



# ARBITRAGE ON THE ENERGY MARKET AND ITS IMPACT ON THE EXHAUSTION OF RESERVE ENERGY

Tim Ritmeester and Hildegard Meyer-Ortmanns

Jacobs University Bremen, Germany

ENERGY 21-Conference, May 30-June 1, 2021

Special Track on Modelling Dynamics of Power Grids (MoDyPoG/ENERGY 2021)  
talk based on T. Ritmeester and H.M.O., *Physica A* **573**, (2021) 125927~1-19.

## FURTHER REFERENCES

- D. Challet, M. Marsili, G. Ottino, *Physica A* 332 (2004) 469.
- D. Challet, M. Marsili, Y.-C. Zhang, *Physica A* 276 (1) (2000) 284.
- A. Coolen, *The Mathematical Theory of Minority Games*, Oxford University Press (2005).

## Current research interests:

- Complex systems with methods from nonlinear dynamics and statistical physics,
- in particular the role of stochastic fluctuations,
- trade-offs between costs and precision , or costs and risk
- applications to power grids

## I. MOTIVATION

How to prevent the kind of blackout events that almost occurred in June 2019 in Germany caused by strong fluctuations in the prices for reserve energy and their likely abuse.

Energy differences between production and consumption and their forecasts are balanced via trade on the energy market.

**Energy transition**  $\longrightarrow$  **Reorganization of the energy market** towards a decentralized structure

$\longrightarrow$  **Options of abuse**

We distinguish two energy markets:

the **reserve energy market**

TSOs (transmission system operators)

the **intraday energy market**

BRPs (balancing responsible parties)

Price for reserve power is determined by merit order.

Usually: price  $R$  for reserve energy  $>$  price  $I$  on the intraday market.

Occasionally:

$R < I \longrightarrow$  option for arbitrage

**Arbitrage** = practice of taking advantage of a price difference between 2 or more markets without risk.

(corresponds to the concept of frustration in physics, more familiar from spin glasses (Mack, 1981))

Here: arbitrage if  $R < I$  and more reserve energy is bought by the BRPs than is accessible and sold at a higher price on the intraday market , or if  $R > I$  and more energy is bought on the intraday market than required and fed into the grid

Incentives for BRPs to behave as arbitrageurs amounts to a minority mechanism.

Uncertainty on who participates in the minority, but BRPs should anti-coordinate to the average behavior of retailers.

## II. OUR CONTRIBUTION: MAP OF THE BEHAVIOR OF ARBITRAGEURS TO A MINORITY GAME

### Parameters:

**W**: total amount of power available for arbitrage

**w<sub>i</sub>**: contribution to W of player i

**I** : price on the intraday market

**R** : price on the reserve energy market

**η** : external noise to capture ongoing fluctuations in the energy balance

**P** : parameterizes the amount of available information :  $\mu = 1, 2, \dots, P$

To each value of  $\mu$  assign uniformly and randomly a value of  $a = \pm 1 \rightarrow 2^P$  possible strings, pool of strategies

$a_i = +1$ : agent sells energy on the intraday market and buys reserve energy: profit if  $R < I$

$a_i = -1$ : agent buys energy on the intraday market and feeds too much into the grid: profit if  $R > I$

**Arbitrage** =  $A := \sum w_i a_i$ : = total imbalance of power, caused by the actions of all agents, adds upon other sources of imbalance  $\eta$ , so that

$$u_i = a_i (I - R(A + \eta)).$$

What are the decisions  $a_i = \pm 1$  based upon? On  **$S \geq 1$  strategies**  $s$  with

$$s_i(\mu) = a_i \in \{\pm 1\}, \quad \mu \in \{1, \dots, P\}$$

**Choice of strategies:** constant over time, not necessarily homogeneous over the agents, choice either stochastic (+1 with prob  $p$  and -1 with prob  $1-p$ ) or deterministic from a subset of the whole pool.

Agents can switch between them during a **learning phase** according to the **score**:

$$U^{t+1}_{s_i} = U^t_{s_i} + s^i(\mu^t) [I - R(A^t + \eta^t)]$$

Evaluate the score (success) in the past, keep track of their evaluations. Choose the strategy at time  $t+1$ , which is the best in **hindsight** according to this evaluation.

**Observables:**  $\langle A \rangle$ ,  $\sigma_A^2 = \langle A^2 \rangle - \langle A \rangle^2$

as a function of  $N$ ,  $P$ , risk aversion  $\varepsilon$ , price function  $R$  and noise  $\eta$

**Methods:** **Analytical** derivation of bounds on the fluctuations

**Agent-based modelling** in the following steps:

- **Initialization:** choice of parameters and functions
- **Learning phase:**  
all agents simultaneously update their evaluation  $U^{t+1}_{s_i}$ , based on the measured value  $A^t = \sum w_i a_i^t$  to determine the best strategy in hindsight  $s_{best}^i$ , leading to a decision at time  $t+1$ :  

$$a_i(t+1) = s_{best}^i(\mu) \text{ with } \mu \text{ at } t+1 \text{ randomly uniformly chosen to be the same for all agents and the best strategy the one with the highest score } U^{t+1}_{s_i}$$

Repeat the learning steps until convergence
- **Measurement of observables:**  $\langle A \rangle = \sum_{t=t_0}^{t_0+T} A^t = \sum w_i a_i^t$  with  $a_i = s_{best}^i(\mu)$ , histogram of  $A$ , fluctuations  $\sigma_A$
- **Gain statistics:** repeat the whole procedure with new initialization of strategies 100 times

### III. RESULTS

#### 1. Analytical bounds on the variance:

Nash equilibria accessible in limiting cases of perfect anti-coordination or no anti-coordination.

For the case of no anti-coordination, a Nash equilibrium is achieved if assuming  $p_i=p$  for all agents  $i$ , and  $p$  the prob to choose  $a_i=+1$  and  $1-p$  the prob to choose  $a_i=-1$ .

$p$  should be chosen such that  $\langle 1 - R(A+\eta) \rangle = 0$ . The variance is then given as:

$$\sigma_A^2 \equiv \langle A^2 \rangle - \mu_A^2 = \frac{W^2}{N/X} \times (1 - \mu_A^2/W^2) = \mathcal{O}(N).$$

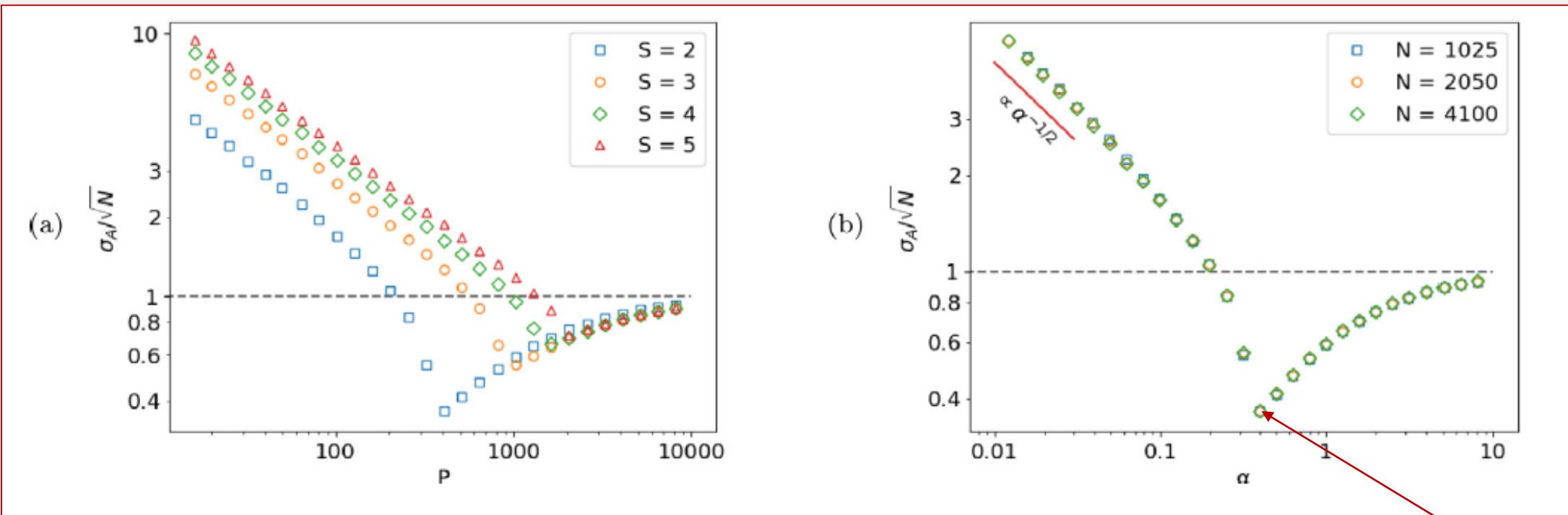
with  $W \equiv \sum_i w_i$  the total power available for arbitrage and

$X \equiv (\frac{1}{N} \sum_i w_i^2) / (\frac{1}{N} \sum_i w_i)^2$  a measure for the non-uniformity of  $P(w_i)$ .

$N/X$  effective number of agents contributing to the fluctuations

## 2. Results from agent-based modelling

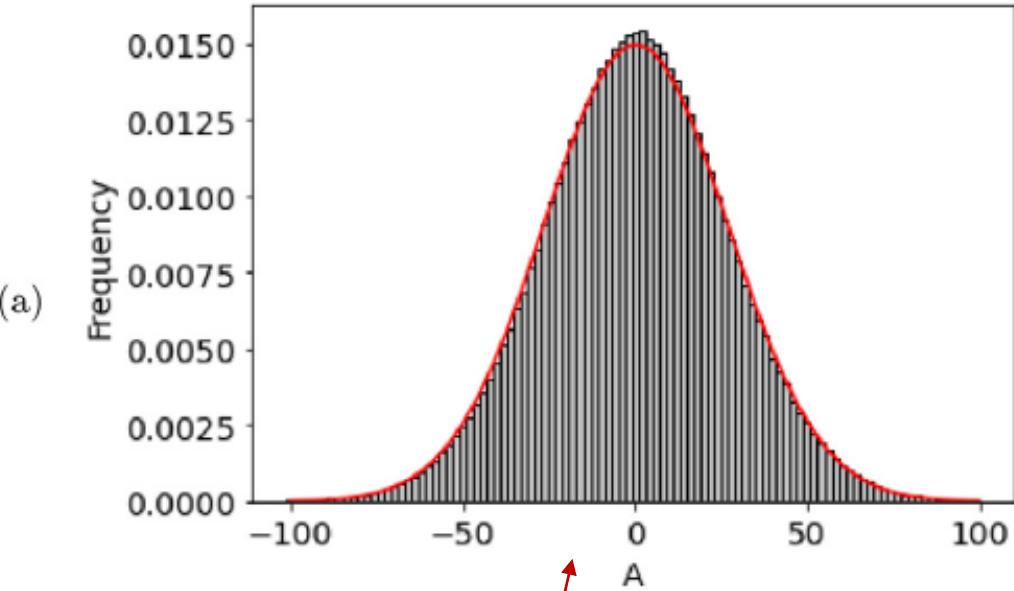
Fluctuations for different  $S$  and different  $N$ , counterpart of replica symmetry breaking



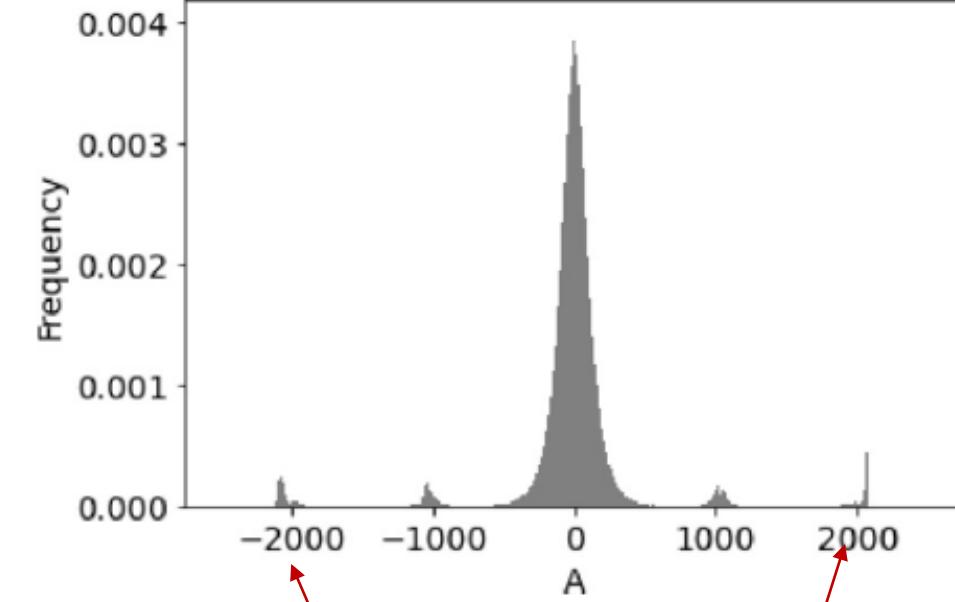
$\alpha = P/N$  is the driving parameter,

Note the scaling of  $\sigma_A \approx N$

# Histograms of arbitrage above and below the phase transition at critical $\alpha_c$



(a)



(b)

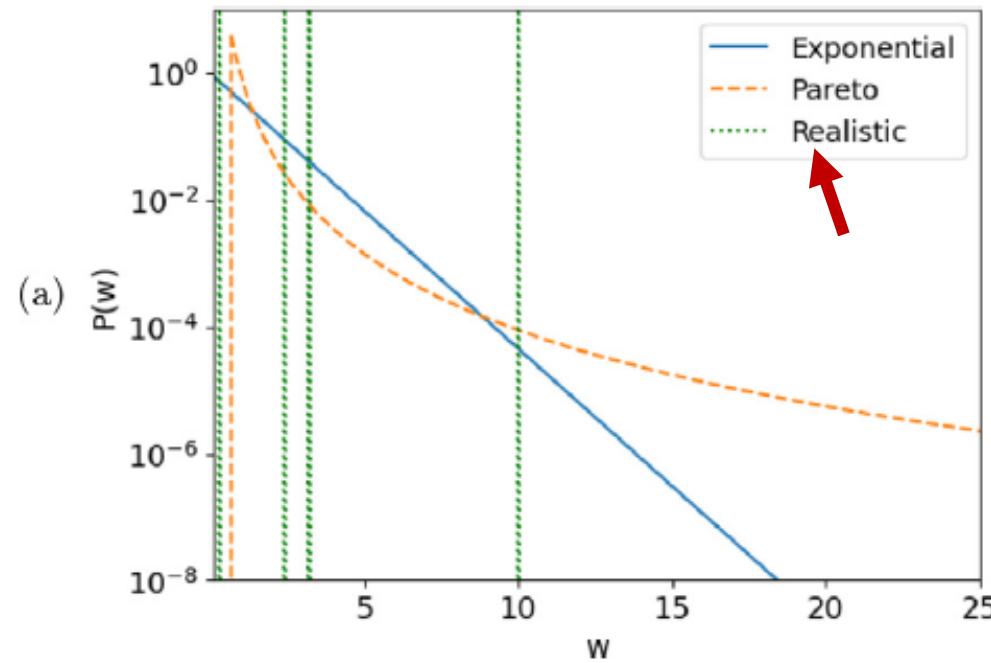
**Fig. 5.** Histograms of  $A^t$  (after convergence) for (a)  $\alpha = 1 > \alpha_c$  and (b)  $\alpha \approx 0.0076 < \alpha_c$ ;  $N = 4100$ , averages over  $10^6$  time-steps. For  $\alpha < \alpha_c$  (b), the distribution is strongly non-Gaussian, in contrast to the  $\alpha > \alpha_c$  case (a).

Symmetric phase

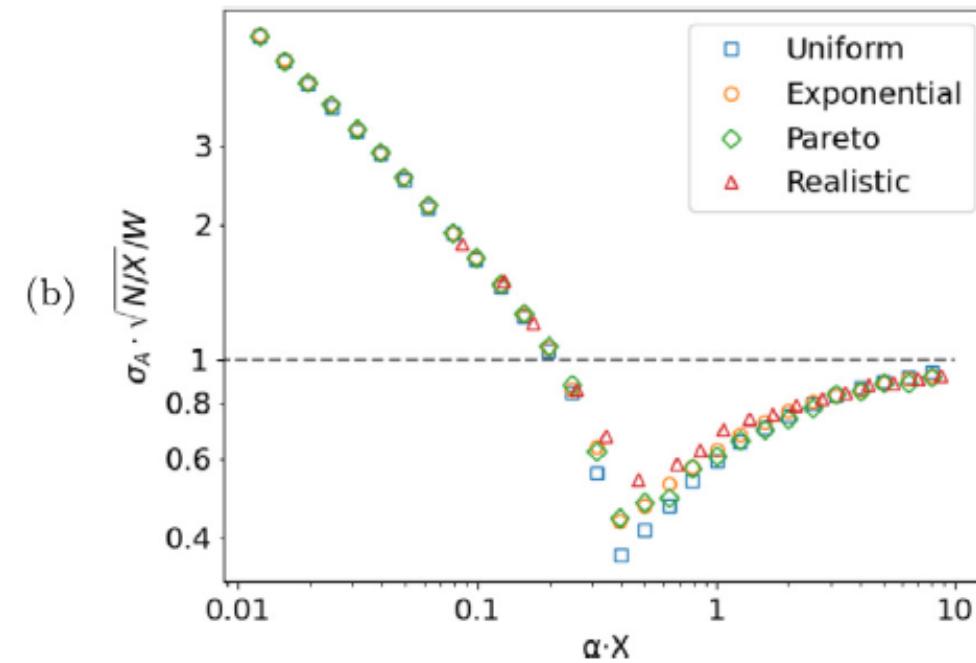
Outliers in the broken phase may lead to exhaustion of reserve energy.

# Non-uniform participation of arbitrageurs, characterized by $P(w)$

Different  $P(w)$

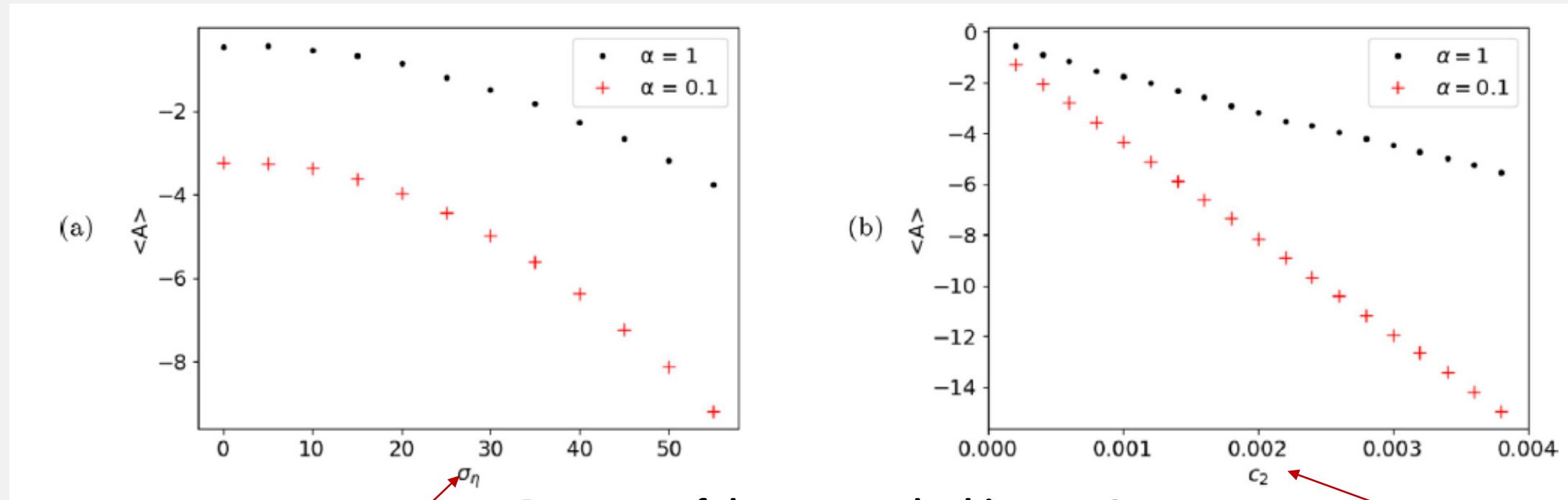


Stable underlying structure of the phase transition



Rescaling of the axes determined by the analytical bound for  $\sigma_A$  works reasonably well.

# Nonlinear price function of reserve power in combination with external noise $\eta$



Decrease of the expected arbitrage  $\langle A \rangle$  as

the external noise increases

the nonlinearity, parameterized by  $c_2$  increases

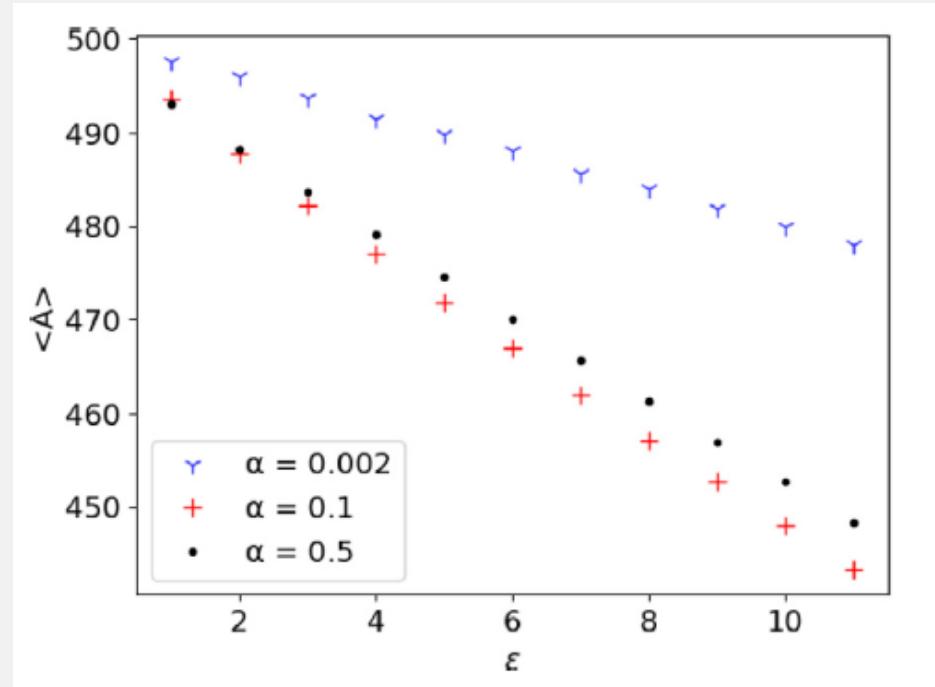
Note, for a nonlinear price function

$$\langle R(A + \eta) \rangle = \int_{-\infty}^{\infty} dx R(x) P(A + \eta = x)$$

We assume, for example,  $R(A + \eta) = I + c_1(A + \eta - A^*) + c_2(A + \eta - A^*)^2$  where  $A^*$  solves  $I - R(A^* + \eta) = 0$ .

Essential that the second derivative of the price function is positive to reduce the amount of arbitrage.

## What is the impact of risk aversion?



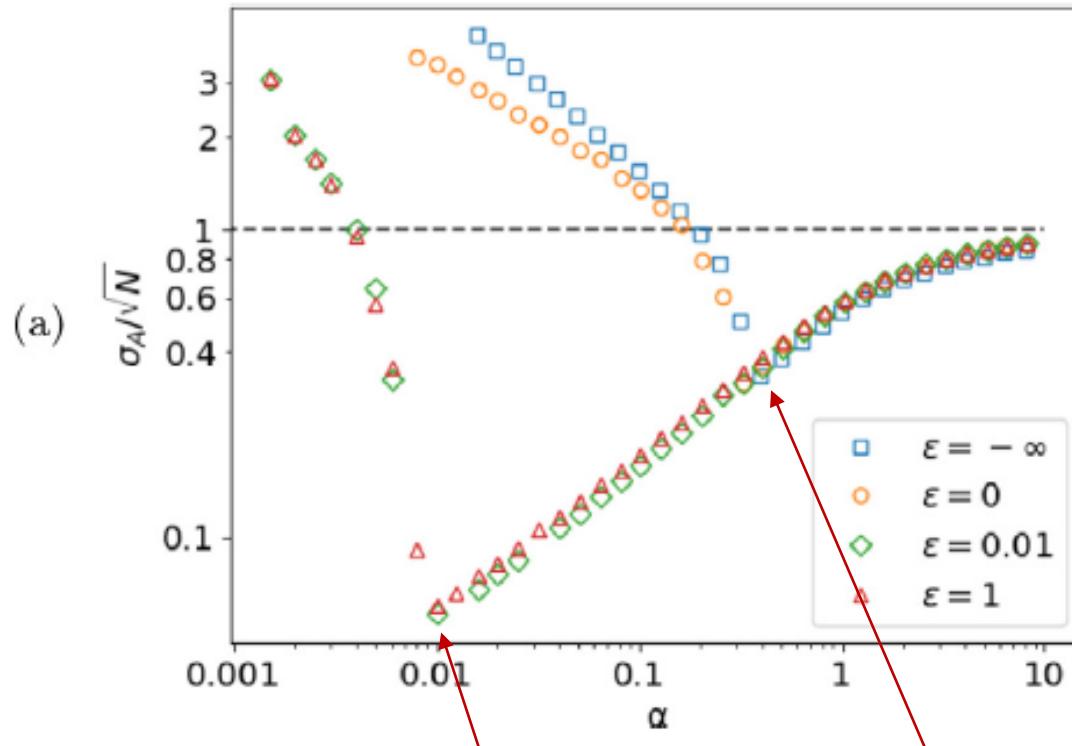
$N = 2000, S = 2, I = 500$ , for different values of  $\alpha$ .

Get involved in arbitrage only if the expected profit

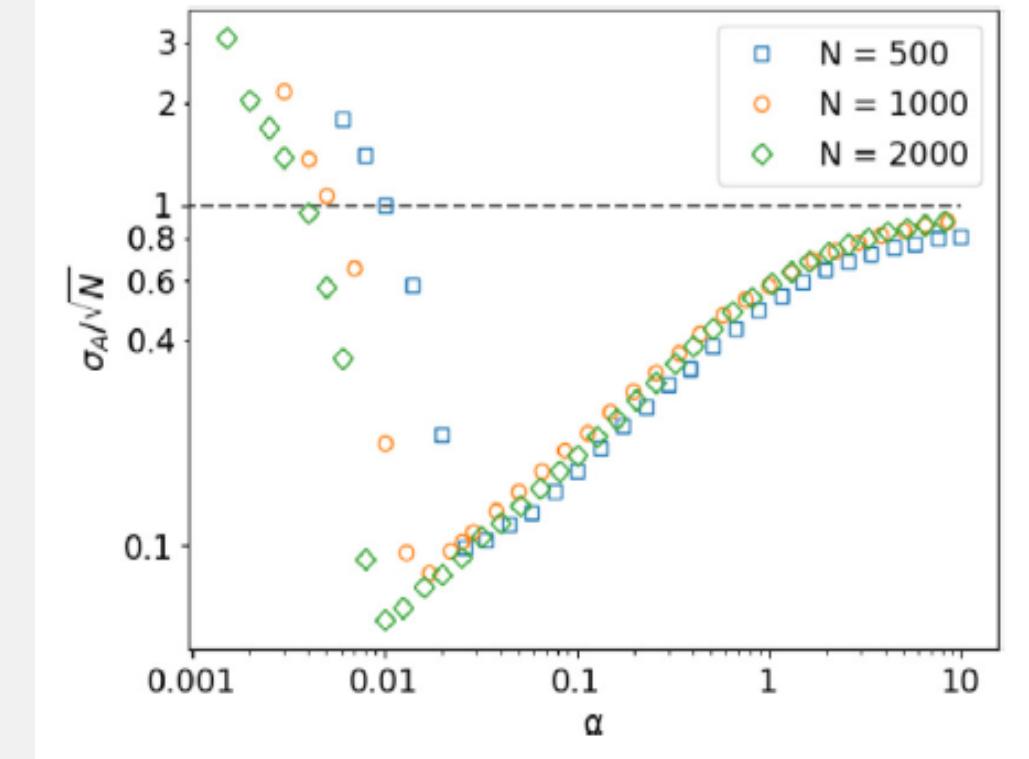
$$\langle a_i \rangle (I - R(A + \eta)) \geq \epsilon_i .$$

In this case a number of agents refrain from getting involved at all.

# Further effects of risk aversion: Shift of the transition point, N-dependence



The higher the risk aversion, the more the phase transition gets shifted towards lower  $\alpha$ -values with a low level of fluctuations.



Sensitive N-dependence in the low- $\alpha$  phase

Whom to make preferably risk averse: those BRPs that trade with large volumes of power.

### Economic measures:

- via tuning the reserve power price between some cutoffs
- by a suitable choice of the intraday price
- by a suitable choice of the size of the market (N) (volatility may increase  $\approx N$  )

### Statutory measures

- introducing risk aversion via penalties
- particularly for high-weight agents

## V. SUMMARY

**Physics perspective of the energy market is useful for**

- uncovering collective effects like the phase transition underlying the nonmonotonous behavior of fluctuations (and their growth with N) and outliers in A for small  $\alpha < \alpha_c$
- analyzing the effect of a nonlinear price function in the presence of external fluctuations on  $\langle A \rangle$
- identifying the main troublemakers in events like those in 2019 of almost-blackouts
- supporting market design such as the choice of N.