

Dual Decomposition Method for Reliable Allocations of Wireless Network Resources

E. Laitinen *, I. Konnov **, A. Kashuba **

*University of Oulu, Finland

** Kazan Federal University, Russia

MOBILITY 2021, May 30 to June 03, 2021 - Valencia, Spain



Short resume of the presenter

- ▶ Name: Erkki Laitinen, e-mail: erkki.laitinen@oulu.fi
- ▶ Education: PhD, Appl. Math.
- ▶ Affiliation:
 - ▶ University of Oulu, Res. Unit of Math. Sci., P.O.Box 8000, FI-90014 University of Oulu
 - ▶ University of Jyväskylä, Dept. of Math. Inf. Tech., P.O. Box 35, 40014 University of Jyväskylä
- ▶ Fields of research: numerical analysis; numerical methods; modelling and simulation; optimization and control. Applications to metal industry; telecommunication industry; forest industry



Introduction

- ▶ The current development of information technologies and telecommunications gives rise to new control problems related to efficient transmission of information and allocation of limited network resources.
- ▶ At the same time, wireless networks should be reliable with respect to various attacks. These attacks may lead to degrading the network performance and Quality of Service, as well as losing important data, reputations, and revenue.
- ▶ In this paper, we consider the problem of allocation of link resources in telecommunication networks with respect to both utility and reliability goals.

Problem formulation

- ▶ We consider the telecommunication network where the reliability factor should be taken into account. Namely, we associate the reliability to each arc flow and determine $\mu_l(f_l)$ as the non-reliability of the l -th arc having the flow f_l for $l \in L$. Then $\sum_{l \in L} \mu_l(f_l)$ is the total network non-reliability and we formulate the network manager problem as follows:

$$\max \rightarrow \sum_{i \in I} u_i(x_i) - \sum_{l \in L} \mu_l(f_l), \quad (1)$$

subject to

$$\sum_{i \in I_l} x_i = f_l, \quad l \in L; \quad (2)$$

$$0 \leq f_l \leq c_l, \quad l \in L; \quad (3)$$

$$0 \leq x_i \leq \alpha_i, \quad i \in I. \quad (4)$$

If the functions $u_i(x_i)$ and $-\mu_l(f_l)$ are concave, this is a convex optimization problem with the polyhedral feasible set.



Solution method

By duality, we can replace problem (1)–(4) with the dual unconstrained optimization problem:

$$\min \rightarrow \varphi(y), \quad (5)$$

where

$$\varphi(y) = \max_{x \in X, f \in F} L(x, f, y). \quad (6)$$

where

$$L(x, f, y) = \sum_{i \in I} u_i(x_i) - \sum_{l \in L} \mu_l(f_l) + \sum_{l \in L} y_l \left(\sum_{i \in I_l} x_i - f_l \right)$$

is the Lagrange function of the problem.



- ▶ Calculation of its value and its gradient is rather simple and decomposed into independent solution of single-dimensional problems.
- ▶ The convexity and concavity properties enable us to apply the usual Uzawa gradient method to find a solution of the dual problem (5):

$$y^{k+1} = y^k - \lambda_k \varphi'(y^k), \lambda_k > 0.$$

Computational experiments

- ▶ The method was implemented in C++ with a PC with the following facilities: Intel(R) Core(TM) i7-4500, CPU 1.80 GHz, RAM 6 Gb.
- ▶ In our example we used quadratic functions of non-reliability of arcs (QuadA)

$$\mu_l(f_l) = \mu_{1,l}f_l^2 + \mu_{0,l}f_l, \quad \mu_{1,l}, \mu_{0,l} > 0, l \in L,$$

and logarithmic functions of utility of origin-destination pairs (LogC)

$$u_i(x_j) = u_{2,i} \ln(u_{0,i} + u_{1,i}x_j), \quad u_{j,i} > 0, j = 0, \dots, 2, i \in I,$$

- ▶ The notations in the table are:
 1. ε is the accuracy of finding solution of the problem,
 2. $T_{\varepsilon,1}$ and $T_{\varepsilon,100}$ are the time (in seconds) of the method with the starting point e and $100e$, respectively,
 3. $I_{\varepsilon,1}$ and $I_{\varepsilon,100}$ are the numbers of iterations spent searching for a solution to the problem with the starting point e and $100e$, respectively.



In the Table, we give the results for the case where $|I| = 620$, $|L| = 310$ and for different values ε . $|L|$ is a number of transmission links (arcs) and $|I|$ is a number of selected pairs of origin-destination vertices within a fixed time period.

Table: Computations for $|I| = 620$, $|L| = 310$ (QuadA-LogC)

ε	$T_{\varepsilon,1}$	$I_{\varepsilon,1}$	$T_{\varepsilon,100}$	$I_{\varepsilon,100}$
10^{-1}	0.016	50	0.078	200
10^{-2}	0.028	57	0.094	220
10^{-3}	0.031	70	0.125	248
10^{-4}	0.047	91	0.172	407