Power grids: Small Signal Stability vs. Dynamical Parameters

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References

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Melvyn obtained his master degree and PhD in Theoretical Physics at the Swiss Federal Institute of Technology in Lausanne (EPFL) respectively in 2016 and 2020. He is currently working as a postdoc researcher at the University of Geneva (UNIGE). His research focuses on complex network-coupled dynamical systems and the identification of their local/global vulnerabilities against external perturbations. He also recently developed methods for inferring coupling network from time-series and for locating line and node disturbances in diffusively coupled agents.
Motivation: Energy Transition

Traditional transmission power grid
Motivation: Energy Transition

Traditional transmission power grid
Motivation: Energy Transition

Future transmission power grid
Motivation: Energy Transition

Traditional transmission power grid

50/60Hz

50/60Hz

50/60Hz

50/60Hz

50/60Hz
Motivation: Energy Transition

Future transmission power grid

50/60Hz
Motivation: Energy Transition

How do inertia and damping affect performances?
Swing equations in the lossless line approximation

Voltage phase dynamics is given by

\[ m_i \ddot{\omega}_i + d_i \dot{\omega}_i = P_i - \sum_j b_{ij} \sin(\theta_i - \theta_j), \quad i \in \text{Generators}, \tag{1} \]

\[ d_i \dot{\omega}_i = P_i - \sum_j b_{ij} \sin(\theta_i - \theta_j), \quad i \in \text{Loads}. \tag{2} \]

\[ b_{ij} : \text{line capacity.} \]
\[ m_i : \text{inertia.} \]
\[ d_i : \text{damping.} \]
\[ \omega_i = \dot{\theta}_i. \]
Swing equations in the lossless line approximation:
The common assumption on dynamical parameters

\[ \gamma^{-1} d_i \dot{\omega}_i + d_i \omega_i = P_i - \sum_j b_{ij} \sin(\theta_i - \theta_j). \]

\( b_{ij} \): line capacity.
\( m_i \): inertia.
\( d_i \): damping.
\( \omega_i = \dot{\theta}_i. \)

Usual assumptions that allow analytical treatment: inertia-to-damping constant ratio \( \gamma^{-1} = m_i/d_i, \forall i. \)
Swing equations in the lossless line approximation: The common assumption on dynamical parameters

\[ \gamma^{-1} d_i \dot{\omega}_i + d_i \omega_i = P_i - \sum_{j} b_{ij} \sin(\theta_i - \theta_j) . \]

\( b_{ij} \): line capacity.
\( m_i \): inertia.
\( d_i \): damping.
\( \omega_i = \dot{\theta}_i \).

Usual assumptions that allow analytical treatment: inertia-to-damping constant ratio \( \gamma^{-1} = m_i / d_i \), \( \forall i \).
We also take this assumption... but eventually say something about realistic power networks!
Robustness Assessment

Quadratic performance metrics: $\mathcal{H}_2$ norms → Quantify the amplitude of the transient response following a disturbance.

$$\theta_i(t), \omega_i(t)$$

$$\theta_i^{(0)}, \omega_i^{(0)}$$
Robustness Assessment

- Quadratic performance metrics: $\mathcal{H}_2$ norms.

$\theta_i(t), \omega_i(t)$

Performance vs. Topology $\rightarrow$ Generalized Kirchhoff indices $Kf_n$ and resistance Centralities $C_n(k)$.

MT, Coletta, Jacquod *Physical review letters* **120** (8), 084101 (2018)
Response to Perturbations: Linearization

Swing equations in the lossless line approximation:

\[
\gamma^{-1} d_i \ddot{\theta}_i + d_i \dot{\theta}_i = P_i - \sum_j b_{ij} \sin(\theta_i - \theta_j).
\]

Linear response: Perturbation of the injected/consumed powers.

- \( P_i(t) = P_i^{(0)} + \delta P_i(t) \rightarrow \theta_i(t) = \theta_i^{(0)} + \delta \theta_i(t) \):

\[
\gamma^{-1} \delta \ddot{\varphi}(t) + \delta \dot{\varphi}(t) = D^{-1/2} \delta P(t) - D^{-1/2} \mathbb{I}_s(\{\theta_i^{(0)}\}) D^{-1/2} \delta \varphi(t),
\]

where \( \delta \varphi(t) = D^{1/2} \delta \theta(t) \).
Response to Perturbations: Linearization

Swing equations in the lossless line approximation:

\[ \gamma^{-1} d_i \ddot{\theta}_i + d_i \dot{\theta}_i = P_i - \sum_j b_{ij} \sin(\theta_i - \theta_j). \]

Linear response: Perturbation of the injected/consumed powers.

- \( P_i(t) = P_i^{(0)} + \delta P_i(t) \rightarrow \theta_i(t) = \theta_i^{(0)} + \delta \theta_i(t): \)

\[ \gamma^{-1} \delta \ddot{\phi}(t) + \delta \dot{\phi}(t) = D^{-1/2} \delta P(t) - D^{-1/2} \mathbb{L}(\{\theta_i^{(0)}\}) D^{-1/2} \delta \phi(t), \]

\( \mathbb{L}(\{\theta_i^{(0)}\}) \): the weighted Laplacian matrix,

\[ [D^{-1/2} \mathbb{L} D^{-1/2}]_{ij} = \begin{cases} -\frac{b_{ij}}{\sqrt{d_i d_j}} \cos(\theta_i^{(0)} - \theta_j^{(0)}), & i \neq j, \\ \frac{1}{d_i} \sum_k b_{ik} \cos(\theta_i^{(0)} - \theta_k^{(0)}), & i = j. \end{cases} \]
Linear response: Perturbation of the injected/consumed powers.

- \( P_i(t) = P_i^{(0)} + \delta P_i(t) \rightarrow \theta_i(t) = \theta_i^{(0)} + \delta \theta_i(t) : \)

\[
\gamma^{-1} \delta \ddot{\varphi}(t) + \delta \dot{\varphi}(t) = D^{-1/2} \delta P(t) - D^{-1/2} \mathbb{I}_n(\{\theta_i^{(0)}\}) D^{-1/2} \delta \varphi(t),
\]

Solution:

\[
\delta \varphi_i(t) = \sum_{\alpha} \gamma e^{-\frac{\gamma - \Gamma_{\alpha}}{2} t} \int_0^t e^{\Gamma_{\alpha} t_1} \times \int_0^{t_1} [D^{-1/2} \delta P(t_2)]^\top u_{\alpha} D e^{-\frac{\gamma - \Gamma_{\alpha}}{2} t_2} dt_2 dt_1 u_{\alpha, i} \quad (3)
\]
Fluctuating Power Generation

Time-correlated power fluctuations:

\[ \langle \delta P_i \rangle = 0, \quad \langle \delta P_i(t) \delta P_j(t') \rangle = \delta_{ij} \delta P_0^2 \exp\left[-|t - t'|/\tau_0\right]. \]

Primary control effort:

\[ \mathcal{P}(T) = \lim_{T \to \infty} T^{-1} \int_0^T (\omega^\top - \bar{\omega}^\top) D(\omega - \bar{\omega}) \, dt, \]
\[ = \lim_{T \to \infty} T^{-1} \int_0^T (\delta \varphi^\top - \bar{\delta \varphi}^\top)(\delta \varphi - \bar{\delta \varphi}) \, dt. \]

Linear system $\rightarrow$ analytical solution!
Primary control effort:

\[ \overline{P}^\infty = \sum_{\alpha \geq 2} \sum_{i \in N_n} \delta P_{0i}^2 u_{\alpha,i}^D \frac{\lambda_{D,i}^\tau \gamma^{-1} \tau_0^{-1}}{\lambda_{\alpha}^{\tau_0} + 1} d_i^{-1}, \]

with \( \lambda_{\alpha}^{D} \) the eigenvalue associated with the eigenvector \( u_{\alpha}^{D} \) of \( L_{D} \) of the form

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Primary Control Effort

Short noise correlation time: \( \tau_0 \ll \gamma^{-1}, \lambda_{\alpha}D^{-1} \)

\[
\bar{P}^\infty = \tau_0 \sum_{i \in N_n} \delta P^2_{0i} \left(1/m_i - 1/\sum_j m_j\right).
\]

No dependence on damping nor network connectivity!

Long correlation time: \( \tau_0 \gg \gamma^{-1}, \lambda_{\alpha}D^{-1} \)

\[
\bar{P}^\infty = \tau_0^{-1} \sum_{\alpha \geq 2} \sum_{i \in N_n} \frac{\delta P^2_{0i} u_{\alpha,i}^D 2d_i^{-1}}{\lambda_{\alpha}^D} .
\]

No dependence on inertia!
Primary Control Effort

**Short noise correlation time:** $\tau_0 \ll \gamma^{-1}, \lambda_{\alpha} D^{-1}$

$$\overline{P}^\infty = \tau_0 \sum_{i \in N_n} \delta P_{0i}^2 \left(1/m_i - 1/\sum_j m_j\right).$$

No dependence on damping nor network connectivity!

**Long correlation time:** $\tau_0 \gg \gamma^{-1}, \lambda_{\alpha} D^{-1}$

$$\overline{P}^\infty = \tau_0^{-1} \sum_{\alpha \geq 2} \sum_{i \in N_n} \delta P_{0i}^2 u_{\alpha,i} D^2 d_i^{-1} \lambda_{\alpha}^{-1}. $$

No dependence on inertia!

Realistic high-voltage power networks: $\lambda_{\alpha} D^{-1} < 0.5s$ and $\gamma^{-1} \approx 2.5s$.

Renewable power sources fluctuate on time scales of few seconds.
Numerical Validation

IEEE 118-Bus Test Case:

(a) $\chi^{D_{\tau_0}} \ll 1$

(b) $\chi^{D_{\tau_0}} \gg 1$

Numerical Validation

PanTaGruEl:

(a) $\bar{\rho}_{\text{num}}$ vs $\bar{\rho}_{\text{th}}$

- $\gamma \tau_0 = 4$
- $\gamma \tau_0 = 40$
- $m/d \neq \gamma^{-1}$

(b) $u_{2,i}^D$ and $u_{3,i}^D$ with color scale

Conclusion

Description of realistic power networks

- Consider $D^{-1/2}L D^{-1/2}$ instead of $M^{-1/2}L M^{-1/2}$,
- Time-correlated noise instead of white-noise,
→ Primary control effort for power networks with inhomogeneous dynamical parameters.
- Inertia does not impact much primary control effort.
→ Focus on damping/control.

References