Reasoning with Exceptions in Contextualized Knowledge Repositories

Loris Bozzato

Data and Knowledge Management Research Unit,
Fondazione Bruno Kessler - Trento, Italy
bozzato[at]fbk.eu

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Joint work with...

DKM and PDI units @ Fondazione Bruno Kessler:
- Luciano Serafini
- Martin Homola
- Mathew Joseph
- Francesco Corcoglioniti
- Chiara Ghidini
- Andrei Tamilin
- Gaetano Calabrese

Institut für Informationssysteme @ TU Wien:
- Thomas Eiter
Reasoning in context

Classic example: Magic Box [Ghidini and Giunchiglia, 2001]
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Contextual AI theory principia: [McCarthy, 1993]

- Every formula is asserted in a context

- Context are first class logical objects (formulas can predicate about contexts)

- Knowledge propagates across contexts
Contextual AI theory principia: [McCarthy, 1993]

- Every formula is asserted in a context
  
  “In FIFA World Cup 2006, the Winner is Italy.”
  “In FIFA World Cup 2010, the Winner is Spain.”

- Context are first class logical objects
  (formulas can predicate about contexts)

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Contextual AI theory principia: [McCarthy, 1993]

- Every formula is **asserted in a context**
  - “In FIFA World Cup 2006, the Winner is Italy.”
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- Context are **first class logical objects**
  (formulas can predicate about contexts)
  - “Context FifaWC10 is about FifaWorldCup in year 2010”
  - “Context Football9810 is about Football in years 1998-2010”
  - “Football9810 is more general than FifaWC10”

- **Knowledge propagates across contexts**
Contextual AI theory principia: [McCarthy, 1993]

- Every formula is **asserted in a context**
  - “In FIFA World Cup 2006, the Winner is Italy.”
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- Context are **first class logical objects**
  (formulas can predicate about contexts)
  - “Context FifaWC10 is about FifaWorldCup in year 2010”
  - “Context Football9810 is about Football in years 1998-2010”
  - “Football9810 is more general than FifaWC10”

- Knowledge propagates **across contexts**
  - “Every Winner in FifaWC06 is a QualifiedTeam in FifaWC10”
Theory of contexts: Context as a Box

Idea [Benerecetti et al., 2000]

- A context is a logical theory...
- ...associated to a region in a contextual space
Theory of contexts: Context as a Box

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- A context is a **logical theory**...
- ...associated to a region in a contextual space

\[ \text{HostTeam} \sqsubseteq \text{QualifiedTeam} \]

\[ \ldots \]

\[ \text{Winner}(\text{team_spain}) \]

\[ \text{RunnerUp}(\text{team_holland}) \]

\[ \ldots \]

\[ \text{playsFor}(\text{buffon}, \text{team_italy}) \]

\[ \text{playsFor}(\text{cannavaro}, \text{team_italy}) \]

\[ \ldots \]
Theory of contexts: Context as a Box

Idea [Benerecetti et al., 2000]

- A context is a logical theory...
- ...associated to a region in a contextual space

\[
\begin{align*}
\text{time}(C, 2010), & \text{ location}(C, \text{South_Africa}), \text{ topic}(C, \text{FIFA_WC}) \\
C = & \\
\text{HostTeam} \sqsubseteq & \text{QualifiedTeam} \\
\ldots & \\
\text{Winner}(\text{team_spain}) & \\
\text{RunnerUp}(\text{team_holland}) & \\
\ldots & \\
\text{playsFor}(\text{buffon, team_italy}) & \\
\text{playsFor}(\text{cannavaro, team_italy}) & \\
\ldots & 
\end{align*}
\]
Theory of contexts: Context as a Box

Idea [Benerecetti et al., 2000]

- A context is a **logical theory**...
- ...associated to a region in a **contextual space**
Motivation: contexts and SW data

Need for context in Semantic Web

- Most of Semantic Web data holds in specific contextual space (time, location, topic...)
- No explicit support for reasoning with context sensitive knowledge in Semantic Web languages

→ Current practice:
  Contextual information often “handcrafted” in implementation
Example: current context implementation

Freebase: context representation for events

\[
\langle \text{fb:base.x2016fifaeurocupfrance.} \\
\text{euro\_cup\_team.qualified\_as} \rangle
\]

represents:
Example: current context implementation

Freebase: context representation for events

<fb:base.x2016fifaeurocupfrance.
euro_cup_team.qualified_as>

represents:

- a context dependent relation: euro_cup_team.qualified_as
Example: current context implementation

Freebase: context representation for events

\(<\text{fb:base.x2016fifaeurocupfrance.}\>

\text{euro\_cup\_team.qualified\_as}\)

represents:

- a context dependent relation: \text{euro\_cup\_team.qualified\_as}
- in the context identified by:
  - Time: 2016
Example: current context implementation

Freebase: context representation for events

<fb:base.x2016fifaeurocupfrance.
euro_cup_team.qualified_as>

represents:

- a context dependent relation: `euro_cup_team.qualified_as`
- in the context identified by:
  - Time: `2016`
  - Topic: `fifaeurocup`
Example: current context implementation

Freebase: context representation for events

```xml
<fb:base.x2016fifaeurocupfrance.
    euro_cup_team.qualified_as>
```

represents:

- a context dependent relation: `euro_cup_team.qualified_as`
- in the context identified by:
  - Time: 2016
  - Topic: `fifaeurocup`
  - Location: `france`
Example: current context implementation

Freebase: context representation for events

\[<\text{fb:base.x2016fifa-euro-cup-france.euro-cup-team.qualified-as}>\]

represents:

- a context dependent relation: `euro_cup_team.qualified_as`
- in the context identified by:
  - Time: 2016
  - Topic: `fifa-euro-cup`
  - Location: `france`

- Context information encoded in the link is **implicit knowledge**!
- No way to **uniformly retrieve and reason** over such information
Example: current context implementation

Freebase: context representation for events

<fb:base.x2016fifaeurocupfrance.
euro_cup_team.qualified_as>
represents:

- a context dependent relation: `euro_cup_team.qualified_as`
- in the context identified by:
  - Time: 2016
  - Topic: fifaeurocup
  - Location: france

Context information encoded in the link is implicit knowledge!
No way to uniformly retrieve and reason over such information

Context representation for Semantic Web data needs a well-defined theory of contexts
Contextualized Knowledge Repository (CKR)

- DL based framework for representation and reasoning with contextual knowledge in the Semantic Web
- **Contextual theory**: based on formal AI theories of context
  

Other DL contextual frameworks:

[Bao et al., 2010, Klarman and Gutiérrez-Basulto, 2011, Straccia et al., 2010].
Contextual modelling needs

From study on typical use of context in Semantic Web data:

**Requirements**

- **Statement contextualization**: associate context to facts
- **Symbols locality**: local meaning for symbols
- **Cross-context TBox statements**: knowledge relations across contexts
- **Complex contextualization**: more than one contextual values to facts
- **Modularity**: separation of knowledge in independent modules
- **Unified reasoning and query**: inference and query use context structure

...
Contextual modelling needs

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Requirements

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- ...

➡️ Definition of “contextual primitives” of CKR
(e.g. cross-context statements ➔ *eval* operator, complex contextualization ➔ c.classes and modules . . . )
## CKR objectives

A general **formalism and tool** for the **representation and reasoning** with contextual knowledge in the Semantic Web.

- **Theory**: based on formal theories of context from AI
- **Implementation**: built over state of the art tools
- **Evaluation**: for performance and ease of modeling

---

**Plan**

1. Tailor a logic of context in AI for Semantic Web needs
2. Provide an axiomatization of this new logic
3. Define reasoning services
4. Implement the theory on a platform
5. Evaluate by representation adequacy and performance
CKR objectives and plan

CKR objectives

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Plan

1. Tailor a **logic of context** in AI for Semantic Web needs
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Overview

1. CKR model
2. Reasoning
3. Implementation on RDF
4. Defeasible axioms
5. Contextual hierarchies
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CKR structure

Global context

Local modules

Global context

(Local) contexts

Metaknowledge:
structure of contexts, context classes, relations, modules and attributes

Global object knowledge:
knowledge shared by all contexts

(Object) contexts

Knowledge distributed across different modules

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Global context

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(Local) contexts
**Global context**

- **Metaknowledge:** structure of contexts, context classes, relations, modules and attributes
- **Global object knowledge:** knowledge shared by all contexts

---

**Global object knowledge**

- $A \subseteq B$, $B \subseteq C$, ...
- $R \subseteq S$, ...
- $A(a)$, $B(a)$, ...
- $R(a, b)$, $S(a, c)$ ...

---

**Local contexts**

(Local) contexts
**Global context**

- **Metaknowledge:** structure of contexts, context classes, relations, modules and attributes
- **Global object knowledge:** knowledge shared by all contexts

---

**(Local) contexts**

- **Object knowledge with references:**
  local knowledge with references to value of predicates in other contexts
- **Knowledge distributed across different modules $K_m$**
Basic modeling language: description logic SROIQ-RL,
- SROIQ-RL is a restriction of SROIQ
- It corresponds to the syntax of the OWL-RL profile of OWL-2

SROIQ-RL

C := A | \{a\} | C_1 \cap C_2 | C_1 \cup C_2 | \exists R. C_1 | \exists R. \{a\} | \exists R. \top
D := A | D_1 \cap D_2 | \neg C_1 | \forall R. D_1 | \exists R. \{a\} | \leq [0, 1] R. C_1 | \leq [0, 1] R. \top

TBox axioms: C \sqsubseteq D \quad \text{ABox axioms: } D(a), R(a, b)

Example

- CulturalEvent \sqsubseteq Event, SportsEvent \sqsubseteq Event
- Event \sqsubseteq \exists \text{mod.}\{m\_event\}
- VolleyA1Competition(A1_2012-13), SportiveTourist(volley_fan_01)
Metalanguage $\mathcal{L}_\Gamma$

Metavocabulary $\Gamma$: Contexts structure objects

- **N**: context names (match1, volley_season2013)
- **M**: module names (m_match1, m_event)
  with role $\text{mod} : N \times M$
- **C**: context classes (Event, VolleyMatch)
  with $\text{Ctx} \in C$: class of all contexts
- **R**: contextual relations (hasSubEvent, covers)
- **A**: contextual attributes (time, location, topic)
- $D_A$ attribute values of $A \in A$ (2013, trento, sport)

Metalanguage $\mathcal{L}_\Gamma$: DL language over $\Gamma$
Object language $\mathcal{L}_\Sigma$

Object vocabulary $\Sigma$: domain vocabulary

Eval expression

For $X$ a concept or role expression in $\Sigma$, $C$ a concept expression in $\Gamma$

$$eval(X, C)$$

“The interpretation of $X$ in all the contexts of type $C$”

Idea: “imports” meaning of $X$ from all contexts in $C$
Object language $\mathcal{L}_\Sigma$

Object vocabulary $\Sigma$: domain vocabulary

Eval expression

For $X$ a concept or role expression in $\Sigma$, $C$ a concept expression in $\Gamma$

$$\text{eval}(X, C)$$

“The interpretation of $X$ in all the contexts of type $C$”

Idea: “imports” meaning of $X$ from all contexts in $C$

Object language with references $\mathcal{L}_\Sigma^e$: $\mathcal{L}_\Sigma$ with eval expressions
Object language $\mathcal{L}_\Sigma$

Object vocabulary $\Sigma$: domain vocabulary

Eval expression
For $X$ a concept or role expression in $\Sigma$, $C$ a concept expression in $\Gamma$

$eval(X, C)$

“The interpretation of $X$ in all the contexts of type $C$”

Idea: “imports” meaning of $X$ from all contexts in $C$

Object language with references $\mathcal{L}^e_\Sigma$: $\mathcal{L}_\Sigma$ with eval expressions
Contextualized Knowledge Repository (CKR):

$$\mathcal{K} = \langle \mathcal{G}, \{K_m\}_{m \in M}\rangle$$

- $\mathcal{G}$ contains
  - metaknowledge axioms in $\mathcal{L}_\Gamma$
  - global object axioms in $\mathcal{L}_\Sigma$

- for every module name $m \in M$,
  $K_m$ contains object axioms with references in $\mathcal{L}_\Sigma^e$
Tourism example:

- **Idea**: Tourism recommendation for events in Trentino
- **Structure of contexts** represents **events** and **tourists information**

→ **Task**: find interesting events on the base of tourists’ preferences

We model this domain in a CKR $\mathcal{K}_{tour} = \langle \mathcal{G}, \{K_m\}_{m \in M} \rangle$
Tourism example: CKR structure

G

Event

CulturalEvent
Concert

trento_cuneo_120922
modena_trento_130112

SportEvent
VolleyMatch

trento_latina_130203

VolleyA1
Competition

campionato_A1_2012-13

Tourist

SportiveTourist
volley_fan_01

CulturalTourist

hasParentEvent
Tourism example: CKR structure

Event
- CulturalEvent
  - Concert
- SportEvent
  - VolleyMatch
  - Competition

Tourist
- SportiveTourist
- CulturalTourist

Campionato_A1_2012-13
- Volley_fan_01
- Trento_cuneo_120922
- Modena_trento_130112
- Trento_latina_130203

m_event
m_sport_ev
m_v_match
m_tourist
m_sp_tourist
m_tourist01

hasParentEvent
Tourism example: CKR structure

![Diagram of CKR structure]

**Concepts:**
- Event
  - CulturalEvent
  - SportEvent
  - Concert
  - VolleyMatch
  - Competition
- Tourist
  - SportiveTourist
  - CulturalTourist

**Examples:**
- trento_cuneo_120922
- modena_trento_130112
- trento_latina_130203
- campionato_A1_2012-13

**Key Events:**
- Kevent
- Ksport_ev
- Kv_match
- Kmatch1
- Ktourist01

**HasParentEvent:**
- m_event
- m_sport_ev
- m_v_match
- m_tourist
- m_sp_tourist
- m_match1
- m_match2
- m_match3

**Additional:**
- L. Bozzato (DKM - FBK)
- CKR Tutorial
- eKNOW20 17/75
Tourism example: some modules contents

In $K_{v\_match}$:
- $\text{HomeTeam} \sqsubseteq \text{Team}$
- $\text{HostTeam} \sqsubseteq \text{Team}$
- $\text{Winner} \sqsubseteq \text{Team}$
- $\text{Loser} \sqsubseteq \text{Team}$

In $K_{match2}$:
- $\text{HomeTeam}(\text{casa\_modena\_volley})$
- $\text{HostTeam}(\text{itas\_trentino\_volley})$
- $\text{Winner}(\text{casa\_modena\_volley})$
- $\text{Loser}(\text{itas\_trentino\_volley})$
Tourism example: some modules contents

In $K_{v\_match}$:
- $HomeTeam \sqsubseteq Team$
- $HostTeam \sqsubseteq Team$
- $Winner \sqsubseteq Team$
- $Loser \sqsubseteq Team$

In $K_{match2}$:
- $HomeTeam(casa\_modena\_volley)$
- $HostTeam(itas\_trentino\_volley)$
- $Winner(casa\_modena\_volley)$
- $Loser(itas\_trentino\_volley)$

... 

In $K_{sport\_ev}$: “Winners of major volley matches are top teams”

$$eval(Winner, VolleyMatch \sqcap \exists hasParentEvent.\text{VolleyA1Competition}) \sqsubseteq TopTeam$$

In $K_{sp\_tourist}$: “Top teams are preferred teams”

$$eval(TopTeam, SportEvent) \sqsubseteq PreferredTeam$$
CKR interpretation

Idea

CKR interpretations are two layered interpretations

CKR interpretation $\mathcal{I} = \langle \mathcal{M}, \mathcal{I} \rangle$

- $\mathcal{M}$ is a DL interpretation over $\Gamma \cup \Sigma$
- For every $x \in \text{Ctx}^{\mathcal{M}}$, $\mathcal{I}(x)$ is a DL interpretation over $\Sigma$
  - $\Delta^\mathcal{I}(x) = \Delta^{\mathcal{M}}$
  - for $a \in \text{NI}_\Sigma$, $a^\mathcal{I}(x) = a^{\mathcal{M}}$

Interpretation of eval: $\text{eval}(X, C)^{\mathcal{I}}(x) = \bigcup_{e \in \text{C}^{\mathcal{M}}} X^{\mathcal{I}(e)}$
CKR model $\mathcal{I} \models \mathcal{K}$

$\mathcal{I} = \langle \mathcal{M}, \mathcal{I} \rangle$ is a CKR model of $\mathcal{K}$ if:

- for $\alpha \in \mathcal{L}_\Sigma \cup \mathcal{L}_\Gamma$ in $\mathcal{G}$, $\mathcal{M} \models \alpha$
- for $\langle x, y \rangle \in \text{mod}^\mathcal{M}$ with $y = m^\mathcal{M}$, $\mathcal{I}(x) \models K_m$
- for $\alpha \in \mathcal{G} \cap \mathcal{L}_\Sigma$ and $x \in \text{Ctx}^\mathcal{M}$, $\mathcal{I}(x) \models \alpha$
Tourism example: semantics

Suppose we have $\mathcal{I} = \langle \mathcal{M}, \mathcal{I} \rangle$ s.t. $\mathcal{I} \models \mathcal{R}_{tour}$.

For each match $\text{matchN}$, its KB is:

$K(\text{matchN}^\mathcal{M}) = K_{\text{event}} \cup K_{\text{sport\_ev}} \cup K_{\text{v\_match}} \cup K_{\text{matchN}}$
Tourism example: semantics

Suppose we have $\mathcal{I} = \langle \mathcal{M}, \mathcal{I} \rangle$ s.t. $\mathcal{I} \models \mathcal{R}_{\text{tour}}$.

For each match $\text{matchN}$, its KB is:

$$K(\text{matchN}^\mathcal{M}) = K_{\text{event}} \cup K_{\text{sport}_e} \cup K_{\text{v_match}} \cup K_{\text{matchN}}$$

$\text{VolleyMatch} \sqsupseteq$

$\exists \text{hasParentEvent}.\text{VolleyA1Competition} = \text{TopMatch}$

$\text{eval}(\text{Winner}, \text{TopMatch}) \subseteq \text{TopTeam} \in K_{\text{sport}_e}$
Tourism example: semantics

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For each match $\text{matchN}$, its KB is:

$K(\text{matchN}^\mathcal{M}) = K_{\text{event}} \cup K_{\text{sport}_\text{ev}} \cup K_{\text{v_match}} \cup K_{\text{matchN}}$

$\text{VolleyMatch} \sqsubseteq \exists \text{hasParentEvent.}\text{VolleyA1Competition} = \text{TopMatch}$

$\text{eval}(\text{Winner, TopMatch}) \subseteq \text{TopTeam} \in K_{\text{sport}_\text{ev}}$

$\text{eval}(\text{Winner, TopMatch})^{\mathcal{I}(\text{matchN})} \subseteq \text{TopTeam}^{\mathcal{I}(\text{matchN})}$
Tourism example: semantics

Suppose we have $\mathcal{I} = \langle \mathcal{M}, \mathcal{I} \rangle$ s.t. $\mathcal{I} \models \mathcal{R}_{tour}$.

For each match $\text{matchN}$, its KB is:

$K(\text{matchN}^M) = K_{\text{event}} \cup K_{\text{sport}_ev} \cup K_{\text{v_match}} \cup K_{\text{matchN}}$

$\text{VolleyMatch} \sqcap \exists \text{hasParentEvent}.\text{VolleyA1Competition} = \text{TopMatch}$

$eval(\text{Winner}, \text{TopMatch}) \subseteq \text{TopTeam} \in K_{\text{sport}_ev}$

$\bigcup_{e \in \text{TopMatch}^M} \text{Winner}^\mathcal{I}(e) \subseteq \text{TopTeam}^\mathcal{I}(\text{matchN})$
Tourism example: semantics

Suppose we have $\mathcal{I} = \langle \mathcal{M}, \mathcal{I} \rangle$ s.t. $\mathcal{I} \models \mathcal{R}_{tour}$.

For each match $\text{match}_N$, its KB is:

$K(\text{match}_N^M) = K_{\text{event}} \cup K_{\text{sport}_ev} \cup K_{\text{v_match}} \cup K_{\text{match}_N}$

$\text{VolleyMatch} \sqcap \exists \text{hasParentEvent.\text{VolleyA1Competition}} = \text{TopMatch}$

$\text{eval}(\text{Winner, TopMatch}) \subseteq \text{TopTeam} \in K_{\text{sport}_ev}$

$$\bigcup_{e \in \{\text{match}_2, \text{match}_3\}} \text{Winner}^{\mathcal{I}(e)} \subseteq \text{TopTeam}^{\mathcal{I}(\text{match}_N)}$$
Tourism example: semantics

Suppose we have $\mathcal{I} = \langle \mathcal{M}, \mathcal{I} \rangle$ s.t. $\mathcal{I} \models \mathcal{R}_{\text{tour}}$.

For each match $\text{matchN}$, its KB is:

$K(\text{matchN}^\mathcal{M}) = K_{\text{event}} \cup K_{\text{sport_ev}} \cup K_{\text{v_match}} \cup K_{\text{matchN}}$

$\forall \text{VolleyMatch} \ni \exists \text{hasParentEvent.VolleyA1Competition} = \text{TopMatch}$

$\text{eval}(\text{Winner, TopMatch}) \subseteq \text{TopTeam} \in K_{\text{sport_ev}}$

\{ itas_trentino, casa_modena \} \subseteq \text{TopTeam}^\mathcal{I}(\text{matchN})
Tourism example: semantics

Suppose we have \( \mathcal{I} = \langle \mathcal{M}, \mathcal{I} \rangle \) s.t. \( \mathcal{I} \models \mathcal{K}_{\text{tour}} \).

For the context of \textit{volley\_fan}:

\[ K(\text{volley\_fan}^\mathcal{M}) = \mathcal{K}_{\text{tourist}} \cup \mathcal{K}_{\text{sp\_tourist}} \cup \mathcal{K}_{\text{tourist01}} \]
Tourism example: semantics

Suppose we have $\mathcal{I} = (\mathcal{M}, \mathcal{I})$ s.t. $\mathcal{I} \models \mathcal{K}_{\text{tour}}$.

For the context of \texttt{volley\_fan}:

$K(\text{volley\_fan}^\mathcal{M}) = K_{\text{tourist}} \cup K_{\text{sp\_tourist}} \cup K_{\text{tourist01}}$

$\text{eval}(\text{TopTeam, SportEvent}) \sqsubseteq \text{PreferredTeam} \in K_{\text{sp\_tourist}}$
Suppose we have $\mathcal{I} = \langle M, \mathcal{I} \rangle$ s.t. $\mathcal{I} \models \mathcal{K}_{\text{tour}}$.

For the context of \texttt{volley\_fan}:

$K(\text{volley\_fan}^M) = K_{\text{tourist}} \cup K_{\text{sp\_tourist}} \cup K_{\text{tourist01}}$

$\text{eval}(\text{TopTeam}, \text{SportEvent}) \sqsubseteq \text{PreferredTeam} \in K_{\text{sp\_tourist}}$

$\text{eval}(\text{TopTeam}, \text{SportEvent}) \mathcal{I}(\text{volley\_fan}) \sqsubseteq \text{PreferredTeam} \mathcal{I}(\text{volley\_fan})$
Suppose we have $\mathcal{I} = \langle \mathcal{M}, \mathcal{I} \rangle$ s.t. $\mathcal{I} \models \mathcal{K}_{\text{tour}}$.

For the context of `volley_fan`:

$$K(\text{volley}_\text{fan}^M) = K_{\text{tourist}} \cup K_{\text{sp}_\text{tourist}} \cup K_{\text{tourist01}}$$

eval(\text{TopTeam, SportEvent}) \sqsubseteq \text{PreferredTeam} \\
\in K_{\text{sp}_\text{tourist}} \\
\bigcup_{e \in \text{SportEvent}^M} \text{TopTeam}^{\mathcal{I}(e)} \\
\sqsubseteq \text{PreferredTeam}^{\mathcal{I}(\text{volley}_\text{fan})}$$
Tourism example: semantics

Suppose we have $\mathcal{I} = \langle \mathcal{M}, \mathcal{I} \rangle$ s.t. $\mathcal{I} \models \mathcal{K}_{\text{tour}}$.

For the context of \texttt{volley\_fan}:

\[ K(\texttt{volley\_fan}^\mathcal{M}) = K_{\text{tour}} \cup K_{\text{sp\_tourist}} \cup K_{\text{tourist01}} \]

\[ \text{eval}(\text{TopTeam, SportEvent}) \subseteq \text{PreferredTeam} \subseteq K_{\text{sp\_tourist}} \]

\[ \bigcup_{e \in \{\text{match\_1, match\_2, match\_3}\}} \text{TopTeam}^{\mathcal{I}(e)} \]

\[ \subseteq \text{PreferredTeam}^{\mathcal{I}(\texttt{volley\_fan})} \]
Tourism example: semantics

Suppose we have $\mathcal{I} = \langle M, \mathcal{I} \rangle$ s.t. $\mathcal{I} \models K_{tour}$.

For the context of $\text{volley\_fan}$:

$K(\text{volley\_fan}^M) = K_{tourist} \cup K_{sp\_tourist} \cup K_{tourist01}$

$\text{eval}(\text{TopTeam}, \text{SportEvent}) \subseteq \text{PreferredTeam} \in K_{sp\_tourist}$

$\{\text{itas\_trentino}, \text{casa\_modena}\}$

$\subseteq \text{PreferredTeam}^{\mathcal{I}(\text{volley\_fan})}$
Summary:

- **Two-layered** DL knowledge base
- General context structure (extending [Serafini and Homola, 2012])
- *eval* operator: knowledge propagation across contexts
- Model theoretic DL semantics
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Reasoning tasks

Satisfiability

Instance query answering

Boolean conjunctive query answering
### Reasoning tasks

#### Satisfiability

- Does a given CKR $\mathcal{K}$ have some CKR model?

#### Instance query answering

#### Boolean conjunctive query answering
Reasoning tasks

Satisfiability

- Does a given CKR $\mathcal{K}$ have some CKR model?

Instance query answering

- Given a CKR $\mathcal{K}$, an assertion $\alpha$, a context $c$ of $\mathcal{K}$
- Does $\mathcal{K}$ entail $\alpha$ at $c$ (denoted $\mathcal{K} \models c : \alpha$), i.e., does $\mathcal{I}(c^\mathcal{M}) \models \alpha$ hold for every CKR model $\mathcal{I}$ of $\mathcal{K}$?

Boolean conjunctive query answering
Reasoning tasks

**Satisfiability**
- Does a given CKR \( \mathcal{K} \) have some CKR model?

**Instance query answering**
- Given a CKR \( \mathcal{K} \), an assertion \( \alpha \), a context \( c \) of \( \mathcal{K} \)
- Does \( \mathcal{K} \) entail \( \alpha \) at \( c \) (denoted \( \mathcal{K} \models c : \alpha \)), i.e., does \( \mathcal{I}(c^\mathcal{M}) \models \alpha \) hold for every CKR model \( \mathcal{I} \) of \( \mathcal{K} \)?

**Boolean conjunctive query answering**
- Given a CKR \( \mathcal{K} \) and a formula \( q = \exists y \gamma(y) \), where \( \gamma(y) = c_1 : \alpha_1, \ldots, c_n : \alpha_n \), the \( c_i \) are contexts and the \( \alpha_i \) atoms that may contain variables
- Does \( \mathcal{K} \) entail \( q \) (denoted \( \mathcal{K} \models q \)), i.e., does for every CKR model \( \mathcal{I} \) of \( \mathcal{K} \), some variable assignment \( \sigma \) to \( y \) exists s.t. \( \mathcal{I}(c_i^\mathcal{I}), \sigma \models \alpha_i \) for every \( i \)?
Materialization calculus:

- Calculus for instance checking in OWL RL CKR
- Extension to the CKR structure of materialization calculus for OWL EL of [Krötzsch, 2010]
- Formalizes the operation of forward closure in implementation

Idea

Composed by 3 kinds of rule sets:

- Input rules $I$: translation of DL axioms to datalog atoms
- Deduction rules $P$: forward inference rules
- Output rules $O$: translation for DL proved ABox assertion
## Translation rules

### Input rules \( I \)

- Rule 1: \( I_{\text{glob}} \):
  \[ c \in N \Rightarrow \{ \text{inst}(c, \text{Ctx}, g_{m}) \} \]

- Rule 2: \( I_{\text{loc}} \):
  \[ \text{eval}(A, C) \sqsubseteq B \Rightarrow \{ \text{subEval}(A, C, B, c) \} \]

### Deduction rules \( P \)

- Rule 1: \( P_{\text{rl}} \):
  \[ \text{subClass}(y, z, c), \text{inst}(x, y, c) \rightarrow \text{inst}(x, z, c) \]

- Rule 2: \( P_{\text{loc}} \):
  \[ \text{subEval}(a, c_1, b, c), \text{inst}(c', c_1, g_m), \text{inst}(x, a, c') \rightarrow \text{inst}(x, b, c) \]

### Output rules \( O \)

- Rule 1: \( O \):
  \[ \{ \text{inst}(a, A, c) \} \Rightarrow c : A(a) \]

- Rule 2: \( O \):
  \[ \{ \text{triple}(a, R, b, c) \} \Rightarrow c : R(a, b) \]
Translation rules

Input rules $I$

$I_{rl}$: $SROTQ$-RL input rules

\[ c : A(a) \Rightarrow \{ \text{inst}(a, A, c) \} \quad c : A \sqsubseteq B \Rightarrow \{ \text{subClass}(A, B, c) \} \]
Translation rules

Input rules $I$

$I_{rl}$: $SROIQ$-RL input rules

$c : A(a) \Rightarrow \{ \text{inst}(a, A, c) \}$

$c : A \sqsubseteq B \Rightarrow \{ \text{subClass}(A, B, c) \}$

Deduction rules $P$

$P_{rl}$: $SROIQ$-RL deduction rules

$subClass(y, z, c), \text{inst}(x, y, c) \rightarrow \text{inst}(x, z, c)$

Output rules $O$

$O \{ \text{inst}(a, A, c) \} \Rightarrow c : A(a) \Rightarrow \{ \text{triple}(a, R, b, c) \} \Rightarrow c : R(a, b)$
## Translation rules

### Input rules $I$

$I_{rl}$: **SROIQ-RL input rules**

- $c : A(a) \Rightarrow \{ \text{inst}(a, A, c) \}$
- $c : A \sqsubseteq B \Rightarrow \{ \text{subClass}(A, B, c) \}$

$I_{glob}$: **Global input rules**

- $c \in \mathbf{N} \Rightarrow \{ \text{inst}(c, \text{Ctx}, \text{gm}) \}$
- $C \in \mathbf{C} \Rightarrow \{ \text{subClass}(C, \text{Ctx}, \text{gm}) \}$

### Deduction rules $P$

$P_{rl}$: **SROIQ-RL deduction rules**

- $\text{subClass}(y, z, c), \text{inst}(x, y, c) \Rightarrow \text{inst}(x, z, c)$

### Output rules $O$

- $\{ \text{inst}(a, A, c) \} \Rightarrow c : A(a)$
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## Translation rules

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**$I_{loc}$**: Local input rules

- $c : \text{eval}(A, C) \sqsubseteq B \Rightarrow \{ \text{subEval}(A, C, B, c) \}$

### Deduction rules $P$

**$P_{rl}$**: $SROIQ$-RL deduction rules

- $\text{subClass}(y, z, c), \text{inst}(x, y, c) \rightarrow \text{inst}(x, z, c)$

**$P_{loc}$**: Local deduction rules

- $\text{subEval}(a, c_1, b, c), \text{inst}(c', c_1, \text{gm}), \text{inst}(x, a, c') \rightarrow \text{inst}(x, b, c)$

### Output rules $O$

- $\{ \text{inst}(a, A, c) \} \Rightarrow c : A(a)$
- $\{ \text{triple}(a, R, b, c) \} \Rightarrow c : R(a, b)$
Translation rules

**Input rules** \( I \)

\( I_{rl} \): \( SROIQ \)-RL input rules
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**Deduction rules** \( P \)

\( P_{rl} \): \( SROIQ \)-RL deduction rules
\[ \text{subClass}(y, z, c), \text{inst}(x, y, c) \rightarrow \text{inst}(x, z, c) \]

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\[ \text{subEval}(a, c_1, b, c), \text{inst}(c', c_1, \text{gm}), \text{inst}(x, a, c') \rightarrow \text{inst}(x, b, c) \]

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\[ \{ \text{inst}(a, A, c) \} \Rightarrow c : A(a) \quad \{ \text{triple}(a, R, b, c) \} \Rightarrow c : R(a, b) \]
Translation process

1. Global program $PG(\emptyset)$: translation for global context
Translation process

1. Global program $PG(\mathcal{G})$: translation for global context
2. Computation of local knowledge bases $K_c$ for each context $c$ in $\mathcal{G}$
Translation process

1. Global program $PG(\mathcal{G})$: translation for global context
2. Computation of local knowledge bases $K_c$ for each context $c$ in $\mathcal{G}$
3. Local programs $PC(c)$: translation for local contexts
Translation process

1. Global program $PG(\mathcal{G})$: translation for global context
2. Computation of local knowledge bases $K_c$ for each context $c$ in $\mathcal{G}$
3. Local programs $PC(c)$: translation for local contexts
4. CKR program $PK(\mathcal{K})$: union of global and local programs
Consider CKR $\mathcal{K}$ where the axioms are in a **normal form**

- Needed for universal encoding: e.g., $A_1 \cap A_2 \cap \cdots \cap A_n \subseteq B$

### Translation completeness

1. $\mathcal{K} \models c : \alpha$ iff $PK(\mathcal{K}) \models O(\alpha, c)$ (axiom $\alpha$ in context $c$)

2. $\mathcal{K} \models \exists y \gamma(y)$ iff $PK(\mathcal{K}) \models O(\exists y \gamma(y))$ (boolean conjunctive queries)
Reasoning

Summary:

- Instance checking procedure for CKRs in OWL RL
- Calculus based on a translation to datalog
- Formalizes forward closure in implementation
Overview

1. CKR model
2. Reasoning
3. Implementation on RDF
4. Defeasible axioms
5. Contextual hierarchies
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SPRINGLES: implementation on SPARQL

**Semantic Web languages**
- **RDF**: representation for data
- **OWL**: representation for schema
- **SPARQL**: query on RDF data

**CKR implementation**
- Contexts as OWL/RDF repositories
- Reasoning rules as SPARQL queries
CKR implementation on top of SPRINGLES:
SParql-based Rule Inference over Named Graphs Layer Extending Sesame

SPRINGLES features:
- transparent/on-demand closure materialization based on rules
- rules encoded as SPARQL queries on Named Graphs (NG)
- customizable rule evaluation strategy

Why SPRINGLES:
- no inference over NGs in RDF stores

Why SPARQL:
- exploits optimized query engines
- can scale to large KBs (cf. RETE)
SPRINGLES rules and evaluation strategy

SPRINGLES rule

Forward SPARQL-based rules of the form:

\[
<\text{rule-name}> a \text{ spr:Rule; spr:head"""" < graphpattern > """"; spr:body"""" < sparqlquery > """".}
\]

SPRINGLES evaluation strategy

Composition of SPRINGLES primitives:

- parallel rule evaluation
- sequence
- fixpoint
- repeat
CKR ruleset and evaluation strategy

Ruleset

Translation to SPRINGLES rules of materialization calculus rules:

```
:pel-c-subc a spr:Rule ;
  spr:head "" GRAPH ?mx { ?x rdf:type ?z } "" ;
  spr:body "" GRAPH ?m1 { ?y rdfs:subClassOf ?z }
    GRAPH ?m2 { ?x rdf:type ?y }
    GRAPH sys:dep { ?mx sys:derivedFrom ?m1,?m2 }
  FILTER NOT EXISTS {
    GRAPH ?m0 { ?x rdf:type ?z }
    GRAPH sys:dep { ?mx sys:derivedFrom ?m0 }
  } "" .
```

Evaluation strategy

- Associate inferred graph to `ckr:global`
- By fixpoint, compute OWL RL and global closure on `ckr:global`
- Compute modules associated to each context
- Create local graphs for contexts and for inference
- Evaluate local rules for OWL RL on context graphs
Implementation on RDF

Current CKR implementations:

- **CKR prototype:**
  1st implementation on Sesame/OWLIM [Tamilin et al., 2010]

- **CKR on SPRINGLES:** SPARQL-based forward rules on named graphs over Sesame [Bozzato and Serafini, 2013]

- **CKRew:** CKR datalog rewriter [Bozzato et al., 2018a]

- **CKR on RDFpro:**
  SPARQL rules for RDF processor [Schuetz et al., 2020]
Evaluation

Findings [Bozzato et al., 2013, Bozzato and Serafini, 2014]

- **Modelling:**
  - **Language:** CKR model reduces redundancy, easier references
  - **Model:** CKR uses less symbols than Flat modelling
  - **Query:** CKR performs better on context-based queries

- **Reasoning:**
  - **Scalability:** influenced by expressivity and number of contexts
  - **Propagation:** CKR connections outperform flat replication
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1. CKR model
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Need for defeasibility in contexts

CKR structure: two layers

- **Global context:**
  Structure of contexts and object knowledge shared by all contexts

- **(Local) contexts:**
  Local object knowledge (with references)
Need for defeasibility in contexts

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\[
\begin{align*}
\text{Bird} & \sqsubseteq \text{Fly} \\
\text{Horse} & \sqsubseteq \neg \text{Fly}
\end{align*}
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Need for defeasibility in contexts

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- **(Local) contexts:**
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- Bird ⊑ Fly
- Horse ⊑ ¬Fly

- greek_myths
- Horse(pegasus), Fly(pegasus)

We want to specify that certain global axioms are defeasible: they hold globally, but allow exceptional instances in local contexts.
Need for defeasibility in contexts

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Horse ⊑ ¬Fly

greek_myths
Horse(pegasus), Fly(pegasus)
Horse(pedasus)
```
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\[
\text{greek\_myths}
\]

\[
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\text{Horse(pegasus), Fly(pegasus)} \\
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greek_myths
Horse(pegasus), Fly(pegasus)
Horse(pedasus), ¬Fly(pedasus)

→ We want to specify that certain global axioms are defeasible: they hold globally, but allow exceptional instances in local contexts
CKR extension for defeasibility:
AI Journal (257):72-126, 2018 [Bozzato et al., 2018a]

- Syntax and semantics of an extension of CKR with defeasible axioms in global context
- Define reasoning problems:
  - extended CKR satisfiability
  - CKR axiom entailment \( \mathcal{R} \models c : \alpha \)
    \( \alpha = \text{Fly(pegasus)} \)
  - CKR conjunctive query answering \( \mathcal{R} \models \exists y \gamma(y) \)
    \( \gamma(y) = \text{greek_myths: Horse(pegasus), hasFeature(pegasus, y), Wing(y)} \)

- Characterize their computational cost (complexity)
- Extend datalog translation for OWL RL based CKR with rules for the translation of defeasible axioms
- Prototype implementation for CKR datalog rewriter
Notable aspects

Interesting points of our work:

- Expressive means for defeasibility on structured KBs in DL
  - defeasibility in contextual systems
  - non-monotonic reasoning in DLs

- Reason by cases: conflicts in overridings not ruled by “preference”

- Inheritance of properties: no “exceptional” elements

- Translation to datalog extends monotonic materialization calculus
We extend the type of axioms appearing in global object knowledge:

Defeasible axiom $\alpha$ of $\mathcal{G}$: $D(\alpha) \in \mathcal{G}$ for $\alpha \in \mathcal{L}_\Sigma$

“$\alpha$ propagates to local contexts, but admits exceptional instances”

DL language $\mathcal{L}_\Sigma^D \subseteq \mathcal{L}_\Sigma$ with defeasible axioms
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DL language $\mathcal{L}^D_\Sigma$ $\mathcal{L}_\Sigma$ with defeasible axioms
Semantics: clashing assumptions

<table>
<thead>
<tr>
<th>Idea</th>
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<tbody>
<tr>
<td>Exception of axiom instances modelled as <strong>clashing assumptions</strong> $\langle \alpha, e \rangle$</td>
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Semantics: clashing assumptions

Idea

- Exception of axiom instances modelled as clashing assumptions $\langle \alpha, e \rangle$

  "In context $c$, ignore instance $e$ in evaluation of $\alpha$" $\langle (Cheap \sqsubseteq Interesting), fbmatch \rangle$
Semantics: clashing assumptions

Idea

- Exception of axiom instances modelled as **clashing assumptions** \( \langle \alpha, e \rangle \)
  
  "In context c, ignore instance e in evaluation of \( \alpha \)"

- **Clashing assumption** \( \langle \alpha, e \rangle \):
  
  assumption that e is exceptional for \( \alpha \)

- **CAS-interpretation** \( \mathcal{I}_{CAS} = \langle \mathcal{M}, \mathcal{I}, \chi \rangle \):
  
  \( \chi(c) \): set of clashing assumptions of context c
Semantics: clashing assumptions

Idea

- Exception of axiom instances modelled as clashing assumptions $\langle \alpha, e \rangle$
  “In context c, ignore instance e in evaluation of $\alpha$” $\langle (Cheap \sqsubseteq Interesting), fbmatch \rangle$

- Clashing assumption $\langle \alpha, e \rangle$:
  assumption that e is exceptional for $\alpha$

- CAS-interpretation $I_{CAS} = \langle M, I, \chi \rangle$:
  $\chi(c)$: set of clashing assumptions of context $c$

CAS-model $I_{CAS} \models K$

$I_{CAS}$ is a CAS-model for $K$ if:

- $M \models \alpha$, for every $\alpha \in G$ strict or defeasible

L. Bozzato (DKM - FBK)
Semantics: clashing assumptions

Idea

- Exception of axiom instances modelled as clashing assumptions \( \langle \alpha, e \rangle \)
  “In context c, ignore instance e in evaluation of \( \alpha \)” \( \langle (Cheap \sqsubseteq Interesting), fbmatch \rangle \)

- Clashing assumption \( \langle \alpha, e \rangle \):
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  \( \chi(c) \): set of clashing assumptions of context \( c \)

CAS-model \( \mathcal{I}_{CAS} \models \mathcal{K} \)

\( \mathcal{I}_{CAS} \) is a CAS-model for \( \mathcal{K} \) if:

- \( M \models \alpha \), for every \( \alpha \in \mathcal{G} \) strict or defeasible
- \( \mathcal{I}(x) \models K_m \), if \( m \) is a module of context \( x \)
- \( \mathcal{I}(x) \models \alpha \), for every \( \alpha \in \mathcal{G} \) strict
Semantics: clashing assumptions

Idea

- Exception of axiom instances modelled as *clashing assumptions* \( \langle \alpha, e \rangle \)
  
  “In context c, ignore instance e in evaluation of \( \alpha \)” \( \langle (Cheap \sqsubseteq Interesting), fbmatch \rangle \)

- Clashing assumption \( \langle \alpha, e \rangle \):
  
  assumption that \( e \) is exceptional for \( \alpha \)

- CAS-interpretation \( \mathcal{I}_{CAS} = \langle M, \mathcal{I}, \chi \rangle \):
  
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- \( \mathcal{I}(x) \models K_m \), if \( m \) is a module of context \( x \)
- \( \mathcal{I}(x) \models \alpha \), for every \( \alpha \in \mathcal{G} \) strict
- for every \( D(\alpha) \in \mathcal{G} \), if \( \langle \alpha, e \rangle \notin \chi(x) \), then \( \mathcal{I}(x) \models \alpha(e) \)
Idea

- Assumptions must be justified by local assertions in a clashing set $S$
  
  “In context $c$, $\alpha(e) \cup S$ is unsatisfiable”
Idea

Assumptions must be justified by local assertions in a clashing set $S$

“In context $c$, $\alpha(e) \cup S$ is unsatisfiable” \{Cheap(fbmatch), $\neg$Interesting(fbmatch)\}
Semantics: justification

**Idea**
- Assumptions must be justified by local assertions in a clashing set $S$.
  
  \[
  \text{``In context } c, \alpha(e) \cup S \text{ is unsatisfiable}'' \quad \{\text{Cheap}(fbmatch), \neg \text{Interesting}(fbmatch)\}
  \]

**Justification**

$I_\chi = \langle M, I, \chi \rangle$ is justified, if for every context $x \in \text{Ctx}^M$ and clashing assumption $\langle \alpha, e \rangle \in \chi(x)$,

1. some clashing set $S = S_{\langle \alpha, e \rangle, x}$ exists such that $I(x) \models S_{\langle \alpha, e \rangle, x}$, and
2. for every model $I_\chi' = \langle M', I', \chi \rangle$ of $\mathcal{K}$ that is NI-congruent with $I_\chi$ (i.e., $c^M = c^{M'}$ for every individual name $c$), $I'(x) \models S_{\langle \alpha, e \rangle, x}$

→ Justified if, for every clashing assumption $\langle \alpha, e \rangle$, we have a factual evidence $S$ of its local unsatisfiability.

Moreover, this factual evidence is a logical consequence (provable).
Semantics: CKR model

Idea

- CKR models are interpretation where all c. assumptions are justified

CKR model $\mathcal{I} \models \mathcal{K}$

$\mathcal{I} = \langle M, I \rangle$ is a CKR model of $\mathcal{K}$, if some $\mathcal{I}_{\text{CAS}} = \langle M, I, \chi \rangle$ is a justified CAS-model of $\mathcal{K}$
Examples

- **CAS-model:** $\mathcal{I}_{CAS} = \langle \mathcal{M}, \mathcal{I}, \chi \rangle$
  with $\langle (\text{Cheap} \sqsubseteq \text{Interesting}), \text{fbmatch} \rangle \in \chi(\text{cultural_tourist})$

- **Justification:** $S = \{ \text{Cheap(fbmatch)}, \neg \text{Interesting(fbmatch)} \}$
CAS-model: $\mathcal{I}_{CAS} = \langle \mathcal{M}, \mathcal{I}, \chi \rangle$
with $\langle (\text{Cheap} \sqsubseteq \text{Interesting}), \text{fbmatch} \rangle \in \chi(\text{cultural_tourist})$

Justification: $S = \{ \text{Cheap(fbmatch)}, \neg \text{Interesting(fbmatch)} \}$

$\mathcal{I}_{CAS} \not\models \text{Interesting(fbmatch)}$ but $\mathcal{I}_{CAS} \models \text{Interesting(market)}$ and $\mathcal{I}'_{CAS} \models \text{Interesting(market)}$ for each $\mathcal{I}'_{CAS}$ NI-congruent with $\mathcal{I}_{CAS}$
Examples

\[ D(\text{WorkingBefore} \sqsubseteq \text{WorkingNow}) \]

\[
\begin{align*}
\text{emp}_2017 & : \text{WorkingNow}(\text{alice}), \\
& \quad \text{WorkingNow}(\text{bob}), \\
& \quad \text{WorkingNow}(\text{charlie}) \\
\text{emp}_2018 & : \text{eval}(\text{WorkingNow}, \text{emp}_2017) \\
& \quad \sqsubseteq \text{WorkingBefore}, \\
& \quad \lnot \text{WorkingNow}(\text{charlie})
\end{align*}
\]

**CAS-model:**

\[ I_{\text{CAS}} = \langle M, I, \chi \rangle \]

with \ \langle (\text{WorkingBefore} \sqsubseteq \text{WorkingNow}), \text{charlie} \rangle \in \chi(\text{emp}_2018)

**Justification:**

\[ S = \{ \text{WorkingBefore}(\text{charlie}), \lnot \text{WorkingNow}(\text{charlie}) \} \]
Examples

\[ D(\text{WorkingBefore} \sqsubseteq \text{WorkingNow}) \]

\[ \text{emp}_{2017} \]
- WorkingNow(alice),
- WorkingNow(bob),
- WorkingNow(charlie)

\[ \text{emp}_{2018} \]
- \text{eval}(\text{WorkingNow}, \text{emp}_{2017}) \sqsubseteq \text{WorkingBefore},
- \neg\text{WorkingNow}(\text{charlie})
- WorkingNow(alice),
- WorkingNow(bob)

\[ \text{CAS-model: } \mathcal{I}_{\text{CAS}} = \langle \mathcal{M}, \mathcal{I}, \chi \rangle \]
with \( \langle (\text{WorkingBefore} \sqsubseteq \text{WorkingNow}), \text{charlie} \rangle \in \chi(\text{emp}_{2018}) \)

\[ \text{Justification: } S = \{ \text{WorkingBefore}(\text{charlie}), \neg\text{WorkingNow}(\text{charlie}) \} \]

\[ \mathcal{I}_{\text{CAS}} \not\models \text{WorkingNow}(\text{charlie}) \text{ but } \]
\[ \mathcal{I}_{\text{CAS}} \models \text{WorkingNow}(\text{alice}) \text{ and } \mathcal{I}_{\text{CAS}} \models \text{WorkingNow}(\text{bob}) \]
Reasoning tasks

1. **CKR satisfiability** (does \( \mathcal{K} \) have a CKR model)
2. **CKR axiom entailment** \( \mathcal{K} \models c : \alpha \)
3. **CKR conjunctive query answering** \( \mathcal{K} \models \exists y \gamma(y) \)

Main complexity results

- Deciding whether \( \mathcal{K} \) has some CKR-model is **NP-complete**
- Deciding \( \mathcal{K} \models c : \alpha \) is **coNP-complete**
- Deciding \( \mathcal{K} \models \exists y \gamma(y) \) is **\( \Pi_2^p \)-complete**
Extended CKR translation to datalog

Main Idea

- extend the materialization calculus for instance checking in [Bozzato and Serafini, 2013]
- add rules for overriding
- use a fixed set of rules and provide $R$ etc as data
- requires a normal form for $R +$ slight restrictions on $D(\alpha)$

Program Structure

Composed by 3 kinds of rule sets:

- Input rules $I$: translation of DL axioms to Datalog atoms
- Deduction rules $P$: forward inference rules
- Output rules $O$: translation for DL proved ABox assertion

$\rightarrow I$ and $P$, contain “overriding rules” to treat defeasible propagation
**Defeasibility rules**

\[ I_D: \text{Defeasibility input rules (overriding conditions)} \]

\[
D(A \sqsubseteq B) \Rightarrow \\
\{ \text{ovr}(\text{subClass}, x, A, B, c) \leftarrow \text{ninstd}(x, B, c), \text{instd}(x, A, c), \text{prec}(c, g). \} \\
\text{where ninstd}(x, B, c) \text{ represents } \neg \text{instd}(x, B, c) \\
\]

\[ P_D: \text{Defeasibility deduction rules (defeasible propagation)} \]

\[
\text{instd}(x, z, c) \leftarrow \text{subClass}(y, z, g), \text{instd}(x, y, c), \text{prec}(c, g), \\
\text{not ovr}(\text{subClass}, x, y, z, c). \\
\]
Defeasibility rules

**I_D**: Defeasibility input rules (overriding conditions)

\[ D(\text{Cheap} \sqsubseteq \text{Interesting}) \Rightarrow \{ \text{ovr}(\text{subClass}, x, \text{Cheap}, \text{Interesting}, c) \leftarrow \text{ninstd}(x, \text{Interesting}, c), \text{instd}(x, \text{Cheap}, c), \text{prec}(c, g). \} \]

where \( \text{ninstd}(x, B, c) \) represents \( \neg \text{instd}(x, B, c) \)

**P_D**: Defeasibility deduction rules (defeasible propagation)

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\[ \neg \text{ovr}(\text{subClass}, x, y, z, c). \]

\[ \text{PK}(K) | = \text{ovr}(\text{subClass}, fbmatch, Cheap, Interesting, c) \]
\[ \text{but} \text{PK}(K) \not| = \text{ovr}(\text{subClass}, market, Cheap, Interesting, c) \]
\[ \text{thus} \text{PK}(K) | = \text{instd}(\text{market}, Interesting, c) \]
**Defeasibility rules**

$I_D$: Defeasibility input rules (overriding conditions)

\[
D(A \sqsubseteq B) \Rightarrow \\
\{ \text{ovr}(\text{subClass}, x, A, B, c) \leftarrow \text{ninstd}(x, B, c), \text{instd}(x, A, c), \text{prec}(c, g). \} \\
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\text{instd}(x, \text{Cheap}, c), \text{prec}(c, g). \} \\
\]

\[
\rightarrow PK(\mathcal{K}) \models \text{ovr}(\text{subClass}, \text{fbmatch}, \text{Cheap}, \text{Interesting}, c) \text{ but} \\
PK(\mathcal{K}) \not\models \text{ovr}(\text{subClass}, \text{market}, \text{Cheap}, \text{Interesting}, c) \text{ thus} \\
PK(\mathcal{K}) \models \text{instd}(\text{market}, \text{Interesting}, c)
\]
Defeasibility rules: negative literals

Disjunctive information

Negative rule for $A \cap B \subseteq C$:
\[
ninstd(x, y_1, c) \lor ninstd(x, y_2, c) \leftarrow subConj(y_1, y_2, z, c), ninstd(x, z, c).
\]

- needed for completeness of justifications
- in practice, may generate large number of models
- is in general not sufficient to derive all negative consequences
Defeasibility rules: negative literals

Disjunctive information

Negative rule for $A \sqcap B \sqsubseteq C$:

$$\text{ninstd}(x, y_1, c) \lor \text{ninstd}(x, y_2, c) \leftarrow \text{subConj}(y_1, y_2, z, c), \text{ninstd}(x, z, c).$$

- needed for completeness of justifications
- in practice, may generate large number of models
- is in general not sufficient to derive all negative consequences

Solution: contradiction testing

$$\mathcal{K} \models c : \neg p(e) \text{ iff } \mathcal{K}' = \mathcal{K} \cup \{c : p(e)\} \text{ is unsatisfiable}$$

- use $\text{nlit}(p, e)$ to represent negative literals
- use $\text{unsat}(\text{nlit}(p, e))$ for unsatisfiability with $p(e)$
- use $\text{test}(\text{nlit}(p, e))$ and $\text{test\_fails}(\text{nlit}(p, e))$ for test environment for $\text{nlit}(p, e)$ and for test failure, resp.
Defeasibility rules: contradiction tests

**Contradiction testing: example rules**

- **Instantiate the test.** E.g., for atomic inclusions:
  
  \[
  \text{test}(\text{nlit}(x,z,c)) \leftarrow \text{def\_subclass}(y,z), \text{instd}(x,y,c,\text{main}), \text{prec}(c,g).
  \]

- **Exclude overriding, if the test fails.**
  E.g., for the `subClass` overriding,
  \[
  \leftarrow \text{test\_fails}(\text{nlit}(x,z,c)), \text{ovr}(\text{subClass},x,y,z,c).
  \]

- **Determine if test fails**
  i.e., no clashes (= instances unsat) are found:
  \[
  \text{test\_fails}(\text{nlit}(x,z,c)) \leftarrow \\
  \quad \text{instd}(x,z,c,\text{nlit}(x,z,c)), \text{not unsat}(\text{nlit}(x,z,c)).
  \]

- **Generate test environment for each negative literal:**
  e.g., for assertions
  \[
  \text{instd}(x_1,y_1,c,t) \leftarrow \text{instd}(x_1,y_1,c,\text{main}), \text{test}(t). \\
  \quad \text{instd}(x,z,c,\text{nlit}(x,z,c)) \leftarrow \text{test}(\text{nlit}(x,z,c)).
  \]
Translation process

1 Global program $PG(\emptyset)$: translation for global context
Translation process

1. Global program $PG(\mathcal{G})$: translation for global context
2. Computation of local knowledge bases $K_c$ for each context $c$ in $\mathcal{G}$
Translation process

1. Global program $PG(\mathcal{G})$: translation for global context
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3. Local programs $PC(c)$: translation for local contexts
Translation process

1. Global program $PG(\mathcal{G})$: translation for global context
2. Computation of local knowledge bases $K_c$ for each context $c$ in $\mathcal{G}$
3. Local programs $PC(c)$: translation for local contexts
4. CKR program $PK(\mathcal{K})$: union of global and local programs

Translation Correctness

For a $\mathcal{K}$ in normal form

1. $\mathcal{K}$ entails $c : \alpha$ iff $PK(\mathcal{K}) \models O(\alpha, c)$ (axiom $\alpha$ in context $c$)
2. $\mathcal{K} \models \exists y \gamma(y)$ iff $PK(\mathcal{K}) \models O(\exists y \gamma(y))$ (Boolean conjunctive queries)
Prototype implementation:

- Extends basic translation of OWL RL ontologies to 2 layer CKR structure
- **Input:** OWL files for global context and knowledge modules
- **Output:** datalog translation for CKR program
Translation process implementation:

CKRew translation process

Prototype and examples available at: [http://ckrew.fbk.eu/](http://ckrew.fbk.eu/)
Other approaches

- **Normality in DLs**: cf. [Britz and Varzinczak, 2016]
  no complex contextual structure with contextual reasoning inside modules
Other approaches

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  no complex contextual structure with contextual reasoning inside modules

- **Typicality in DL**: $\mathcal{ALC} + T_{\text{min}}$ [Giordano et al., 2013]
  - defeasible membership similar to typical instances of $C (TC)$
  - model-based, our approach is syntax-sensitive
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  similar notion of abnormality, model based minimization
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  similar notion of abnormality, model based minimization

- **Nonmonotonic description logic $\mathcal{DL}^N$**: [Bonatti et al., 2015]
  - extends a generic base DL $\mathcal{DL}$ with an operator $NC$ for *normality concepts*
  - *defeasible inclusions (DIs)* $C \sqsubseteq_n D$ between concepts,
  - a polynomial rewriting procedure to base $\mathcal{DL}$
  - can not handle *reasoning by cases* (Nixon Diamond)
Other approaches

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  - *defeasible inclusions* ($DIs$) $C \subseteq^n D$ between concepts,  
  - a polynomial rewriting procedure to base $\mathcal{DL}$  
  - can not handle reasoning by cases (Nixon Diamond)
- **Non-monotonic multi-context systems (MCS)**:  
  [Brewka and Eiter, 2007, Bikakis and Antoniou, 2010]  
  - translate CKR to MCS with bridge rules
Justifiable exceptions

Summary:

- Extension of CKR semantics to represent clashing assumptions and justifications
- Extension of CKR datalog translation with defeasible propagation
- CKR\text{ew} datalog rewriter implementation

Reasoning in $\mathcal{EL}_\bot$ and $DL-Lite_\mathcal{R}$

Introduce problem of reasoning with existential axioms and exceptions

- CKR in $\mathcal{EL}_\bot$ [Bozzato et al., 2019c]
- Justifiable exceptions in $DL-Lite_\mathcal{R}$ KB [Bozzato et al., 2019b]
Overview

1. CKR model
2. Reasoning
3. Implementation on RDF
4. Defeasible axioms
5. Contextual hierarchies
Overview

1. CKR model
2. Reasoning
3. Implementation on RDF
4. Defeasible axioms
5. Contextual hierarchies
Limits of the model

CKR with Justifiable Exceptions

- **Global context:**
  Structure of contexts and object knowledge shared by all contexts
  Defeasible axioms: allow exceptional instances in local contexts

- **(Local) contexts:** Local object knowledge (with references)

![Diagram showing the structure of contexts and object knowledge]

- $D(Cheap \sqsubseteq Interesting)$
  - Cheap(fbmatch), Cheap(market)

- Cultural tourist
  - $\neg Interesting(fbmatch)$
  - Interesting(market)

- Sportive tourist
  - Interesting(fbmatch)
  - Interesting(market)

Limited to 2 level hierarchy
No further refinements allowed
Limits of the model

CKR with Justifiable Exceptions

- Global context: Structure of contexts and object knowledge shared by all contexts
  Defeasible axioms: allow exceptional instances in local contexts

- (Local) contexts: Local object knowledge (with references)

```
D(Cheap ⊑ Interesting)
Cheap(fbmatch), Cheap(market)

cultural_tourist
¬Interesting(fbmatch)
Interesting(market)

sportive_tourist
Interesting(fbmatch)
Interesting(market)
```

- Limited to 2 level hierarchy
- No further refinements allowed (e.g. sportive_cultural_tourist)
Proposal: contextual hierarchies

Idea

- Allow local defeasible axioms
- Contexts organized in a coverage hierarchy
- Axiom preference defined by context position: “more specific axioms are stronger”
Proposal: contextual hierarchies

Idea
- Allow local defeasible axioms
- Contexts organized in a coverage hierarchy
- Axiom preference defined by context position: “more specific axioms are stronger”

⇒ sCKR with ranked contextual hierarchies [Bozzato et al., 2018b]
- Syntax and semantics for simple CKRs with ranked contextual hierarchies
- Study of reasoning problems and their complexity
- Extended datalog translation for OWL-RL based sCKR with rules for model preference (weak constraints)
Simple CKR: idea

sCKR: idea

- **Global context**: poset representing context hierarchy
- **Local contexts**: local context KBs with defeasible axioms

→ Simplifies presentation of coverage, representable in “regular” CKR
Coverage and language

Context names: $\mathbf{N} \subseteq \mathbf{NI}$

Coverage: strict partial order $\prec \subseteq \mathbf{N} \times \mathbf{N}$

if $c_1 \prec c_2$, $c_2$ covers $c_1$ (i.e. $c_2$ is more general than $c_1$)

Contextual language $\mathcal{L}_N^D$

DL language $\mathcal{L}$ extended with:

- eval expressions: $\text{eval}(X, c)$ ("the interpretation of $X$ in context $c$")
- defeasible axioms: $D(\alpha)$ for $\alpha \in \mathcal{L}$
Simple Contextualized Knowledge Repository (sCKR):

\[ \mathcal{K} = \langle \mathcal{C}, K_N \rangle \]

- \( \mathcal{C} \) is a poset \((N, \prec)\)
- \( K_N = \{ K_c \}_{c \in N} \) for every context name \( c \in N \),
- \( K_c \) is a local DL knowledge base over \( L^D_N \)
Example: introduction

Example of coverage structure defined by contextual dimensions [Serafini and Homola, 2012]

- A large organization has different policies with respect to:
  - local branches (location dimension)
  - time period (time dimension)

- Active in different fields:
  Electronics (E), Robotics (R), Musical instruments (M)

- A local Supervisor (S) can manage only one of the fields
Example: dimensions

```
Year Semester

2018 S1 S2 ...

Time

Location

World Continent Country

world EU IT
```

L. Bozzato (DKM - FBK)
Example: hierarchy and local contexts

\[ C_{(2018,\text{world})} : \{ M \cap E \sqsubseteq \bot, M \cap R \sqsubseteq \bot, E \cap R \sqsubseteq \bot \} \]

\[ C_{(2018,\text{EU})} : \{ D(S \sqsubseteq E) \} \]

\[ C_{(2018,\text{IT})} : \{ D(S \sqsubseteq M) \} \]

\[ C_{(S1,\text{IT})} : \{ S(i), R(i) \} \quad C_{(S2,\text{IT})} : \{ S(i) \} \]
Example: hierarchy and local contexts

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\[ C_{(S1,\text{IT})} : \{ S(i), R(i) \} \quad C_{(S2,\text{IT})} : \{ S(i) \} \]
### Idea

Hierarchies with a notion of **level**

### Ranked hierarchy

A contextual hierarchy $\mathcal{C} = (\mathbb{N}, \prec)$ is **ranked** iff, for every root context $r \in \mathcal{C}$ and every context $c$ with $c \prec r$, all paths from $c$ to $r$ have the same length.
Ranked hierarchies

Idea
Hierarchies with a notion of level

Ranked hierarchy
A contextual hierarchy \( \mathcal{C} = (N, \prec) \) is ranked iff, for every root context \( r \in \mathcal{C} \) and every context \( c \prec r \), all paths from \( c \) to \( r \) have the same length

Level function: \( l : N \rightarrow \mathbb{N} \)

\[
l(c) = \begin{cases} 
0, & \text{if } c \text{ is root} \\
1 + \max(\{l(c') \mid c \prec c'\}), & \text{otherwise}
\end{cases}
\]
Ranked hierarchies

Idea
Hierarchies with a notion of level

Ranked hierarchy
A contextual hierarchy \( \mathcal{C} = (\mathbb{N}, \prec) \) is ranked iff, for every root context \( r \in \mathcal{C} \) and every context \( c \) with \( c \prec r \), all paths from \( c \) to \( r \) have the same length.

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1 + \max(\{l(c') \mid c \prec c'\}), & \text{otherwise}
\end{cases}
\]

Example: products of ranked dimension hierarchies
(like our example hierarchy in previous slide...)
sCKR interpretation

Idea
Set of interpretations for each local context

sCKR interpretation $\mathcal{I}$

- $\mathcal{I} = \{\mathcal{I}(c)\}_{c \in \mathbb{N}}$
- For $c, c' \in \mathbb{N}$, $\mathcal{I}(c)$ is a DL interpretation:
  - $\Delta^{\mathcal{I}(c)} = \Delta^{\mathcal{I}(c')}$
  - For $a \in NI$, $a^{\mathcal{I}(c)} = a^{\mathcal{I}(c')}$
Clashing assumptions

- **CAS-interpretation** $\mathcal{I}_{CAS} = \langle \mathcal{I}, \chi \rangle$:
  - $\chi(c)$: set of clashing assumptions of context $c$

**CAS-model** $\mathcal{I}_{CAS} \models \mathcal{K}$

$\mathcal{I}_{CAS}$ is a CAS-model for $\mathcal{K}$ if:

- $\mathcal{I}(c') \models K_c$, if $c' \preceq c$
- for every $D(\alpha) \in K_c$, $\mathcal{I}(c) \models \alpha$
- for every $D(\alpha) \in K_c$ and $c' \prec c$, if $\langle \alpha, e \rangle \notin \chi(c')$, then $\mathcal{I}(c') \models \alpha(e)$
Clashing assumptions

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  - $\chi(c)$: set of clashing assumptions of context $c$

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- For every $D(\alpha) \in K_c$, $\mathcal{I}(c) \models \alpha$
- For every $D(\alpha) \in K_c$ and $c' \prec c$, if $\langle \alpha, e \rangle \notin \chi(c')$, then $\mathcal{I}(c') \models \alpha(e)$

**Justification**

$\mathcal{I}_\chi = \langle \mathcal{I}, \chi \rangle$ model of $\mathcal{K}$ is **justified**, if for every context $c \in N$ and clashing assumption $\langle \alpha, e \rangle \in \chi(c)$,

1. some clashing set $S = S_{\langle \alpha, e \rangle, c}$ exists such that $\mathcal{I}(c) \models S_{\langle \alpha, e \rangle, c}$, and
2. for every model $\mathcal{I'}$ of $\mathcal{K}$ that is NI-congruent with $\mathcal{I}_\chi$, $\mathcal{I'}(c) \models S_{\langle \alpha, e \rangle, c}$
**Assumption profile and ordering**

**Idea**

- We want to give priority to more specific axioms
- Maximize the level of overridden axioms
- Order models using level of clashing assumptions

**Global profile** $p(\chi)$: vector $(l_n, \ldots, l_0)$, each $l_i$ is the number of clashing assumptions for axioms at level $i$

**Ordering** $p(\chi) < p(\chi')$: lexicographical ordering
  e.g. $(0,1,0,1) < (0,1,5,0)$
sCKR model

**Idea**

sCKR models are justified and “maximize the rank” of overridings

**Model preference:**

\( \mathcal{I}_\chi = \langle \mathcal{I}, \chi \rangle \) is preferred to \( \mathcal{I}'_\chi = \langle \mathcal{I}, \chi' \rangle \) iff \( p(\chi) < p(\chi') \)
sCKR model

**Idea**

sCKR models are justified and “maximize the rank” of overridings

**Model preference:**

\[ \mathcal{I}_\chi = \langle \mathcal{I}, \chi \rangle \text{ is preferred to } \mathcal{I}'_\chi = \langle \mathcal{I}, \chi' \rangle \text{ iff } p(\chi) < p(\chi') \]

**sCKR model** \(\mathcal{I} \models K\)

\(\mathcal{I}\) is a sCKR model of \(K\) if

- some \(\mathcal{I}^{CAS}\) is a justified CAS-model of \(K\)
- there exists no \(\mathcal{I}'^{CAS}\) that is preferred to \(\mathcal{I}^{CAS}\)
Example: preferred models

\[ C_{(2018,\text{world})} : \{M \sqcap E \subseteq \bot, M \sqcap R \subseteq \bot, E \sqcap R \subseteq \bot\} \]
\[ C_{(2018,\text{EU})} : \{D(S \subseteq E)\} \]
\[ C_{(2018,\text{IT})} : \{D(S \subseteq M)\} \]
\[ C_{(S1,\text{IT})} : \{S(i), R(i)\} \]
\[ C_{(S2,\text{IT})} : \{S(i)\} \]

2 justified models:

\[ \chi_1(C_{(S1,\text{IT})}) = \{\langle S \subseteq E, i \rangle, \langle S \subseteq M, i \rangle\} \]
\[ \chi_2(C_{(S1,\text{IT})}) = \{\langle S \subseteq E, i \rangle, \langle S \sqsubseteq M, i \rangle\} \]
\[ \chi_1(C_{(S2,\text{IT})}) = \{\langle S \subseteq E, i \rangle\} \]
\[ \chi_2(C_{(S2,\text{IT})}) = \{\langle S \sqsubseteq M, i \rangle\} \]
Example: preferred models

\( \mathcal{C}_{(2018, \text{world})} : \{ M \sqcap E \sqsubseteq \bot, M \sqcap R \sqsubseteq \bot, E \sqcap R \sqsubseteq \bot \} \)

\( \mathcal{C}_{(2018, \text{EU})} : \{ D(S \subseteq E) \} \)

\( \mathcal{C}_{(2018, \text{IT})} : \{ D(S \subseteq M) \} \)

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\( \mathcal{C}_{(S2, \text{IT})} : \{ S(i) \} \)

2 justified models:

\[ \chi_1(\mathcal{C}_{(S1, \text{IT})}) = \{ \langle S \sqsubseteq E, i \rangle, \langle S \sqsubseteq M, i \rangle \} \]

\[ \chi_1(\mathcal{C}_{(S2, \text{IT})}) = \{ \langle S \sqsubseteq E, i \rangle \} \]

\[ \chi_2(\mathcal{C}_{(S1, \text{IT})}) = \{ \langle S \sqsubseteq E, i \rangle, \langle S \sqsubseteq M, i \rangle \} \]

\[ \chi_2(\mathcal{C}_{(S2, \text{IT})}) = \{ \langle S \sqsubseteq M, i \rangle \} \]

Profile ordering: \( p(\chi_1) = (0, 1, 2, 0) < p(\chi_2) = (0, 2, 1, 0) \)

→ Model based on \( \chi_1 \) is the preferred model
1. Satisfiability (does $\mathcal{K}$ have a CKR model)
2. Model checking (is $\mathcal{I}_{CAS}$ a model for $\mathcal{K}$)
3. Axiom entailment $\mathcal{K} \models c : \alpha$
4. Conjunctive query answering $\mathcal{K} \models \exists y \gamma(y)$

### Complexity results
- Satisfiability is **NP-complete** (was NP-complete)
- Model checking is **coNP-complete** (was polynomial)
- Axiom entailment is $\Delta^p_2$-complete (was coNP-complete)
- (Boolean) CQ answering is $\Pi^p_2$-complete (was $\Pi^p_2$-complete)
Main idea:

- Materialization calculus for instance checking and CQ answering in sCKR based on $SROIQ$-RL (OWL-RL)
- Extends the datalog translation for CKR with justifiable exceptions in [Bozzato et al., 2018a]
- Interpreted under Answer Set semantics

→ Rules for model preference: weak constraints [Leone et al., 2002]
Overriding level rules

Level preference rules: attach level info to overridings

$$\text{ovrlevel}_{\text{subClass}}(x, A, B, c, n) \leftarrow \text{ovr}(\text{subClass}, x, A, B, c_1, c), \text{level}(c_1, n).$$

Weak constraints: prefer models with ovr. at higher level

$$\sim \text{ovrlevel}_{\text{subClass}}(x, y, z, c, n).$$  \[1 : n\]
Overriding level rules

Level preference rules: attach level info to overridings
\[ \text{ovrlevel}_\text{subClass}(x, A, B, c, n) \leftarrow \text{ovr}(\text{subClass}, x, A, B, c_1, c), \text{level}(c_1, n). \]

Weak constraints: prefer models with ovr. at higher level
\[ \sim \text{ovrlevel}_\text{subClass}(x, y, z, c, n). [1 : n] \]

Weak constraints

- [1 : n]: weight 1, priority level n
- wc interpretation: “minimize weight of violations at higher levels”
  → prefer models with less overridings and at the higher levels
Translation process

1. Global program $PG(\mathcal{C})$: translation for global context $\mathcal{C}$
2. Local programs $PC(c, \mathcal{K})$: translation for local contexts $\mathcal{K}_c$
3. CKR program $PK(\mathcal{K})$: union of global and local programs
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Translation Correctness

1. $\mathcal{K} \models c : \alpha$ iff $PK(\mathcal{K}) \models O(\alpha, c)$ (axiom $\alpha$ in context $c$)
2. $\mathcal{K} \models \exists y \gamma(y)$ iff $PK(\mathcal{K}) \models O(\exists y \gamma(y), c)$ (Boolean CQ in context $c$)
Contextual hierarchies

Summary:
- CKR extension with local defeasible axioms and knowledge propagation across coverage structure
- For ranked hierarchies: global model preference relation
- Datalog translation extending [Bozzato et al., 2018a] for instance checking based on weak constraints

sCKR with general hierarchies [Bozzato et al., 2019a]
- Semantics: local ordering on models
- Reasoning: selection procedure for preferred answer sets
Conclusion

Summary:
- **Contextual model** formalized in DL and AI theory of context
- **Reasoning** formalized as datalog materialization calculus
- Different (RDF based) implementations
- Extension with defeasible global axioms and justifiable exceptions
- Extension with defeasible local axioms in contextual hierarchies

Current and future directions:
- Application to OLAP operations on RDF cubes [Schuetz et al., 2020]
- Extension to different DL languages (see $\mathcal{EL}_\bot$ [Bozzato et al., 2019c])
- Study of alternative translations and implementation (CKRew)
- Different preference relations (e.g. for representation, efficiency)
- Interaction of different contextual relations (e.g. temporal, revision...)
Thank you for listening

Reasoning with Exceptions in Contextualized Knowledge Repositories

Loris Bozzato
bozzato[at]fbk.eu
https://dkm.fbk.eu/

Data and Knowledge Management Research Unit,
Fondazione Bruno Kessler - Trento, Italy


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