Computing Efficiency in Membrane Systems

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His research interests concern the areas of formal languages, molecular computing models, DNA computing, Membrane Computing and Computational Complexity.

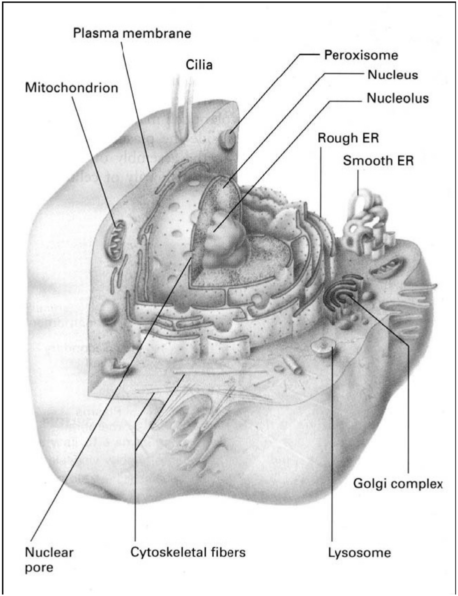
Summary

- Membrane Systems: ideas and definitions
- Membrane Systems with Active Membranes
- Computing Power of Membrane Systems
- Attacking Computationally Hard Problems
- Space Complexity in Membrane Systems with Active Membranes

Membrane Systems: a Bio-inspired computing model

- G. Paun, 1998 (P Systems): computational model inspired from the structure and functioning of the cell
  - Discrete
  - Non-deterministic
  - Maximally parallel application of the rules
- Main components:
  - Cellular structure
  - Chemical substances
  - Cellular reactions
  - Communication of substances

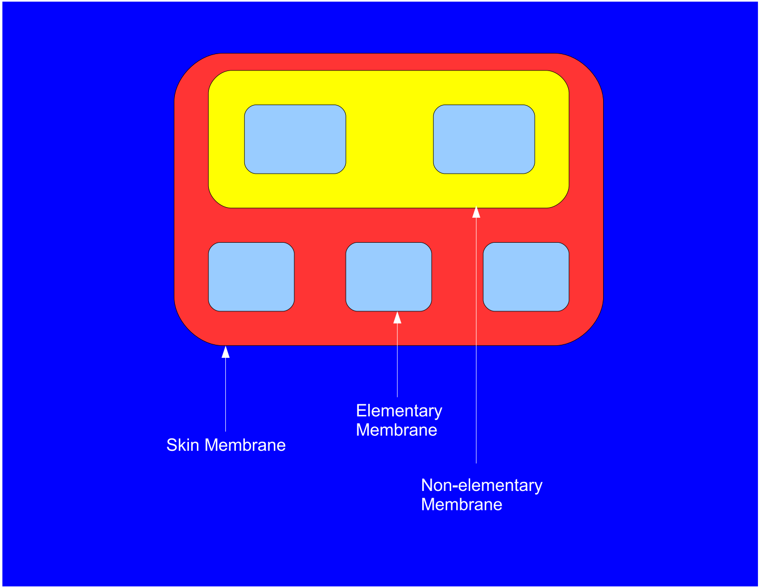
Cell Structure



Membrane structure

- Each membrane defines a REGION (compartment) in the membrane structure
- The most external membrane separates the system and the environment. It is called SKIN
- Some substances are communicated through the membranes
- A membrane is identified by means of a label

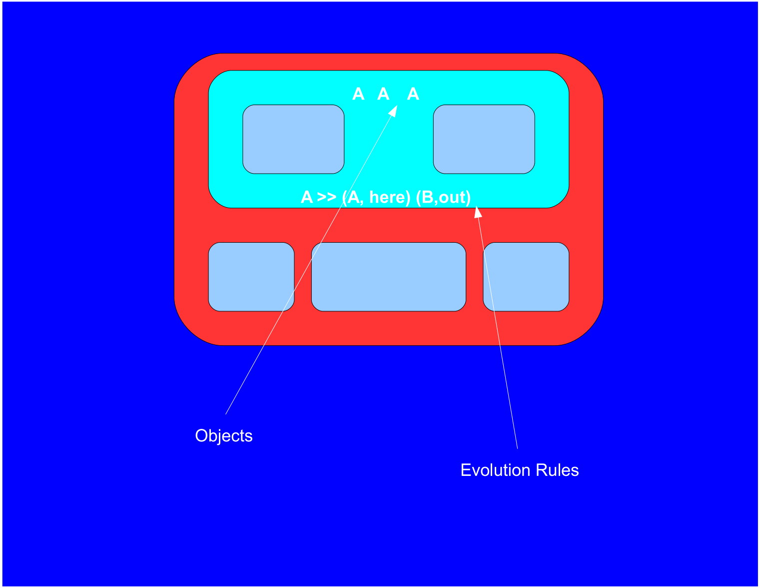
Membrane Structure



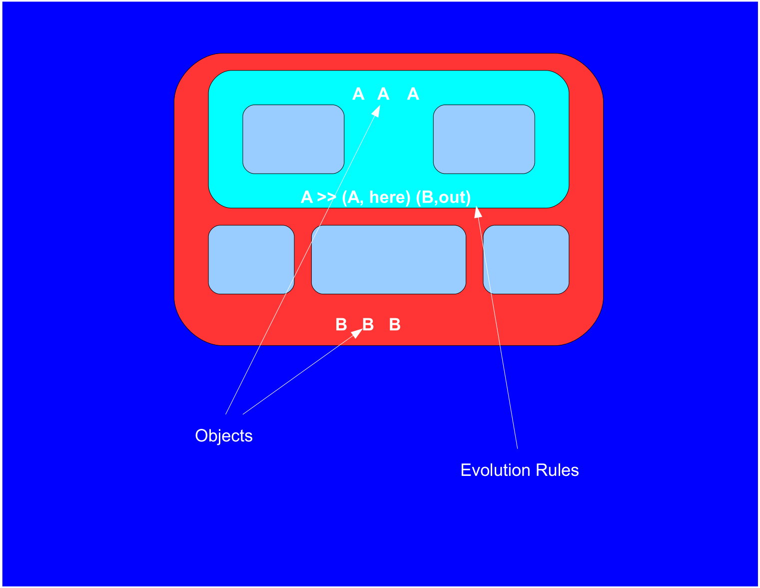
Membrane Systems: Chemicals and Reactions

- Chemicals - Ions, molecules, proteins: multisets of symbols over an alphabet
  - Multiset: each symbol can be present in one or more copies in a region
  - E.g. a^5, b^3, c: five copies of chemical a, three of b, and one of c are present in a region
- A reaction is described by a CF rewriting rule and target indication
  - Chemical on left replaced by chemicals on right
  - Obtained chemicals communicated according to target indication
  - Special Symbol δ: membrane is dissolved
  - E.g. $a \rightarrow (x, here)(y, out)(z, in_3)\delta$

Membrane System



Membrane System



Definition: Membrane System

$$\Pi = (V, \mu, M_1, \ldots, M_n, (R_1, \rho_1), \ldots, (R_n, \rho_n), i_0)$$

- V: Alphabet
- μ: Membrane structure (Ex. $[\ [\ ]_2\ [\ ]_3\ [\ [\ ]_5\ [\ ]_6\ ]_4\ ]_1$)
- M_i: Multisets of symbols (or strings) in V
- R_i: Finite sets of evolution rules $x \to y$, $x \in V^*$, $y = y'$ or $y = y'\delta$ where y' is a string over $(V \times tar)$, $tar \in \{here, out, in_j\}$
- ρ_i: Partial order relations over R_i
- i_0: Output Membrane. If empty, then the output region is the environment

Evolution

- $M_1, \ldots, M_n$: initial configuration
- Rules are applied following the given priorities
- Rules are applied in a non-deterministic way
- All objects evolve in parallel
- All regions evolve in parallel
- Rules can move objects through membranes
  - *here*: the object is not moved
  - *out*: the object is sent to the adjacent external region
  - in_j: the object is sent to the inner membrane with label j

Computation

- **Computation**: Sequence of transitions between two configurations (by means of rules). A computation halts when no further rule can be applied
- **Output**:
  - Objects in i_0 (or outside the skin) when the computation halts
  - $\emptyset$ if the computation never stops

Active Membranes: active role in the computation

- Polarizations: electrical charges (positive +, negative -, or neutral 0) are associated with the membranes.
- Rules are applied according to polarizations
- Membranes can be dissolved
- New membranes can be created by division of existing ones. Objects in the divided membrane are duplicated:
  - Division for elementary membranes
    $[_h \; A \;]_h^{\alpha} \rightarrow [_h \; B \;]_h^{\beta} \;\; [_h \; C \;]_h^{\gamma}$
  - Division for non-elementary membranes
    $[_{h_0} \; [_{h_1} \;]_{h_1}^{+} \ldots [_{h_k} \;]_{h_k}^{+} \; [_{h_{k+1}} \;]_{h_{k+1}}^{-} \ldots [_{h_n} \;]_{h_n}^{-} \;]_{h_0}^{\alpha} \rightarrow$
    $[_{h_0} \; [_{h_1} \;]_{h_1}^{+} \ldots [_{h_k} \;]_{h_k}^{+} \;]_{h_0}^{\beta} \;\; [_{h_0} \; [_{h_{k+1}} \;]_{h_{k+1}}^{-} \ldots [_{h_n} \;]_{h_n}^{-} \;]_{h_0}^{\gamma}$

Definition: Membrane Systems with Active Membranes

$$\Pi = (V, H, \mu, M_1, \ldots, M_n, R)$$

- V: Alphabet
- H: set of labels for membranes
- μ: Membrane structure (Ex. $[\,[\,]_2\ [\,]_3\ [\,[\,]_5\ [\,]_6\ ]_4\ ]_1$)
- M_i: String over V, initial multiset of symbols in region i
- R: Finite sets of evolution rules
- Membranes are marked using polarization: $\{+, -, 0\}$

Developmental Rules

Assume $a \in V, w \in V^*, h \in H, \alpha_i \in \{+, -, 0\}$

- **Object evolution:** $[a \to w]_h^{\alpha_1}$
- **IN communication:** $a\,[\;]_h^{\alpha_1} \to [b]_h^{\alpha_2}$
- **OUT communication:** $[a]_h^{\alpha_1} \to [\;]_h^{\alpha_2}\, b$
- **Dissolution:** $[a]_h^{\alpha_1} \to b$

Division Rules

- **Elementary division:**
  $[a]_h^{\alpha_1} \rightarrow [b]_h^{\alpha_2}\,[c]_h^{\alpha_3}$
- **Non-elementary division:**
  $\left[[\,]_{h_1}^{+} \cdots [\,]_{h_k}^{+} [\,]_{h_{k+1}}^{-} \cdots [\,]_{h_n}^{-}\right]_h^{\alpha_1} \rightarrow$
  $\left[[\,]_{h_1}^{+} \cdots [\,]_{h_k}^{+}\right]_h^{\alpha_2}$
  $\left[[\,]_{h_{k+1}}^{-} \cdots [\,]_{h_n}^{-}\right]_h^{\alpha_3}$
- Non-elementary division: Membranes with neutral polarization are duplicated

Application of the rules

- Maximal parallel semantics
- At each step, each object and membrane can be the subject of only one rule
- If two conflicting rules can be applied: non–deterministic choice
- When a membrane divides, its content is replicated unchanged in the new copy
- OUTPUT: Symbols that exit from the skin in a halting computation

Computing Power of Membrane Systems

- Systems using a single membrane can only generate length sets of context–free languages
- Computing power cannot be extended by using an unlimited number of membranes
- Allowing the dissolution of membranes increases computing power, when at least two membranes are used; universality is not reached.
- To obtain universal systems, further features must be considered: cooperative (non context-free) rules, priorities defining the order of rules application, or structured objects
- Membranes are necessary to reach universality (one membrane does not suffice)

Exploiting Elementary Membrane Division: SAT problem

- SAT - Satisfiability for boolean formulas: a boolean formula Φ in CNF, with
  - n boolean variables $x_1, x_2, \ldots x_n$
  - m clauses
- Question: is there a truth assignment for $x_1, x_2, \ldots x_n$ such that Φ is true?
- Brute force algorithm requires exponential time
- SAT is NP–complete

Solving SAT in linear time

- $[[z_1 \quad a_1 a_2 \ldots a_n]_2^0]_1^0$

Solving SAT in linear time

- $[[z_1 \;\; a_1 a_2 \ldots a_n]_2^0]_1^0$
- $[[z_2 \;\; T_1 \;\; a_2 \ldots a_n]_2^0 \;\; [z_2 \;\; F_1 \;\; a_2 \ldots a_n]_2^0]_1^0$

Solving SAT in linear time

- $[[z_1 \;\; a_1 a_2 \ldots a_n]_2^0]_1^0$
- $[[z_2 \;\; T_1 \;\; a_2 \ldots a_n]_2^0 \;\; [z_2 \;\; F_1 \;\; a_2 \ldots a_n]_2^0]_1^0$
- $[[z_3 \;\; T_1 T_2 \;\; a_3 \ldots a_n]_2^0 \;\; [z_3 \;\; T_1 F_2 \;\; a_3 \ldots a_n]_2^0$
  $[z_3 \;\; F_1 T_2 \;\; a_3 \ldots a_n]_2^0 \;\; [z_3 \;\; F_1 F_2 \;\; a_3 \ldots a_n]_2^0]_1^0$

Solving SAT in linear time

- $[[z_1 \;\; a_1 a_2 \dots a_n]_2^0]_1^0$
- $[[z_2 \;\; T_1 \;\; a_2 \dots a_n]_2^0 \;\; [z_2 \;\; F_1 \;\; a_2 \dots a_n]_2^0]_1^0$
- $[[z_3 \;\; T_1 T_2 \;\; a_3 \dots a_n]_2^0 \;\; [z_3 \;\; T_1 F_2 \;\; a_3 \dots a_n]_2^0$
  $[z_3 \;\; F_1 T_2 \;\; a_3 \dots a_n]_2^0 \;\; [z_3 \;\; F_1 F_2 \;\; a_3 \dots a_n]_2^0]_1^0$
- $\dots$
- $[[z_n \;\; T_1 T_2 \dots T_n]_2^0 \;\; [z_n \;\; T_1 T_2 \dots T_{n-1} F_n]_2^0$
  $\dots \;\; [z_n \;\; F_1 F_2 \dots F_n]_2^0]_1^0$
- In n steps we generate all possible truth assignments

Solving SAT in linear time

- In one step we change the polarization of the membranes using z_n
- $[[T_1\,T_2\ldots T_n]_2^+\;\;[T_1\,T_2\ldots T_{n-1}F_n]_2^+$
  $\ldots\;\;[F_1F_2\ldots F_n]_2^+]_1^0$

Solving SAT in linear time

- In one step we change the polarization of the membranes using z_n
- $[[T_1 T_2 \ldots T_n]_2^+ \ [T_1 T_2 \ldots T_{n-1} F_n]_2^+$
  $\ldots \ [F_1 F_2 \ldots F_n]_2^+]_1^0$
- In one step every symbol T_i (resp. F_i) is replaced by some symbols R_{h_i}
- $1 \leq h_i \leq m$ is the index of a clause satisfied by setting $x_i =$TRUE (resp. $x_i =$FALSE)
- We obtain, for example,
  $[[R_1 R_3 R_1 R_4 \ldots R_6]_2^+ \ [R_7 R_3 R_2 R_3 \ldots R_2]_2^+$
  $\ldots \ [R_2 R_5 R_1 R_5 \ldots R_1]_2^+]_1^0$

Solving SAT in linear time

- In $2m$ steps we check whether or not a membrane contains all R_j, where $1 \leq j \leq m$
- $[[\ldots]_2^- \;\; [\ldots]_2^+ \;\; \ldots \;\; [\ldots]_2^- T \;\; T]_1^0$

Solving SAT in linear time

- In $2m$ steps we check whether or not a membrane contains all R_j, where $1 \leq j \leq m$
- $[[...]_2^- \ [...]_2^+ \ \dots \ [...]_2^- T \ \ T]_1^0$
- After $n + 2m + 2$ steps, eventually a symbol T appears in the skin membrane
- $[[...]_2^- \ [...]_2^+ \ \dots \ [...]_2^- T]_1^- \ \ YES$
- If after exactly $n + 2m + 3$ computation steps we obtain a T, then a symbol YES is sent out through the skin; otherwise a symbol NO is sent out.

Features of the Solution

- Requires linear time
- Requires exponential space
- The solution proposed is said to be **Semiuniform**:
  - Every input instance requires a specific membrane system to be computed
  - Given an input instance x of length n, the membrane system used to solve it can be generated by a deterministic Turing machine in polynomial time w.r.t. n
- The solution is said to be CONFLUENT

Determinism vs Non–determinism

- A Membrane System Π is said to be **deterministic** if there is at most one possible transition from a configuration to the following one, for all possible configurations
- A **non–deterministic** Membrane system Π is said to be **confluent** if the computations of Π are either all accepting or all rejecting. Such a system *accepts* in the former case and *rejects* in the latter
- When not all computations necessarily agree on the result, the system is called **non-confluent**. Non-confluent systems are said to *accept* when there exists an accepting computation, and to *reject* otherwise

Complexity classes for **confluent** Membrane systems

$(N)PMC_T$: languages decided **IN POLYNOMIAL TIME** by (non–)confluent Membrane systems in the class T

- $T = \mathcal{A}M$: systems with both division for elementary and non–elementary membranes
- $T = \mathcal{E}AM$: systems with division for elementary membranes only
- $T = \mathcal{N}AM$: systems without membrane division

Basic properties

- $PMC_T \subseteq NPMC_T$
- $PMC_{\mathcal{N}AM} \subseteq PMC_{\mathcal{E}AM} \subseteq PMC_{\mathcal{A}M}$
- $NPMC_{\mathcal{N}AM} \subseteq NPMC_{\mathcal{E}AM} \subseteq NPMC_{\mathcal{A}M}$

CONFLUENT P systems without division rules

- $P \subseteq PMC_{\mathcal{N}AM}$
  - "Trick": the DTM deciding $L \in P$ is used to solve *DIRECTLY* the problem in polynomial time
  - Then, we build a P system with a single membrane containing either an object *YES*, whenever an input $x \in L$ is given, or *NO*, otherwise. This requires polynomial time.
  - The P system send out the object in a single step

CONFLUENT P systems without division rules

The opposite is also true:

- $PMC_{\mathcal{NAM}} \subseteq P$
  - Idea: simulation of a generic P system Π without membrane division using a DTM M, with a polynomial slowdown
  - We keep track of the NUMBER OF OCCURRENCES of each symbol in each membrane
  - The application of a rule in Π can be simulated by modifying the counters used in M

CONFLUENT systems with elementary division rules only

- We have already seen that SAT is solvable by a family of Membrane systems that make use only of elementary membrane division. It follows: $NP \subseteq PMC_{\mathcal{E}AM}$
- SQRT-3SAT (PP–complete problem) can also be solved by such systems. Hence: $PP \subseteq PMC_{\mathcal{E}AM}$
- Confluent P systems with elementary membrane division can be simulated by Deterministic Turing machines using polinomial space: $PMC_{\mathcal{E}AM} \subseteq PSPACE$

CONFLUENT systems with both types of division rules

- $PMC_{\mathcal{A}M} \subseteq PSPACE$: can be simulated by DTM in polynomial space
- Quantified SAT (QSAT) - SAT using quantifiers: consider a Boolean expression Φ in CNF. Question: $\exists x_1 \forall x_2 \exists x_3 \forall x_4 \dots Q_n x_n \Phi$?
  - QSAT is PSPACE–complete
  - QSAT $\in PMC_{\mathcal{A}M}$
  - $PSPACE \subseteq PMC_{\mathcal{A}M}$
- $PSPACE = PMC_{\mathcal{A}M}$
- What if we remove dissolving action and polarizations? $P = PMC_{\mathcal{A}M}(n\delta, nPol)$!!!

Introducing space complexity classes

- Idea: both objects and membrane need physical space
- Let C_i be a configuration of a P system Π
- *size* $size(C_i)$ is the sum of number of membranes in μ and the total number of objects they contain
- The space required by a halting computation $C = (C_0, C_1, \ldots, C_m)$ of Π is $size(C) = max\{size(C_0), \ldots, size(C_m)\}$
- The *space required by* Π itself is $size(\Pi) =$
  $= max\{size(C) : C \text{ is a halting computation of } \Pi\}$

Some basic result concerning space complexity classes

From results concerning time complexity, it follows immediately:

- $P \subseteq MCSPACE_{\mathcal{N}AM}(O(1))$
- $NP \cup co-NP \subseteq EXPMCSPACE_{\mathcal{E}AM}$
- $PSPACE \subseteq EXPMCSPACE_{\mathcal{A}M}$

Space Complexity Results

- **PSPACE**–complete problem Quantified–3SAT can be solved by Membrane–systems with active membranes using a polynomial amount of space
- Membrane–systems with active membranes using a polynomial amount of space can be simulated by Turing machines using polynomial space
- Hence, **PSPACE** $= PMCSPACE_{\mathcal{AM}}$
- Similarly, **EXPSPACE** $= EXPMCSPACE_{\mathcal{AM}}$
- What about sublinear space?

Sublinear Space Membrane Systems

- Two distinct alphabets: *INPUT* alphabet and *WORK* alphabet
- Input objects cannot be rewritten and do not contribute to the size of a configuration
- Size of a configuration: number of membranes + total number of working objects
- Weaker uniformity condition: *DLOGTIME*-uniformity (*DLOGTIME* Turing machines)

Power of Sublinear Space Membrane Systems

- Idea: compare with logarithmic space Turing machines (or other equivalent models)
- Two problems if we use "standard" techniques:
  - Need for a polynomial number of working objects (violates log–space condition)
  - Need for a polynomial number of rewriting rules (violates uniformity condition)
- Solution: use polarization both to communicate objects and store information

Power of Sublinear Space Membrane Systems

Each Log–space DTM M can be simulated by a *DLOGTIME*-uniform family Π of Membrane systems with active membranes in logarithmic space having:

- A state object $q_{i,w}$: M is in state q, input–head on i-th symbol, work–head on w-th symbol
- $O(\log(n))$ nested membranes (INPUT tape membranes) containing, in the innermost one, the input symbols of M
- $O(\log(n))$ membranes to store the work tape of M (WORK tape membranes).
- Two sets of membranes, which size depends on the dimensions of the input and the working alphabets of M (SYMBOL membranes).

Power of Sublinear Space Membrane Systems

To simulate a computation step of M

- The state object enters the INPUT membranes: the bits corresponding to the actual position of the INPUT head of M are **stored in the polarizations** of the INPUT membranes
- Only the object corresponding to the INPUT symbol actually read can reach the skin
- The state object identifies the symbol actually under the WORK head
- The transition of M can be simulated using the SYMBOLS membranes

Power of Sublinear Space Membrane Systems

- Only a logarithmic number of objects and membranes are required (besides the input objects)
- The family Π is *DLOGTIME*-uniform
- Thus: **L** (class of problems solved by log–space Turing machines) is contained in the class of problems solved by *DLOGTIME*-uniform, log–space Membrane systems with active membranes.

Main resources

- BOOKS:
  - G. Paun, Membrane Computing - An introduction, Springer-Verlag, Berlin, 2002
  - G. Ciobanu, M.J. Perez-Jimenez, G. Paun (Eds), Applications of Membrane Computing, Springer-Verlag, Berlin 2006
  - P. Frisco, Computing with Cells. Advances in Membrane Computing, Oxford University Press, 2009
  - G. Paun, G. Rozenberg, A. Salomaa (eds.), The Oxford Handbook of Membrane Computing, Oxford University Press, 2010
- INTERNET: P systems web page: http://ppage.psystems.eu