Tensor-Based Recursive Least Squares Algorithm with Low Computational Complexity

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Presenter's Biography

- **Current:**
  - **PhD student**
    - @ Doctoral School of Electronics, Telecommunications & Information Technology, University Politehnica of Bucharest since October 2020
    - **Thesis subject:** Efficient algorithms for acoustic applications
    - **Coordinator:** Prof. Constantin Paleologu

- **Past:**
  - **Bachelor's degree** (Valedictorian)
    - @ Telecommunications Technologies and Systems (TST), University Politehnica of Bucharest (2014 - 2018)
    - **Diploma thesis:** Convolutional Neural Networks for Object Segmentation and Tracking in Video Sequences
    - **Coordinators:** Prof. Mihai Ciuc, PhD. Cosmin Toca
  - **Master's degree**
    - @ Advanced Digital Imaging Techniques (TAID), University Politehnica of Bucharest (2018 - 2020)
    - **Dissertation thesis:** Deep neural networks for environmental sounds classification
    - **Coordinators:** Assoc. Prof. Cristian-Lucian Stanciu, Assoc. Prof. Cristian Anghel

Outline

- Introduction
- Multilinear forms
- RLS-based Algorithms
- The exponential weighted **RLS-DCD-T** algorithm
- Experiments
- Conclusions

Introduction

- **Adaptive filters → system identification problems**
- **Targets → convergence rate, accurate estimation, complexity**
- **MISO systems → large parameter space → linearly separable systems**

  (beamforming, nonlinear AEC, channel equalization, source separation)
- **Efficient solutions → tensor-based adaptive filters → decomposition of rank-1 tensors → global solution = combination of shorter adaptive filters**
- **In this work →** exponentially weighted **RLS** (for MISO) **+ DCD** iterations

Multilinear forms

Framework = a real-valued MISO system ⇒ **the output signal:**

$$y(n) = \sum_{l_1=1}^{L_1} \sum_{l_2=1}^{L_2} \dots \sum_{l_N=1}^{L_N} x_{l_1 l_2 \dots l_N}(n) h_{1,l_1} h_{2,l_2} \dots h_{N,l_N},$$

where $\mathbf{h}_i = [h_{i,1} \ h_{i,2} \ \dots \ h_{1,L_i}]^T$ are N individual channels of length L_i

The input signals ⇒ **Tensorial form:** $\chi(n) \in \mathbb{R}^{L_1 \times L_2 \times \cdots \times L_N}$, with $(\chi)_{l_1 l_2 \dots l_N}(n) = x_{l_1 l_2 \dots l_N}(n)$

The output signal becomes: $y(n) = \chi(n) \times_1 \mathbf{h}_1^T \times_2 \mathbf{h}_2^T \times_3 \dots \times_N \mathbf{h}_N^T,$ (mode-N product)

$y(n)$ has a linear function of each $\mathbf{h}_i$, when the other $N-1$ components are fixed

$y(n)$ **is s a multilinear form**

Multilinear forms

- Let us consider the **tensor** $\mathcal{H} \in \mathbb{R}^{L_1 \times L_2 \times \cdots \times L_N}$ with the **elements** $(\mathcal{H})_{l_1,l_2,\ldots,l_N} = h_{1,l_1} h_{2,l_2} \ldots h_{N,l_N}$ such that $\mathcal{H} = \mathbf{h}_1 \circ \mathbf{h}_2 \circ \cdots \circ \mathbf{h}_N$ (outer product)

- Also, $\text{vec}(\mathcal{H}) = \mathbf{h}_N \otimes \mathbf{h}_{N-1} \otimes \cdots \otimes \mathbf{h}_1$ (vectorization; Kronecker product) ⇒ $y(n) = \text{vec}^T(\mathcal{H})\, \text{vec}[\boldsymbol{\chi}(n)]$

- Additionally, $\mathrm{x}(n) = \text{vec}[\boldsymbol{\chi}(n)]$ (input vector of length $L_1 L_2 \ldots L_N$) and $\mathbf{g} = \text{vec}(\mathcal{H})$ (global impulse response of length $L_1 L_2 \ldots L_N$)

- **The output signal becomes:** $y(n) = \mathbf{g}^T \mathbf{x}(n)$

- **The reference signal usually results as:** $d(n) = \mathbf{g}^T \mathbf{x}(n) + w(n)$ (measurement noise)

- **Goal** => identification of the global system $\mathbf{g}$, i.e. the components $\mathbf{h}_i$, $i = 1,2,\ldots,N$

RLS-based Algorithms

The RLS-T algorithm → complexity $\sim \sum_{i=1}^{N} \mathcal{O}\left({L_i}^2\right)$

Kalman gain vectors

$$e_{\hat{\mathbf{h}}_\mathrm{i}}(n) = d(n) - \hat{\mathbf{h}}_i^T(n-1)\mathrm{x}_{\hat{\mathbf{h}}_i}(n)$$

LS error criterion + Minimization of cost functions →

$$\hat{\mathbf{h}}_i\,(n) = \hat{\mathbf{h}}_i\,(n-1) + \mathbf{k}_i(n)\, e_{\hat{\mathbf{h}}_\mathrm{i}}(n)$$

$$\mathrm{x}_{\hat{\mathbf{h}}_i}(n) = \left[\hat{\mathbf{h}}_N(n-1) \otimes \hat{\mathbf{h}}_{N-2}(n-1) \otimes \cdots \otimes \hat{\mathbf{h}}_{i+1}(n-1) \otimes \mathbf{I}_{L_i} \otimes \hat{\mathbf{h}}_{i-1}\,(n-1) \otimes \cdots \otimes \hat{\mathbf{h}}_2(n-1) \otimes \hat{\mathbf{h}}_1(n-1)\right]^T \mathbf{x}(n)$$

$$\mathbf{k}_i(n) = \frac{\mathbf{R}_i^{-1}(n-1)\,\mathrm{x}_{\hat{\mathbf{h}}_i}(n)}{\lambda_i + \mathrm{x}_{\hat{\mathbf{h}}_i}^T(n)\mathbf{R}_i^{-1}(n-1)\,\mathrm{x}_{\hat{\mathbf{h}}_i}(n)}$$

Matrix inversion lemma →

$$\mathbf{R}_i^{-1}(n) = \frac{1}{\lambda_i}\left[\mathbf{I}_{L_i} - \mathbf{k}_i(n)\,\mathrm{x}_{\hat{\mathbf{h}}_i}^T(n)\right]\mathbf{R}_i^{-1}(n-1)$$

Individual forgetting factors

The RLS-DCD-T algorithm → complexity $\sim \sum_{i=1}^{N} \mathcal{O}(L_i)$

DCD iterations

$$\hat{\mathbf{h}}_i\,(n) = \hat{\mathbf{h}}_i\,(n-1) + \mathbf{k}_i(n)\, e_{\hat{\mathbf{h}}_\mathrm{i}}(n) \;\rightarrow\; \hat{\mathbf{h}}_i\,(n) = \hat{\mathbf{h}}_i\,(n-1) + \Delta\hat{\mathbf{h}}_i\,(n)$$

The exponential weighted RLS-DCD-T algorithm

Initialization :

Set $\widehat{\mathbf{h}}_i(0) = \mathbf{0}_{L_i \times 1}, \ \mathbf{r}_i(0) = \mathbf{0}_{L_i \times 1}$

$\mathbf{R}_i^{-1}(0) = \xi_i \mathbf{I}_{L_i}, \ \xi_i > 0, \ \ \lambda_i = 1 - \frac{1}{K_i L_i}, \ K_i \geq 1$

For $n = 1, 2, \ldots,$ number of iterations :

Compute $\mathbf{x}_{\widehat{\mathbf{h}}_i}(n)$

$\mathbf{R}_i^{(0)}(n) = \lambda_i \mathbf{R}_i^{(0)}(n-1) + \mathbf{x}_{\widehat{\mathbf{h}}_i}(n)\mathbf{x}_{\widehat{\mathbf{h}}_i}^{(0)}(n)$ → weighted sample **covariance matrices**

$y_{\widehat{\mathbf{h}}_i}(n) = \widehat{\mathbf{h}}_i^T(n-1)\mathbf{x}_{\widehat{\mathbf{h}}_i}(n)$

$e_{\widehat{\mathbf{h}}_i}(n) = d(n) - y_{\widehat{\mathbf{h}}_i}(n)$ → a-priori **error signals**

$\mathbf{p}_{0,i}(n) = \lambda_i \mathbf{r}_i(n-1) + e_{\widehat{\mathbf{h}}_i}(n)\mathbf{x}_{\widehat{\mathbf{h}}_i}(n)$ → ***cross-correlation*** *vectors*

$\mathbf{R}_i(n)\Delta\mathbf{h}_i(n) = \mathbf{p}_{0,i}(n) \xrightarrow{\text{DCD}} \Delta\widehat{\mathbf{h}}_i(n), \mathbf{r}_i(n)$

$\widehat{\mathbf{h}}_i(n) = \widehat{\mathbf{h}}_i(n-1) + \Delta\widehat{\mathbf{h}}_i(n)$

- Complexity $\sim \sum_{i=1}^{N} \mathcal{O}(L_i)$
- *Only additions and multiplications*
- ***NO** divisions*

Experiment 1

- **Conditions:**

  → Input signals – AR(1) processes; each one is generated by filtering a white Gaussian noise through a first-order system with the pole 0.85

  → The order of the system: $N = 4$

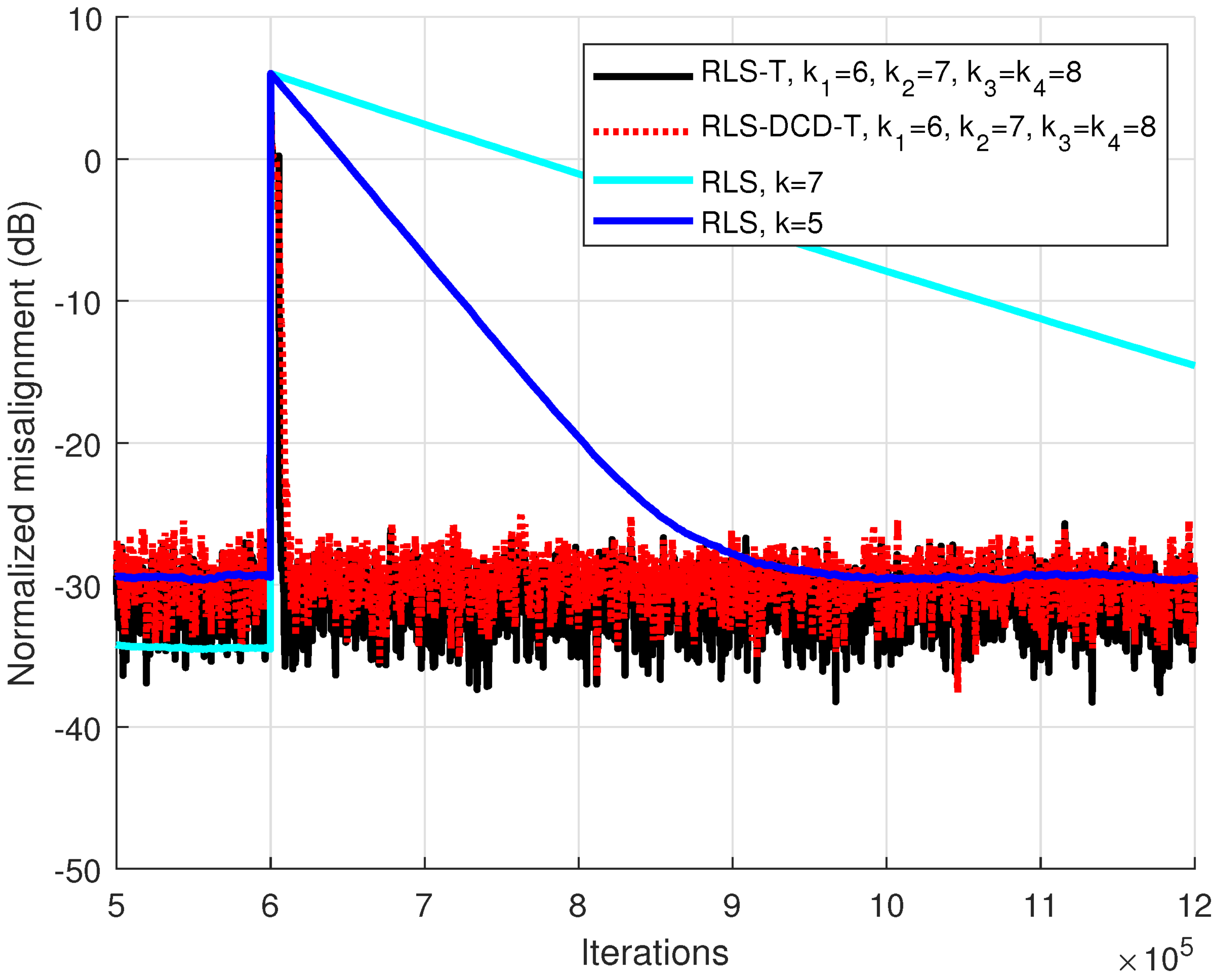
  → Individual impulse response $\mathbf{h}_i$ generated using: $L_1 = 2^4, L_2 = 2^3, \; L_3 = L_4 = 2^2$

  → $\lambda_i = 1 - 1/\left(2^{k_i} L_i\right), \quad$ for $i = 1, 2, \ldots, N$

  → Measure of performance:

$$\mathrm{NM}[\mathbf{g}, \hat{\mathbf{g}}(n)] = \left[\frac{\|\mathbf{g} - \hat{\mathbf{g}}(n)\|_2}{\|\mathbf{g}\|_2}\right]^2 \; [\mathrm{dB}]$$

Experiment 1

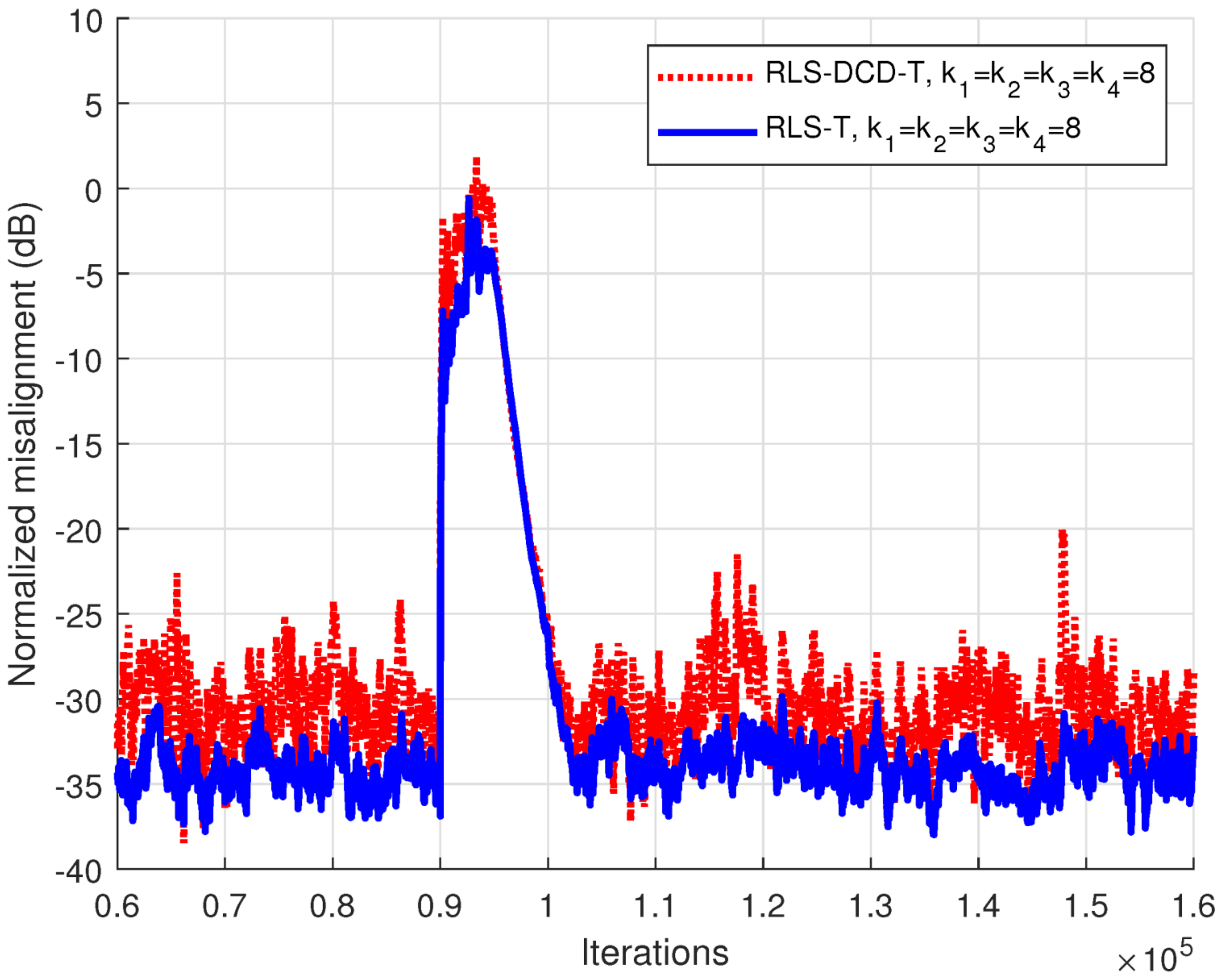


Experiment 2

- **Conditions:**
  - → Input signals – AR(1) processes; each one is generated by filtering a white Gaussian noise through a first-order system with the pole 0.85
  - → Additive noise $w(n)$ between it. 90001 and it. 95000 – WGN, SNR = -15dB
  - → The order of the system: $N = 4$
  - → Individual impulse response $\mathbf{h}_i$ generated using: $L_1 = L_2 = L_3 = L_4 = 2^3$
  - → $\lambda_i = 1 - 1/\left(2^{k_i} L_i\right)$, for $i = 1, 2, \dots, N$
  - → Measure of performance:

$$\mathrm{NM}[\mathbf{g}, \hat{\mathbf{g}}(n)] = \left[\frac{\|\mathbf{g} - \hat{\mathbf{g}}(n)\|_2}{\|\mathbf{g}\|_2}\right]^2 \ [\mathrm{dB}]$$

Experiment 2



Conclusions

- We have introduced a low-complexity RLS-based adaptive algorithm for the identification of unknown systems based on tensorial decompositions.
- The resulting RLS-DCD-T algorithm benefits from the low computational requirements of the DCD iterations and could provide a performance comparable with other versions of tensorial based RLS methods.
- The reduction in complexity for the adaptive filter update process is important.
- The usage of the DCD iterations allows for the coefficient updates to be performed using only bit-shifts and additions.
- **Future work:** robustness improvement analysis with a variable regularization behaviour.

Thank you for your attention!

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