ENERGY 2021, Valencia 1.6.2021

Control of Synchronization Patterns in Complex Dynamical Networks

Eckehard Schöll
Collaborative Research Center SFB 910
Control of Self-Organizing Nonlinear Systems
Technische Universität Berlin
and
Bernstein Center of Computational Neuroscience
and
Potsdam Institute for Climate Impact Research
Germany
schoell@physik.tu-berlin.de
http://www.itp.tu-berlin.de/schoell

1970 studied Physics in Stuttgart + Tübingen
1978 PhD (Maths) Southampton/UK
1981 PhD (Physics) RWTH Aachen/Germany
1983-84 Visiting Ass. Professor (El. Engg) Detroit/USA
1989-2019 Professor of Theoretical Physics TU Berlin

Eckehard Schöll

2000 Visiting Professor Duke University/NC, USA
2004 Visiting Professor London Mathematical Society
2017 Honorary Doctorate Saratov State University/Russia
2019- President of International Physics and Control Society (IPACS)

promotes interaction between researchers in Physics and Control Sciences
organizes PhysCon (Int. Conference on Physics and Control)

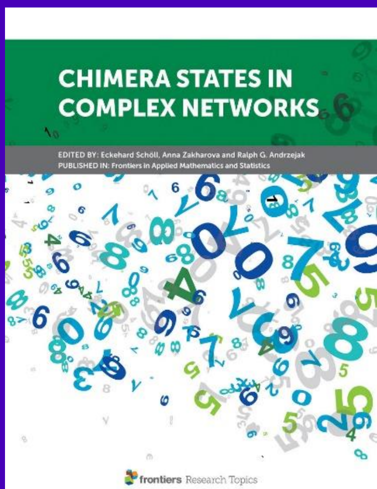
2020- Guest Scientist Potsdam Institute for Climate Impact Research

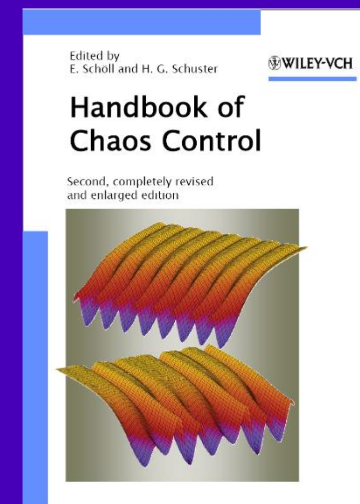
Department of Complexity Science

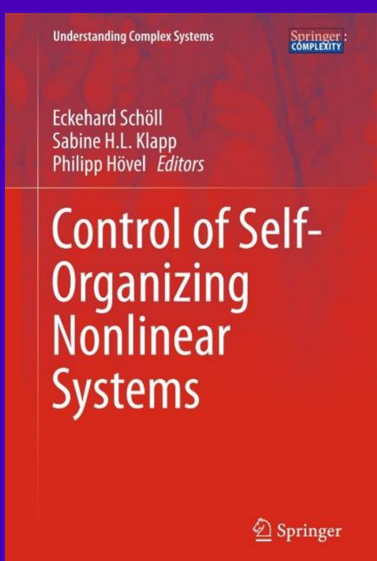
2011-2018 Founder and Chair of Center of Excellence SFB 910
“Control of Self-Organizing Nonlinear Systems”

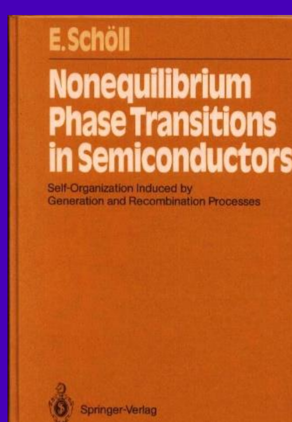
Research Field:
Complex nonlinear dynamical systems

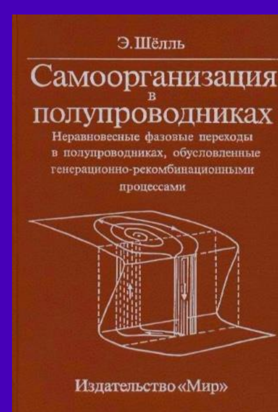
Nonlinear Dynamics and Control:
- Complex networks: brain, power grids
- Synchronization patterns in neural systems
- Dynamics and stability of power grids
- Chaos control, delayed feedback control
- Nonlinear semiconductor laser dynamics
- Growth kinetics of nanostructures and cell populations
- Charge transport and instabilites in semiconductor nanostructures

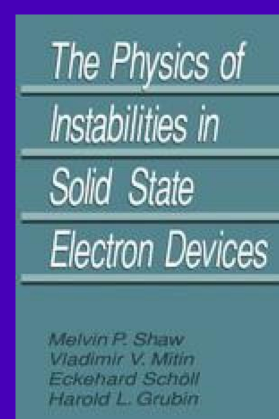


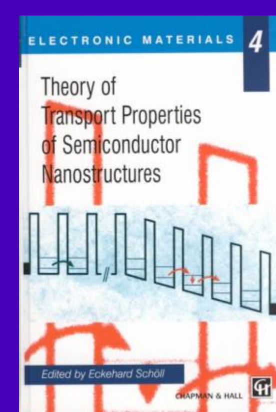


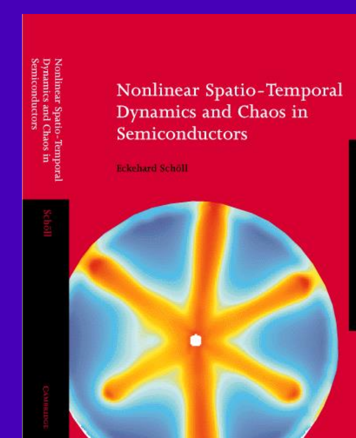








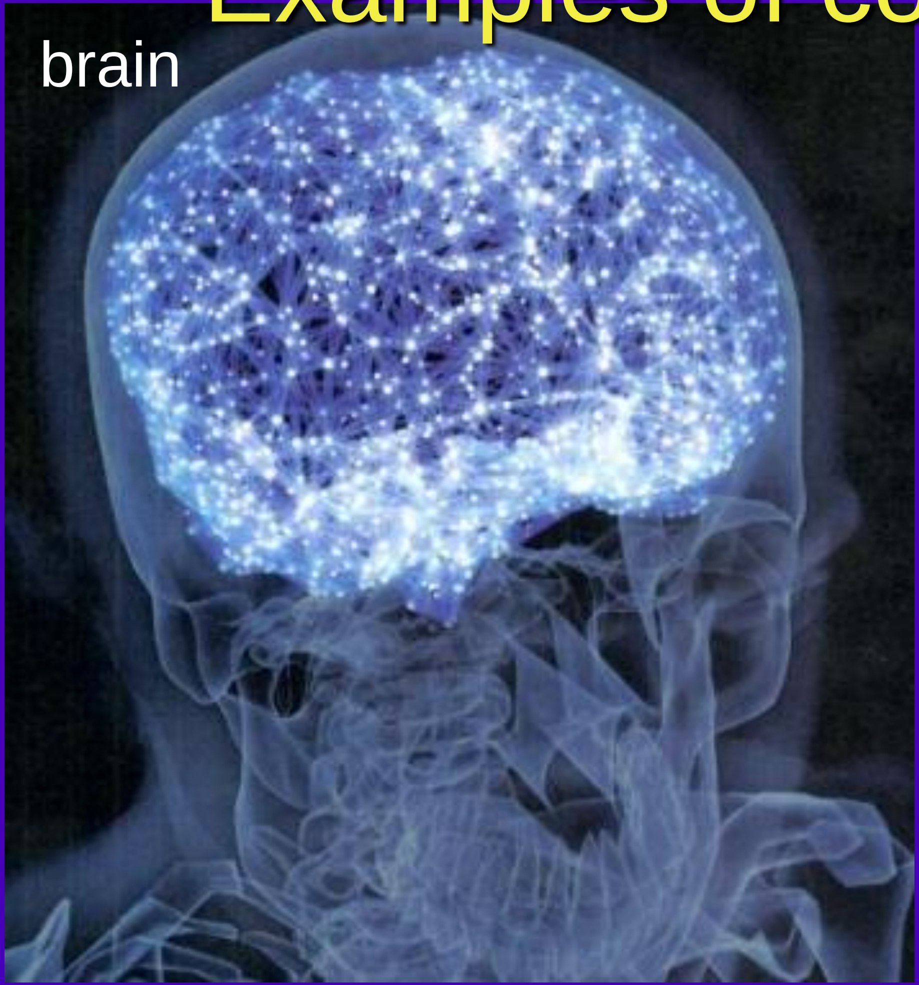




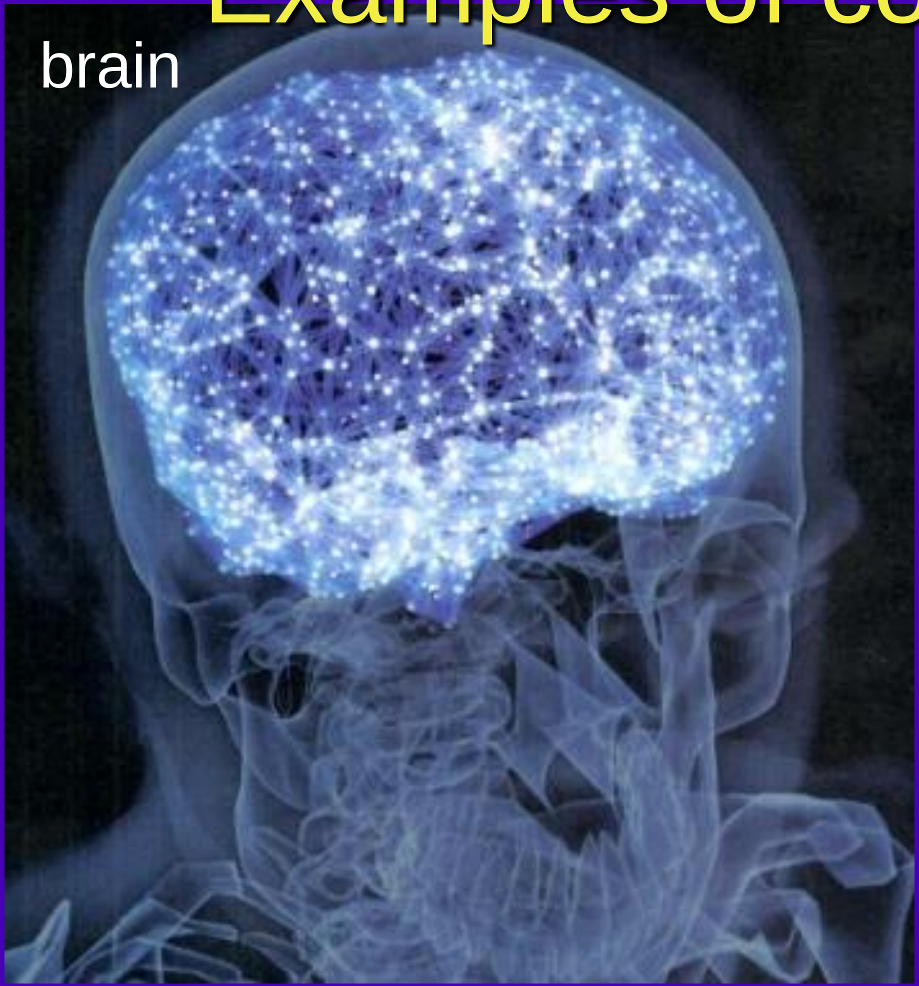
Outline

- Controlling nonlinear dynamics in complex networks
  - Motivation and Introduction: partial synchronization patterns
  - Application: power grids
  - Application: Relay synchronization in the brain
  - Application: Unihemispheric sleep and epileptic seizure

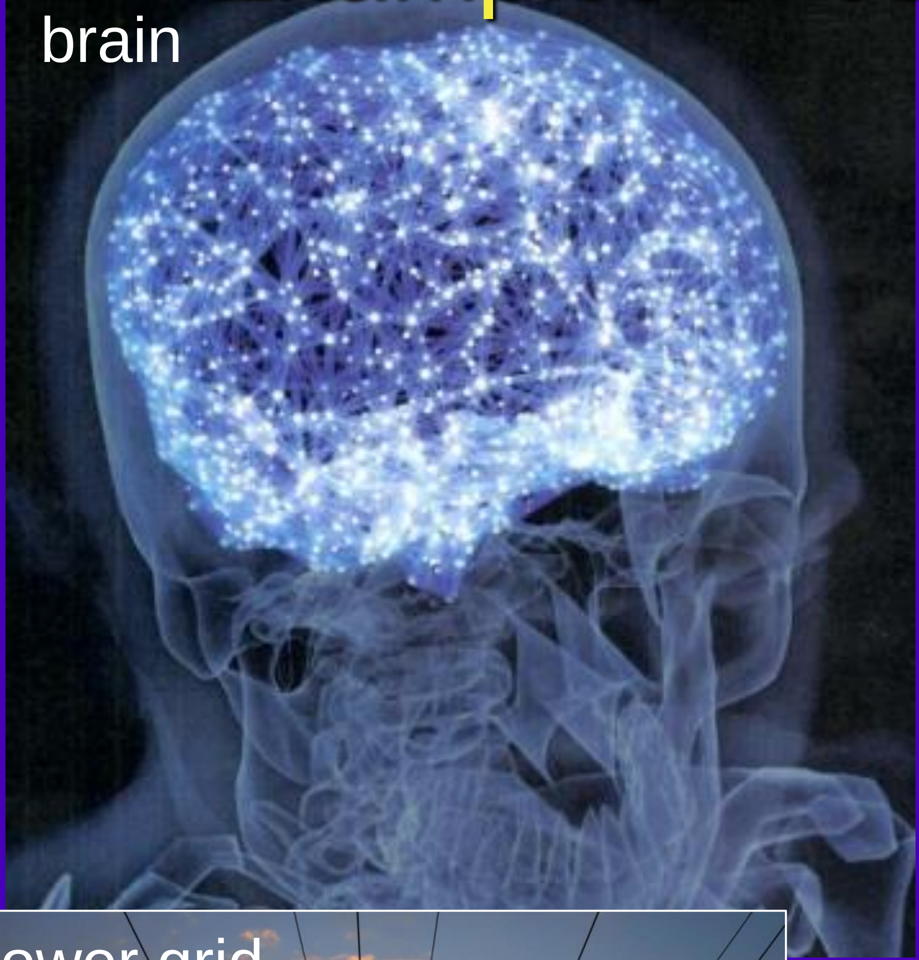
Examples of complex networks



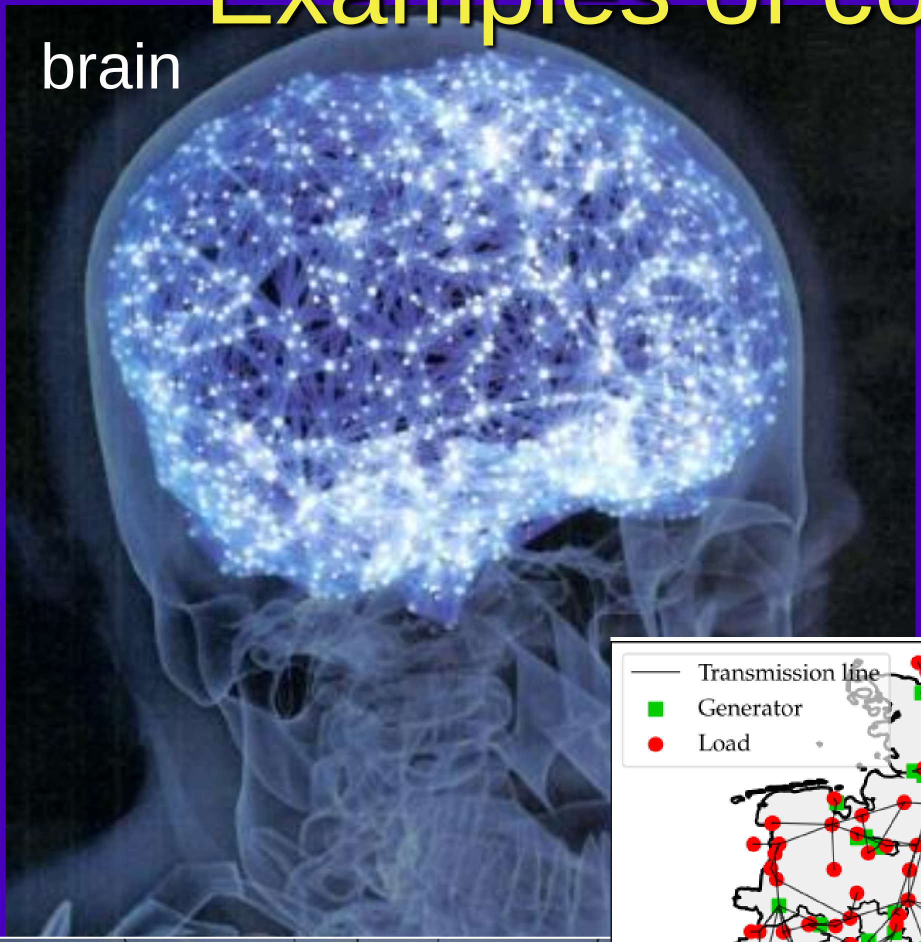
Examples of complex networks

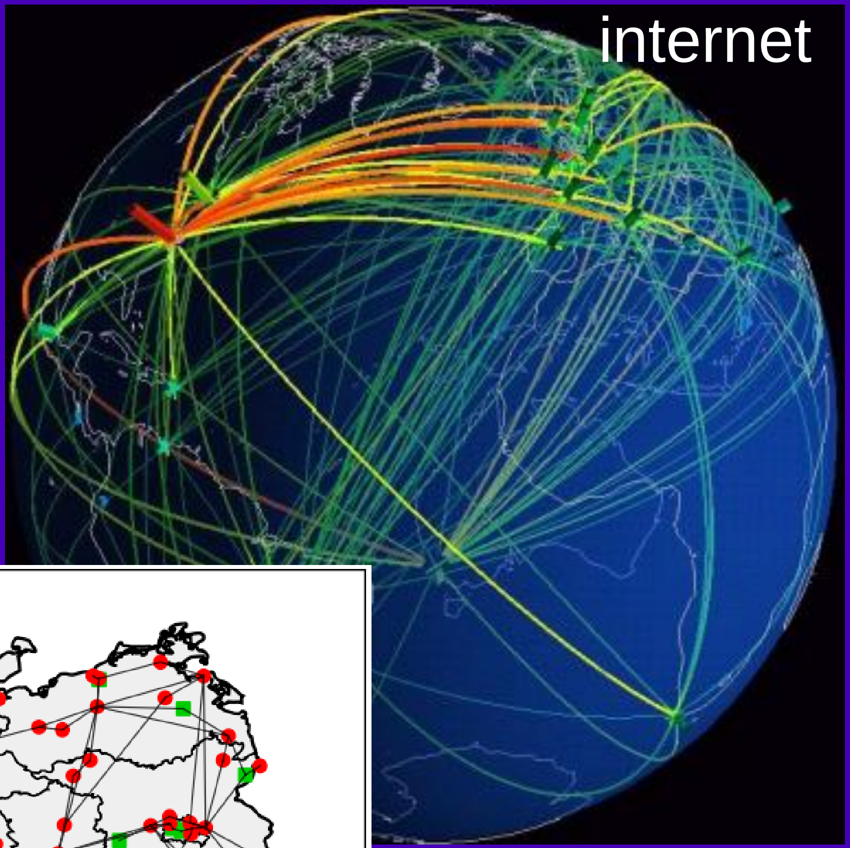


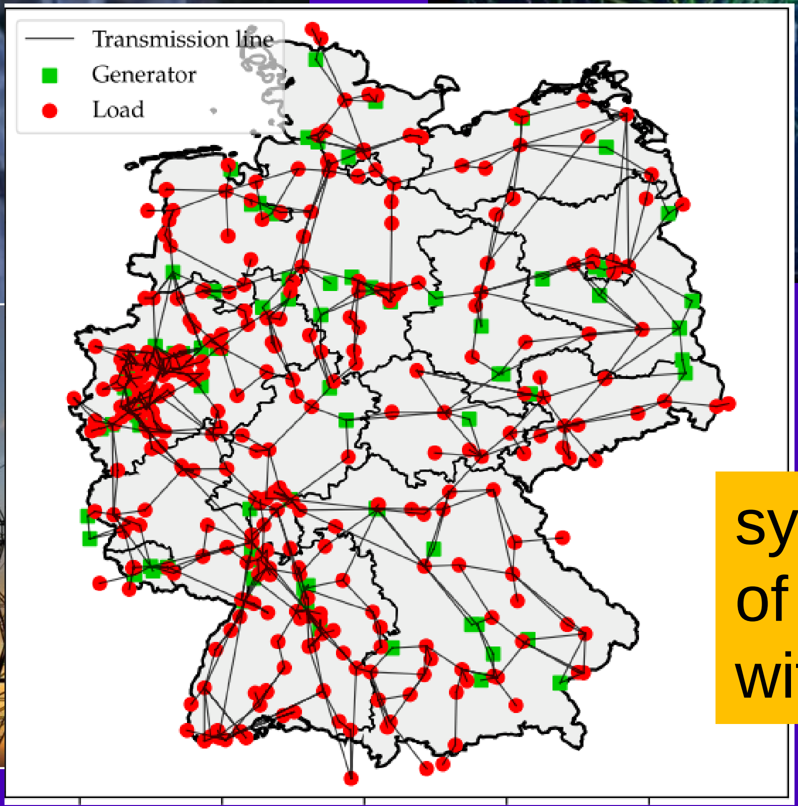
Examples of complex networks



Examples of complex networks

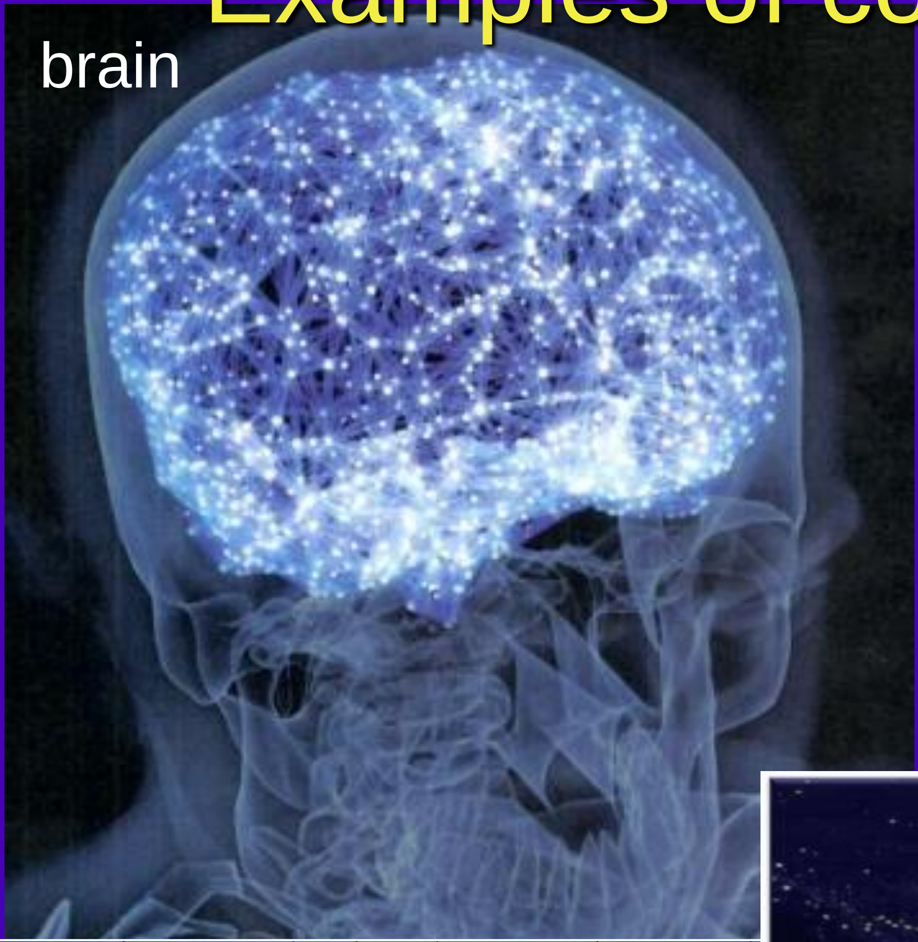


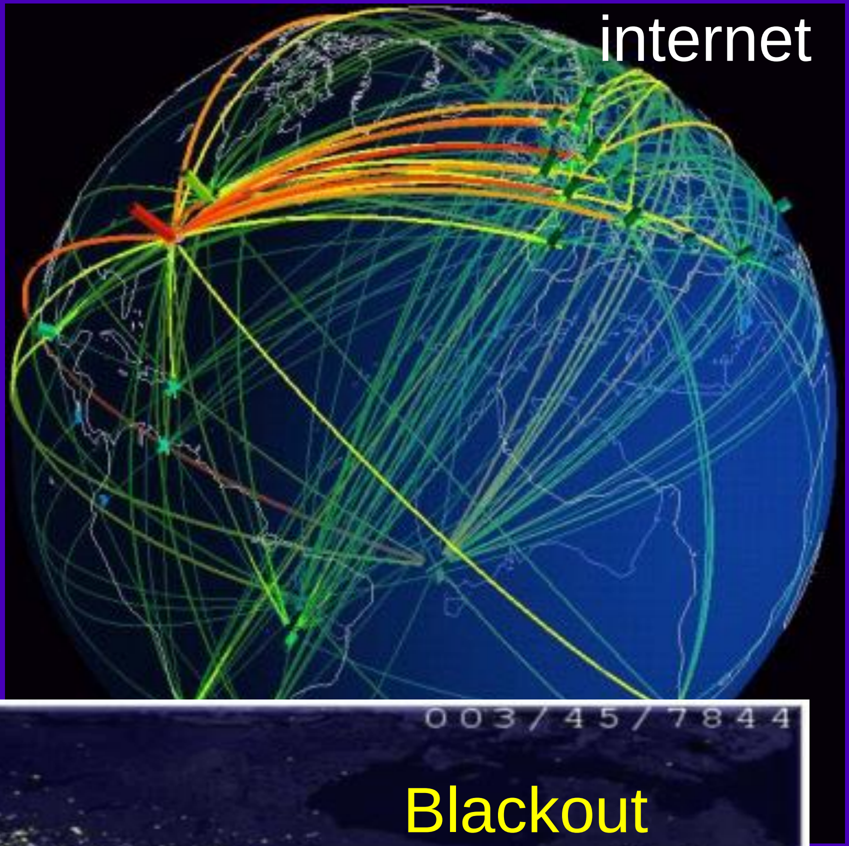


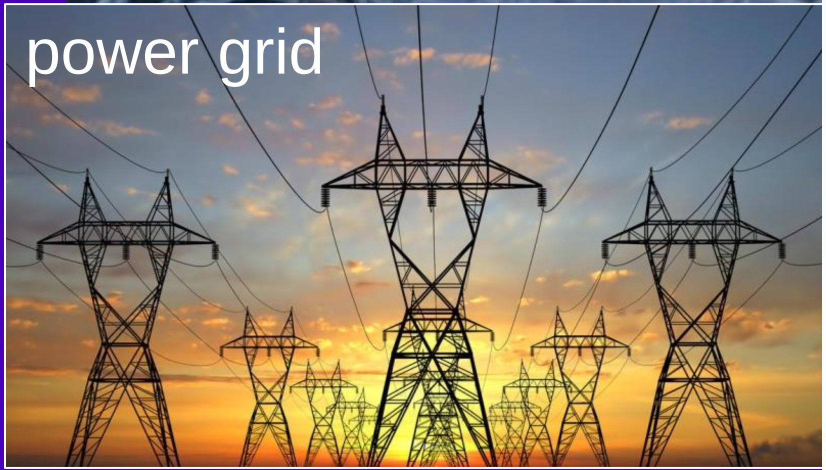


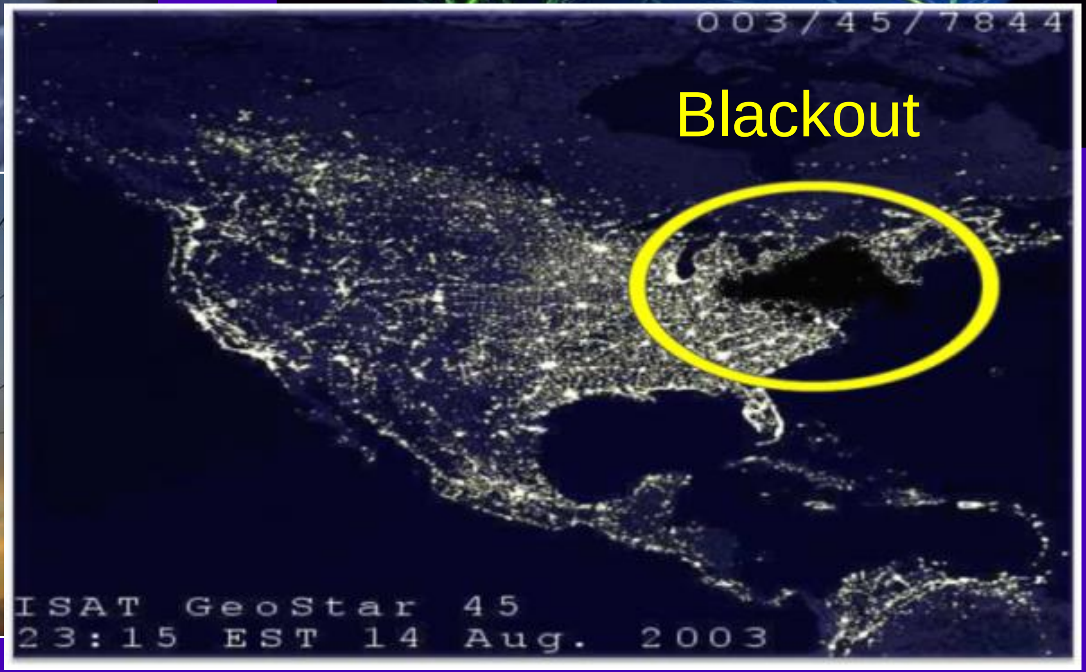
synchronization
of ac voltage
with 50 Hz necessary

Examples of complex networks

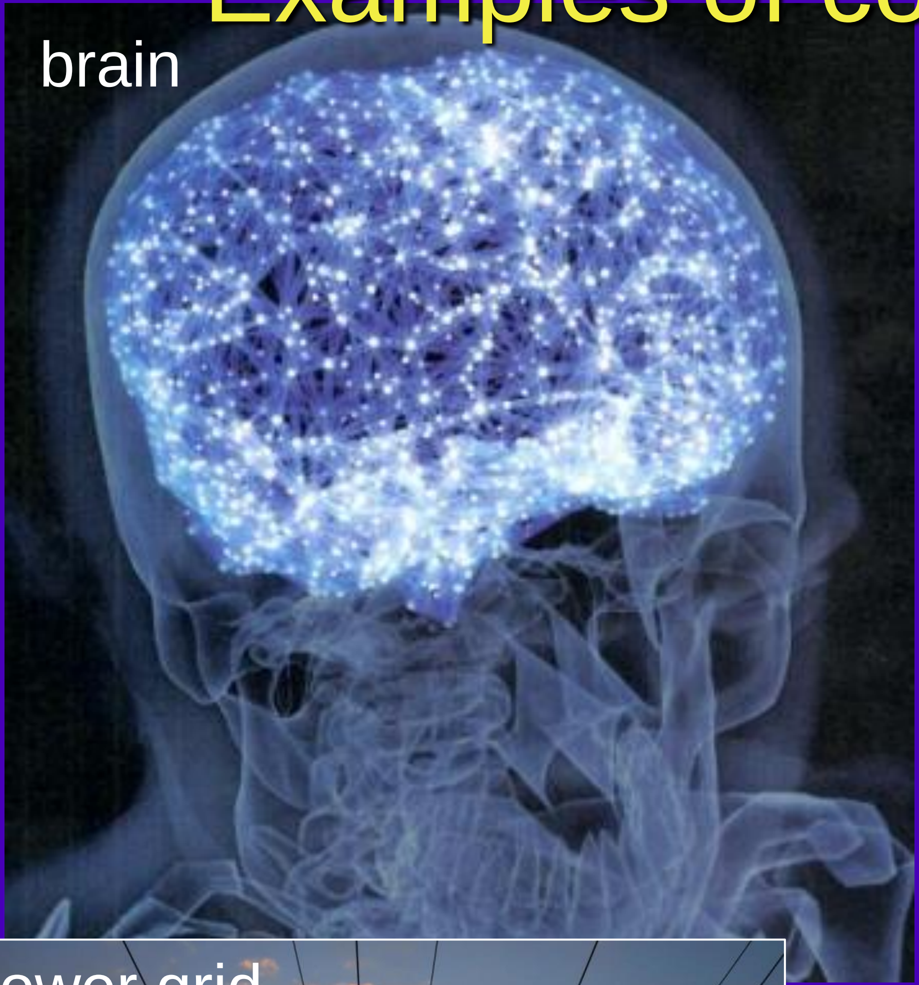








Examples of complex networks



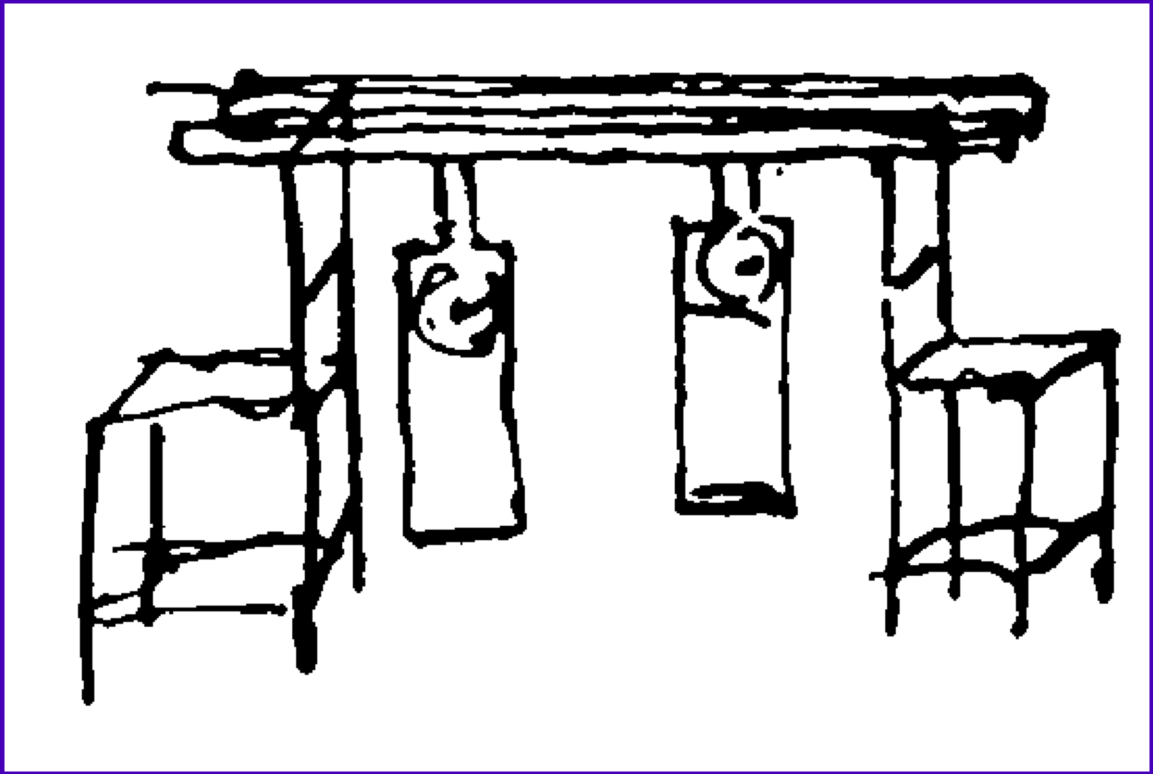
Synchronization and desynchronization

Desirable synchronization:
- Communication networks
- Encrypted communication with chaotic lasers
- Power grids
- Brain: learning and memory

Undesirable synchronization:
- Lateral oscillations of the London Millennium Bridge
- Pathological states in the brain: Parkinson, epileptic seizure

A. Pikovsky, M. G. Rosenblum, and J. Kurths, Synchronization: A Universal Concept in Nonlinear Sciences (Cambridge University Press, 2001).
S. Boccaletti, A. N. Pisarchik, C. I. del Genio, and A. Amann, Synchronization: From Coupled Systems to Complex Networks (Cambridge University Press, 2018).

Synchronization of coupled pendula



Christiaan Huygens: coupled clocks (1656)

Synchronization: Small coupling of nonlinear oscillators induces sync

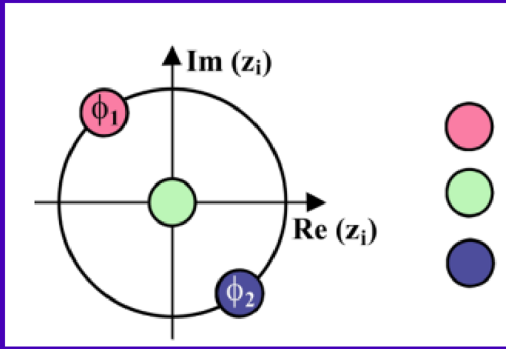
Partial synchronization patterns

Various synchronization patterns:

- complete in-phase synchronization
- group/cluster synchronization T. Dahms, J. Lehnert, E. Schöll: Phys. Rev. E 86, 016202 (2012)
- partial synchronization

  W. Poel, A. Zakharova, E. Schöll:

  PRE 91,022915 (2015)



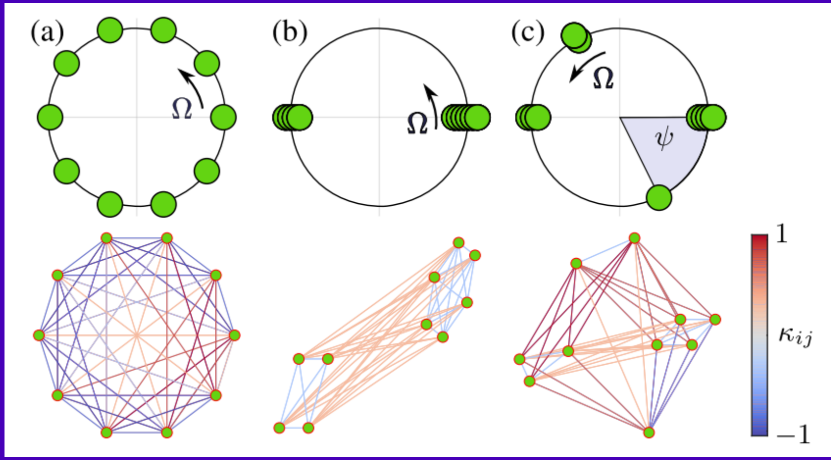
in-phase synchronization
amplitude death
antiphase synchronization

- phase clusters in adaptive networks (synaptic plasticity)

R. Berner, J. Sawicki, E. Schöll,
Phys. Rev. Lett. 124, 088301 (2020)

R.Berner, S.Vock, E.Schöll, S.Yanchuk,
Phys. Rev. Lett. 126, 028301 (2021)

R. Berner, E. Schöll, S. Yanchuk,
SIAM J. Appl.Dyn.Syst. 18, 2227 (2019)



splay antipodal double antipodal

Partial synchronization in networks: Chimeras

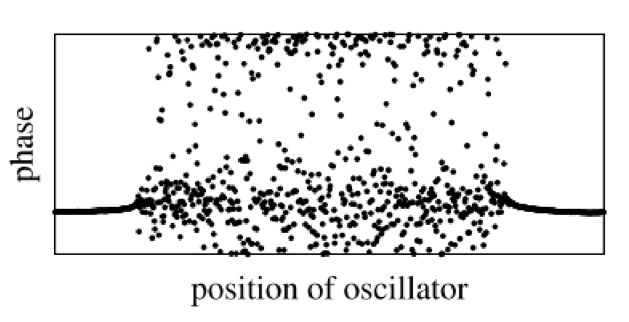
Chimera di Arezzo
400 v. Chr..
National Archeological Museum Florence

Mythological monster, composed of incongruous parts: lion, goat, snake

In complex networks: partial synchronization pattern

Self-organized splitting of network into coexisting domains of synchronized and desynchronized dynamics

Kuramoto and Battogtokh 2002
Abrams and Strogatz 2004



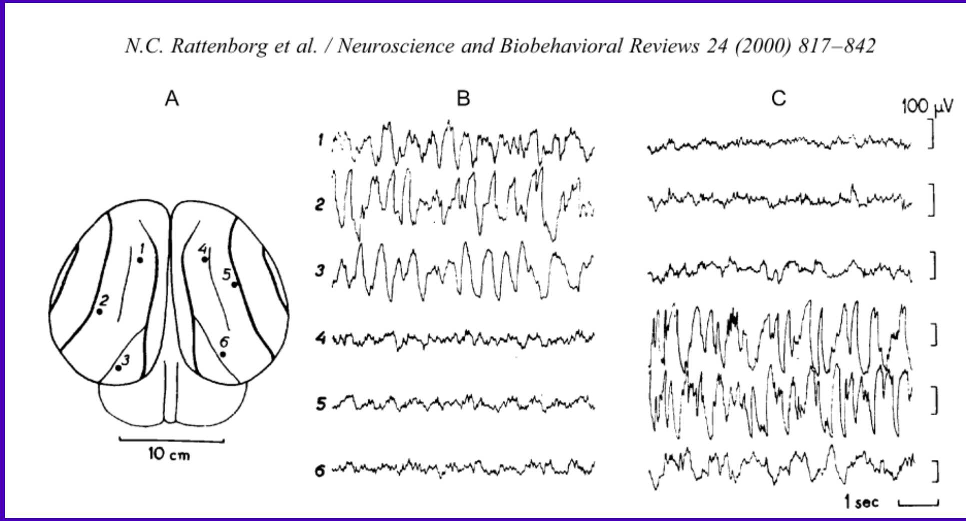
E. Schöll, Eur. Phys. J. – Special Topics 225, 891 (2016): Special Issue on Complex Systems
E. Schöll, A. Zakharova, R. Andrzejak (ed.): Research Topic in Front. Appl. Math. Stat. (2019).

Symmetry-breaking in neuronal systems

- Unihemispheric sleep: some birds and dolphins sleep with one half of their brain, while the other half remains awake

Symmetry-breaking in neuronal systems

- Unihemispheric sleep: some birds and dolphins sleep with one half of their brain, while the other half remains awake



Unihemispheric sleep of bottlenose dolphin (EEG)
B: left,
C: right hemisphere asleep

Symmetry-breaking in neuronal systems

- Unihemispheric sleep: some birds and dolphins sleep with one half of their brain, while the other half remains awake

great frigate bird

ARTICLE

Received 18 Feb 2016 | Accepted 6 Jul 2016 | Published 3 Aug 2016 DOI: 10.1038/ncomms12468 OPEN

Evidence that birds sleep in mid-flight

Niels C. Rattenborg[1,*], Bryson Voirin[1,2,*], Sebastian M. Cruz[3], Ryan Tisdale[1], Giacomo Dell'Omo[4], Hans-Peter Lipp[5,6,7], Martin Wikelski[3,8] & Alexei L. Vyssotski[9]

nature COMMUNICATIONS

Symmetry-breaking in neuronal systems

- Unihemispheric sleep: recently discovered in humans

- humans[1]: first-night effect – our brain stays alert to protect against unknown danger.

[1]M. Tamaki, J. W. Bang, T. Watanabe, Y. Sasaki, Night Watch in One Brain Hemisphere during Sleep Associated with the First-Night Effect in Humans, Curr Biol. 26, 5 (2016)

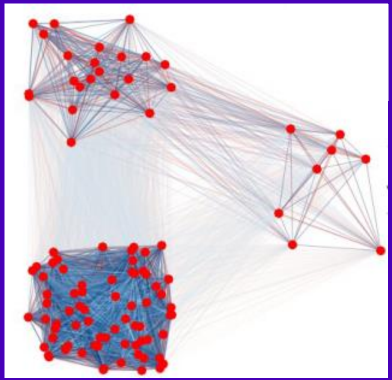
- Epileptic seizure: Chimera as mechanism for termination

Rothkegel and Lehnertz, New J Phys. 16, 055006 (2014)

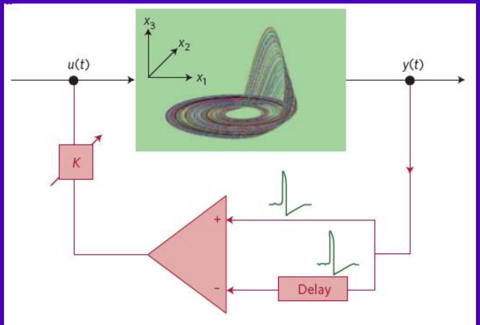
Andrzejak et al., Scientific Reports 6, 23000 (2016)

Control of synchronization patterns

- Multilayer topology: Multiplexing (one layer drives another)
- Modify coupling: Pacemaker (one node drives the others)



- Dynamic links: Adaptive topology (hierarchical multifrequency clusters)
- Proportional to frequency: direct control (stabilize frequency)
- Proportional to frequency difference: difference control (frequency droop control)



- Time delay: delayed feedback control (modify sync patterns)

Why is delay interesting in dynamics?

- Delay increases the dimension of a differential equation to infinity:

$$\dot{x}(t) = -ax(t) + bx(t-\tau)$$

delay τ generates infinitely many eigenmodes

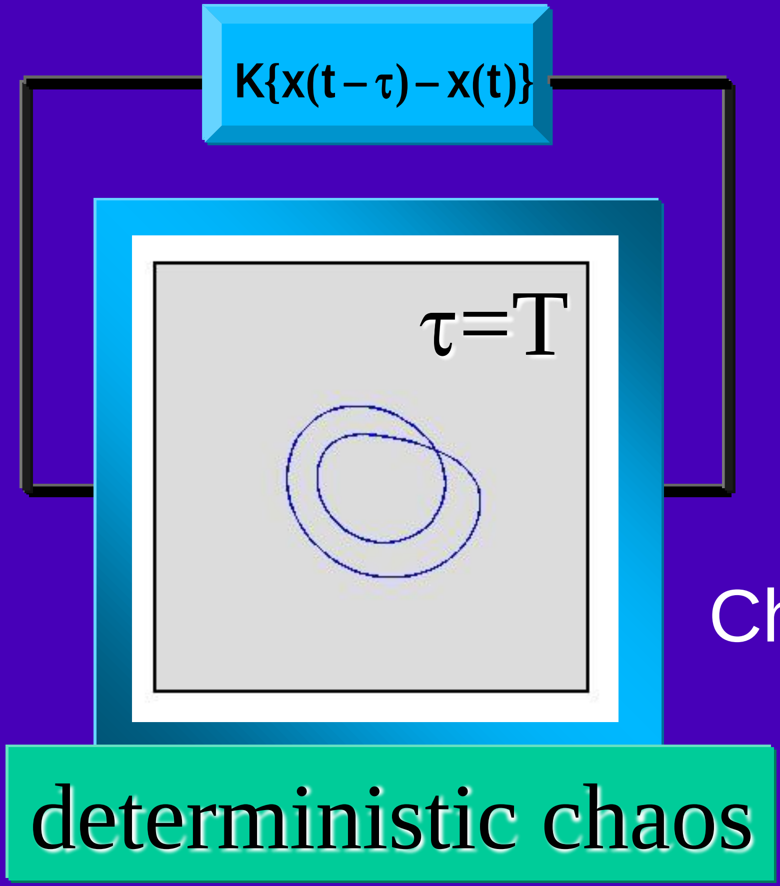
- Simple equations produce very complex behavior:
- delay-induced instabilities
- delay-induced bifurcations
- delay-induced multistability
- stabilization of unstable periodic or stationary states

in networks: delayed coupling

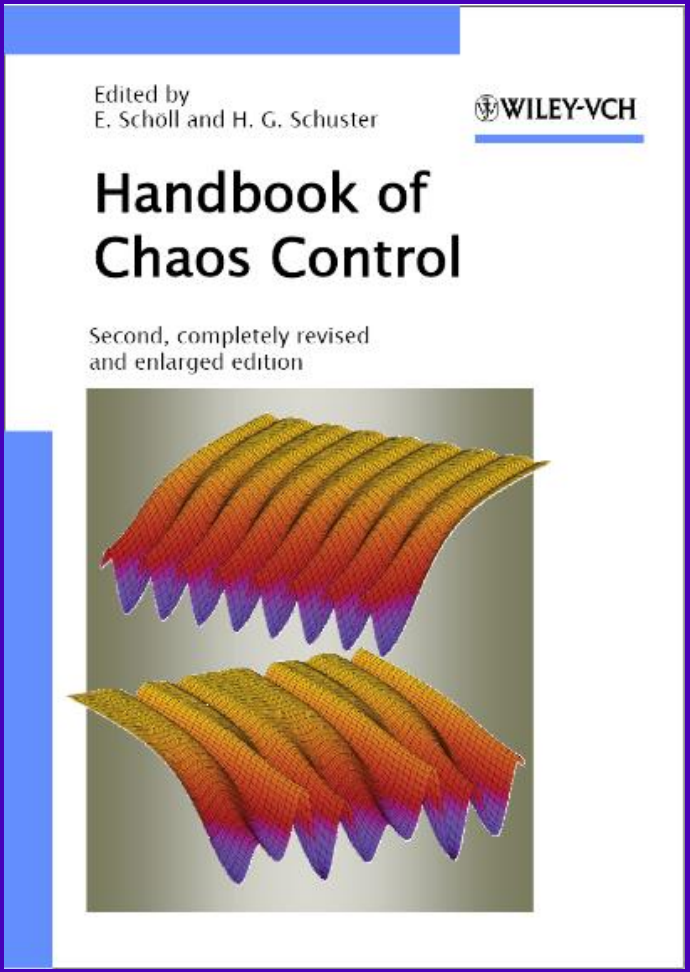
Time-delayed feedback control of nonlinear systems

Stabilisation of unstable periodic orbits (UPOs, dense in the chaotic attractor) or fixed points or synchronization of networks

- Time-delayed feedback (Pyragas 1992):



Chaos control



Synchronization of power grids

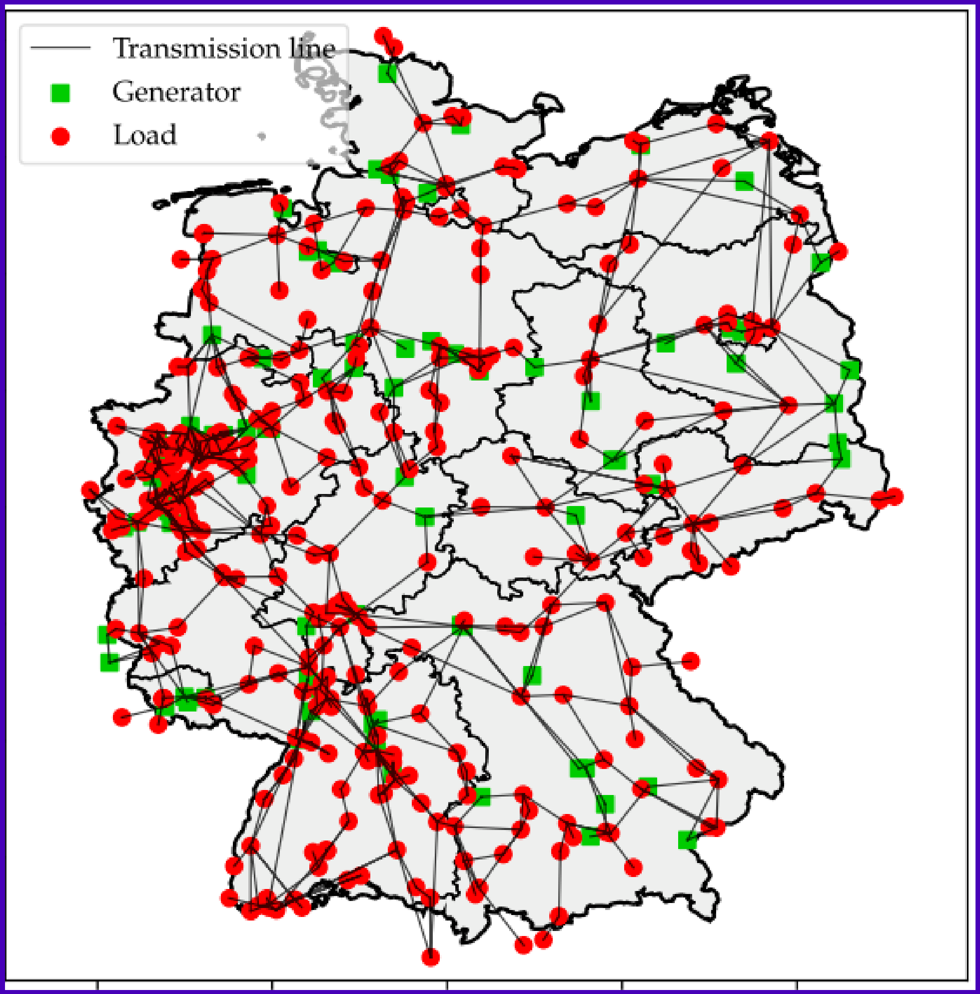
Motivation

- Transition to renewable and sustainable energies
  - stable and efficient operation of the power grid
  - fluctuating power input (wind, solar)

- Complex networks perspective
  - modelling approaches based on simple swing equation
  - interplay of complex topologies and phase oscillator dynamics
  - control of synchronization and stability
  - analysis of cascading failures
  - influence of stochastic fluctuations of generators and loads

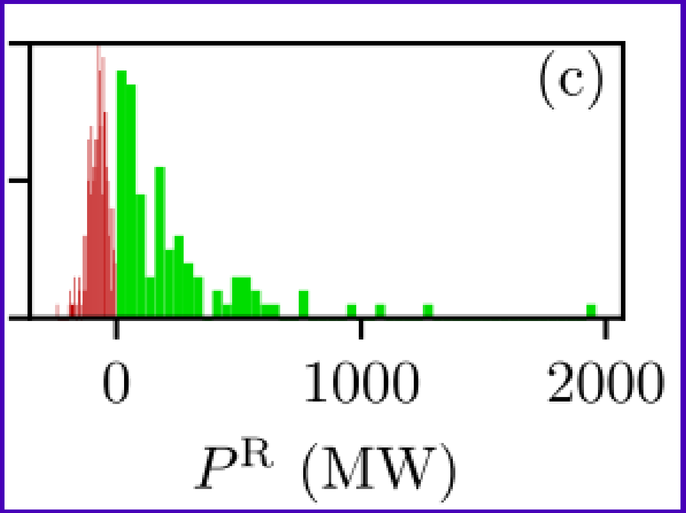
see Special Track on Modelling of Dynamics of Power Grids (MoDyPoG)
Part I: Mo 15:00-17:30   Part II: We 15:00-17:30 Chair: E. Schöll

380 kV high-voltage power grid of Germany



network of generators and loads

Distribution of power
of generators and loads



desired: synchronization
at nominal frequency of 50 Hz

H. Taher, S. Olmi, E. Schöll, Phys. Rev. E 100, 060326 (2019) Talk by L.Tumash (DyMoPoG)
L. Tumash, S. Olmi, E. Schöll, Europhys. Lett. 123, 20001 (2018); Chaos 29, 123105 (2019)
C.H. Totz, S. Olmi, E. Schöll, Phys. Rev. E 102, 022311 (2020) Talk by S.Olmi (DyMoPoG)

Model of power grid

Kuramoto model of coupled phase oscillators with inertia (swing eq.):

$$\ddot{\theta}_i + \alpha\dot{\theta}_i = \frac{P_i}{I_i\omega_G} + \frac{K}{I_i\omega_G}\sum_{j=1}^{N} A_{ij}\sin(\theta_j - \theta_i)$$

N coupled synchronous machines (generators or loads),
$\theta_i(t)$ phase, $\dot{\theta}_i(t) = \frac{d\theta_i}{dt}$ instantaneous frequency, ω_G reference frequency,
power generation ($P_i > 0$) and consumption ($P_i < 0$), power balance $\Sigma P_i = 0$
K transmission capacity, A_{ij} adjacency matrix, α dissipation parameter,
I_i moment of inertia

Perturbation of frequency synchrony:
→ time delayed feedback control
(delay time τ, feedback gain g_i)

$$-\frac{g_i\alpha}{\tau}[\theta_i(t) - \theta_i(t-\tau)]$$

H. Taher, S. Olmi, E. Schöll, Phys. Rev. E 100, 060326 (2019)

Solitary nodes of power grid

Standard deviation of frequencies

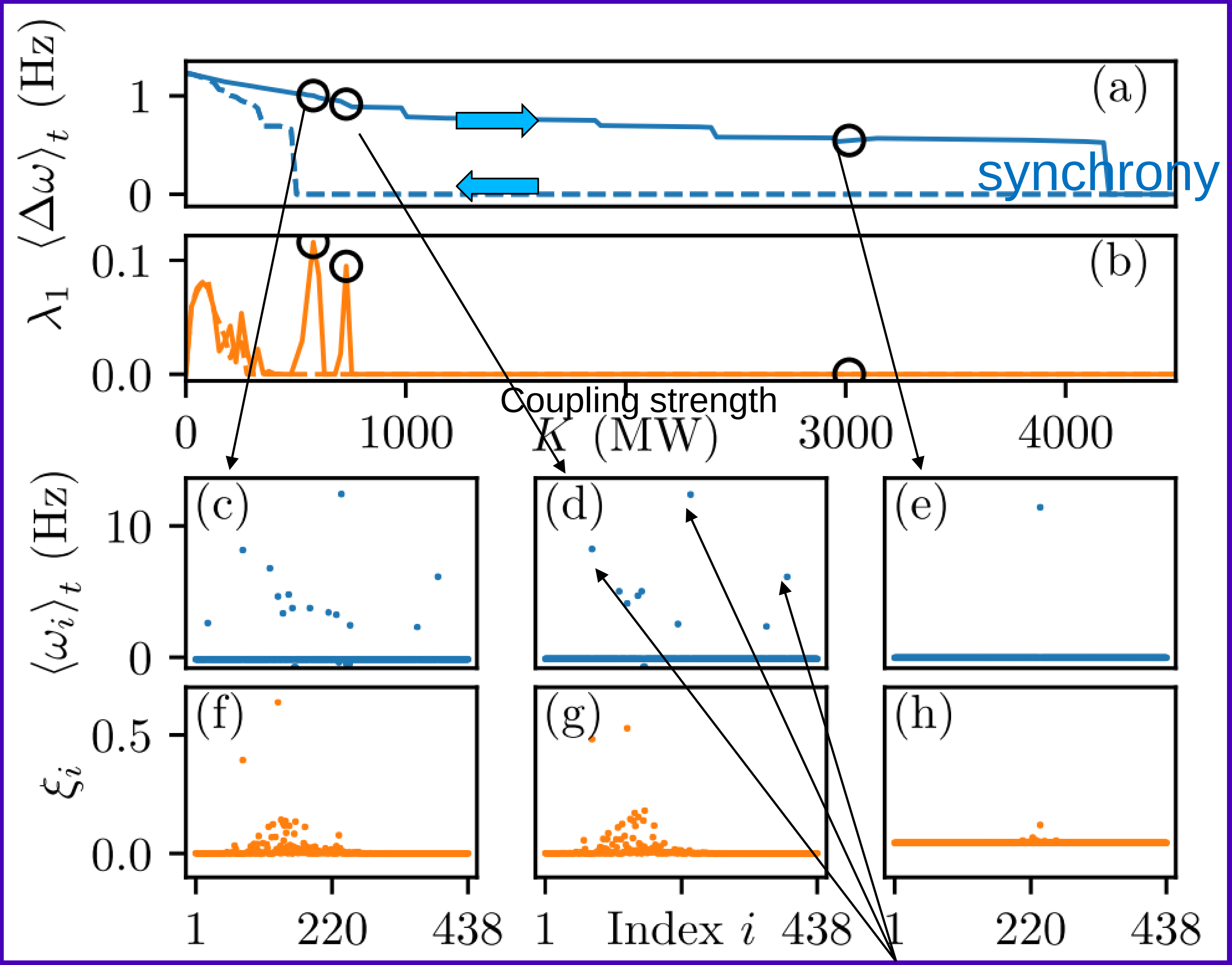
$$\Delta\omega(t) \equiv \frac{1}{N}\sqrt{\sum_{i=1}^{N}[\omega_i(t) - \bar{\omega}(t)]^2}$$

Largest Lyapunov exponent

Average frequency

$$\langle\omega_i\rangle_t := \frac{1}{\tau}\int_{t-\tau}^{t}\dot{\theta}_i(t')dt'$$

components of Lyapunov vector



$<>_t$ time averages over 100s

solitary nodes (to be controlled)

H. Taher, S. Olmi, E. Schöll, Phys. Rev. E 100, 060326 (2019)

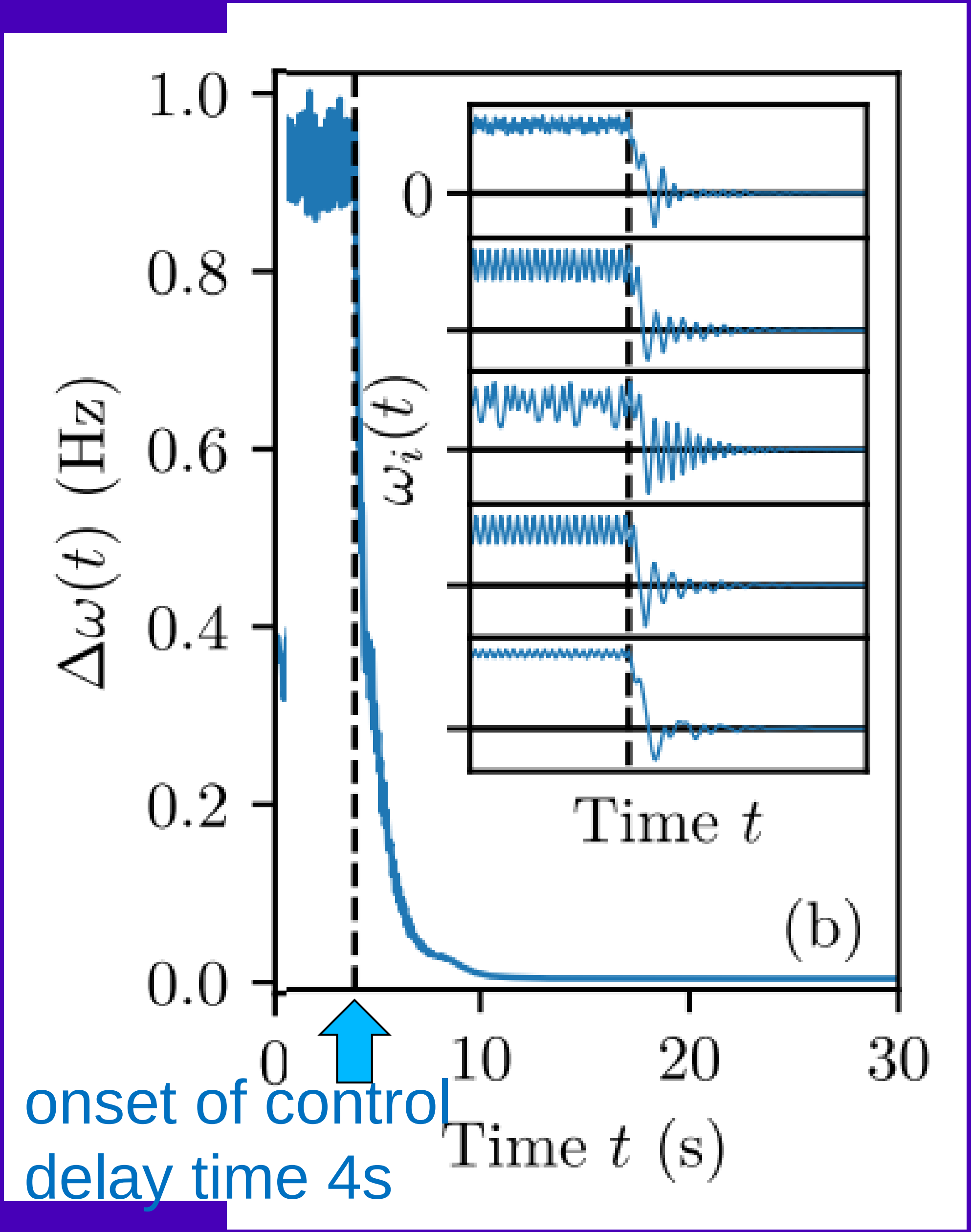
Time delayed feedback control of synchrony

Control of a subset of nodes:
solitary nodes (i=1, …, 11)

Standard deviation of frequencies

$$\Delta\omega(t) \equiv \frac{1}{N}\sqrt{\sum_{i=1}^{N}[\omega_i(t) - \bar{\omega}(t)]^2}$$

conclusion:
re-synchronization after a few s



onset of control
delay time 4s

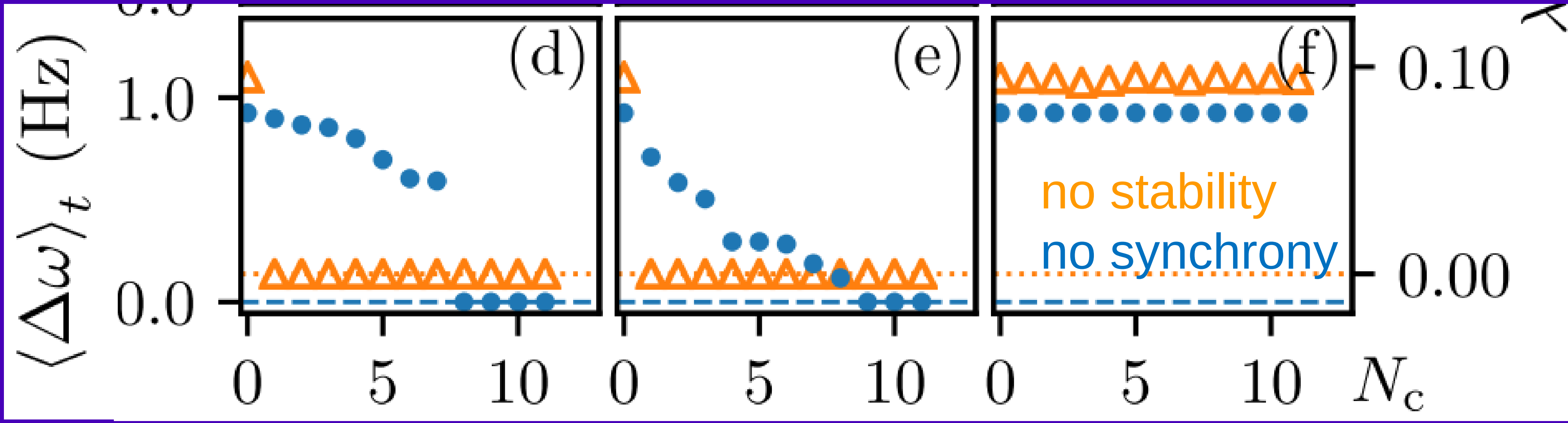
H. Taher, S. Olmi, E. Schöll, Phys. Rev. E 100, 060326 (2019)

Efficiency of control

Control of a subset of nodes only:

Solitary nodes sorted in descending order of Lyapunov vector ξ_i $|\langle\omega_i\rangle_t|$

Randomly picked nodes



blue: time-averaged standard frequency deviation orange: largest Lyapunov exponent

vs. number of controlled nodes

Conclusion: (d) 1 node (e) 1 node sufficient for stability

8 nodes 9 nodes sufficient for frequency synchrony

H. Taher, S. Olmi, E. Schöll, Phys. Rev. E 100, 060326 (2019)

Extension to swing equation with voltage dynamics

phase θ_i
voltage E_i
complex $\mathbf{E}_i = E_i e^{i\theta_i}$

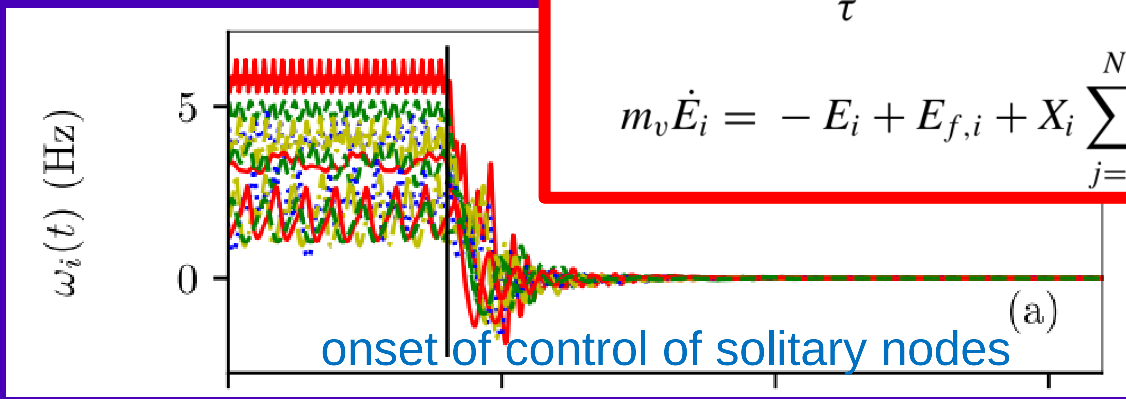
$$\ddot{\theta}_i + \alpha\dot{\theta}_i = \frac{P_i}{I_i\omega_G} + \frac{K}{I_i\omega_G}\sum_{j=1}^{N} A_{ij}E_iE_j\sin(\theta_j - \theta_i)$$

$$m_v\dot{E}_i = -E_i + E_{f,i} + X_i\sum_{j=1}^{N} A_{ij}E\,j\cos(\theta_j - \theta_i),$$

K. Schmietendorf, J. Peinke, R. Friedrich, O. Kamps, Eur. Phys. J. ST 223, 2577 (2014)

with time-delayed feedback control

$$\ddot{\theta}_i + \alpha\dot{\theta}_i = \frac{P_i}{I_i\omega_G} + \frac{K}{I_i\omega_G}\sum_{j=1}^{N} A_{ij}E_iE_j\sin(\theta_j - \theta_i)$$
$$- \frac{g_i\alpha}{\tau}[\theta_i(t) - \theta_i(t-\tau)],$$
$$m_v\dot{E}_i = -E_i + E_{f,i} + X_i\sum_{j=1}^{N} A_{ij}E\,j\cos(\theta_j - \theta_i)$$

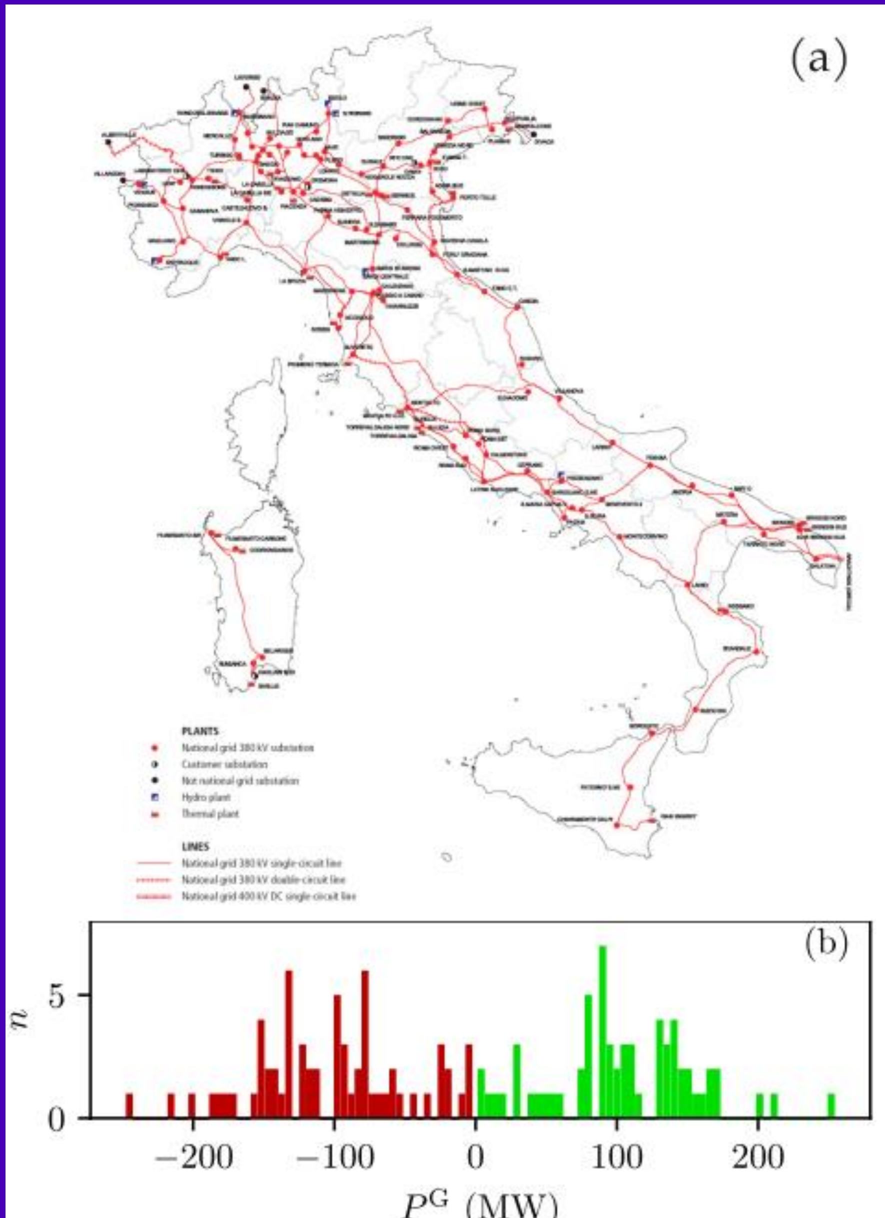


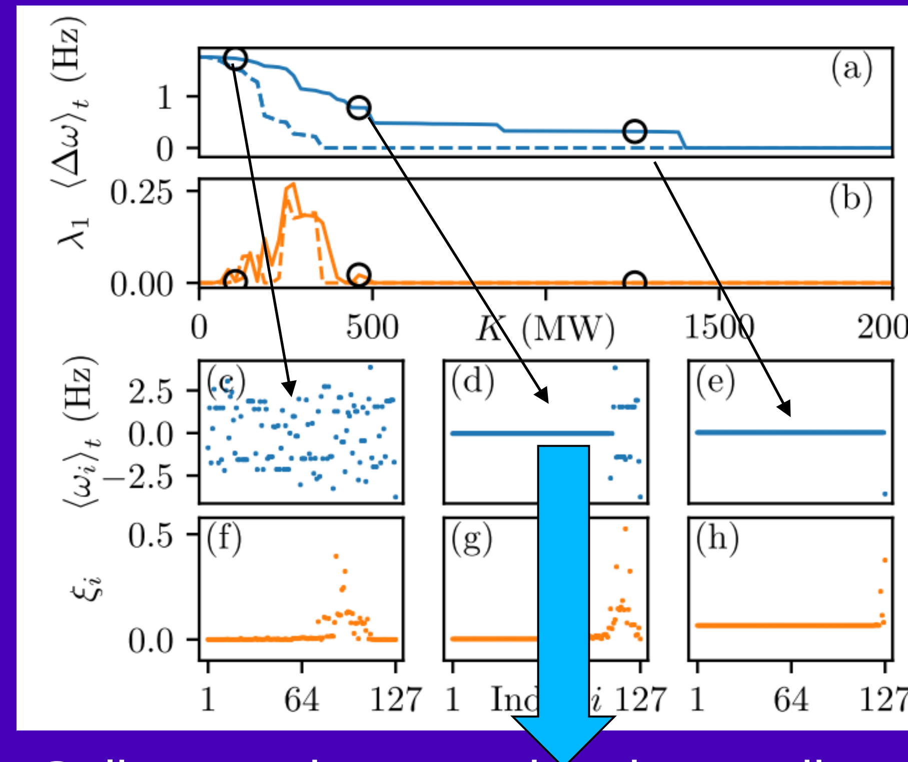
Time-delayed feedback control term represents running frequency average:

$$\langle\omega_i\rangle_t := \frac{1}{\tau}\int_{t-\tau}^{t}\dot{\theta}_i(t')dt' = \frac{1}{\tau}[\theta_i(t) - \theta_i(t-\tau)].$$

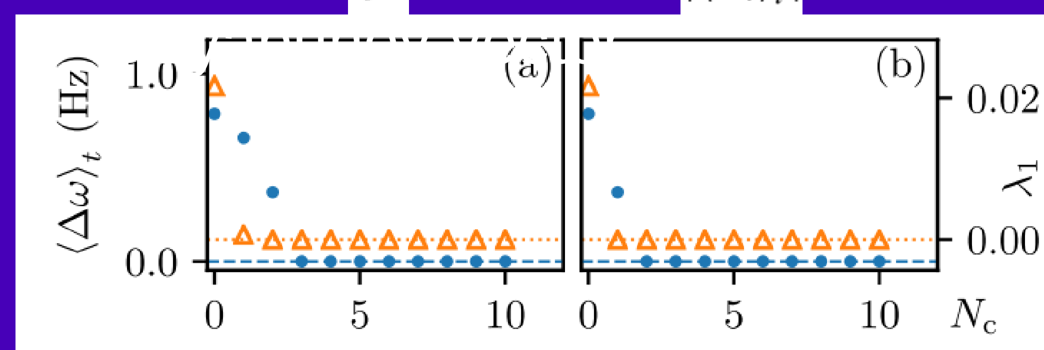
H. Taher, S. Olmi, E. Schöll, Phys. Rev. E 100, 060326 (2019)

Italian grid





Solitary nodes sorted in descending order of ξ_i $|\langle \omega_i \rangle_t|$



H. Taher, S. Olmi, E. Schöll, Phys. Rev. E 100, 060326 (2019)

Partial synchronization patterns in diluted synthetic grid

upsweep - downsweep

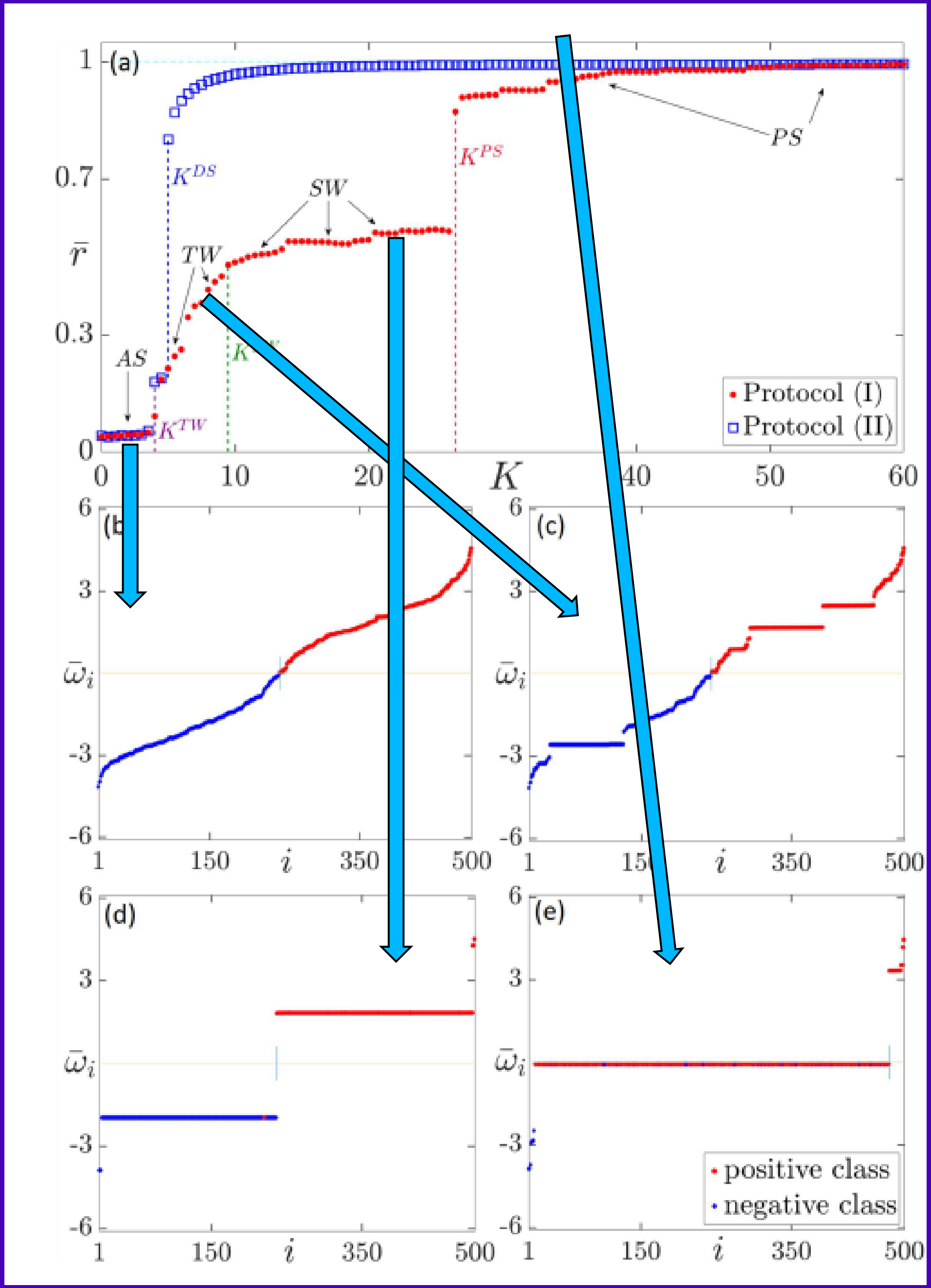
Hysteresis

global order parameter (time-averaged)

mean phase velocity

asynchronous (K=2)

standing wave (K=25)



dilution p=0.2, constant node degree

traveling wave (K=5)

(almost) synchronized (K=33)

L. Tumash, S. Olmi, E. Schöll, Chaos 29, 123105 (2019)

Power grid models ⟺ adaptive neural networks

adaptive network of Kuramoto phase oscillators: neuronal synaptic plasticity

$$\dot{\phi}_i = \omega_i + \sum_{j=1}^{N} a_{ij}\kappa_{ij} f(\phi_i - \phi_j)$$

$$\dot{\kappa}_{ij} = -\epsilon \left(\kappa_{ij} + g(\phi_i - \phi_j)\right),$$

adaptive coupling weights κ_{ij}

phase oscillators with inertia:

$$M\ddot{\phi}_i + \gamma\dot{\phi}_i = P_i + \sum_{j=1}^{N} a_{ij} h(\phi_i - \phi_j)$$

equivalent to

$$\dot{\phi}_i = \omega_i + \psi_i,$$

$$\dot{\psi}_i = -\frac{\gamma}{M}\left(\psi_i - \frac{1}{\gamma}\sum_{j=1}^{N} a_{ij} h(\phi_i - \phi_j)\right)$$

with $\psi_i = \sum_{j=1}^{N} a_{ij}\chi_{ij}$

$$\dot{\phi}_i = \omega_i + \sum_{j=1}^{N} a_{ij}\chi_{ij},$$

$$\dot{\chi}_{ij} = -\epsilon \left(\chi_{ij} + g(\phi_i - \phi_j)\right)$$

pseudo coupling weights

R. Berner, S. Yanchuk, E. Schöll: What adaptive neuronal networks teach us about power grids, Phys.Rev.E 103, 042315 (2021)

Talk by R.Berner (DyMoPoG) We 17:00

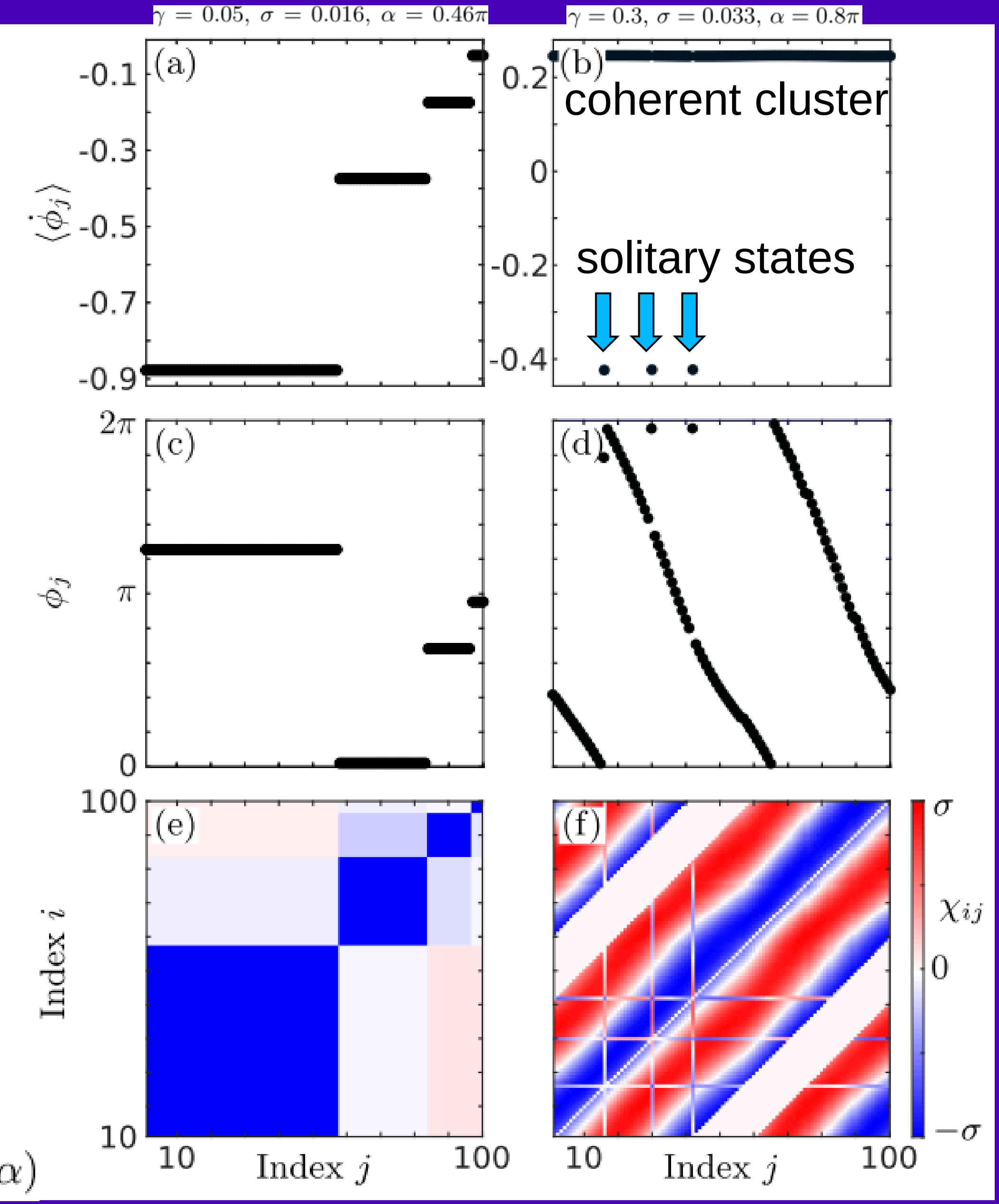
What adaptive networks teach us about power grids

Hierarchical multicluster states:

Multifrequency clusters

left column
4-cluster state of in-phase synchronous clusters

right column
hierarchical mixed type multicluster of large splay cluster and small in-phase cluster



$$h(\phi) = -\sigma\gamma\sin(\phi+\alpha)$$

globally coupled nonlocally coupled ring

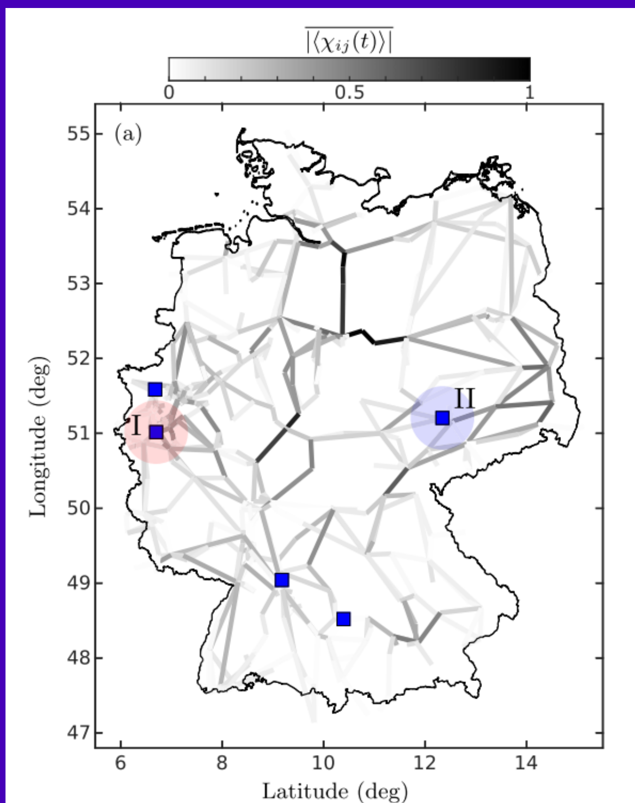
R. Berner, S. Yanchuk, E. Schöll:
What adaptive neuronal networks teach us about power grids, Phys.Rev.E 103, 042315 (2021)

Solitary states in power grids

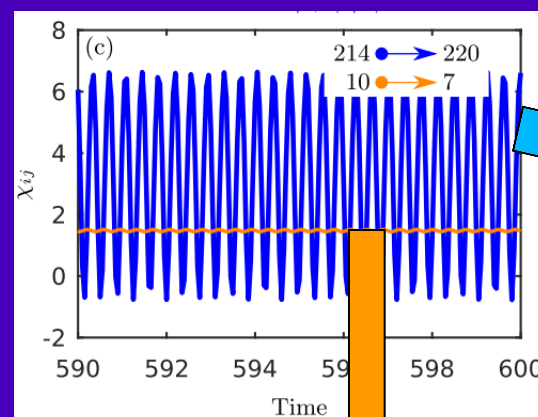
Pseudo coupling weights χ_{ij} = power flow from node j to node i

average power flow

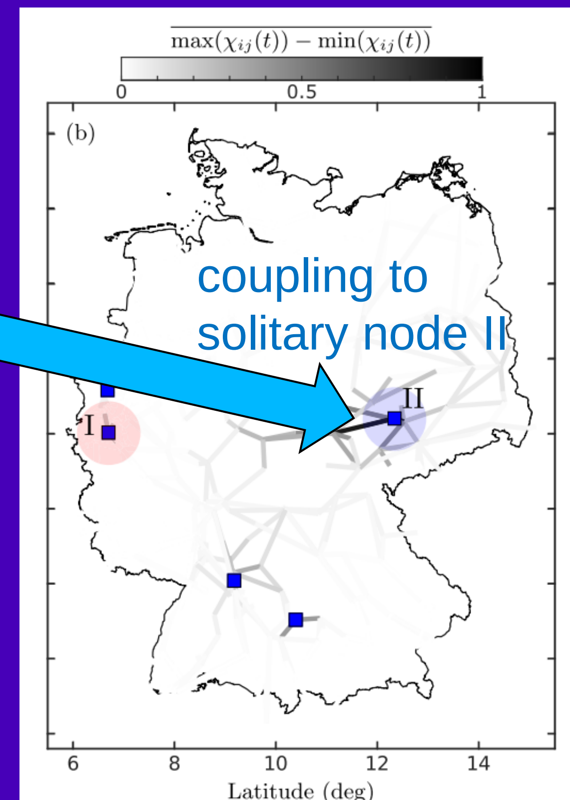
temporal power flow range



oscillating power flow



coupling between
non-solitary nodes



coupling to
solitary node II

spreading of flow variations
critical lines

R. Berner, S. Yanchuk, E. Schöll:
What adaptive neuronal networks teach us about power grids, Phys.Rev.E 103, 042315 (2021)

Synchronization
in brain dynamics

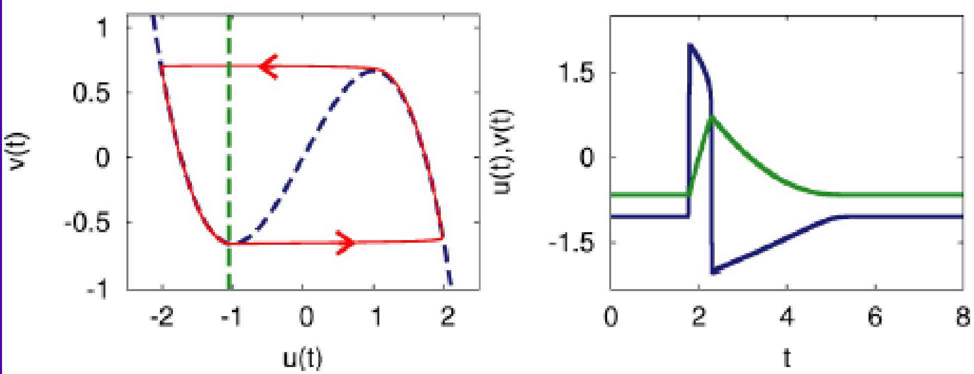
Neural networks: FitzHugh-Nagumo system

coupled amplitude-phase dynamics

The FitzHugh-Nagumo model for neuronal activity

with activator u, inhibitor v: $\mathbf{x} = (u, v)^T$

$$\mathbf{F}(\mathbf{x}) = \begin{pmatrix} \frac{1}{\epsilon}(u - \frac{u^3}{3} - v) \\ u + a \end{pmatrix}, \quad \mathbf{H} = \begin{pmatrix} 1/\epsilon & 0 \\ 0 & 0 \end{pmatrix}$$



- operation in the excitable regime
- uncoupled neurons rest in fixed point
- operation in the oscillatory regime (a < 1)
- uncoupled: oscillates periodically

FitzHugh-Nagumo (FHN) network: single layer

$$\varepsilon \frac{du_k}{dt} = u_k - \frac{u_k^3}{3} - v_k + \frac{\sigma}{2P} \sum_{j=k-P}^{k+P} [b_{uu}(u_j - u_k) + b_{uv}(v_j - v_k)]$$

$$\frac{dv_k}{dt} = u_k + a_k + \frac{\sigma}{2P} \sum_{j=k-P}^{k+P} [b_{vu}(u_j - u_k) + b_{vv}(v_j - v_k)] .$$

σ – coupling strength (control parameter!)

$r = P/N$ – coupling radius (control parameter!)

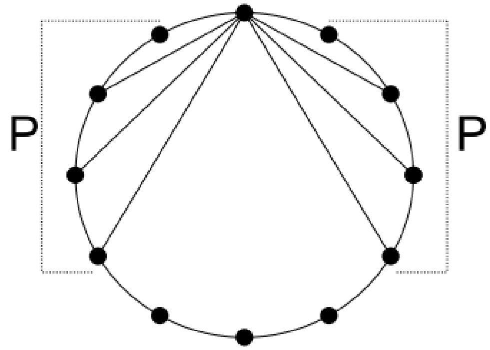
ε – small parameter

$a_k,\ k = 1, ..., N$ – threshold parameters, $a_k \equiv a$

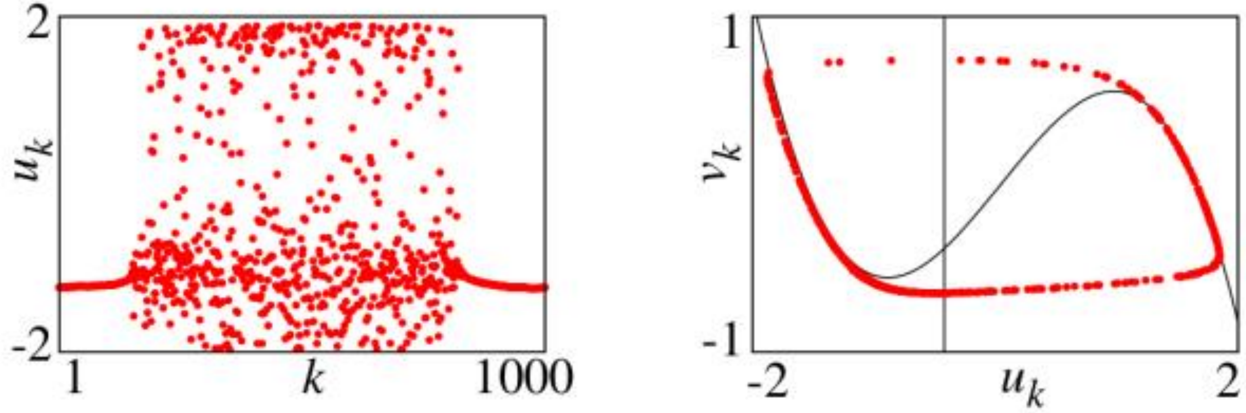
Local interaction matrix:

$$B = \begin{pmatrix} b_{uu} & b_{uv} \\ b_{vu} & b_{vv} \end{pmatrix} = \begin{pmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{pmatrix}, \quad \phi \in [-\pi, \pi)$$

Nonlocal (intermediate) coupling



Chimera states in FHN networks



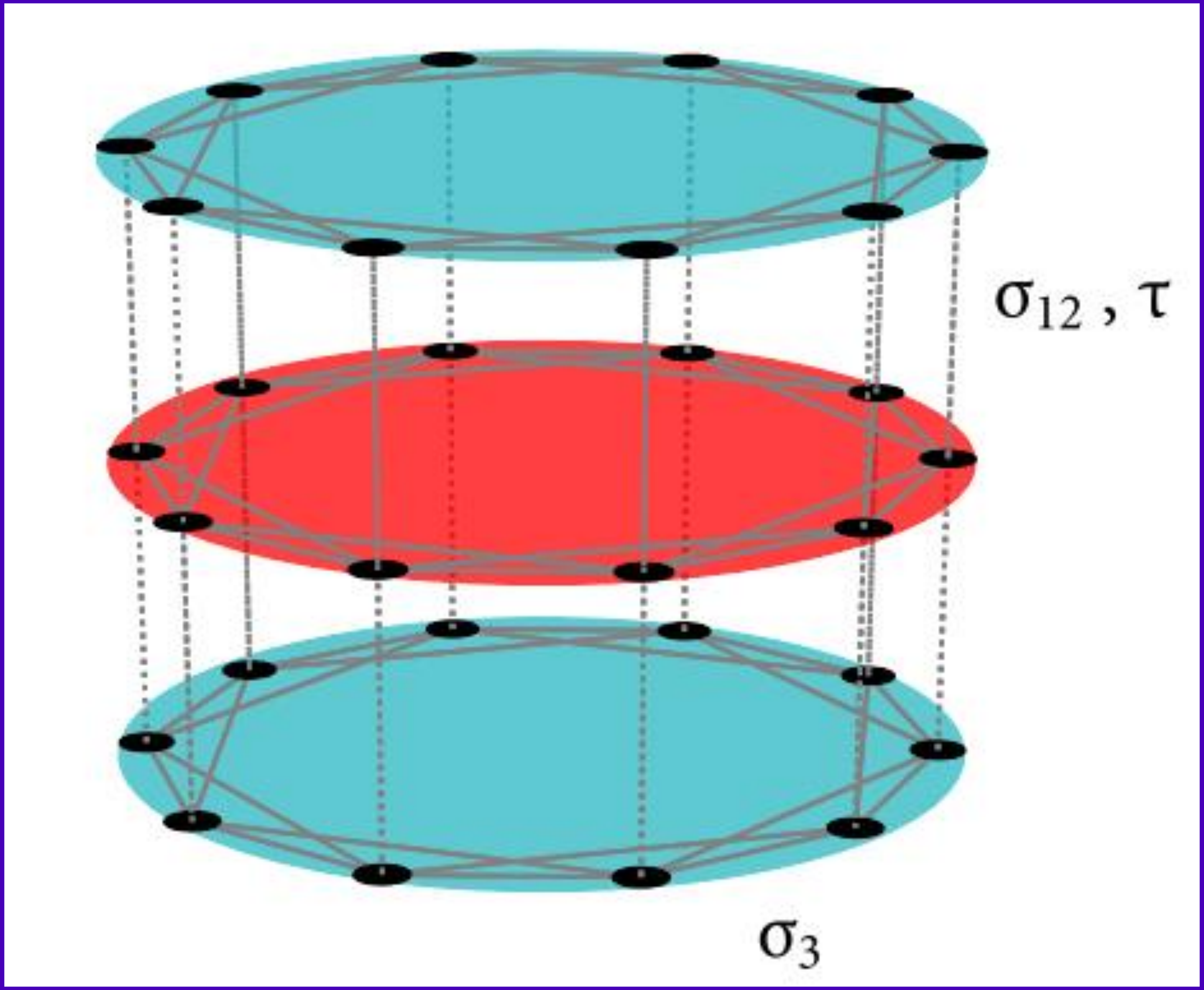
System parameters:

$N = 1000$– large system
$r = 0.35$– intermediate coupling radius
$\sigma = 0.1$– small coupling strength
$a = 0.5, \quad \phi = \pi/2 - 0.1$

I. Omelchenko, O. E. Omel'chenko, P. Hövel, and E. Schöll, Phys. Rev. Lett. **110**, 224101 (2013).

Relay synchronization in multilayer networks

3-layer multiplex network



delayed coupling

Relay synchronization of complex partial synchronization patterns?

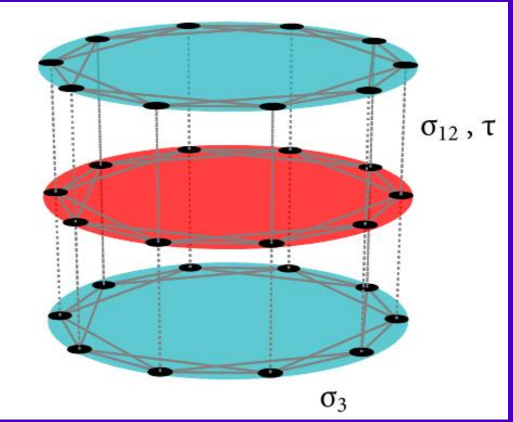
J. Sawicki, I. Omelchenko, A. Zakharova, E. Schöll: PRE 98, 062224 (2018)

FHN model + 3-layer multiplex network

$$\dot{\boldsymbol{x}}_i^m(t) = \boldsymbol{F}(\boldsymbol{x}_i^m(t)) + \frac{\sigma_m}{2R_m}\sum_{j=i-R_m}^{i+R_m} \boldsymbol{H}[\boldsymbol{x}_j^m(t) - \boldsymbol{x}_i^m(t)] + \sum_{l=1}^{3}\sigma_{ml}\boldsymbol{H}[\boldsymbol{x}_i^l(t-\tau) - \boldsymbol{x}_i^m(t)]$$

nonlocal intra-layer coupling

delayed inter-layer coupling (weak)



$$\sigma = \begin{pmatrix} 0 & \sigma_{12} & 0 \\ \frac{\sigma_{12}}{2} & 0 & \frac{\sigma_{32}}{2} \\ 0 & \sigma_{32} & 0 \end{pmatrix}$$

Local inter-layer synchronization error:

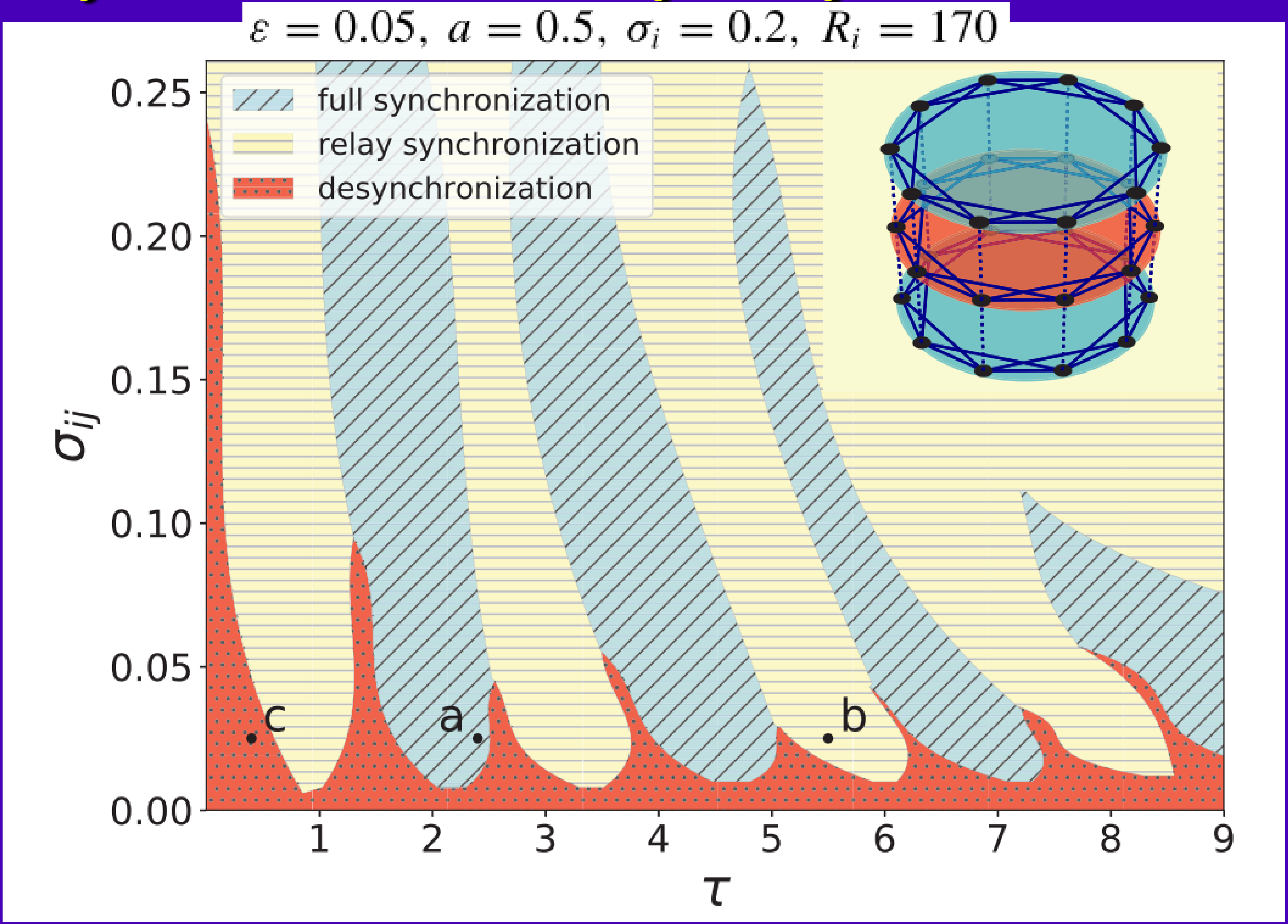
$$E_k^{ij} = \lim_{T\to\infty}\frac{1}{T}\int_0^T \left\|\mathbf{x}_k^j(t) - \mathbf{x}_k^i(t)\right\| dt$$

Global inter-layer synchronization error:

$$E^{ij} = \lim_{T\to\infty}\frac{1}{NT}\int_0^T \sum_{k=1}^{N}\left\|\mathbf{x}_k^j(t) - \mathbf{x}_k^i(t)\right\| dt$$

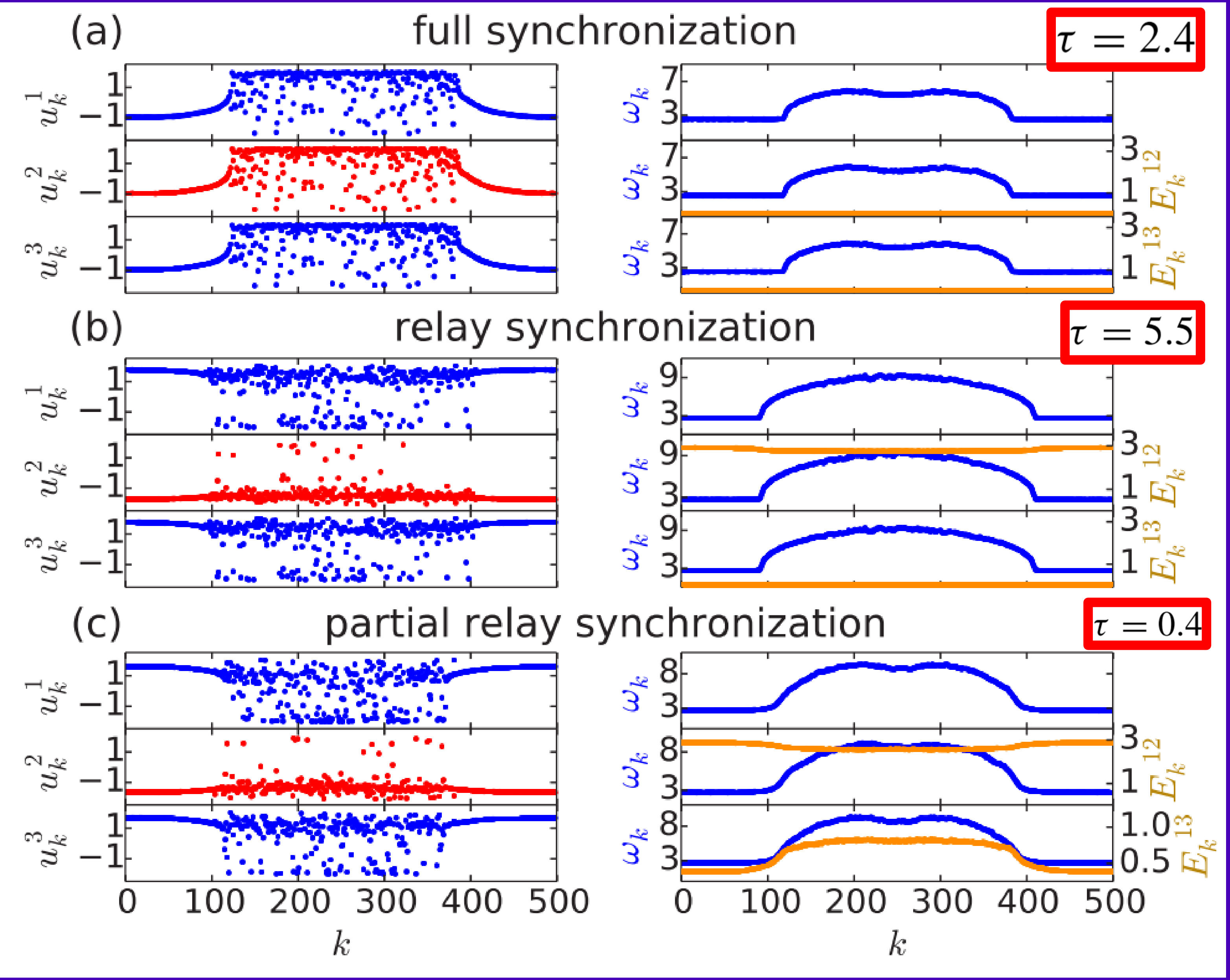
J. Sawicki, I. Omelchenko, A. Zakharova, E. Schöll: PRE 98, 062224 (2018)

Relay and full inter-layer synchronization



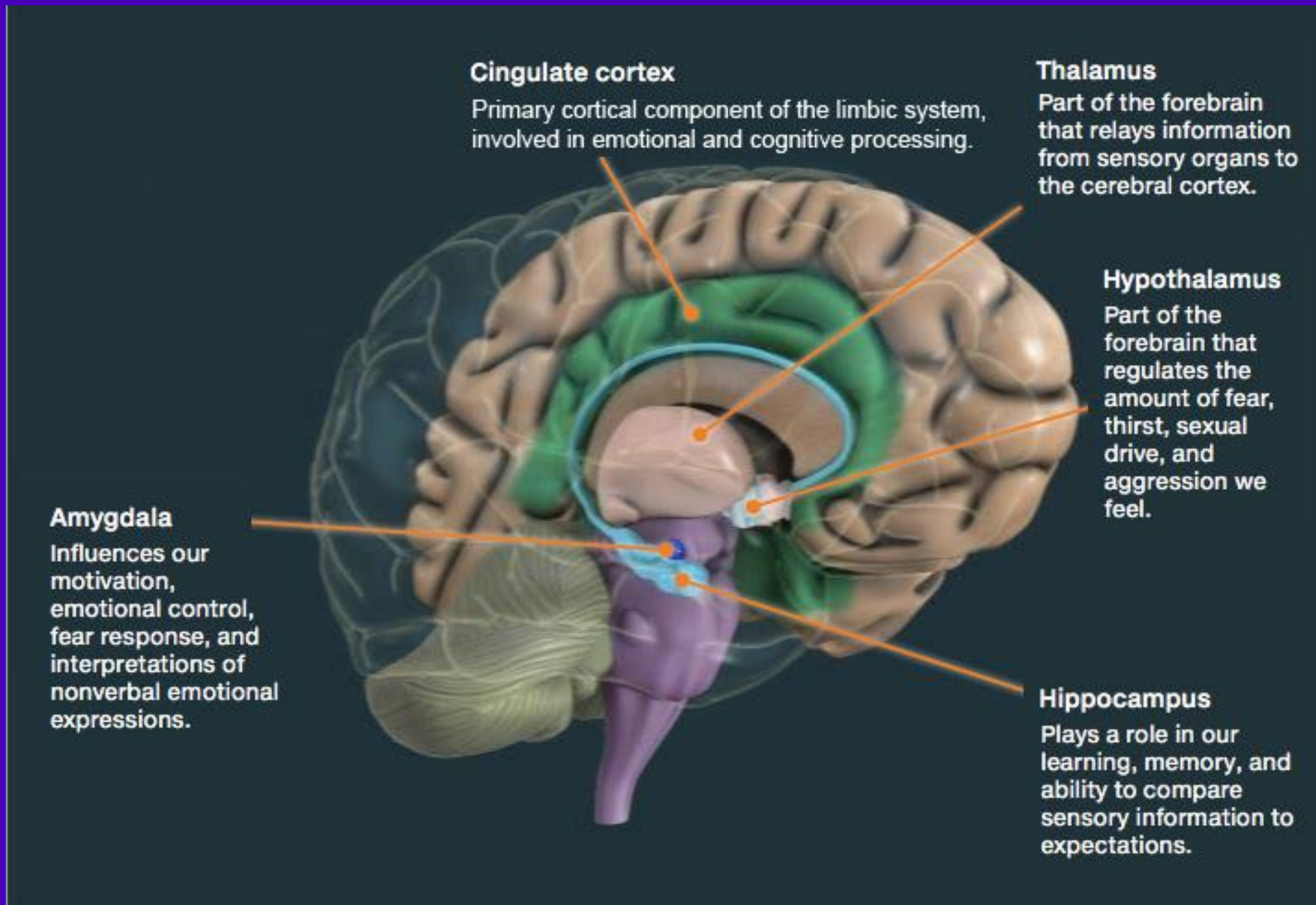
J. Sawicki, I. Omelchenko, A. Zakharova, E. Schöll: PRE 98, 062224 (2018)

Delay controls partial relay synchronization



J. Sawicki, I. Omelchenko, A. Zakharova, E. Schöll: PRE 98, 062224 (2018)

Relay functions in the brain



Thalamus

Hippocampus

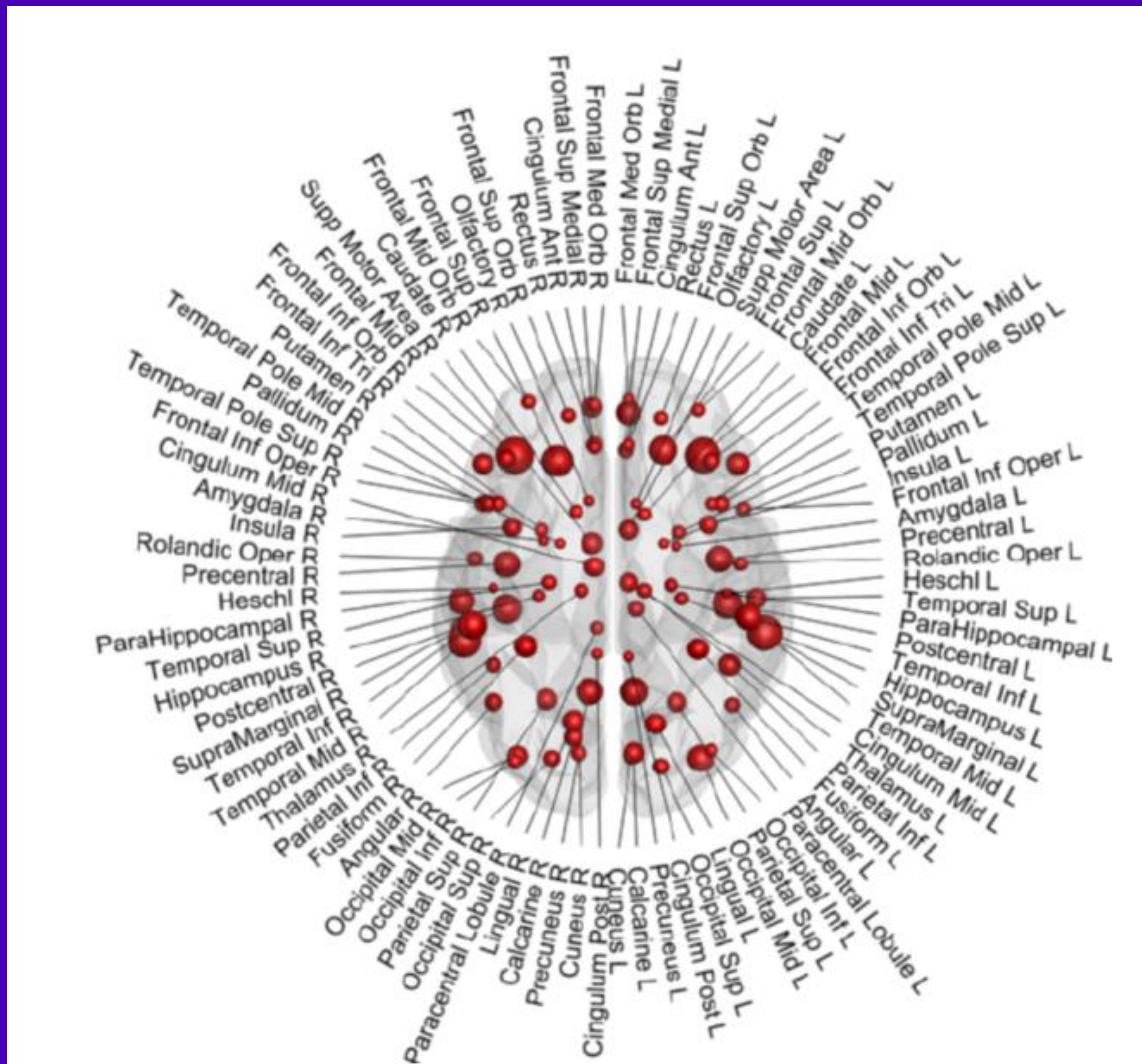
Our model explains complete and partial relay synchronisation

Experiment (mouse): Hippocampus as relay between frontal and visual cortex

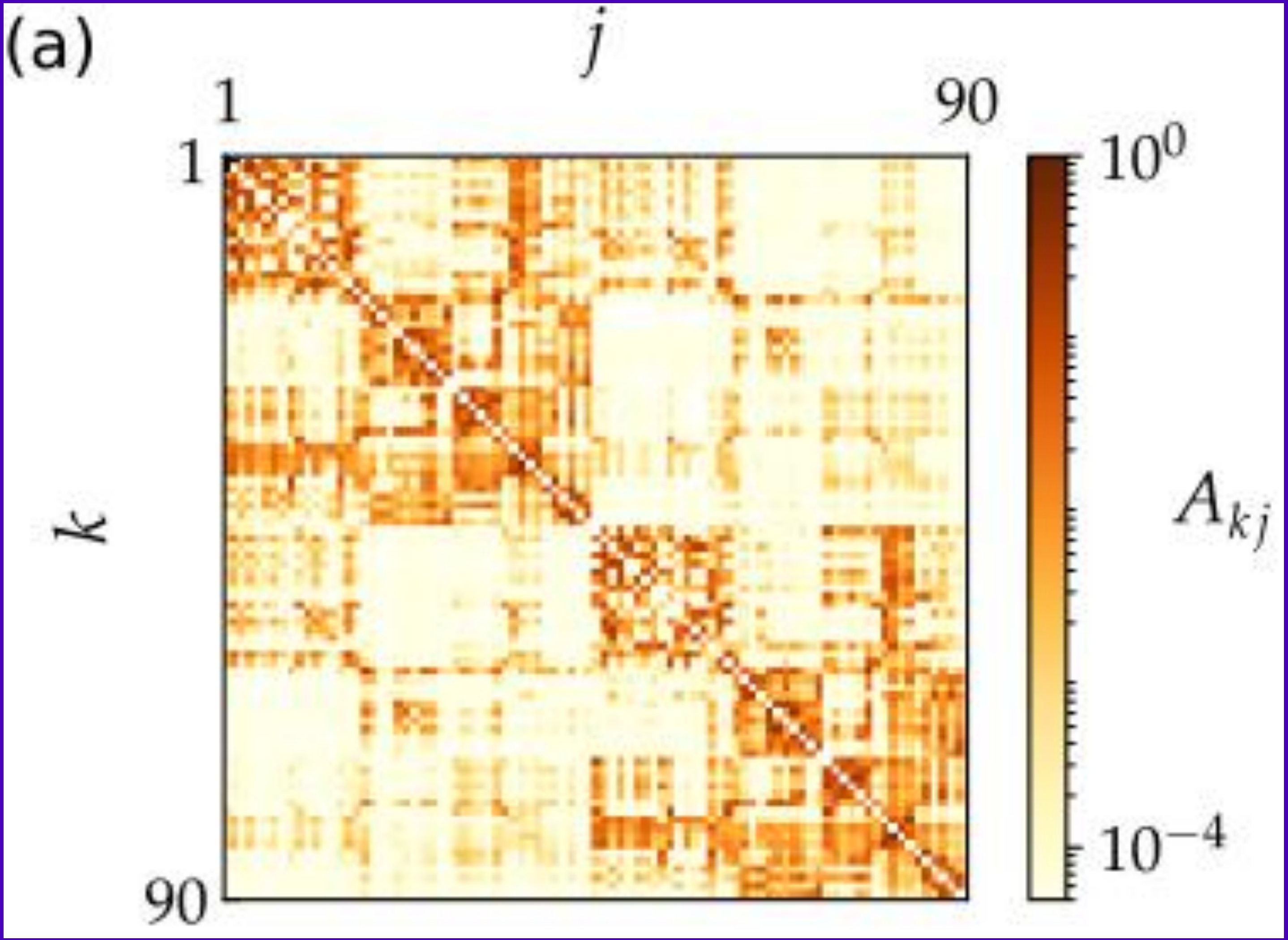
Gollo, L. L., Mirasso, C. R. , Atienza, M., Crespo-Garcia, M. and Cantero, J. L., *Theta Band Zero-Lag Long-Range Cortical Synchronization via Hippocampal Dynamical Relaying*, PLoS ONE **6**, 3, e17756 (2011)

Empirical brain connectivities: unihemispheric sleep

Empirical brain data:
90 areas of Automated Anatomical Labeling atlas



Empirical brain data: empirical connectivity matrix (diffusion-weighted magnetic resonance imaging)



T.Chouzouris, I.Omelchenko, A.Zakharova, J.Hlinka, P.Jiruska, E.Schöll: Chaos 28, 045112 (2018)

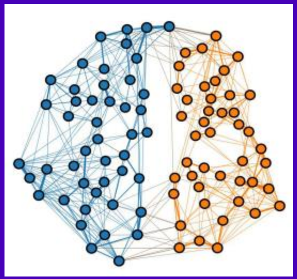
J. Sawicki, E. Schöll: Front. Appl. Math. Stat. 7, 662221 (2021)

Model with empirical brain connectivities

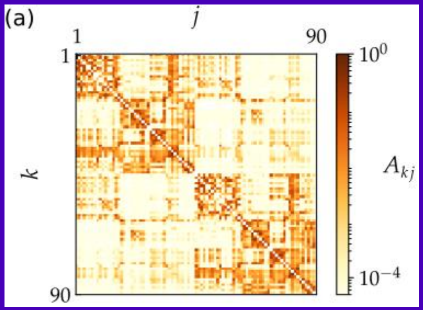
- Unihemispheric sleep: L. Ramlow, J. Sawicki, A. Zakharova, J. Hlinka, J. C. Claussen, E. Schöll: EPL126, 50007 (2019)
  highlighted on phys.org/news/2019-07-unihemispheric-humans.html

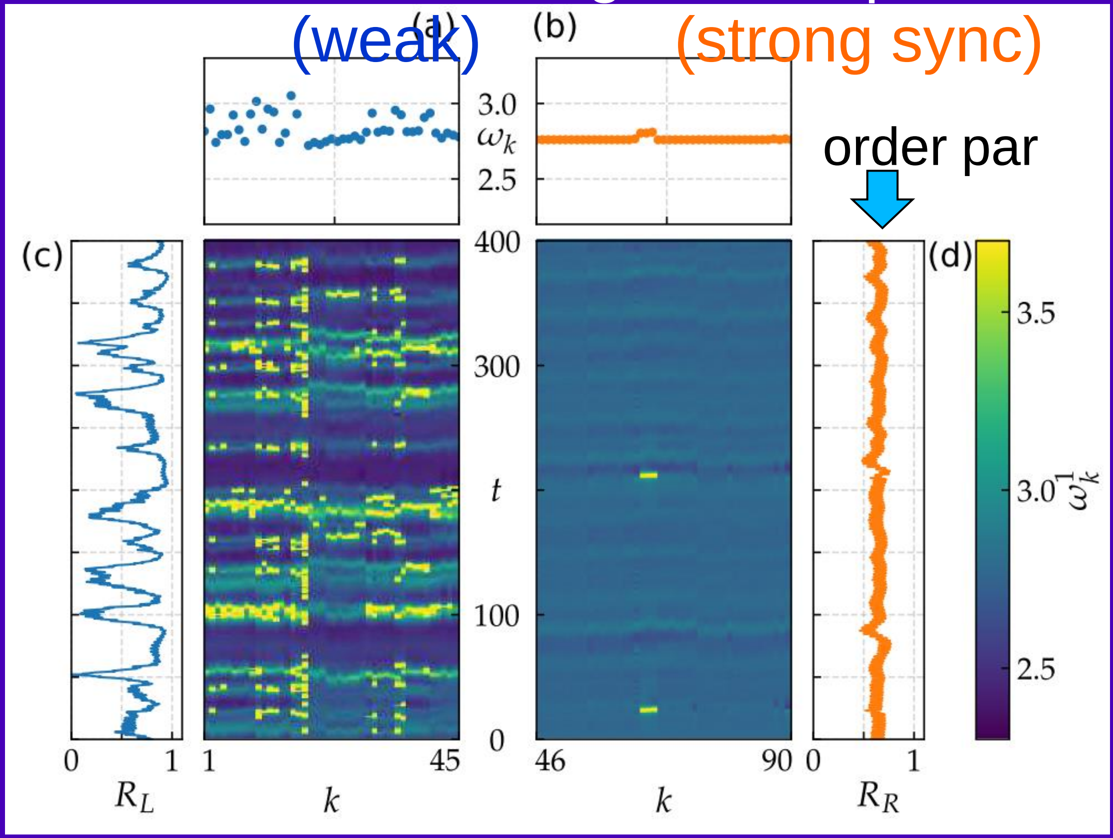
FitzHugh-Nagumo model: left right hemisphere

Anatomical Labeling Atlas



connectivity matrix (DTI-dMRI)

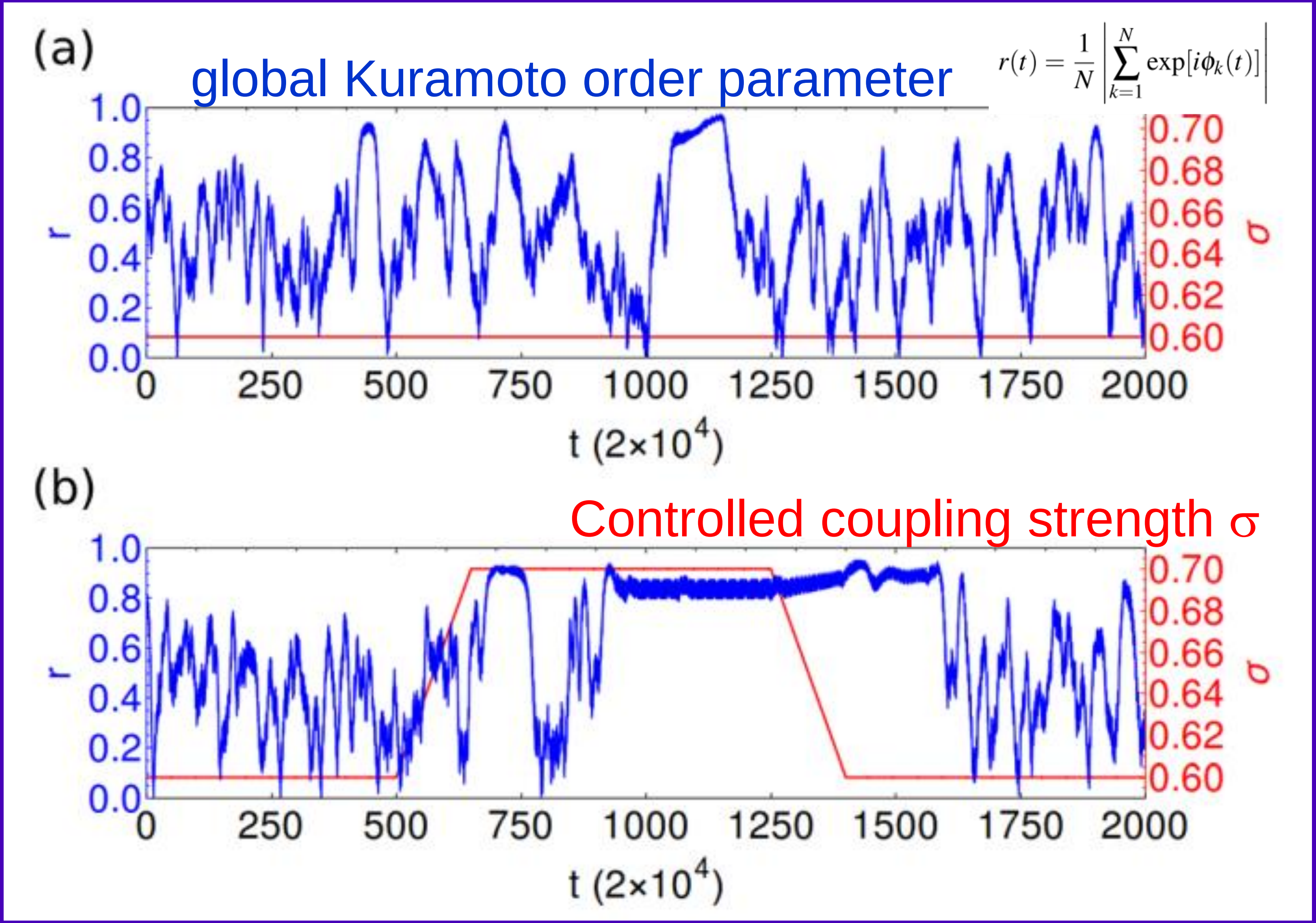




$\sigma = 0.70,\ \varsigma = 0.15$ different intra-/inter-coupling strength

Epileptic-seizure-related synchronization phenomena

Simulation of epileptic seizure: empirical connectivity matrix



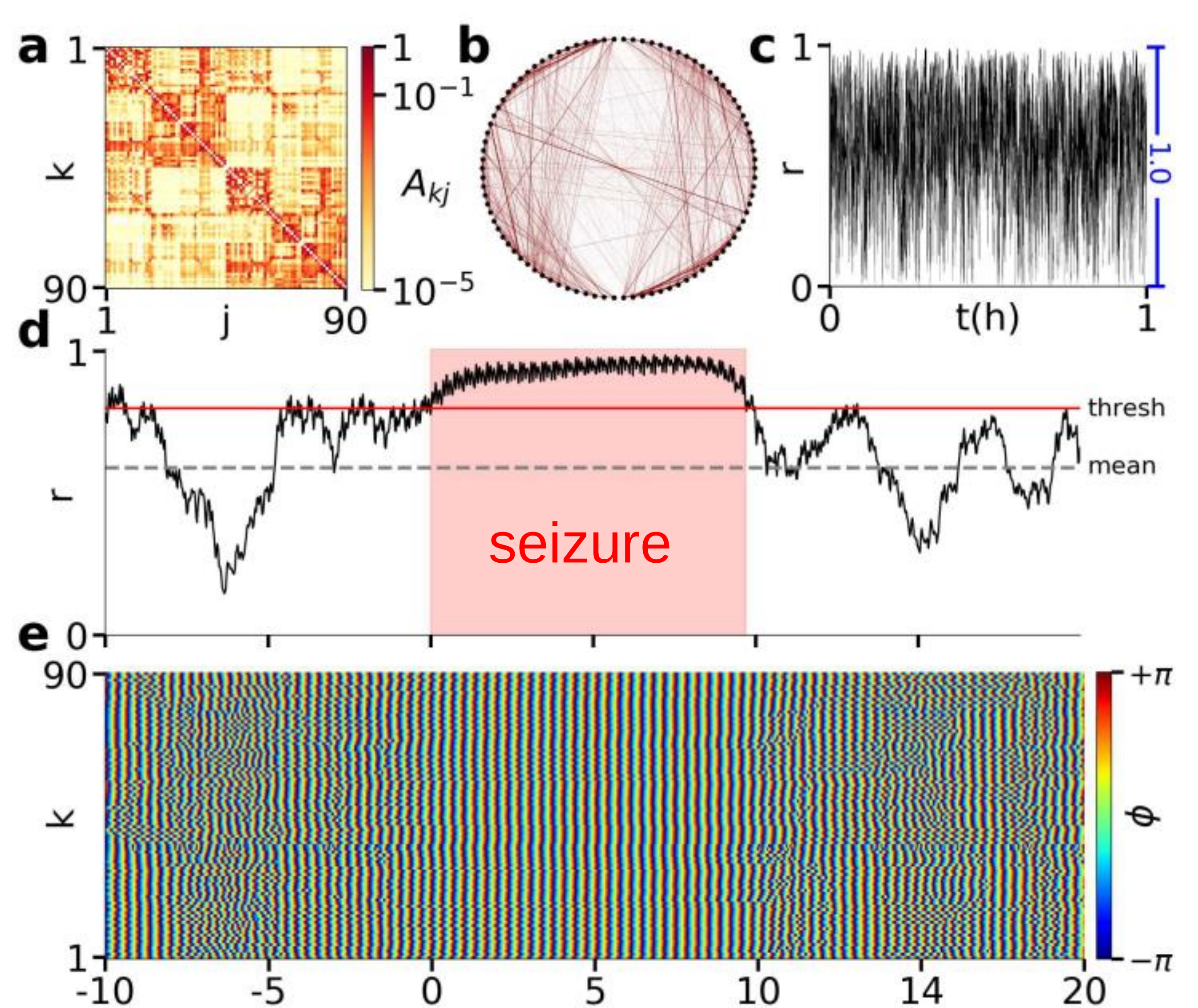
T. Chouzouris, I. Omelchenko, A. Zakharova, J. Hlinka, P. Jiruska, E. Schöll: Chaos 28, 045112 (2018)

Interplay of dynamics and network structure

Empirical connectivity

strong synchronization

$$r(t) = \frac{1}{N}\left|\sum_{k=1}^{N} \exp[i\phi_k(t)]\right|$$

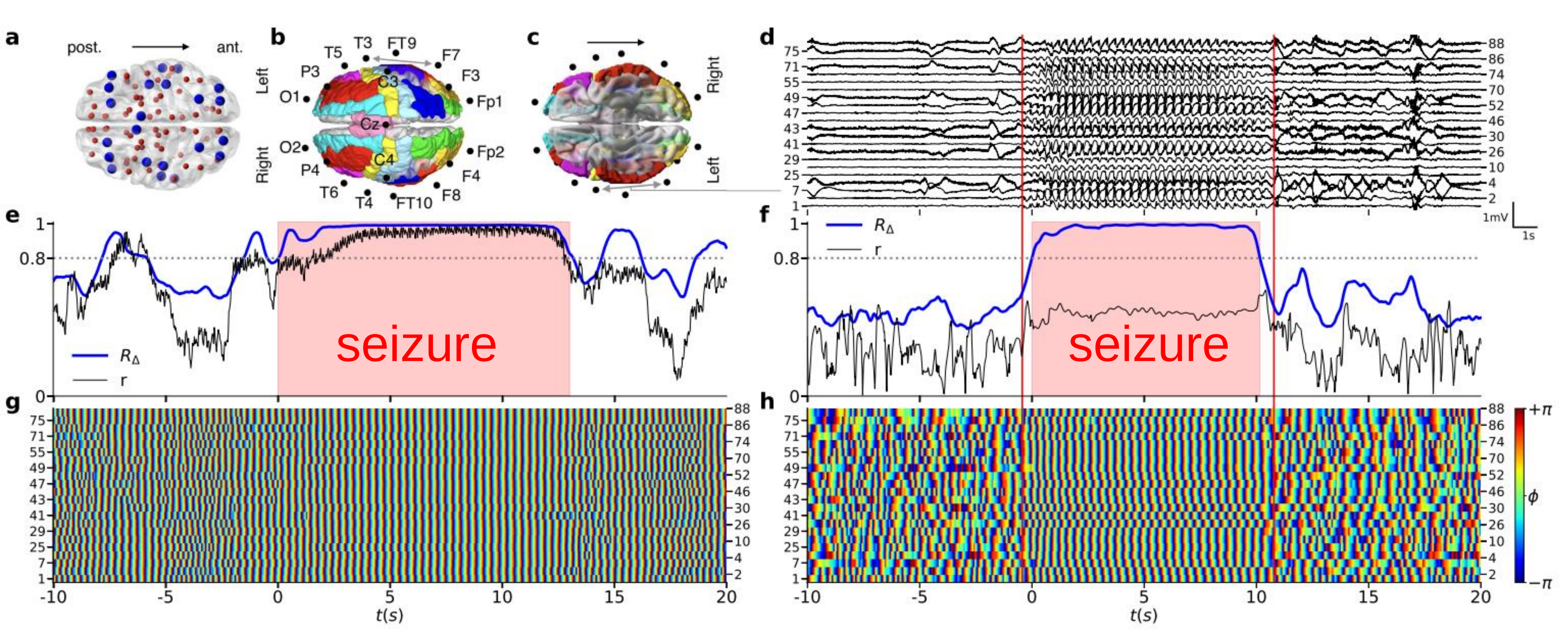


$\varepsilon = 0.05,\ \varphi = \frac{\pi}{2} - 0.1,\ N = 90,\ \sigma = 0.6$
$a = 0.5$

M. Gerster, R. Berner, J. Sawicki, A. Zakharova, A. Skoch, J. Hlinka, K. Lehnertz, E. Schöll, Chaos 30, 123130 (2020), Editor‘s Pick, selected as AIP science highlight

Comparison of simulations and EEG recordings

Simulation with empirical connectivity EEG recordings



High synchronization of neurons in the brain during epileptic seizure:
spontaneous start and spontaneous termination

M. Gerster, R. Berner, J. Sawicki, A. Zakharova, A. Skoch, J. Hlinka, K. Lehnertz, E. Schöll, Chaos 30, 123130 (2020)

Conclusions

- Complex networks perspective:
  interplay of dynamics and network topology

  - Complex real-world network topologies
    simple dynamics of phase oscillators or amplitude-phase

  - Novel control concepts:
    dynamic 2-layer networks, delayed feedback control

  - Bifurcation analysis elucidates instabilities, desynchronization

  - Partial synchronization patterns, e.g., solitary states, multifrequency cluster states, chimera states



jiruska.webp

Thanks to my collaborators:

J.Sawicki R.Berner M.Gerster L.Tumash Chouzouris Ramlow Ruzzene







S.Olmi Omelchenko Taher Claussen Hlinka Lehnertz Provata Andrzejak








Y.Maistrenko O.Omel'chenko M.Wolfrum I.Schneider B.Fiedler S. Yanchuk






A. Fradkov V. Anishchenko A. Zakharova P. Hövel




Thank you!

Partial Synchronization and Chimeras: state of the art

Recent reviews:

- Panaggio and Abrams, Nonlinearity 28, R67 (2015)
- Schöll, EPJ-ST 225, 891(2016), *Special Theme Issue on Complex Systems*
- Schöll, Zakharova, Andrzejak: *Research Topic in Front.Appl.Math.Stat*. (2019)

Sawicki (2020): Schöll, Klapp, Hövel (2016): Zakharova (2020):