A Quantum Statistical Mechanical Modeling Approach to Human Cognitive Interaction with Cyber-Systems

Mark J. Balas
Leland T. Jordan Professor of Control and Dynamical Systems
Mechanical Engineering Department
Texas A&M University
College Station, TX, USA 77845
mbalas@tamu.edu

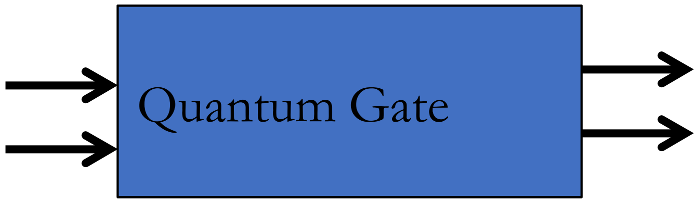
Dr Vinod Gehlot

Prof James Hubbard

Mr. Tristan Griffith

Quantum Computing

A Quantum computer will operate differently from a Classical one.
It will be involved w physical systems on an atomic scale,
eg atoms, photons, trapped ions, or nuclear magnetic moments



Unitary → Reversible

Entanglement with Quantum Environment produces Decoherence

<u>Machine Autonomy</u>

My brain is a Neural Net Processor

Autonomous machines are capable of performing tasks in the world by themselves, without explicit human control.

Examples range from autonomous helicopters to Roomba, the robot vacuum cleaner.

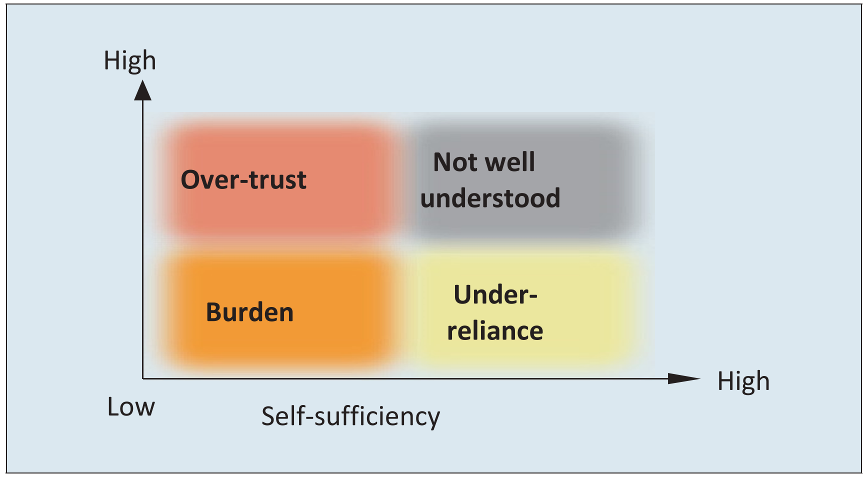
…. George Bekey, <u>Autonomous Robots: From Biological Inspiration to Implementation and Control</u>, MIT Press, 2005

<u>Autonomous Systems</u>: Make Decisions and Operate Independent of Any Human Interference or Effect

Myths

The Seven Deadly Myths of "Autonomous Systems"

Jeffrey M. Bradshaw, Robert R. Hoffman, Matthew Johnson, and David D. Woods,IEEE Intelligent Systems, May-June 2013



Myth 1: "Autonomy" is unidimensional

Myth 2: The conceptualization of "levels of autonomy" is a useful scientific grounding for the development of autonomous system roadmaps.

Myth 3: Autonomy is a widget

Myth 4: Autonomous systems are autonomous.

Myth 5: Once achieved, full autonomy obviates the need for human-machine collaboration.

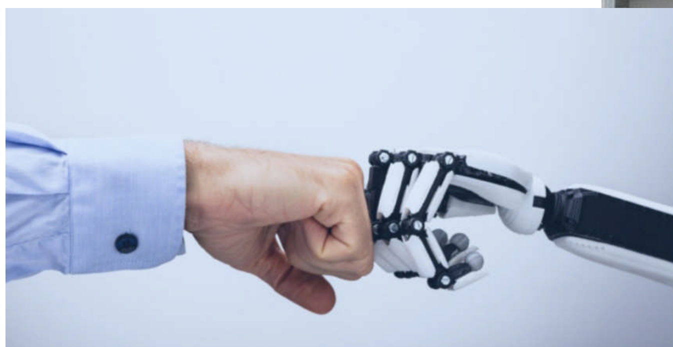
Myth 6: As machines acquire more autonomy, they will work as simple substitutes (or multipliers) of human capability.

Myth 7: "Full autonomy" is not only possible, but is always desirable.

Self Driving Vehicles

More Likely

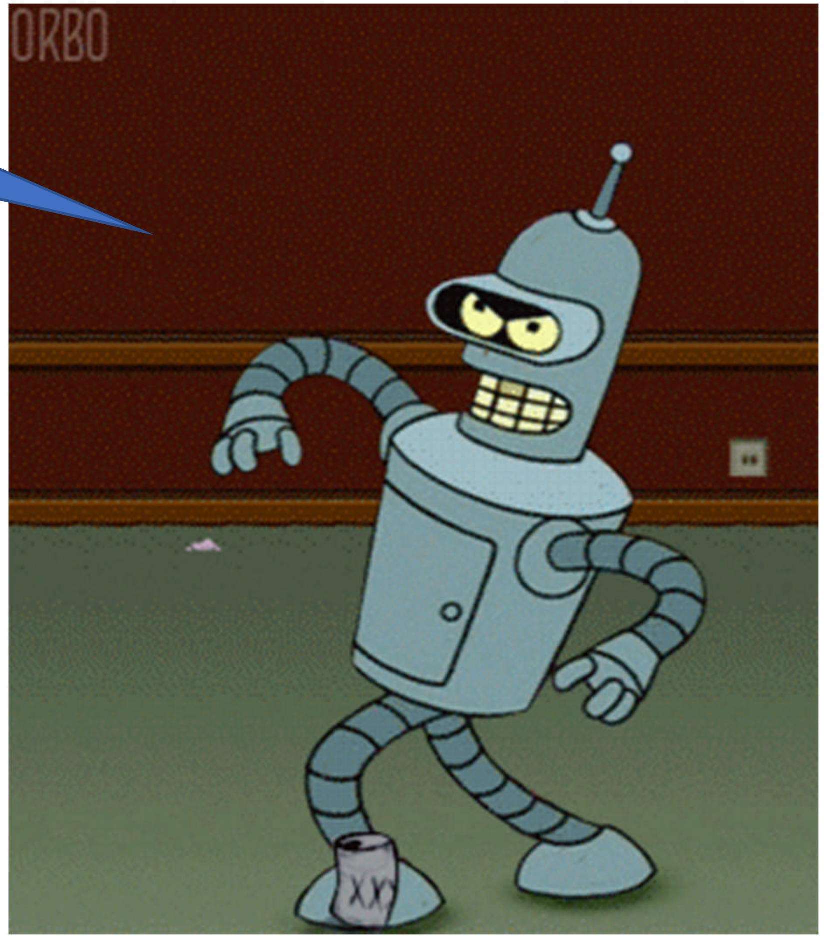
Semi-Autonomous Systems



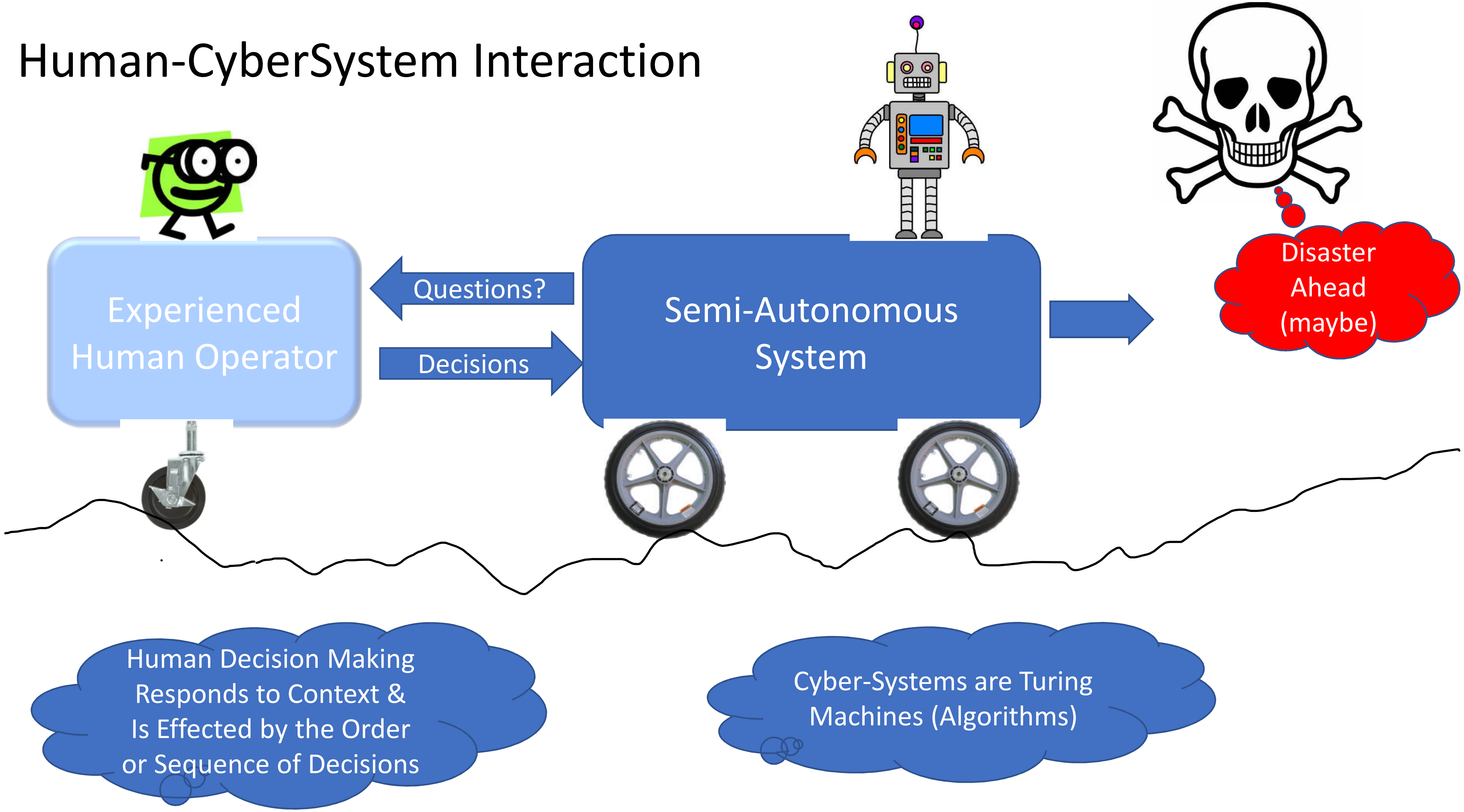
Human-Machine
Collaborations

Human-Machine Barriers

Why Cant Humans Follow the Program



Human-CyberSystem Interaction



Quantum Analysis of Cognition &Decision-Making

1. The Quantum Probability Approach to Human Cognition and Decision- Making is like the Statistical Mechanics approach to Thermodynamics

2. Provides analytical probabilistic understanding to complement time series analysis & data mining based on neural nets and machine learning

3. Uses Special Properties of Quantum Information Systems, e.g. non-commuting probability, entangled bases

4. Benefit: Simulation via Quantum Computing & Computers ??

Introduction

- Achieving full autonomy in cyber-systems is extremely complex and difficult, and very unlikely to be acceptable for reasons that are related to human endeavor and safety. Humans must continue to interact with computer-controlled systems. The only viable system autonomy systems will be one that supports teaming based on a careful foundation of research on human–automation interaction.
- To that end, Verification & Validation for such teamed human computer systems must be able to model to a reasonable extent the human cognitive effects of well-trained operators in decision-making with high performance cyber-systems. Such modeling efforts will never exactly reproduce human cognition in all its variegated complexity, but will model a reasonable likeness that can be usd to predict and improve human-machine performance.
- The mathematics necessary for such a modeling effort is not classical probability. Quantum probability and quantum statistical mechanics is more adept at capturing the unique attributes of human cognition including recognition of context and sequential decision-making. It is well-known that humans recognize and respond fully to these attributes. And the mathematics at the heart of quantum mechanics is indeed able to capture and reproduce these dynamic attributes. However, this is a new aspect of system modeling and most certainly of the V&V of such human-machine systems.

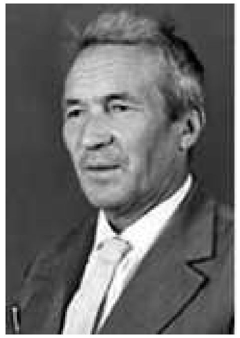
Further

- The fundamental element of quantum statistical mechanics is the quantum density operator. Once determined this operator reveals the statistics of observables in a quantum process which can represent and model human decision-making. As part of this introductory talk we will present some of our research on the estimation and approximation of the quantum density operator from the Liouville-Von Neumann master equation.

- It is critical to retain all the properties of quantum operators during approximation and estimation; so that at any stopping point during the estimation process the result will be a true quantum density operator. The set S of all quantum density operators on the Hilbert space of trace class operators contains all self-adjoint positive semidefinite operators with trace one. This set is contained inside the unit ball and is a closed convex set.

- Once the density operator is known/adequately estimated , the Quantum Statistical Mechanics properties of the Cognition model can be calculated. And recommendatios and predictions can be made about the Human-Machine Interaction.

Classical Probability Theory



Andrei Kolmogorov

Event Space: X

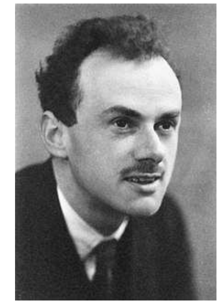
Ω σ-algebra of subsets of X

Probability of event $A \equiv p(A) : 0 \leq p(A) \leq 1,$

$$p(X) = 1, \& \, p(\Phi) = 0, \& \quad p(\bigcup_{i=1}^{\infty} A_i) = \sum_{i=1}^{\infty} p(A_i) \text{ when } A_i \text{ disjoint}$$

Bayes Theorem: $p(A \mid B)\, p(B) = \underbrace{p(A \cap B) = p(B \cap A)}_{Commutes} = p(B \mid A)\, p(A)$

Quantum Probability Theory



Paul Dirac

John Von Neumann

$\underline{\text{Event Space}}: X$ is a Hilbert Space

(Here $\square^{N}$, but usually infinite-dimensional)

$(x, y) \equiv x^{*} y$ inner product & norm $\|x\| \equiv \sqrt{(x, x)}$

$\{\phi_k\}_{k=1}^{N}$ orthonormal basis for $X : (\phi_k, \phi_l) = \delta_{kl}$

$$x = \sum_{k=1}^{N} (\phi_k, x) \underbrace{\phi_k}_{\text{"pure decision state"}} \quad \text{"Mixed Decision Quantum State"}: \|x\|^2 = \sum_{k-1}^{N} |(\phi_k, x)|^2 = 1$$

Defines Quantum State

$\underline{\text{QuantumProbability}}$ (x collapses into sp$\{\phi_k\}$) $\equiv |(\phi_k, x)|^2 = \|P_k\|^2$ where $P_k \equiv (\phi_k, \bullet)\phi_k$

Observables : H (self adjoint $H^{*} = H$)

Dynamics : $i \dfrac{\partial x}{\partial t} \left(= i \dfrac{dx}{dt}\right) = Hx$, $\underline{\text{Schrodinger Wave Equation}}$

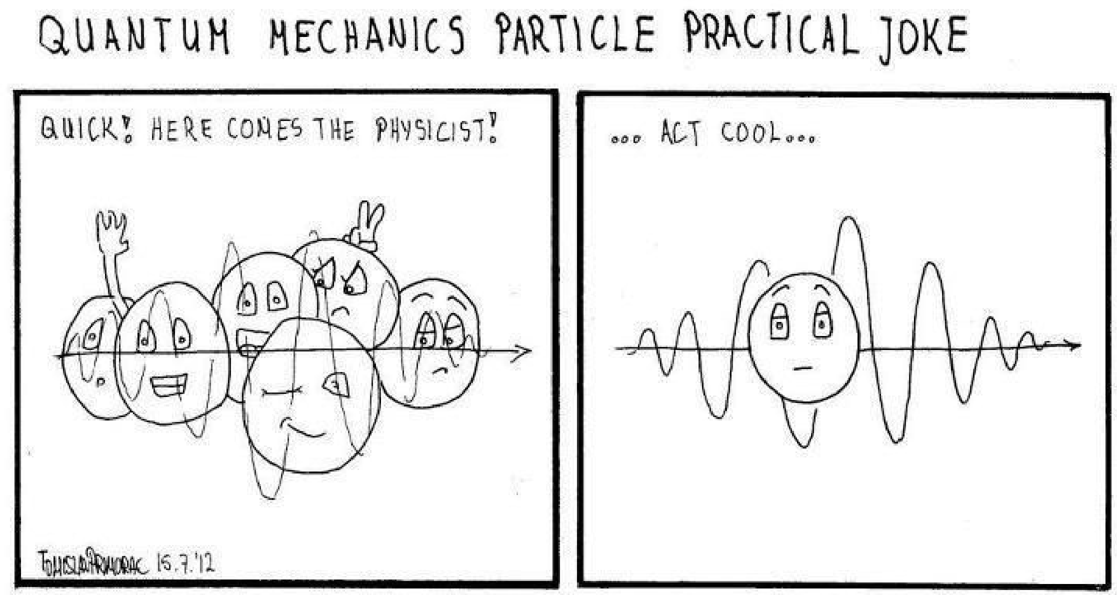
Quantum Spin Example: Qubit

$$X = \mathbb{C}^2 = sp\{\phi_1, \phi_2\}$$

Let the Hamiltonian be

$$H \equiv \sigma_0 + \alpha\sigma_1 = \begin{bmatrix} 1 & -i\alpha \\ i\alpha & 1 \end{bmatrix}; \alpha \text{ real}$$

$$\phi_1 = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ i \end{bmatrix}, \phi_2 = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ -i \end{bmatrix}$$



$$x(t) = e^{-i(1+\alpha)t}(\phi_1, x_0)\phi_1 + e^{-i(1-\alpha)t}(\phi_2, x_0)\phi_2$$

solves the Schrodinger equation ($\hbar \equiv 1$): $i\dfrac{dx}{dt} = Hx; x(0) = x_0$

$$\Rightarrow \ x(t) == e^{-it}\begin{bmatrix} \cos(\alpha t) \\ \sin(\alpha t) \end{bmatrix} \text{ when } x_0 = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \text{,and } \|x(t)\| = \|x_0\| = 1$$

Order Effects: Sequenced Decisions

$X = sp\{\phi_A, \phi_B\}$ Qubit

Orthogonal Projections: $P_1 \equiv \phi_A \phi_A^* \;\&\; P_2 \equiv \frac{1}{2}(\phi_A + \phi_B)(\phi_A + \phi_B)^*$

Let $\phi \equiv \phi_B$:

$$p((2\,|\,1)\phi) \equiv \left\|P_2 P_1 \phi\right\|^2 = \left\|P_2 \phi_A \underbrace{\phi_A^* \phi_B}_{=0}\right\|^2 = 0$$

$$\& \; p((1\,|\,2)\phi) \equiv \left\|P_1 P_2 \phi\right\|^2 = \left\|P_1 \frac{1}{2}(\phi_A + \phi_B)(\phi_A + \phi_B)^* \phi_B\right\|^2$$

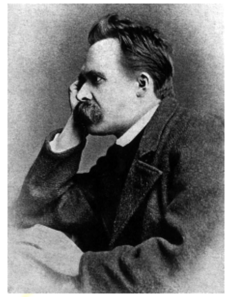
$$= \left\|P_1 \frac{1}{2}(\phi_A + \phi_B)(0+1)\right\|^2 = (\frac{1}{2})^2 \left\|\phi_A \phi_A^* (\phi_A + \phi_B)\right\|^2 = (\frac{1}{2})^2 \left\|\phi_A (1+0)\right\|^2 = (\frac{1}{2})^2 = \frac{1}{4}$$

$$\Rightarrow \; p((2\,|\,1)\phi) \neq p((1\,|\,2)\phi)$$

(Note Projections do not commute: $P_1 P_2 \neq P_2 P_1$)

Quantum Measurement

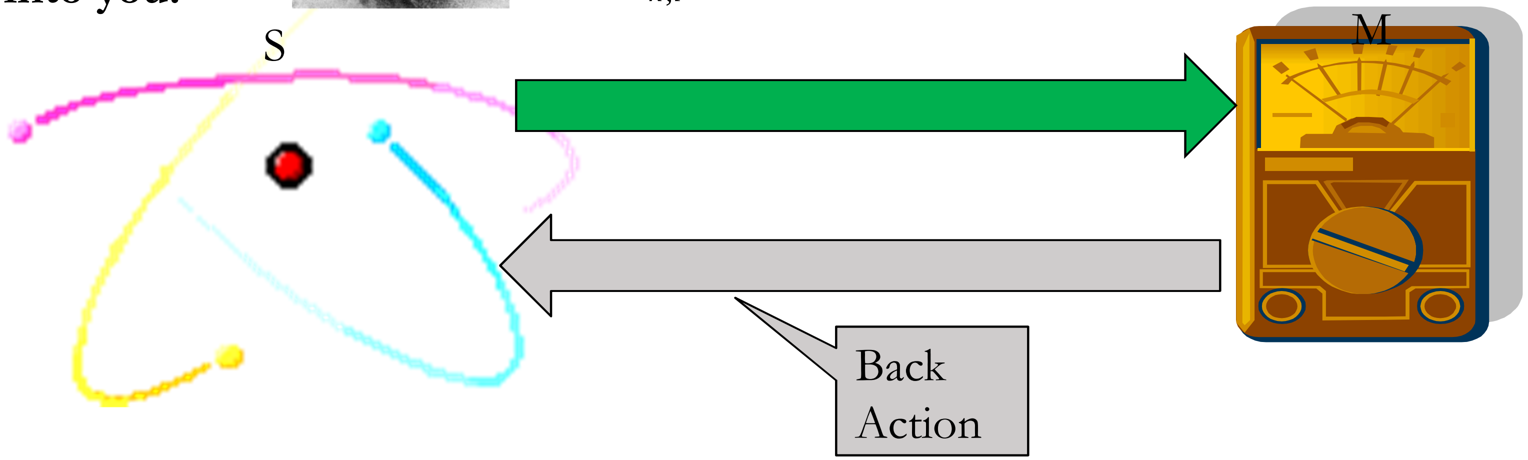
"…for when you gaze long into the abyss. The abyss gazes also into you."



Entanglement

$$X = X_S \otimes X_M$$

$$\phi = \sum_{k,l} \alpha_{kl} (\phi_k^S \otimes \phi_l^M) \neq h \otimes w$$



Heisenberg Uncertainty Principle

Observable $A : X \xrightarrow[\text{self-adjoint}]{\text{bounded}} X$ Hilbert

$$\begin{cases} \text{Mean } \langle A \rangle \equiv Tr(\rho A) \\ \text{Dispersion } \Delta A \equiv \sqrt{Tr(\rho(A - \langle A \rangle)^2)} \end{cases}$$ where ρ is a state or density operator $(\rho > 0 \;\&\; Tr(\rho) = 1)$

Heisenberg Uncertainty Principle : Simultaneous Measurement of $A \,\&\, B$

$$(\Delta A)^2 (\Delta B)^2 \geq \frac{\hbar}{2} |Tr(\rho[A, B])|; \text{ commutator } [A, B] \equiv AB - BA$$

Quantum Statistical Mechanics

Ensemble Behavior
from Multiple Experiments

Quantum Density Operators : $\rho \in \square^{NxN}$

(These carry all the quantum probability information &are now thought of as quantum states

Defining Properties: $\rho^* = \rho$(self-adjoint); $\rho \geq 0$(pos semi-def); $tr\,\rho = 1$

Mixed State: $\rho = \sum_{k=1}^{N} p_k P_k;$

Pure State: $P_k \equiv (\phi_k, \bullet)\phi_k = \phi_k \phi_k^*$

$$Dynamics: \frac{d\rho}{dt} = -i[H, \rho] = -i(\underbrace{H\rho - \rho H}_{L})$$,Quantum Master Equation

Ensemble Averages; Quantum Measurements:

$\text{y}=\langle C \rangle \equiv \text{tr}(\text{C}\rho)$

The Set of All Quantum Density Operators

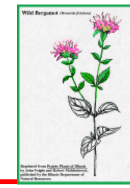
$$S \equiv \{\rho \in \mathbb{C}^{NxN} \mid \rho^* = \rho; \rho \geq 0; tr\rho = 1; tr\rho^2 \leq 1\} \subseteq \text{ Unit Ball in } \mathbb{C}^{NxN}$$

<u>Theorem</u>: S is a <u>closed</u>, <u>convex</u> subset of $\mathbb{C}^{NxN}$, & S is <u>bounded</u> ($S \subseteq$ Unit Ball), where

S <u>closed</u> means: $\forall \{\rho_k\} \subseteq S \ \&\ \rho_k \xrightarrow[k\to\infty]{} \rho \Rightarrow \rho \in S$;

S <u>convex</u> means: $\forall \rho_1, \rho_2 \in S,$ the straight line $\lambda\rho_1 + (1-\lambda)\rho_2 \in S$

Quantum Dynamical System



$$Dynamics: \begin{cases} \dfrac{d\rho}{dt} = -i[H,\rho] = -i(H\rho - \rho H) = -iL\rho \ , \\ \rho(0)=\rho_0 \text{ (unknown)} \end{cases}$$

Quantum Master Equation

Ensemble Averages; Quantum Measurements:

$$y = \langle C \rangle \equiv tr(C\rho)$$

ρ_0

$y(t)$

The available Outputs here are the data from an ensemble of experiments

Note: S is an <u>invariant set</u>: $\rho(0) \in \mathrm{S} \Rightarrow \rho(t) \in \mathrm{S} \, \forall t \geq 0$

"Once a quantum density, always a quantum density"

What is Needed NOW? A Basic Online <u>Linear</u> Estimator

Quantum Density System

$$\begin{cases} \dfrac{\partial \rho}{\partial t} = -iL\rho; L \equiv [H, \rho] \\ y = trC\rho; \rho(0) = \rho_0 \end{cases}$$

y

Available Data

Linear Quantum Density Estimator

$$\begin{cases} \dfrac{\partial \hat{\rho}}{\partial t} = -iL\hat{\rho} + K(y - \hat{y}) \\ \hat{y} = C\hat{\rho}; \hat{\rho}(0) = \hat{\rho}_0 \end{cases}$$

$\hat{\rho}(t) = \rho(t) + e(t)$

$e(t) \xrightarrow[t \to \infty]{} 0$

with exponential rate

But $\hat{\rho}(t)$ <u>does not remain in</u> S even tho it starts there $(\hat{\rho}(0) \in S)$

and it converges to $\rho(t) \in S$

Projection Operator for Closed Convex Sets in Hilbert Space

X Hilbert Space with S closed, convex $\subseteq X$.

$P_S : X \to S :$

$P_S x$ is the (metric) Projection of x onto S when

$$\forall x \in X \quad \|x - P_S x\| = d(x, S) \equiv \min_{z\in S} \|x - z\|$$

Properties of the Projection

1) $P_S(x)$ is defined $\forall x \in X$

2) $P_S(x) = x \Leftrightarrow x \in S$

3) $P_S{}^2 = P_S(idempotent)$

4) $x_* = P_S x \Leftrightarrow \mathrm{Re}(\underbrace{x - x_*}_{Error}, z - x_*) \le 0 \ \forall z \in S$ ("Principle of Orthogonality, sorta")

5) P_S is Lipschitz Continuous, $i.e. \|P_S x - P_S y\| \le \|x - y\| \quad \forall x, y \in X$

But P_S is NOT Linear.

Modified Quantum Estimator

Quantum Density System

$$\begin{cases} \dfrac{\partial \rho}{\partial t} = -iL\rho; L \equiv [H, \rho] \\ y = trC\rho; \rho(0) = \rho_0 \end{cases}$$

y

Linear Quantum Density Estimator

$$\begin{cases} \dfrac{\partial \hat{\rho}}{\partial t} = -iL\hat{\rho} + K(y - \hat{y}) \\ \hat{y} = C\hat{\rho}; \hat{\rho}(0) = \hat{\rho}_0 \end{cases}$$

$\hat{\rho}(t) = \rho(t) + e(t)$

Nonlinear Projection Operator

P_S

$\hat{\hat{\rho}}(t) \equiv P_S \hat{\rho}(t)$

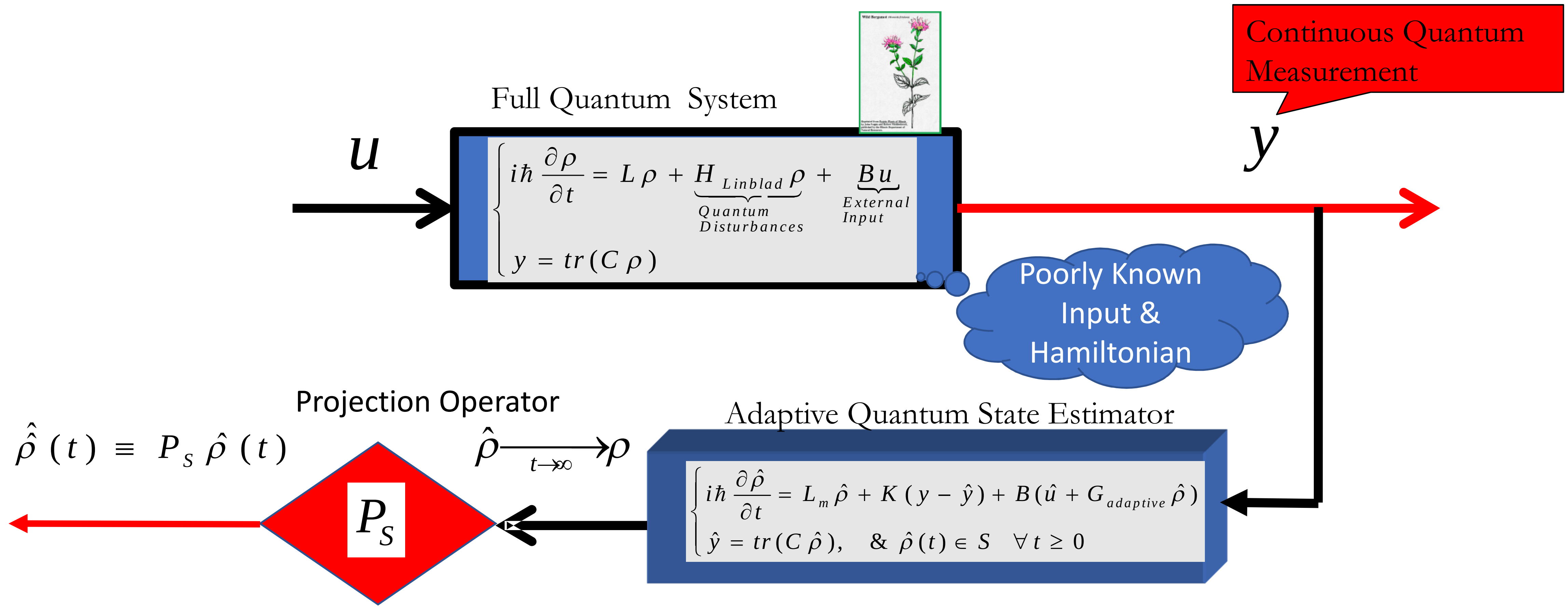
Using the Lipschitz continuity of P_S :

$$\left\| \hat{\hat{\rho}}(t) - \rho(t) \right\| = \left\| P_S \hat{\rho}(t) - \underbrace{P_S \rho(t)}_{\rho(t) \in S} \right\| \leq \left\| \underbrace{\hat{\rho}(t) - \rho(t)}_{e(t)} \right\| = \| e(t) \|$$

where $e(t) \xrightarrow[t \to \infty]{} 0$ with exponential rate set by the original Linear Estimator

And $\hat{\hat{\rho}}(t) \equiv P_S \hat{\rho}(\mathrm{t})$ remains in S $\forall \mathrm{t}$ (and is a Quantum Density) even tho $\hat{\rho}(t)$ does not and it converges to $\rho(t) \in S$

What Is Needed Later: **Adaptive** Quantum State Estimation in Hilbert Space



And $\hat{\hat{\rho}}(t) \equiv P_S\hat{\rho}(t)$ remains in S $\forall$t (and is a Quantum Density) even tho $\hat{\rho}(t)$ does not and it converges to $\rho(t) \in S$

What's The Point ?

We need a better understanding and ability
to make reasonable
Models of the dynamic behavior of human operators
interacting with semi-autonomous cyber-systems.

We think quantum probability theory
And Quantum Statistical Mechanics is a
fundamental place to start.

Our
Quantum Statistical
Mechanics
And Human Cognition
Research Group