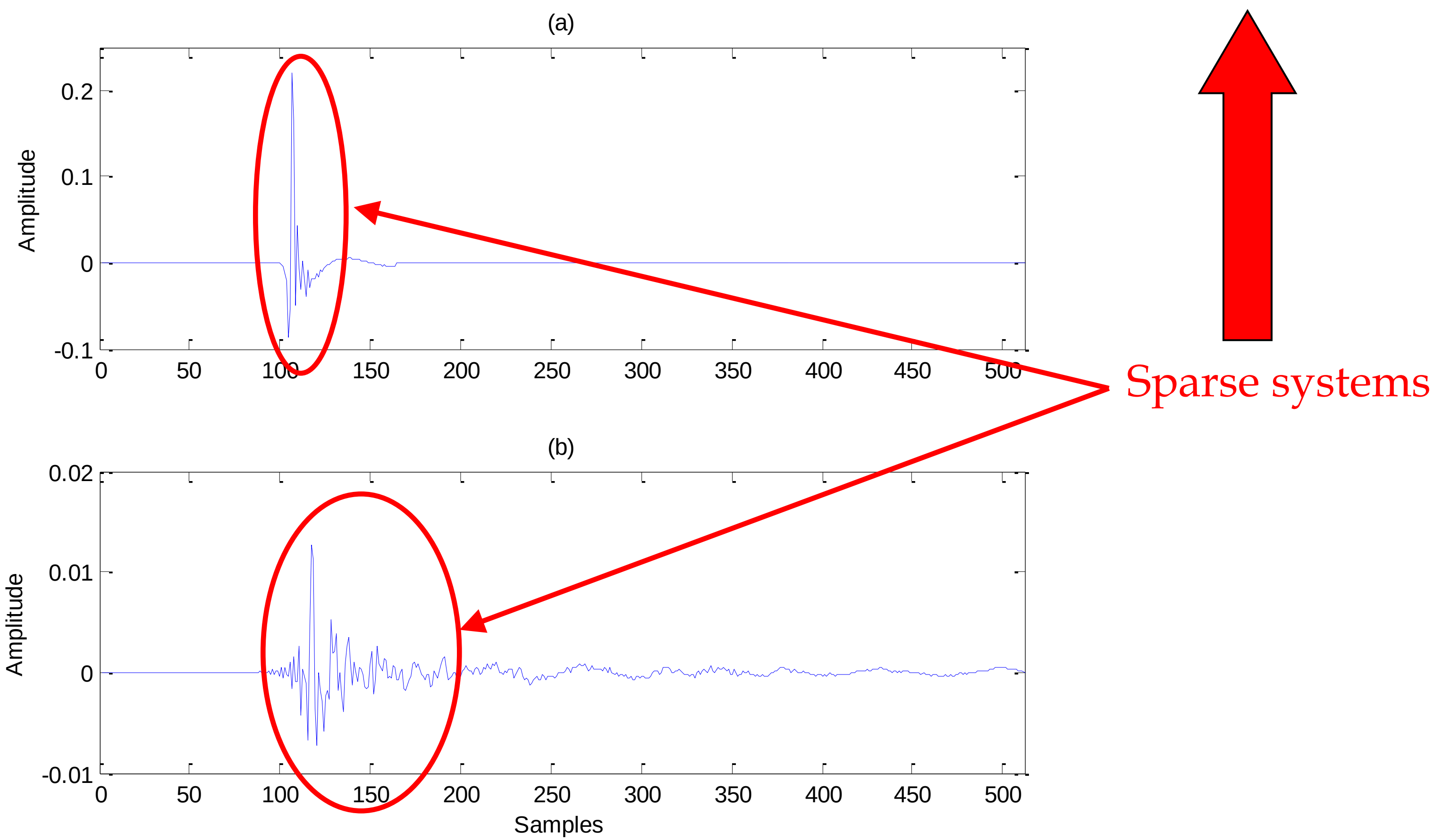
- System identification:

adaptive filter → models → *unknown system*



Sparse system identification

- many systems are *sparse* in nature ➔ low-rank systems



(a) Network impulse response (ITU-T G168/2002); (b) acoustic impulse response.

Sparse system identification (cont.)

- Existing solutions:
  - proportionate algorithms
  - block-sparse algorithms
  - zero-attraction algorithms
  - regularized algorithms (using different norms)
  - variable step-size algorithms

  Update a single filter of length L (usually long)

- Decomposition-based approach:
  - reformulating a high-dimension system identification problem as a combination of low-dimension solutions

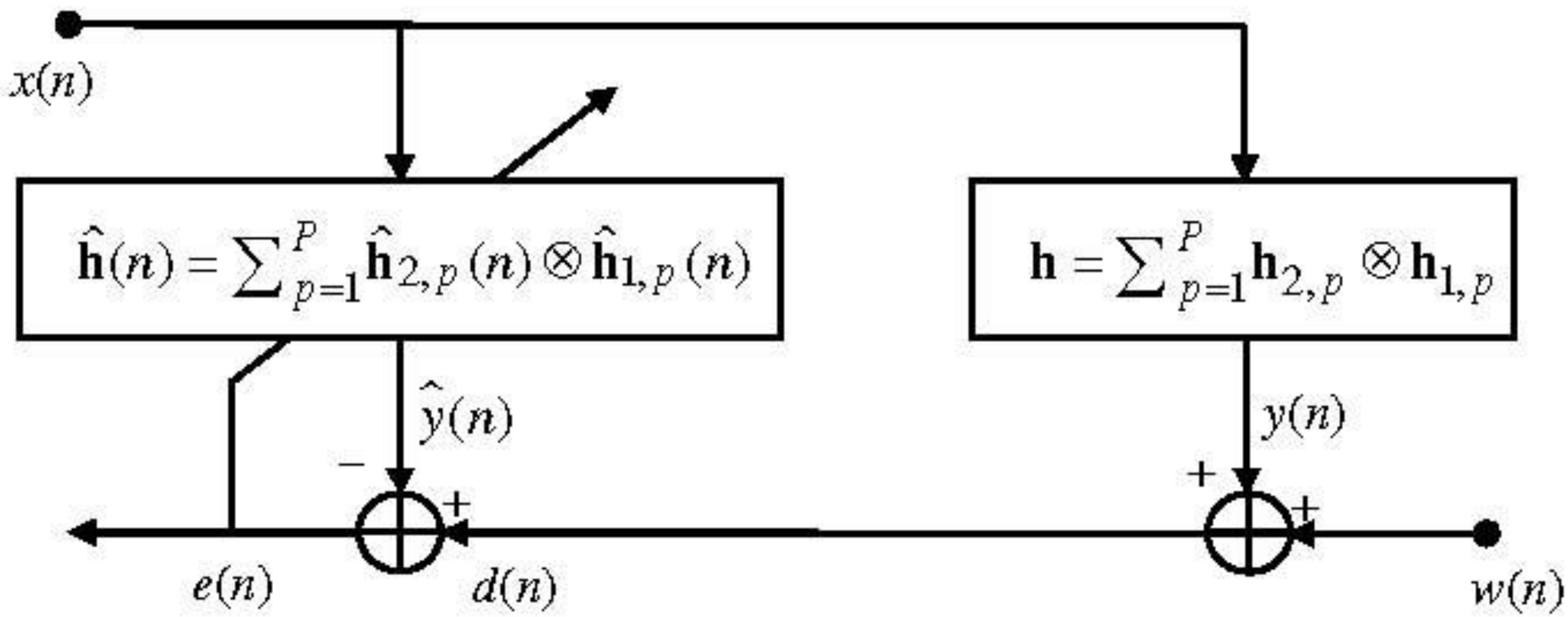
Decomposition-Based Approach

Challenge → identification of long length impulse response

(e.g., network/acoustic echo cancellation)

Solution → decomposition of impulse responses

($\otimes$ → Kronecker product)



Decomposition-Based Approach (cont.)

- Unknown impulse response **h** of length $L = L_1 L_2$
- Low-rank (*sparse*) systems, e.g., echo paths:

$$\mathbf{h} = \sum_{p=1}^{P} \mathbf{h}_{2,p} \otimes \mathbf{h}_{1,p}, \quad P \ll L_2 \quad \Rightarrow \quad \hat{\mathbf{h}}(n) = \sum_{p=1}^{P} \hat{\mathbf{h}}_{2,p}(n) \otimes \hat{\mathbf{h}}_{1,p}(n)$$

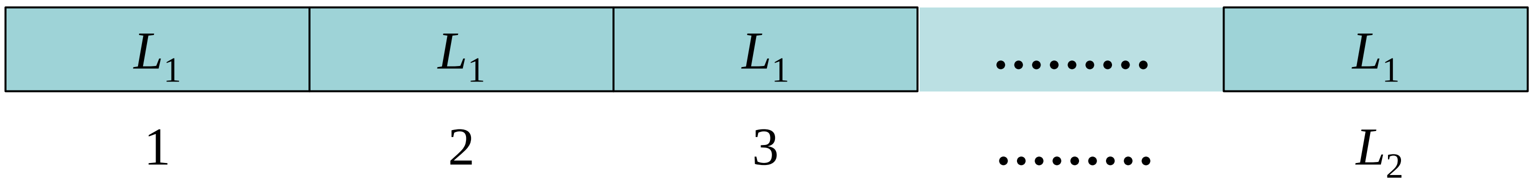
C. Paleologu, J. Benesty, and S. Ciochină, "Linear system identification based on a Kronecker product decomposition," *IEEE/ACM Transactions on Audio, Speech, and Language Processing*, vol. 26, pp. 1793–1808, Oct. 2018.

C. Elisei-Iliescu, C. Paleologu, J. Benesty, C. Stanciu, C. Anghel, and S. Ciochină, "Recursive least-squares algorithms for the identification of low-rank systems," *IEEE/ACM Transactions on Audio, Speech, Language Processing*, vol. 27, pp. 903–918, May 2019.

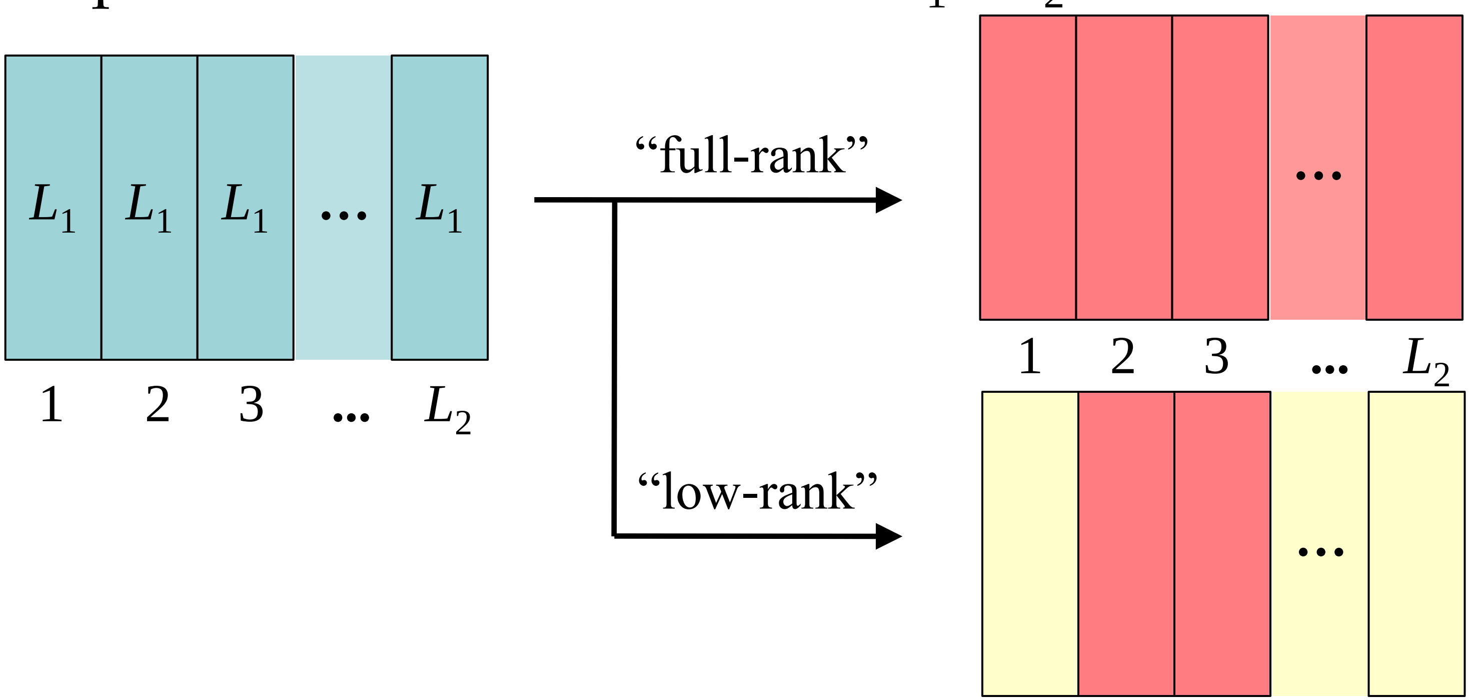
L. M. Dogariu, C. Paleologu, J. Benesty, and S. Ciochină, "An efficient Kalman filter for the identification of low-rank systems," *Signal Processing*, vol. 166, pp. 107239, Jan. 2020.

Decomposition-Based Approach (cont.)

- Unknown impulse response **h** of length $L = L_1 L_2$



- Reshape vector **h** ➔ **H** - matrix $L_1 \times L_2$



Decomposition-Based Approach (cont.)

- Singular value decomposition (SVD) of $\mathbf{H}$

$$\mathbf{H} = \mathbf{U}_1 \boldsymbol{\Sigma} \mathbf{U}_2 = \sum_{l=1}^{L_2} \sigma_l \mathbf{u}_{1,l} \mathbf{u}_{2,l}^T \quad \Leftrightarrow \quad \mathbf{h} = \sum_{l=1}^{L_2} \sigma_l \mathbf{u}_{2,l} \otimes \mathbf{u}_{1,l}$$

singular values (σ_l) — Kronecker product ($\otimes$)

- Low-rank matrix ➔ $\operatorname{rank}(\mathbf{H}) = P \ll L_2 \rightarrow \quad \sigma_l \approx 0, \quad P < l \leq L_2$

➡ $$\mathbf{H} \approx \sum_{p=1}^{P} \sigma_p \mathbf{u}_{1,p} \mathbf{u}_{2,p}^T \quad \Leftrightarrow \quad \mathbf{h} \approx \sum_{p=1}^{P} \sigma_p \mathbf{u}_{2,p} \otimes \mathbf{u}_{1,p}$$

➡ $$\mathbf{h} \approx \mathbf{h}(P) = \sum_{p=1}^{P} \mathbf{h}_{2,p} \otimes \mathbf{h}_{1,p}$$

$$\mathbf{h}_{1,p} = \sqrt{\sigma_p}\, \mathbf{u}_{1,p}$$

$$\mathbf{h}_{2,p} = \sqrt{\sigma_p}\, \mathbf{u}_{2,p}$$

Nearest Kronecker product (NKP) decomposition

Decomposition-Based Approach (cont.)

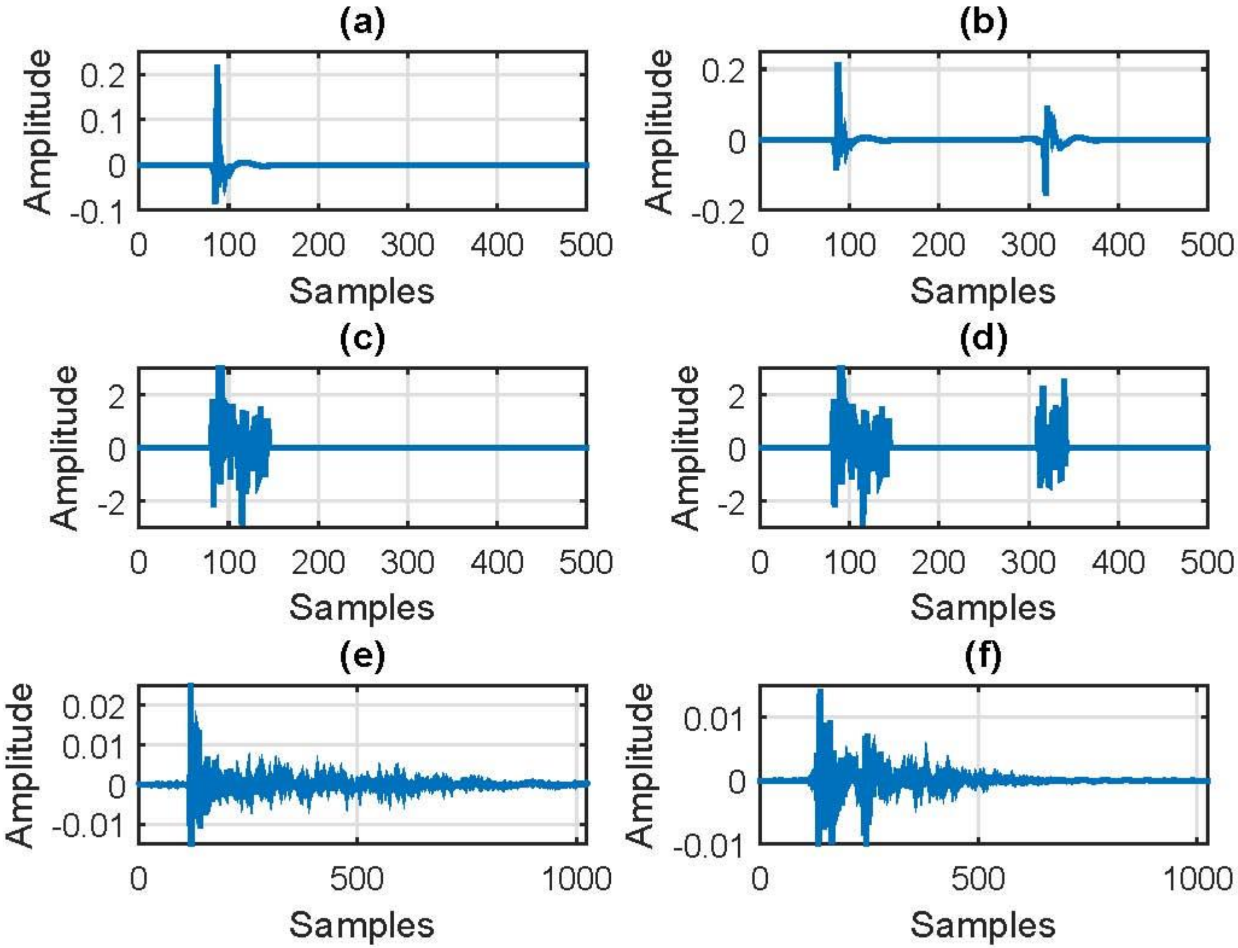


Fig. 1. Impulse responses used in simulations, with $L = 500$ or $L = 1024$.

Decomposition-Based Approach (cont.)

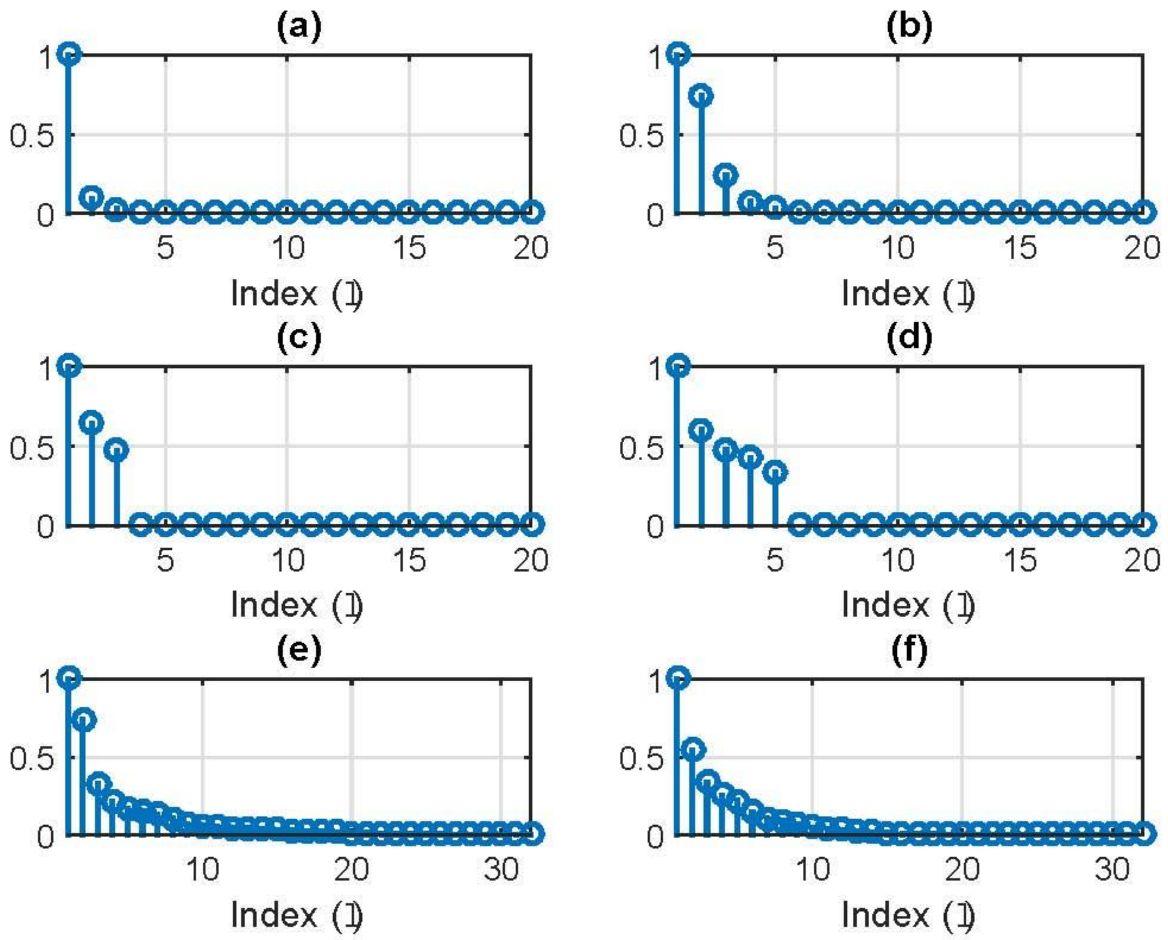


Fig. 2. Corresponding singular values of **H**, for

(a)-(d) $L_1 = 25$ and $L_2 = 20$ and (e), (f) $L_1 = L_2 = 32$.

Decomposition-Based Approach (cont.)

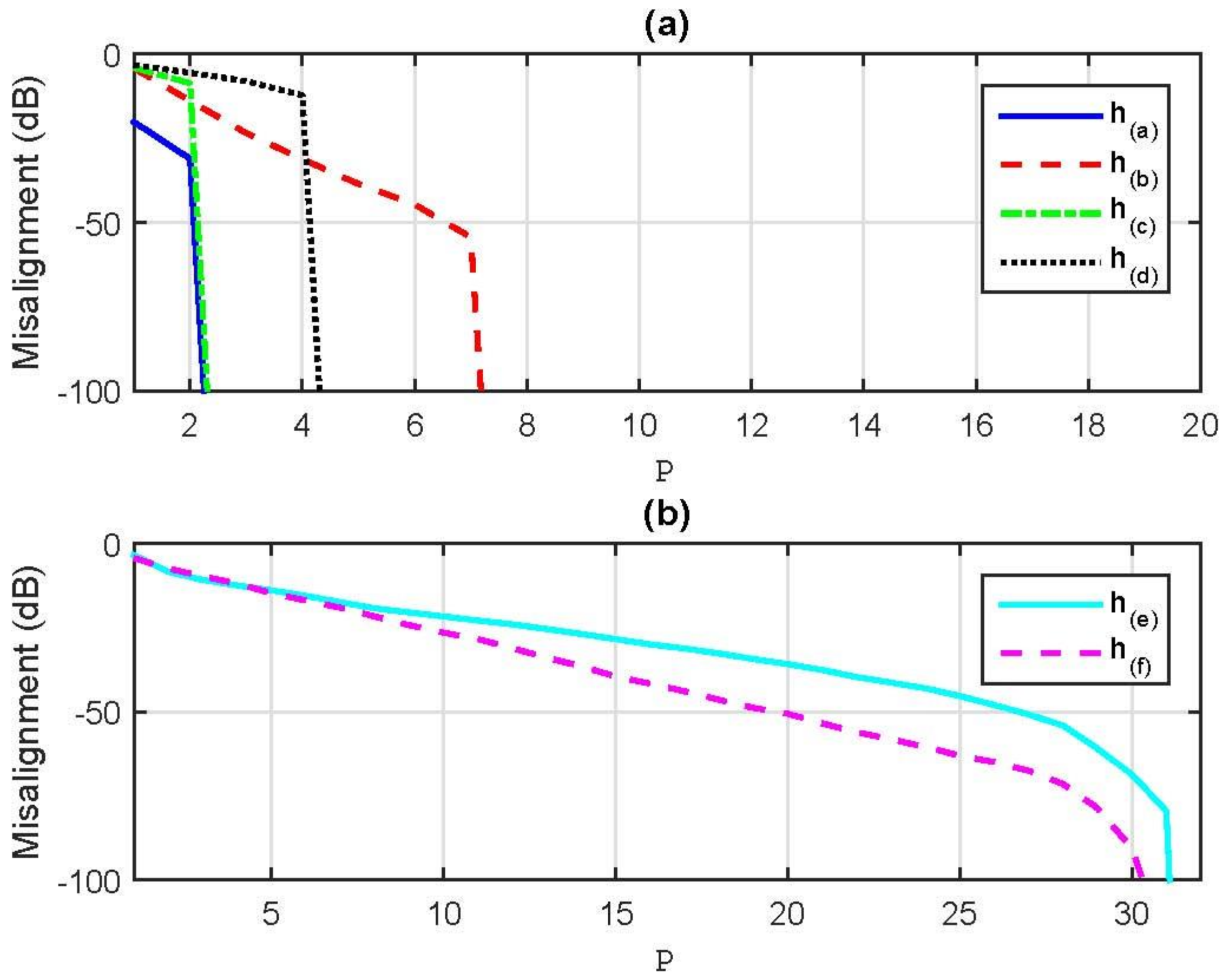
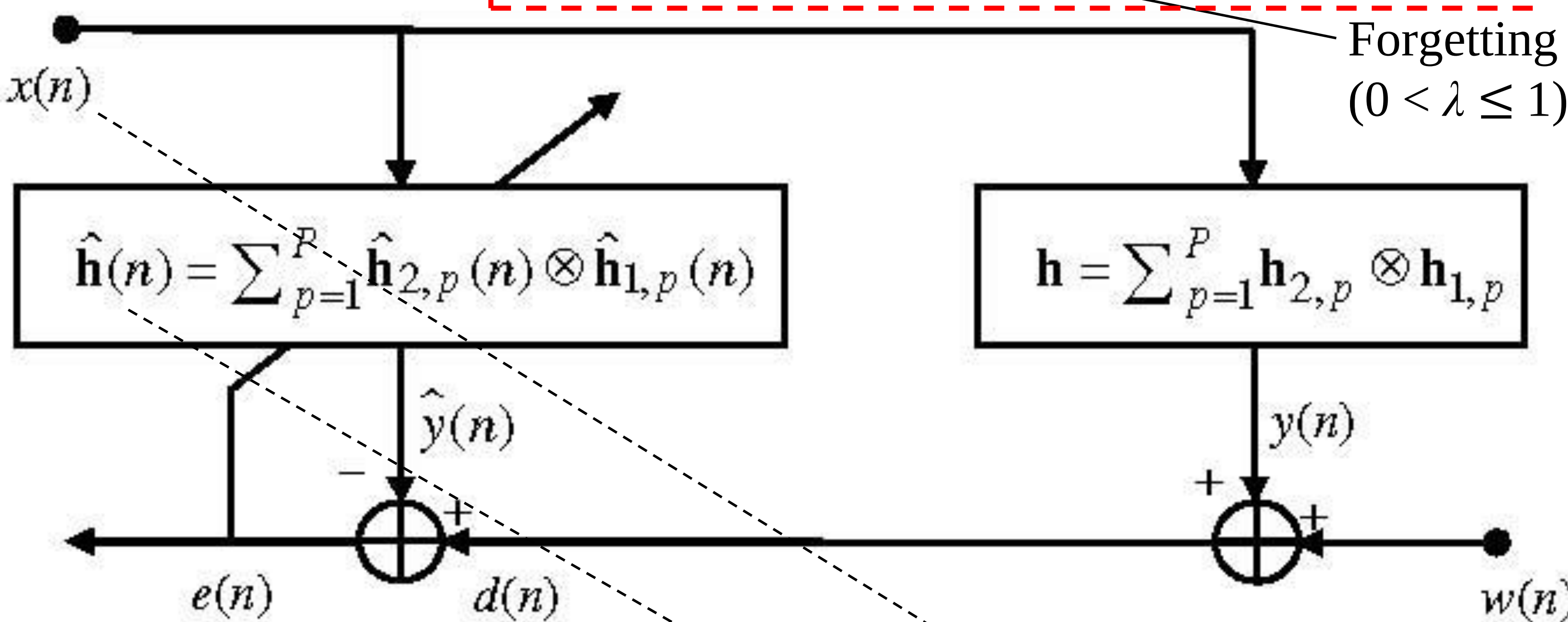


Fig. 3. Normalized misalignment (dB): $20\log_{10}\left[\left\|\mathbf{h}-\mathbf{h}(P)\right\|_2 / \left\|\mathbf{h}\right\|_2\right]$

RLS Algorithm

- Cost function: $J\left[\hat{\mathbf{h}}(n)\right]=\sum_{k=1}^{n}\lambda^{n-k}\left[d(k)-\hat{\mathbf{h}}^{T}(n)x(k)\right]^{2}$

Forgetting factor $(0<\lambda\leq 1)$



- RLS algorithm: $e(n)=d(n)-\hat{\mathbf{h}}^{T}(n-1)\mathbf{x}(n)$

$L=L_1L_2$ → $\hat{\mathbf{h}}(n)=\hat{\mathbf{h}}(n-1)+\mathbf{k}(n)e(n)$

Kalman gain vector (evaluated within the algorithm)

RLS Algorithm (cont.)

$$\hat{\mathbf{h}}(n) = \sum_{p=1}^{P} \hat{\mathbf{h}}_{2,p}(n) \otimes \hat{\mathbf{h}}_{1,p}(n)$$

L → $\hat{\mathbf{h}}(n)$, L_2 → $\hat{\mathbf{h}}_{2,p}(n)$, L_1 → $\hat{\mathbf{h}}_{1,p}(n)$

$L = L_1 L_2 \quad (P \,\square\, L_2)$

- Notation:

$$\underline{\widehat{\mathbf{h}}}_1(n) = \left[\ \widehat{\mathbf{h}}_{1,1}^T(n) \quad \widehat{\mathbf{h}}_{1,2}^T(n) \quad \cdots \quad \widehat{\mathbf{h}}_{1,P}^T(n)\ \right]^T \longrightarrow PL_1 \quad (\square\ L)$$

$$\underline{\widehat{\mathbf{h}}}_2(n) = \left[\ \widehat{\mathbf{h}}_{2,1}^T(n) \quad \widehat{\mathbf{h}}_{2,2}^T(n) \quad \cdots \quad \widehat{\mathbf{h}}_{2,P}^T(n)\ \right]^T \longrightarrow PL_2 \quad (\square\ L)$$

$$\widehat{\mathbf{x}}_{2,p}(n) = \left[\widehat{\mathbf{h}}_{2,p}(n-1) \otimes \mathbf{I}_{L_1}\right]^T \mathbf{x}(n),$$

$$\underline{\widehat{\mathbf{x}}}_2(n) = \left[\ \widehat{\mathbf{x}}_{2,1}^T(n) \quad \widehat{\mathbf{x}}_{2,2}^T(n) \quad \cdots \quad \widehat{\mathbf{x}}_{2,P}^T(n)\ \right]^T$$

$$\widehat{\mathbf{x}}_{1,p}(n) = \left[\mathbf{I}_{L_2} \otimes \widehat{\mathbf{h}}_{1,p}(n-1)\right]^T \mathbf{x}(n),$$

$$\underline{\widehat{\mathbf{x}}}_1(n) = \left[\ \widehat{\mathbf{x}}_{1,1}^T(n) \quad \widehat{\mathbf{x}}_{1,2}^T(n) \quad \cdots \quad \widehat{\mathbf{x}}_{1,P}^T(n)\ \right]^T$$

RLS Algorithm (cont.)

- New cost functions:

$$J_{\underline{\hat{\mathbf{h}}}_2}\left[\underline{\hat{\mathbf{h}}}_1(n)\right] = \sum_{k=1}^{n} \lambda_1^{n-k}\left[d(k) - \underline{\hat{\mathbf{h}}}_1^T(n)\underline{\mathbf{x}}_2(k)\right]^2$$

$$J_{\underline{\hat{\mathbf{h}}}_1}\left[\underline{\hat{\mathbf{h}}}_2(n)\right] = \sum_{k=1}^{n} \lambda_2^{n-k}\left[d(k) - \underline{\hat{\mathbf{h}}}_2^T(n)\underline{\mathbf{x}}_1(k)\right]^2$$

Forgetting factors:

$0 < \lambda_1 \leq 1$

$0 < \lambda_2 \leq 1$

⇒ Bilinear optimization strategy:

⇒ $\underline{\hat{\mathbf{h}}}_2(k)$ is considered fixed for $0 < k \leq n-1$ within the optimization of $\underline{\hat{\mathbf{h}}}_1(n)$

$\underline{\hat{\mathbf{h}}}_1(k)$ is considered fixed for $0 < k \leq n-1$ within the optimization of $\underline{\hat{\mathbf{h}}}_2(n)$

RLS Algorithm (cont.)

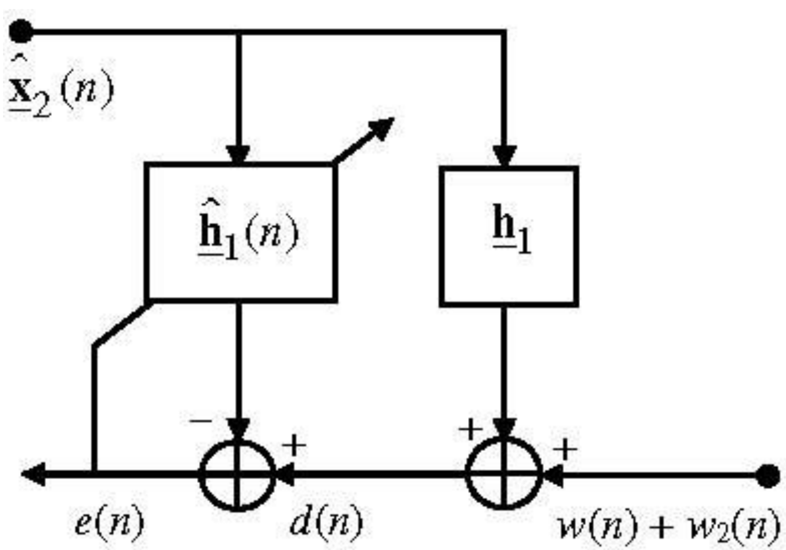
- RLS-NKP algorithm:

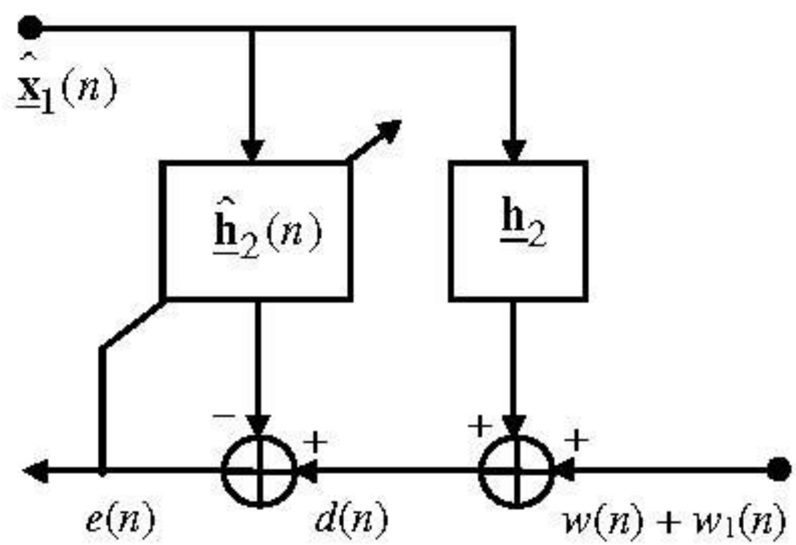
$$e_1(n) = d(n) - \underline{\hat{\mathbf{h}}}_1^T(n-1)\underline{\hat{\mathbf{x}}}_2(n) = e(n)$$

$$e_2(n) = d(n) - \underline{\hat{\mathbf{h}}}_2^T(n-1)\underline{\hat{\mathbf{x}}}_1(n) = e(n)$$

$PL_1 \longrightarrow$ $$\underline{\hat{\mathbf{h}}}_1(n) = \underline{\hat{\mathbf{h}}}_1(n-1) + \mathbf{k}_2(n)e_1(n)$$

$PL_2 \longrightarrow$ $$\underline{\hat{\mathbf{h}}}_2(n) = \underline{\hat{\mathbf{h}}}_2(n-1) + \mathbf{k}_1(n)e_2(n)$$





RLS Algorithm (cont.)

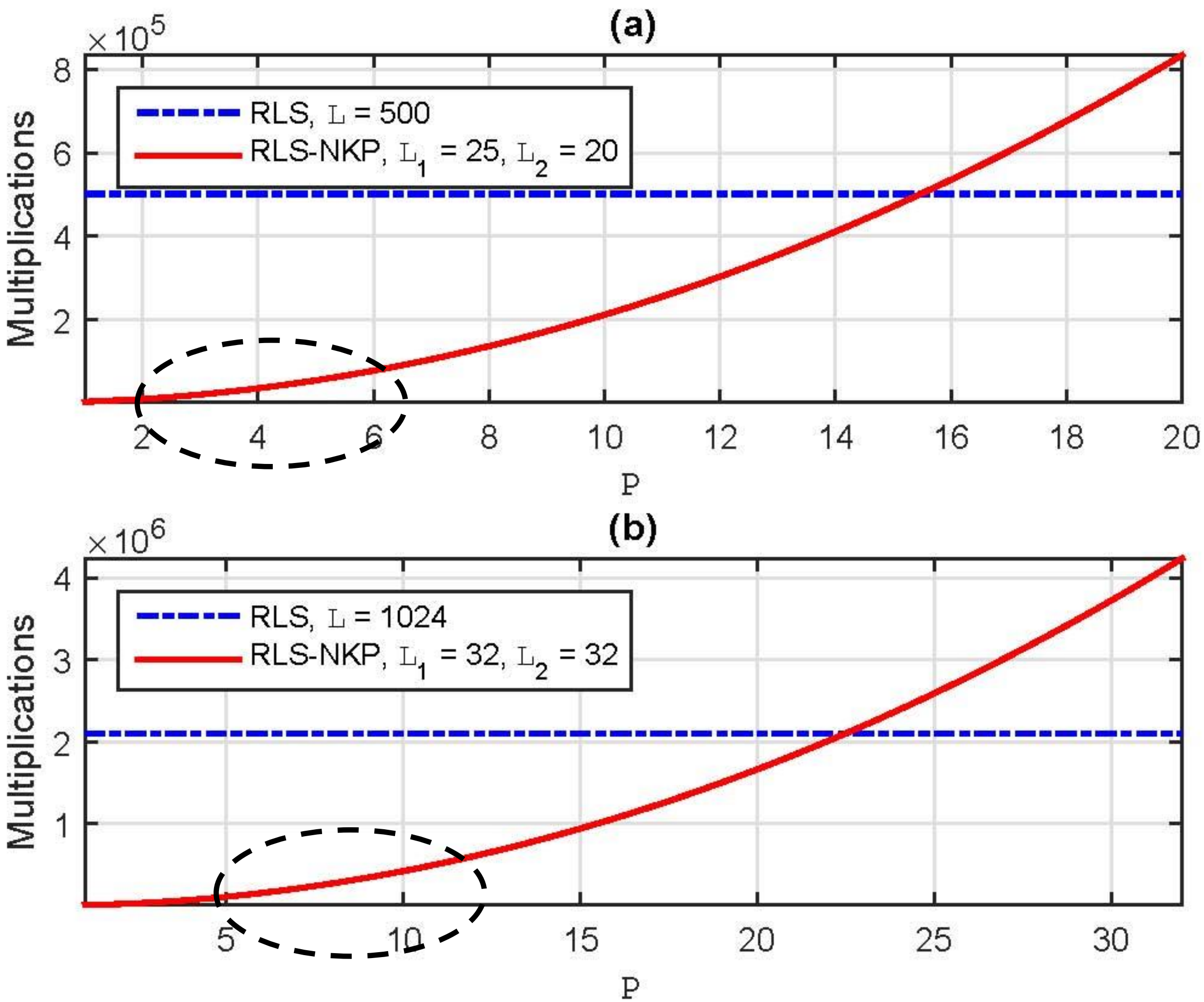


Fig. 4. Computational complexity (no. multiplications): RLS and RLS-NKP.

Simulation Results

- **conditions**

  → **h** from Fig. 1, with L = 500 or L = 1024.

  → NKP decomposition: $L = 500$ ➔ $L_1 = 25$, $L_2 = 20$

  $L = 1024$ ➔ $L_1 = L_2 = 32$

  → input signals – AR1(0.9) process or speech.

  → additive noise $w(n)$ – white Gaussian noise, SNR = 20 dB.

  → performance measure : normalized misalignment (dB).

$$20\log_{10}\left[\left\|\mathbf{h}-\hat{\mathbf{h}}(n)\right\|_2 / \left\|\mathbf{h}\right\|_2\right]$$

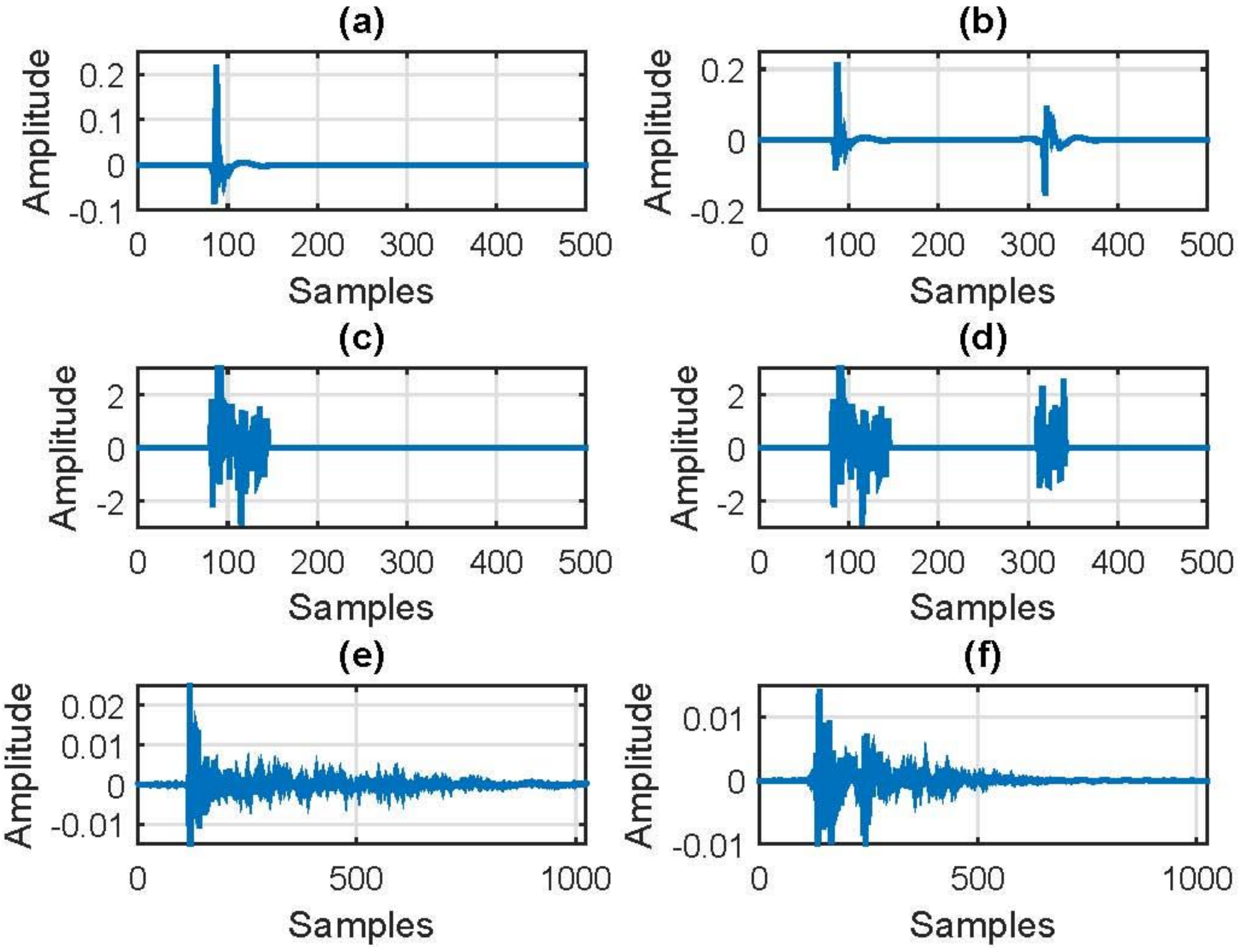
- **algorithms**

  → conventional RLS [Haykin, *Adaptive Filter Theory*, 2002]

  → RLS-DCD [Zakharov *et al.*, *IEEE Trans. Signal Process.*, 2008]

  → RLS-NKP

Simulation Results (cont.)



- Impulse responses used in simulations, with $L = 500$ and $L = 1024$.

Simulation Results (cont.)

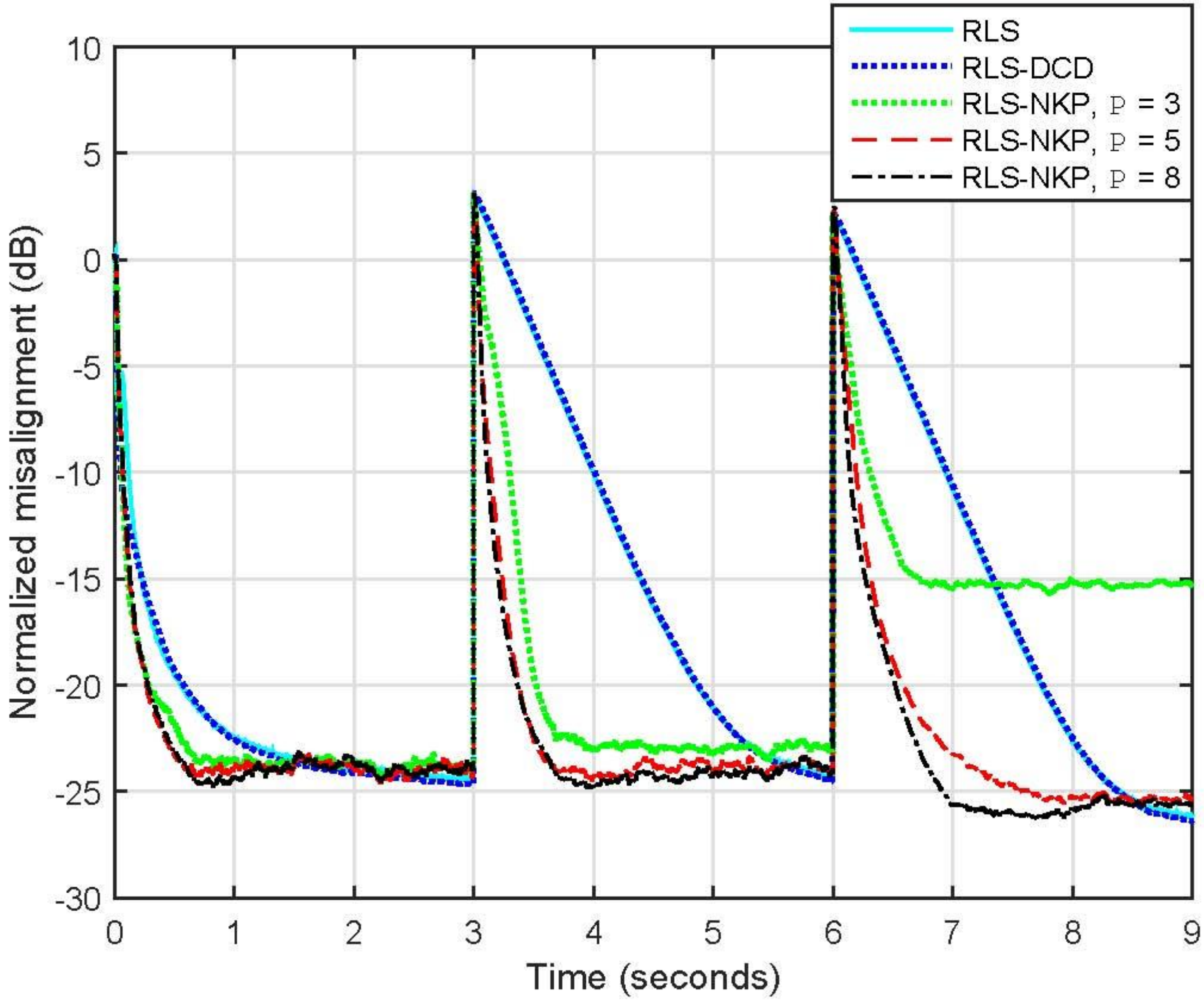


Fig. 5. Performance of the RLS, RLS-DCD, and RLS-NKP algorithms for the identification of the impulse responses from Figs. 1(a) and (b).

Simulation Results (cont.)

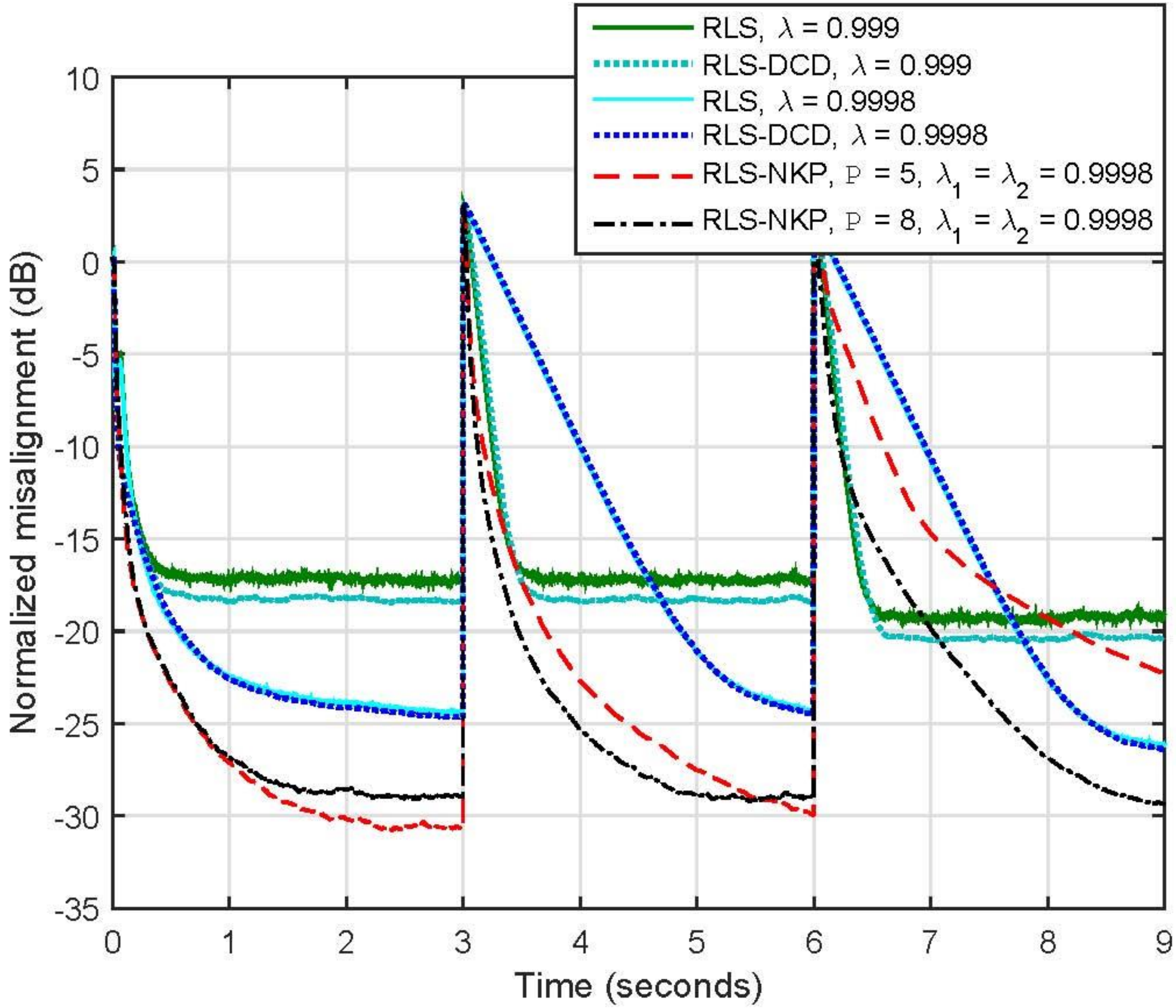


Fig. 6. Performance of the RLS, RLS-DCD, and RLS-NKP algorithms for the identification of the impulse responses from Figs. 1(a) and (b).

Simulation Results (cont.)

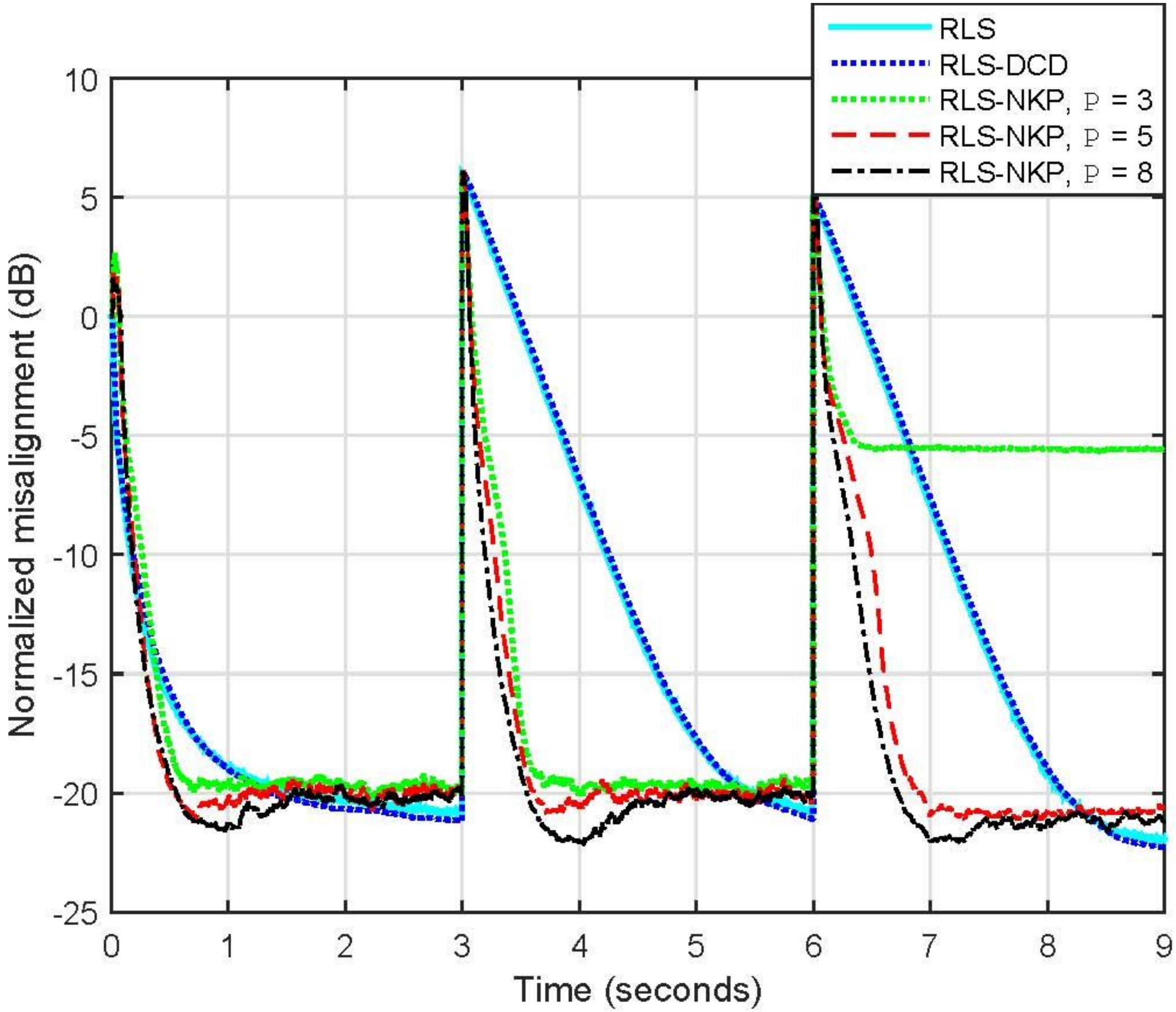


Fig. 7. Performance of the RLS, RLS-DCD, and RLS-NKP algorithms for the identification of the impulse responses from Figs. 1(c) and (d).

Simulation Results (cont.)

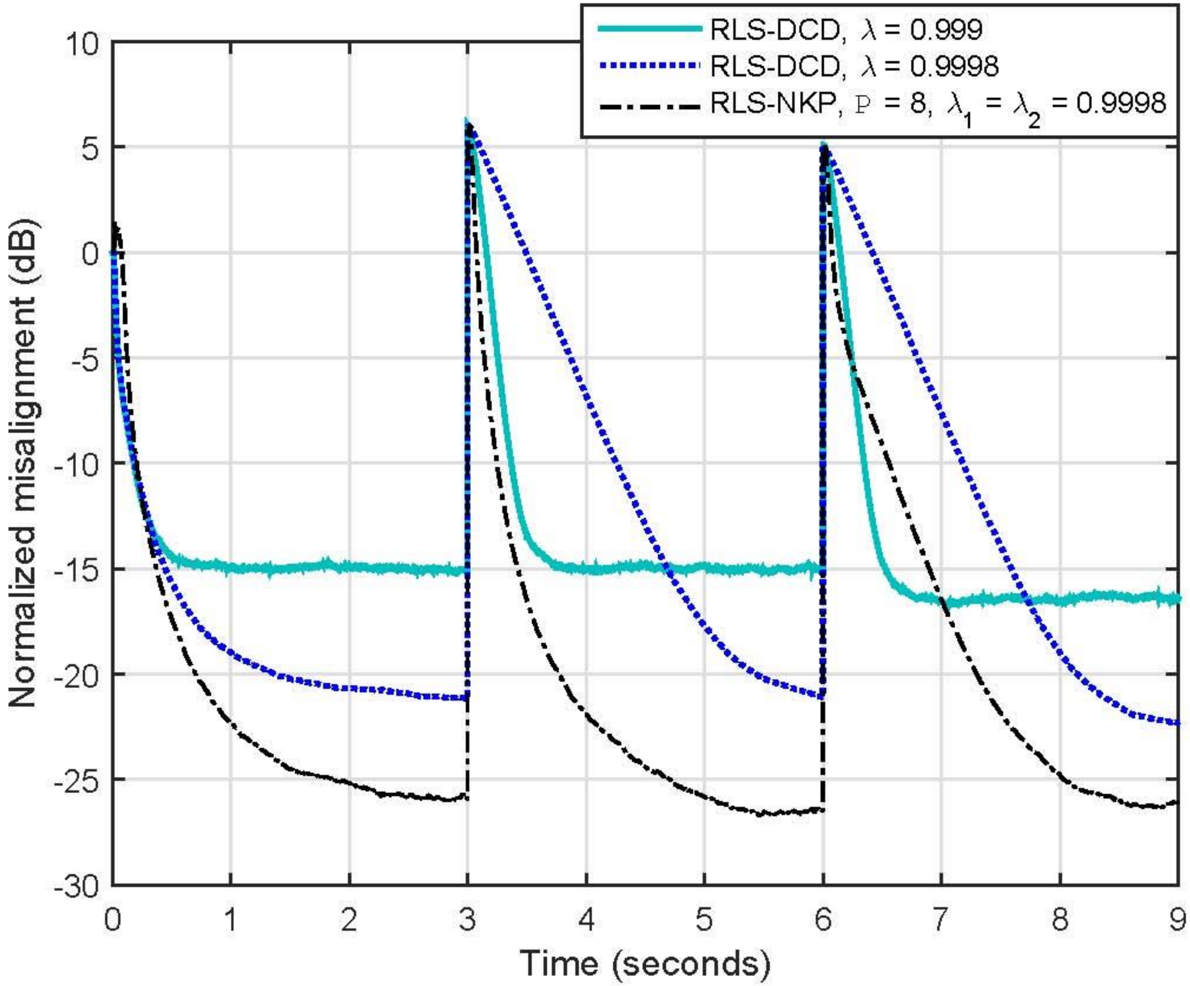


Fig. 8. Performance of the RLS, RLS-DCD, and RLS-NKP algorithms for the identification of the impulse responses from Figs. 1(c) and (d).

Simulation Results (cont.)

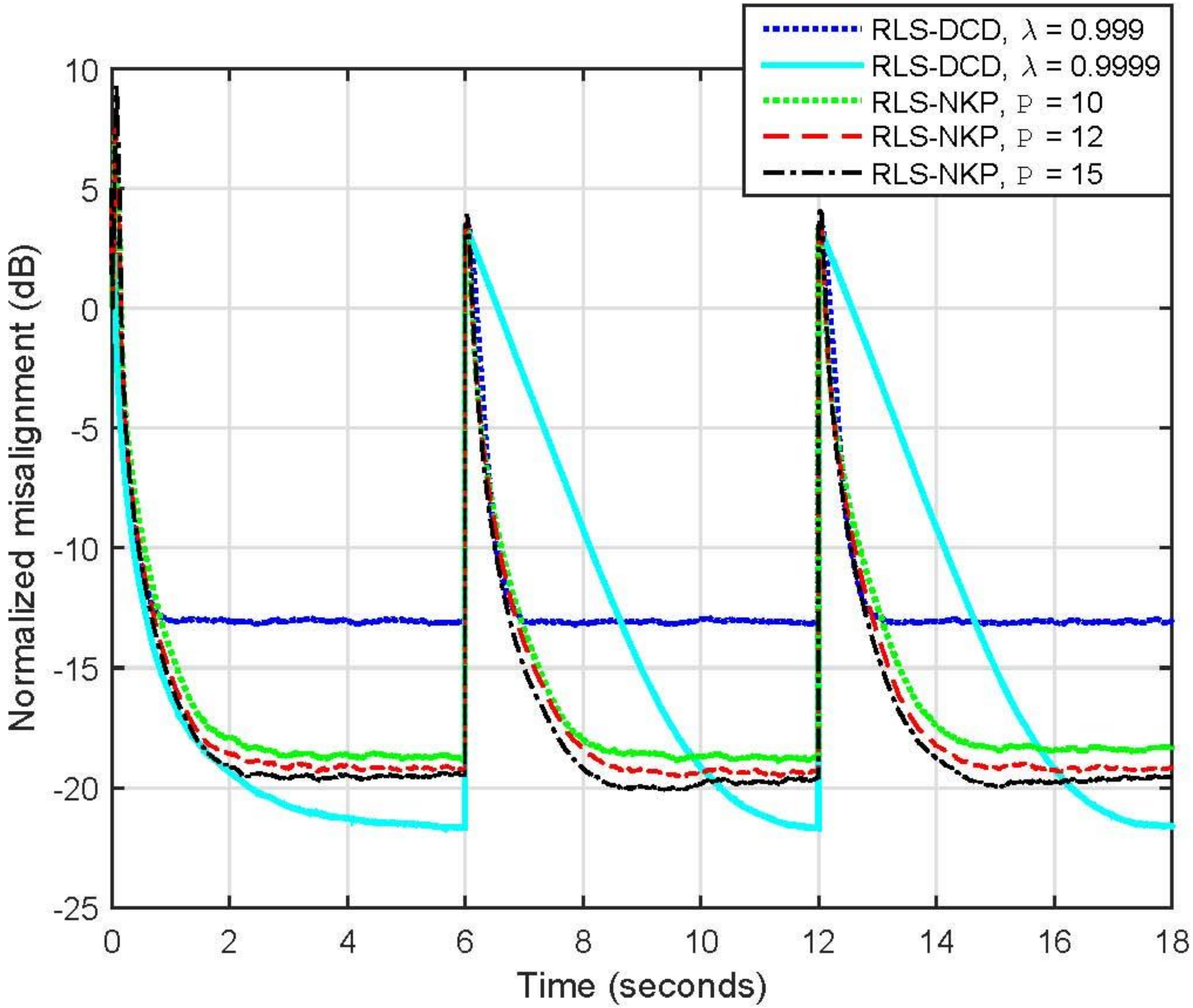


Fig. 9. Performance of the RLS, RLS-DCD, and RLS-NKP algorithms for the identification of the impulse responses from Fig. 1(e).

Simulation Results (cont.)

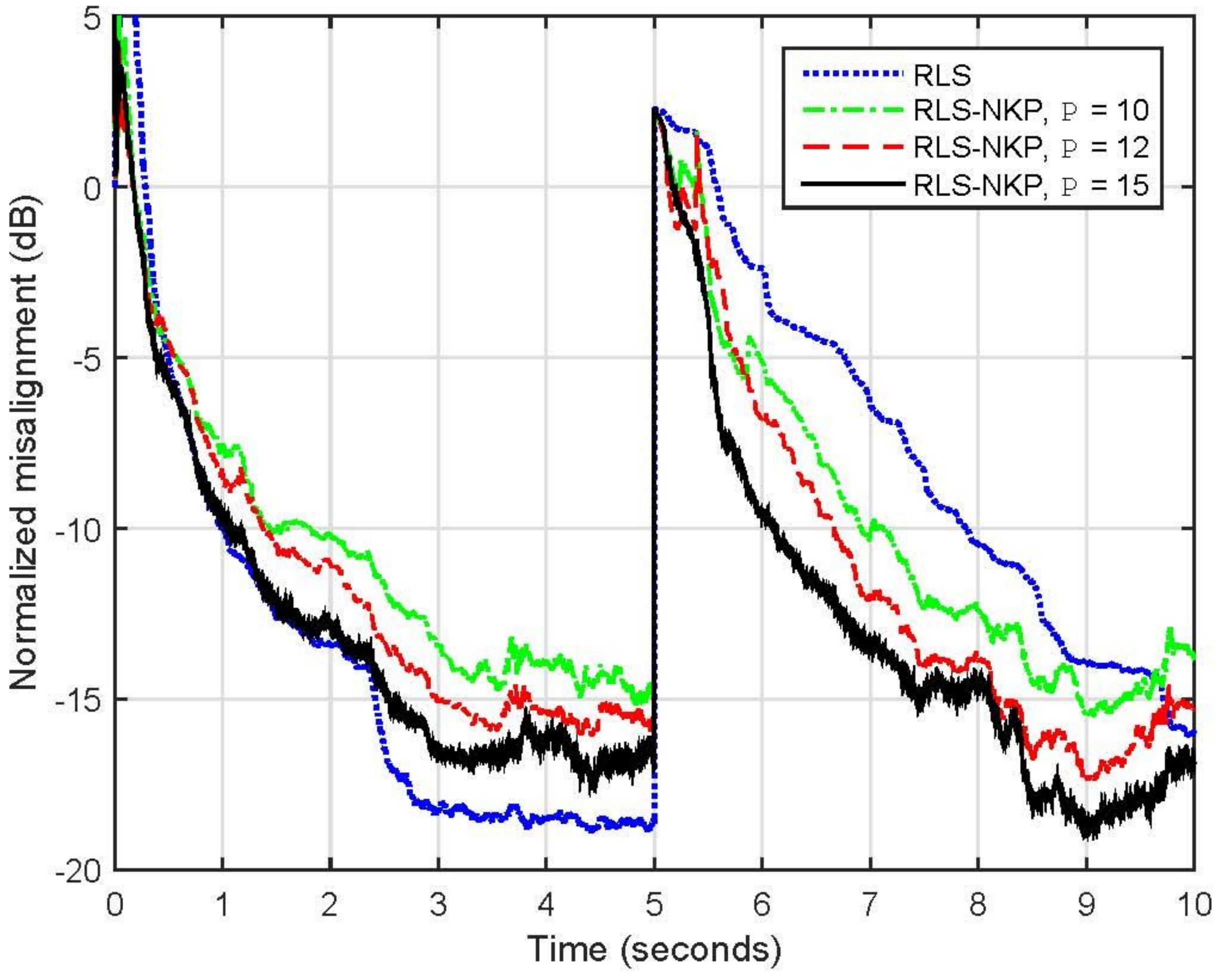


Fig. 10. Performance of the RLS, RLS-DCD, and RLS-NKP algorithms for the identification of the impulse responses from Fig. 1(f).

Conclusions and Perspectives

- Adaptive filtering algorithms exploiting the nearest Kronecker product (NKP) decomposition.
- Efficient solution for the identification of sparse/low-rank systems (e.g., echo paths).
- High-dimension system identification problem → reformulated as a combination of low-dimension solutions (shorter filters).
- Recursive least-squares (RLS)-NKP algorithm → fast convergence & tracking, lower computational complexity.
- RLS-NKP algorithm outperforms the conventional RLS and RLS-DCD algorithms.
- **Future works**: extension to multidimensional case (decomposition based on high-order tensors).

Thank you for your attention!