



# **Pattern Dependent Optimized Mowing of Football Fields with an Autonomous Robot**

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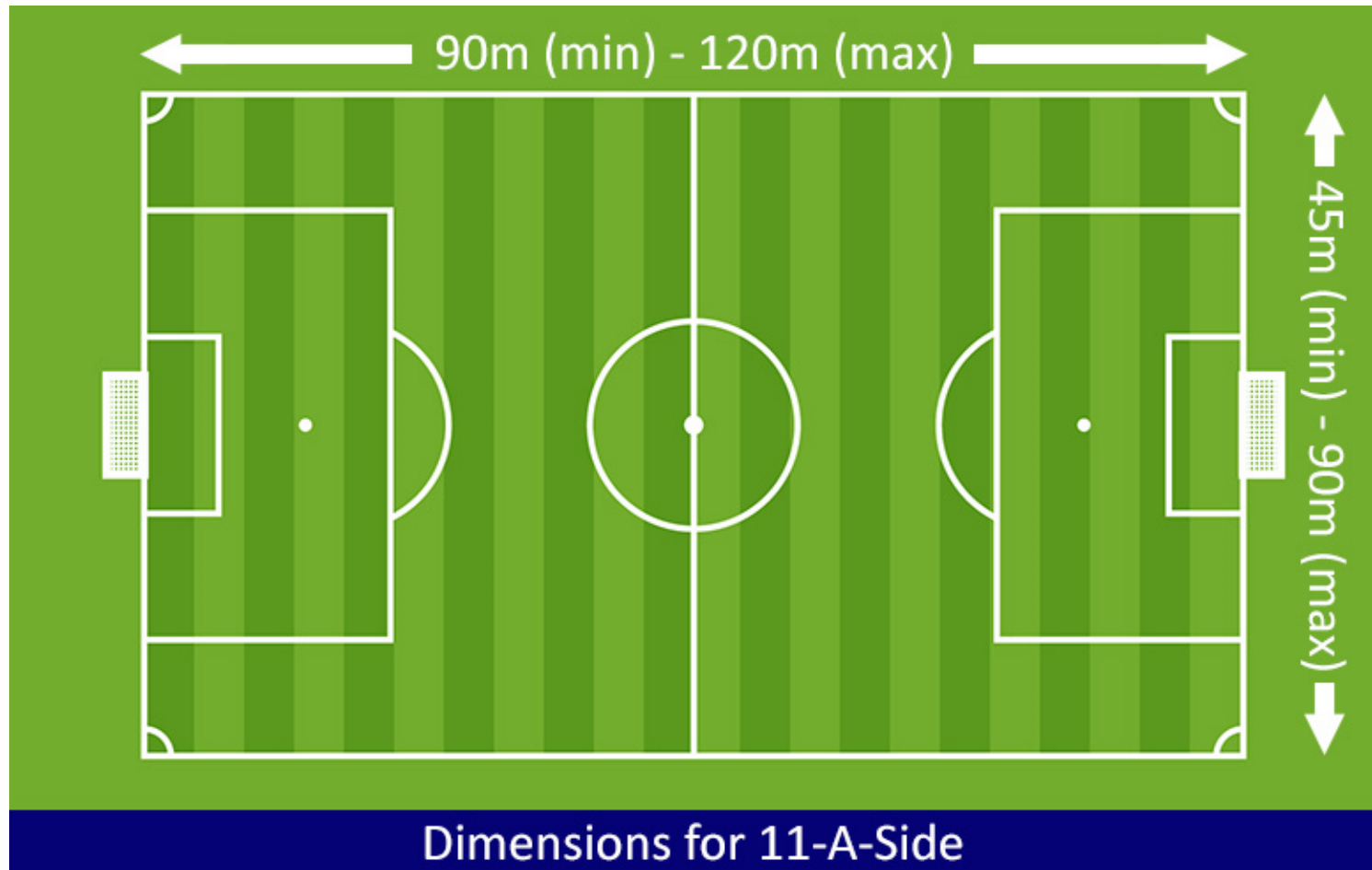
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# Overview

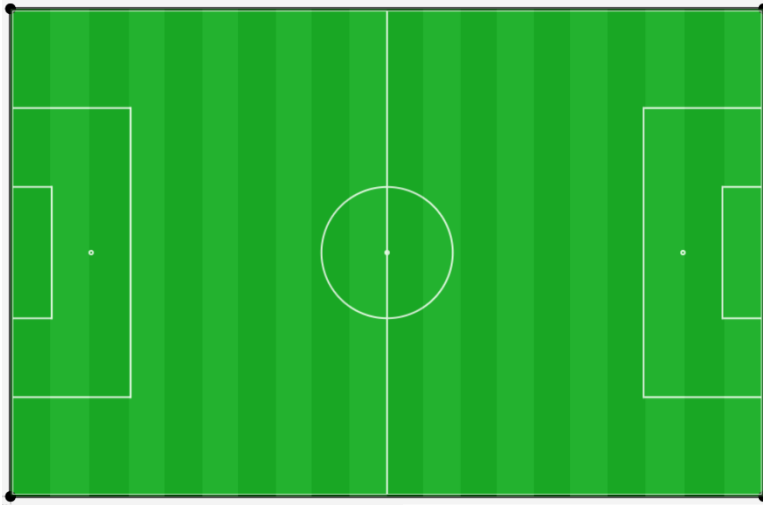
- Football Field Mowing with Pattern
- Approach
- Results
- Conclusion



# Football Field Mowing with Pattern

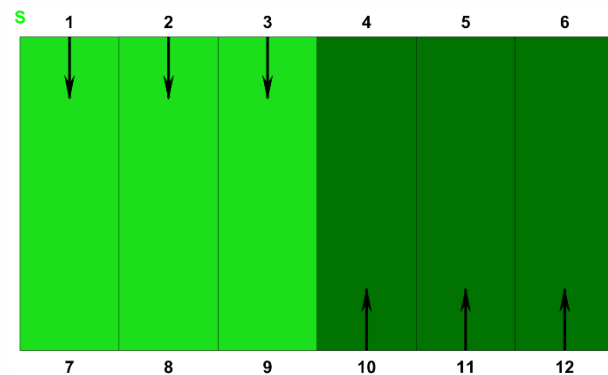


# Football Field Patterns



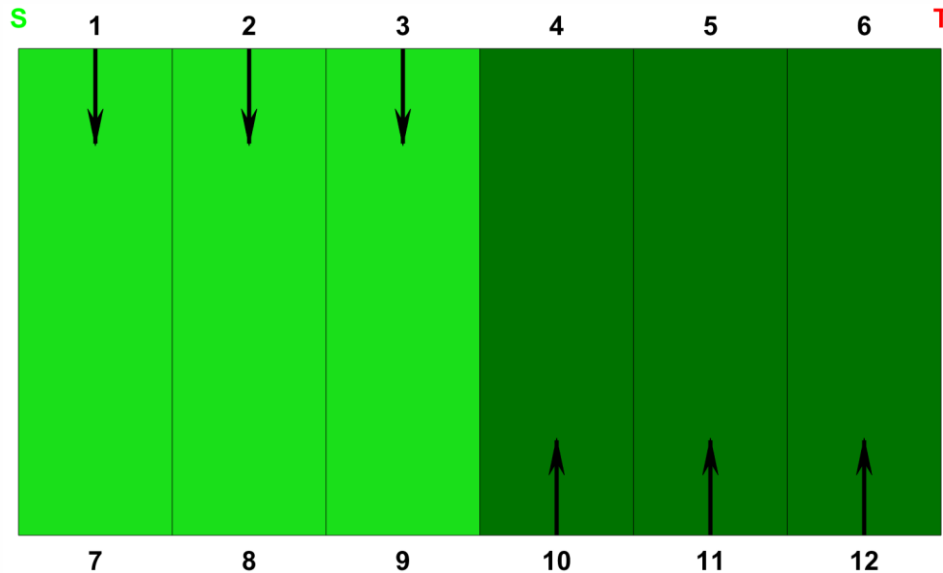
# Cost Matrix

	$S$	1	2	3	4	5	6	7	8	9	10	11	12	$T$
$S$	$\infty$	1	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$
1	$\infty$	$\infty$	166	333	499	642	784	1000	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$
2	$\infty$	166	$\infty$	166	333	475	618	$\infty$	1000	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$
3	$\infty$	333	166	$\infty$	166	309	451	$\infty$	$\infty$	1000	$\infty$	$\infty$	$\infty$	$\infty$
4	$\infty$	499	333	166	$\infty$	142	284	$\infty$	$\infty$	$\infty$	500	$\infty$	$\infty$	1
5	$\infty$	642	475	309	142	$\infty$	142	$\infty$	$\infty$	$\infty$	$\infty$	500	$\infty$	1
6	$\infty$	784	618	451	284	142	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	500	1
7	$\infty$	500	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	166	333	499	642	784	1
8	$\infty$	$\infty$	500	$\infty$	$\infty$	$\infty$	$\infty$	166	$\infty$	166	333	475	618	1
9	$\infty$	$\infty$	$\infty$	500	$\infty$	$\infty$	$\infty$	333	166	$\infty$	166	309	451	1
10	$\infty$	$\infty$	$\infty$	$\infty$	1000	$\infty$	$\infty$	499	333	166	$\infty$	142	284	$\infty$
11	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	1000	$\infty$	642	475	309	142	$\infty$	142	$\infty$
12	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	1000	784	618	451	284	142	$\infty$	$\infty$
$T$	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$



# Dependency Matrix

	<i>S</i>	1	2	3	4	5	6	7	8	9	10	11	12	<i>T</i>
<i>S</i>	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	1	0	0	0	0	0	0	0	0	0	0	0	0	0
2	1	0	0	0	0	0	0	0	0	0	0	0	0	0
3	1	0	0	0	0	0	0	0	0	0	0	0	0	0
4	1	0	0	1	0	0	0	0	0	0	1	0	0	0
5	1	0	0	0	0	0	0	0	0	0	0	1	0	0
6	1	0	0	0	0	0	0	0	0	0	0	0	1	0
7	1	1	0	0	0	0	0	0	0	0	0	0	0	0
8	1	0	1	0	0	0	0	0	0	0	0	0	0	0
9	1	0	0	1	0	0	0	0	0	0	0	0	0	0
10	1	0	0	0	0	0	0	0	0	1	0	0	0	0
11	1	0	0	0	0	0	0	0	0	0	0	0	0	0
12	1	0	0	0	0	0	0	0	0	0	0	0	0	0
<i>T</i>	1	1	1	1	1	1	1	1	1	1	1	1	1	0



# Problem Statement

## Goals

- Mow the field with pattern in **minimum time**
- Mow the pre-defined pattern in the field
- Observe dependency between lanes

## Assumptions

- Lane mowing and lane-to-lane transition time is known and fixed
- Dependencies between the lanes are known
- Mower always starts from S and stops at T
- Each lane has to be mowed exactly once



# Objective Function

$$\min \quad \mathbf{Obj} = \sum_{i=0}^{n-1} \sum_{j=0}^{n-1} d_{ij} * e_{ij}.$$





# Constraints

$$\sum_{j=1}^{n-1} d_{Sj} = 1 \quad (1)$$

$$\sum_{i=0}^{n-1} d_{iT} = 1 \quad (2)$$

$$d_{j+N,j} - \left( \sum_{i=1}^N d_{j,i} \right) = 0 \quad \forall j = 1 \dots N \quad (3)$$

$$\left( \sum_{i=0}^N d_{i,j} \right) - d_{j,j+N} = 0 \quad \forall j = 1 \dots N \quad (4)$$

$$d_{j-N,j} - \left( \sum_{i=N+1}^{n-1} d_{j,i} \right) = 0 \quad \forall j = N+1 \dots 2N \quad (5)$$

$$d_{S,j} + \left( \sum_{i=N+1}^{2N} d_{i,j} \right) - d_{j,j-N} = 0 \quad \forall j = N+1 \dots 2N \quad (6)$$



# Constraints

$$u_{i-1} - u_{j-1} + (n - 1) \times d_{i,j} < n - 2 \quad (7)$$

$$u_{j-1} - u_{i-1} < 0 \quad (8)$$



# Results

Lanes	Status	Obj	Gap(%)	Time (sec)
16	Optimal	41474	0.0	00.13
40	Optimal	231185	0.0	0.9
68	Optimal	539995	0.0	1.0
96	Optimal	2113341	0.0	0.5
146	Optimal	5166207	0.0	499.7
192	Optimal	6761169	0.0	75.3

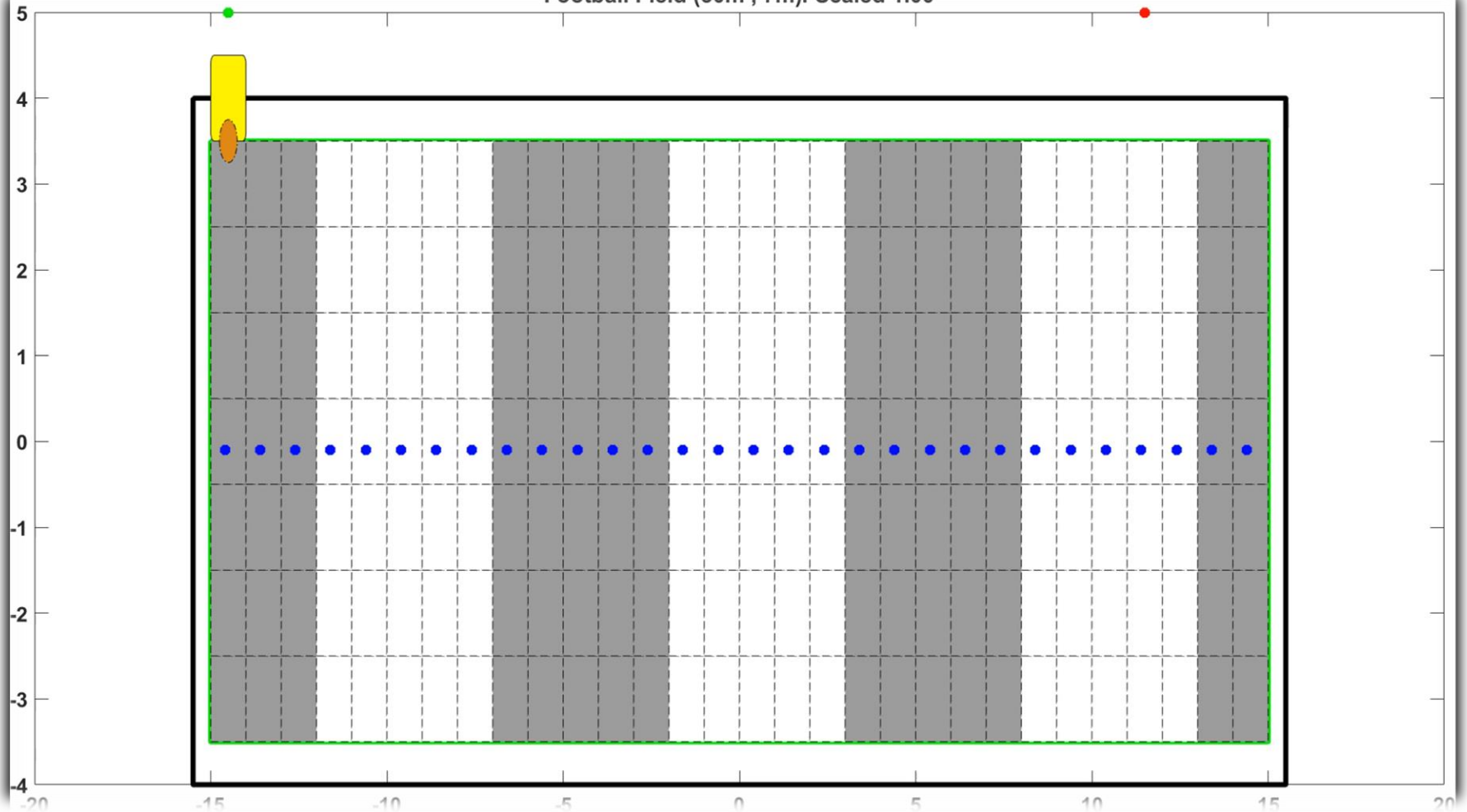
## Configuration setup

- Java - Gurobi 8.0 optimizer
- Intel(R) Core(TM) i7-5600U
- CPU 4-Core 2.60 GHz
- 8GB RAM
- Windows 10 (64-bit)



# Results

Football Field (30m , 7m). Scaled 1.00





# Conclusion

- Optimizing the path of an autonomous football field mowing robot
- A pattern is simultaneously mowed together with respecting the lane dependencies
- **Future Work**
  - Optimizing path with more than one difference between top- and bottom-entering lanes



# Thank you

