Data-Assisted Distributed Stabilization of Interconnected Linear Multiagent Systems without Persistency of Excitation

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Brief resume of the presenter

Margareta Stefanovic received a Ph.D. degree in Electrical Engineering (Control Systems) from the University of Southern California and is currently an Associate Professor of Electrical Engineering at the University of Denver. Her main research interests are in the areas of robust adaptive control of uncertain systems, and distributed control of multi-agent systems. She is an Editor-at-Large for Journal of Intelligent and Robotic Systems and an Associate Editor of ISA Transactions. Dr. Stefanovic is a Senior Member of the Institute of Electrical and Electronics Engineers (IEEE).

Outline

- Problem Statement
- Main Results
- Summary

Multiagent systems (MAS): typical scenarios

Physically decoupled MAS

Agent model: $\dot{x}_i(t) = A\, x_i(t) + B\, u_i(t)$

where $i \in \{1,2,\ldots,N\}$.

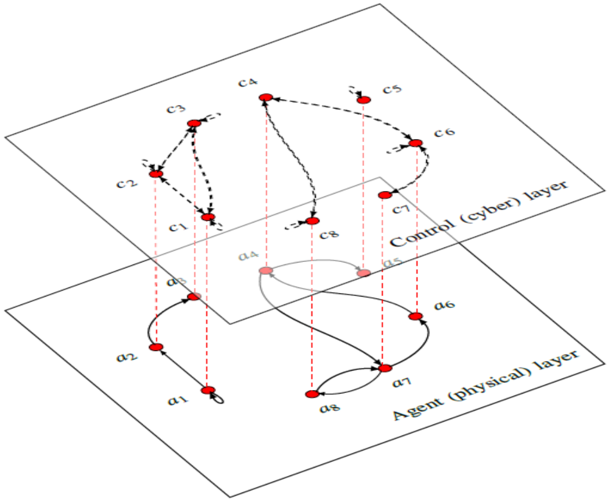
Leaderless consensus problem:

$$\lim_{t\to\infty} (x_i(t) - x_j(t)) = \mathbf{0} \qquad \forall i,j \in \{1,2,\ldots,N\}$$

Leader-follower consensus problem:

$$\lim_{t\to\infty} x_i(t) = x_0(t) \qquad \forall i,j \in \{1,2,\ldots,N\}$$

Interconnected MAS



Agent model: $\dot{x}_i = A\, x_i + B_m u_i + B_m f_i(z_i) + B_u g_i(y_i)$

(generally nonlinear dynamics) where

$z_i = C_{zi} \sum_{j\in\mathcal{N}_i^{az}} a_{ij}^{az} x_i \quad \rightarrow \quad$ matched scenario, $\mathcal{G}_{az}$

$y_i = C_{yi} \sum_{j\in\mathcal{N}_i^{ay}} a_{ij}^{ay} x_i \quad \rightarrow \quad$ unmatched scenario, $\mathcal{G}_{ay}$

Problem Statement: Focus of this work

Linear Physically Interconnected MAS

- Stabilization objective

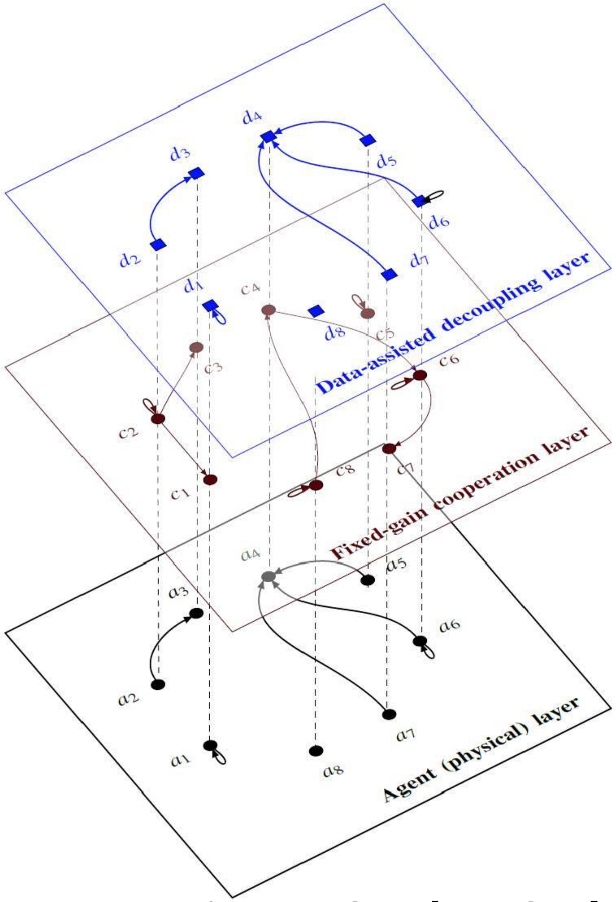
$$x_i(t) \xrightarrow[t\to\infty]{} \mathbf{0} \qquad \forall i \in \{1,2,\dots,N\}$$
$$\text{exp}$$

- Agent model: $\dot{x}_i = A\,x_i + Bu_i + BC_i \sum_{j\in\mathcal{N}_i^a} a_{ij}^a x_i$

- **Assumptions** over the agent layer:

1. (A, B) is stabilizable & known
2. B is full column rank
3. Agent physical interconnection layer $\mathcal{G}_a$ ($\mathcal{N}_i^a$ and $a_{ij}^a \in \mathbb{R}$) is **known** and equal to $\mathcal{G}_d$ (data-assisted communication topology).
4. $\mathcal{N}_i^a$ may include self-loops; a_{ij}^a can be + or –
5. C_i are completely **unknown**

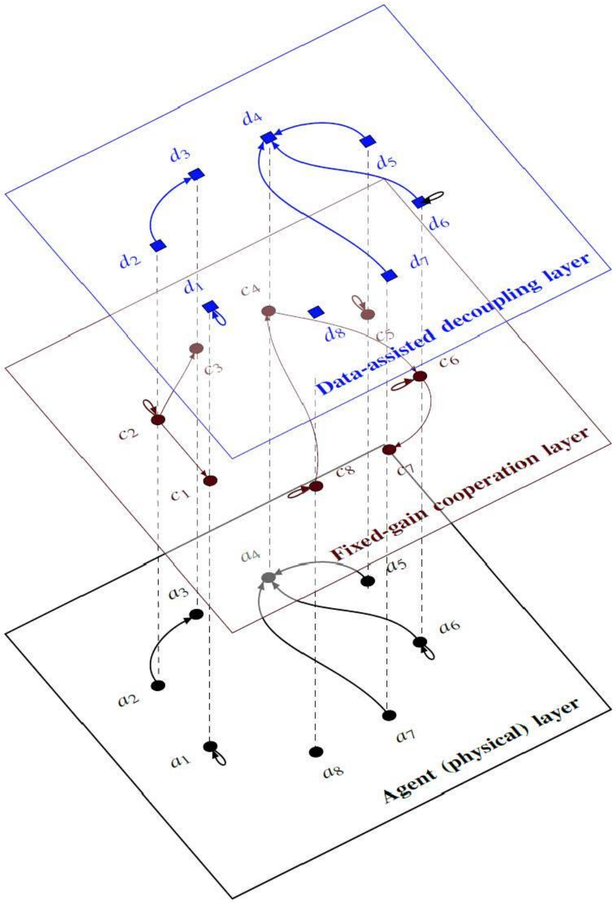
Three-layer (closed-loop) interconnected MAS



Cooperation topology digraph $\mathcal{G}_c$: modified Laplacian matrix $\mathcal{H}_c = L_c + S_c$ where $S_c = \text{diag}\{s_i^c\}$, $s_i^c > 0$ (selfloop), otherwise $s_i^c = 0$.

Problem: Distributed Stabilization w/ Data-assisted Decoupling

Three-layer (closed-loop) interconnected MAS



Distributed Stabilization with Data-Assisted Decoupling

- Agent model:

$$\dot{x}_i = A\, x_i + Bu_i + BC_i \sum_{j\in\mathcal{N}_i^a} a_{ij}^a x_i \qquad \text{or}$$

$$\dot{x}_i(t) = A\, x_i(t) + Bu_i(t) + B \sum_{j\in\mathcal{N}_i^a} a_{ij}^a (I_{n_u} \otimes x_j^T(t))\theta_i^\star$$

$\theta_i^\star = vec(C_i^T)$: Vector of unknown constants (interconnection strengths)

- A two-component distributed stabilization protocol:

$$u_i(t) = u_{ci}(t) + u_{di}(t)$$

- Cooperation protocol (fixed-gain):

$$u_{ci}(t) = K\left(\sum_{j\in\mathcal{N}_i^c} a_{ij}^c \left(x_i(t) - x_j(t)\right) + s_i^c x_i(t)\right) \quad \forall\, i$$

- Data assisted decoupling protocol:

$$u_{di}(t) = -\sum_{j\in\mathcal{N}_i^a} a_{ij}^a (I_{n_u} \otimes x_j^T(t))\, \hat{\theta}_i(t) \qquad \forall\, i \in \mathcal{S}_a$$

$\mathcal{S}_a$: Set of nodes affected by some neighbor agents' dynamics over the agent layer

At least one self-loop assumed to exist in each connected component over Gc

Problem: Distributed Stabilization w/ Data-assisted Decoupling

Distributed Stabilization with Adaptive Decoupling

- Agent model:

$$\dot{x}_i(t) = A\, x_i(t) + Bu_i(t) + B \sum_{j \in \mathcal{N}_i^a} a_{ij}^a (I_{n_u} \otimes x_j^T(t)) \theta_i^\star$$

$\theta_i^\star = vec(C_i^T)$: Vector of unknown constants

- A two-component distributed stabilization protocol:

$$u_i(t) = u_{ci}(t) + u_{di}(t)$$

- Cooperation protocol:

$$u_{ci}(t) = K \left(\sum_{j \in \mathcal{N}_i^c} a_{ij}^c \left(x_i(t) - x_j(t) \right) + s_i^c x_i(t) \right)$$

- Adaptive decoupling protocol:

$$u_{di}(t) = - \sum_{j \in \mathcal{N}_i^a} a_{ij}^a (I_{n_u} \otimes x_j^T(t))\, \hat{\theta}_i(t)$$

- Main control objective:

$$\lim_{t \to \infty} x_i(t) = \mathbf{0} \qquad \forall i \in \{1,2,\dots,N\}$$

- Estimation objective:

$$\lim_{t \to \infty} \hat{\theta}_i(t) = \theta_i^\star \qquad \forall i \in \mathcal{S}_{ai}$$

- PE condition?

$$\int_t^{t+T_j} x_{fj}(\tau) x_{fj}^T(\tau) d\tau \geq \gamma_j I_{n_x} > \mathbf{0} \qquad \forall\, x_{fj}(t) \neq \mathbf{0} \quad \text{and} \quad \forall t \geq 0$$

to be satisfied by each neighbor $j \in \mathcal{N}_i^a$.

Definition 1: Collective FE Condition

- Standard condition in adaptive control for parameter convergence: PE condition

$$\int_t^{t+T_j} x_{fj}(\tau)x_{fj}^T(\tau)d\tau \geq \gamma_j I_{n_x} > \mathbf{0} \qquad \forall\, x_{fj}(t) \neq \mathbf{0} \quad \text{and} \quad \forall t \geq 0$$

to be satisfied by each neighbor $j \in \mathcal{N}_i^a$.

- Proposed *Collective Finite Excitation* (cFE):

$$\sum_{l=1}^{n_{D_i}} \int_{t_{l_s}}^{t_{l_e}} \sum_{j\in N_i^a} a_{ij}^a x_j(\tau)\, d\tau \int_{t_{l_s}}^{t_{l_e}} \sum_{j\in N_i^a} a_{ij}^a x_j^T(\tau)\, d\tau \geq \gamma_i I_{n_x} > \mathbf{0}$$

to be satisfied by a collection of all neighbors $j \in \mathcal{N}_i^a$ for each agent *i* whose dynamics are affected by other neighbors.

n_{Di} = number of non-overlapping, possibly non-contiguous time intervals over which data is collected by agent i.

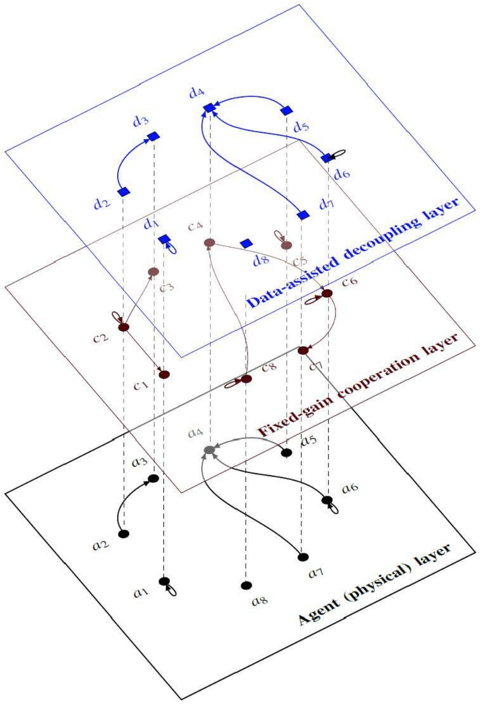
↓

Instead of the restrictive PE, we make the cFE assumption in this work

Outline

- Problem Statement
- Main Results
  - Cooperation Protocol
  - Data-assisted Decoupling Protocol
- Summary

Main: Preliminary Steps



- Closed-loop agent dynamics:

$$\dot{x}_i = \overbrace{Ax_i + B\sum_{j\in\mathcal{N}_i^a}(I_{n_u}\otimes x_j^T)\theta_i^\star}^{\text{Agent layer}} + \overbrace{BK\Big(\sum_{j\in\mathcal{N}_i^c} a_{ij}^c(x_i - x_j) + s_i^c x_i\Big)}^{\text{Fixed-gain cooperation layer}} - \overbrace{B\sum_{j\in\mathcal{N}_i^a}(I_{n_u}\otimes x_j^T)\hat{\theta}_i}^{\text{Data-assisted decoupling layer}}$$

Or

$$\dot{x}_i = \overbrace{Ax_i + BK\Big(\sum_{j\in\mathcal{N}_i^c} a_{ij}^c(x_i - x_j) + s_i^c x_i\Big)}^{\text{Fixed part}} - \overbrace{B\sum_{j\in\mathcal{N}_i^a}(I_{n_u}\otimes x_j^T)\tilde{\theta}_i}^{\text{Adaptive part}}$$

where $\tilde{\theta}_i(t) = \widehat{\theta}_i(t) - \theta_i^\star$.

Main: Preliminary Steps

The fixed part of the dynamics

$$\dot{x}_i = \overbrace{Ax_i + BK\big(\sum_{j\in\mathcal{N}_i^c} a_{ij}^c(x_i - x_j) + s_i^c x_i\big)}^{\text{Fixed part}} \overbrace{- B\sum_{j\in\mathcal{N}_i^a}(I_{n_u}\otimes x_j^T)\tilde{\theta}_i}^{\text{Adaptive part}}$$

Is rewritten as

$$\dot{x}_i = \overbrace{Ax_i(t) + \kappa B v_i(t)}^{\text{networked nominal dynamics}} + \overbrace{BK\left(\sum_{j\in\mathcal{N}_i^c} a_{ij}^c\left(x_i(t) - x_j(t)\right) + \left(s_i^c - \kappa\right)x_i(t)\right)}^{\text{fictitious modeling uncertainty}}$$

$v_i = Kx_i$ - virtual decoupled cooperation signal

κ - positive real scalar to be designed

Main: Preliminary Steps

Fact 1:

There is a positive definite matrix $\Delta=\mathrm{diag}\{\delta_i\}$ such that $\Delta\mathcal{H}_c + \mathcal{H}_c^T\Delta \succcurlyeq \mathbf{0}$ where $\delta_i = \frac{\delta_i^n}{\delta_i^d} > 0$ with $col\{\delta_i^n\} = (\mathcal{H}_c^{-1})^T\mathbf{1}_N$ and $col\{\delta_i^d\} = \mathcal{H}_c^{-1}\mathbf{1}_N$.

Main: Fixed Gain Cooperation Protocol

- Consider the following networked nominal dynamics:

$$\dot{x}_i{}' = A x_i{}' + \kappa B v_i{}'$$

$\kappa > 0$: A design scalar.

- Let $W_x, W_v \succ \mathbf{0}$ be two design weighting matrices (arbitrary).

Design Procedure:

- Choose a non-symmetric control topology with $\mathcal{H}_c$ as its modified Laplacian matrix. Choose $\kappa > 0$ such that $\Delta\mathcal{H}_c + \mathcal{H}_c^T\Delta \succcurlyeq 2\kappa\Delta$.
- Find the solution $v_i{}' = K x_i{}'$ of the following LQR problem. Then, K is a candidate stabilization gain to be used in the cooperation protocol.

$$V\big(x'{}_i(0)\big) = \min_{v_i' \in \mathcal{U}_{vi}} \int_0^\infty \big(x_i'^T W_x x_i' + v_i'^T W_v v_i{}'\big) d\tau$$
$$s.t. \qquad \dot{x}_i{}' = A x_i{}' + \kappa B v_i{}'$$

Main: Fixed Gain Cooperation Protocol

The stabilization gain is $K = -\kappa {W_v}^{-1} B^T P$ where P solves the Algebraic Riccati Equation (ARE): $\mathrm{A}^{\mathrm{T}} P + PA + W_x - \kappa^2 PB{W_v}^{-1} B^T P = \mathbf{0}$

(The cooperation protocol is given by $\boldsymbol{u_{ci}} = \boldsymbol{K}\left(\sum_{j \in \mathcal{N}_i^c} \boldsymbol{a_{ij}^c}\left(\boldsymbol{x_i} - \boldsymbol{x_j}\right) + \boldsymbol{s_i^c x_i}\right)$

The agent model is $\dot{x}_i = A\, x_i + Bu_i + BC_i \sum_{j \in \mathcal{N}_i^a} a_{ij}^a x_i$).

$W_x, W_v, \succ \mathbf{0}$ are two completely arbitrary design weighting matrices, so the tuning rules of thumb can be useful.

Main: Data-assisted Decoupling Protocol

- Proposed distributed adaptation rule:

$$\dot{\hat{\theta}}_i = \Gamma_i\left(R_i^T(t)B^TPx_i - \gamma_{Di}\left(D_i(t_{n_{Di}})\hat{\theta}_i(t) - \sum_{l=1}^{n_{D_i}} R_{gi}^T(t_l)B^T\Delta_{gi}(t_l)\right)\right)$$

Where $R_i = \sum_{j\in\mathcal{N}_i^a} a_{ij}^a(I_{n_u}\otimes x_j^T(t))$ and Γ_i, γ_{Di} are design parameters (p.d. matrices and positive scalars, resp.). $t_{n_{Di}}$ = time associated with the sample n_{Di} .

Integration of the agent dynamics yields

$$\Delta_{gi}(t) \coloneqq x_i(t) - x_i(t-\delta t_i) - s_{gi}(t) = BR_{gi}(t)\theta_i^*$$

Where $$s_{gi}(t) = Ax_{gi}(t) + Bu_{gi}(t)$$

$x_{gi}(t) = \int_{max(t-\delta t_i,0)}^{t} x_i(\tau)d\tau$ and $u_{gi}(t) = \int_{max(t-\delta t_i,0)}^{t} u_i(\tau)d\tau$.

Data collection matrices: $D_i(n_{Di}) = \sum_{l=1}^{n_{D_i}} R_{gi}^T(t_l)B^TBR_{gi}(t_l)$ are updated only at 'acceptable' excitation sample times defined as

$\lambda_{min}\left(D_i(n_{Di})\right) > \lambda_{min}\left(D_i(n_{Di}-1)\right)$

(The decoupling protocol is given by $\boldsymbol{u_{di}(t) = -\sum_{j\in\mathcal{N}_i^a} a_{ij}^a(I_{n_u}\otimes x_j^T(t))\,\hat{\theta}_i(t)}$)

Main: Data-assisted Decoupling Protocol

- Assumption 1

For each agent $i \in \mathcal{S}_a$, there exists a finite n_{Di} such that the collective FE condition in Definition 1 is gradually satisfied over a finite time interval $[t_{si}^{start}, t_{si}]$.

- Properties
  - The data collection matrix $D_i(n_{Di}) = \sum_{l=1}^{n_{D_i}} R_{gi}^T(t_l) B^T B R_{gi}(t_l)$ satisfies, for each $i \in \mathcal{S}_a$:
    - 1) $D_i(n_{Di}) \succcurlyeq \mathbf{0}$ for $\forall t \geq 0$,
    - 2) $D_i(n_{Di}) \succ \mathbf{0}$ for all $t \geq t_{si}$ where $t_{si} > \delta t_i$
    - 3) $\lambda_{min}\big(D_i(n_i')\big) > \lambda_{min}\big(D_i(n_{Di})\big)$ using each new sample $n_i' > n_{Di}$

Main: Data-assisted Decoupling Protocol

Aggregated cooperation signal $u_c = \left(\mathcal{H}_c \otimes K\right) x$

leads to the representation of a 3-layer closed loop interconnected MAS:

$$\dot{x} = \bar{A}x + \kappa \bar{B}v + \kappa \bar{B}\bar{E}_c v + \bar{B}u_d + \bar{B}\bar{C}\bar{\mathcal{A}}_a x$$

Where

$$\bar{E}_c = \left(\left(\frac{\mathcal{H}_c}{\kappa} - I_N\right) \otimes I_{n_u}\right), \quad \bar{\mathcal{A}}_a = \mathcal{A}_a \otimes I_{n_x}, \quad \bar{C} = diag\left\{C_i\right\}$$

We define

$$\Omega_b(t) = x^T(t)\bar{P}x(t) + \frac{1}{\lambda_{\min}(\Gamma)}\left\|\tilde{\theta}(t)\right\|^2$$

$$\rho_\omega = \min\left(\frac{\lambda_{\min}(W_{xK})}{\lambda_{\max}(P)}, 2\gamma_D^{\min}\lambda_{\min}(\Gamma)\lambda_{\min}\left(D(n_D)\right)\right)$$

Where $W_{xK} = W_x + K^T W_V K \succ 0, \quad \gamma_D^{\min} = \min_i\left\{\gamma_{D_i}\right\}, \quad \forall i \in \mathcal{S}_a$

Main: Data-assisted Decoupling Protocol

- **Theorem 1:** In the proposed multilayer interconnected MAS under the data assisted stabilization protocol, the following is guaranteed:
  1. All trajectories $x_i(t)$ and $\tilde{\theta}_i(t)$ are bounded $\forall t \geq 0$,
  2. All trajectories $x_i(t)$ and $\tilde{\theta}_i(t)$ exponentially converge to the origin for $t \geq t_s = \max_i \{t_{si}\}$ where t_{si} defined in Assumption 1,
  3. The upper bounds on all trajectories are given by:

$$\|x_i(t)\| \leq \begin{cases} \sqrt{\frac{1}{\lambda_{min}(P)}\Omega_b(0)}, & \forall 0 \leq t < t_s \\ \sqrt{\frac{\exp^{-\rho_\omega(t-t_s)}}{\lambda_{min}(P)}\Omega_b(t_s)}, & \forall t \geq t_s \end{cases}$$

$$\|\tilde{\theta}_i(t)\| \leq \begin{cases} \sqrt{\lambda_{max}(\Gamma)\Omega_b(0)}, & \forall 0 \leq t < t_s \\ \sqrt{\lambda_{max}(\Gamma)\exp^{-\rho_\omega(t-t_s)}\Omega_b(t_s)}, & \forall t \geq t_s \end{cases}$$

- Proof makes use of the Lyapunov function

$$\Omega\left(x,\tilde{\theta}\right)=\bar{V}(x)+\sum_{i\in S_a}\tilde{\theta}_i^T\Gamma^{-1}\tilde{\theta}_i>0,\quad \bar{V}(x)=x^T\bar{P}x$$

leading to

$$\dot{\Omega}\leq -x^TW_{xK}x-2\sum_{i\in S_a}\gamma_{Di}\tilde{\theta}_i^TD_i\left(t_{n_{Di}}\right)\tilde{\theta}_i\leq 0$$

Hence, all states and parameter estimates are bounded at all times, even when cFE is not satisfied.

- When $t \geq t_s$

$$\dot{\Omega} \leq -\frac{\lambda_{\min}(W_{xK})}{\lambda_{\max}(P)} V(x) - 2\alpha \sum_{i \in S_a} \tilde{\theta}_i^T \Gamma^{-1} \tilde{\theta}_i$$

Where $\alpha = \gamma_D^{\min} \lambda_{\min}(\Gamma) \lambda_{\min}\big(D(n_D)\big) > 0.$

Using $\rho_\omega = \min\left(\frac{\lambda_{\min}(W_{xK})}{\lambda_{\max}(P)}, 2\gamma_D^{\min} \lambda_{\min}(\Gamma) \lambda_{\min}\big(D(n_D)\big) \right)$

We find $\dot{\Omega} \leq -\rho_\omega \Omega$ thus

$$\Omega(x(t), \tilde{\theta}(t)) \leq e^{-\rho_\omega (t - t_s)} \Omega(x(t_s), \tilde{\theta}(t_s))$$

- Over $0 \leq t \quad t_s$ we find

$$\|x(t)\| \leq \sqrt{\frac{1}{\lambda_{min}(P)}\left(x^T(0)\bar{P}x(0) + \frac{\|\tilde{\theta}(0)\|^2}{\lambda_{min}(\Gamma)}\right)}$$

$$\|\tilde{\theta}(t)\| \leq \sqrt{\lambda_{max}(\Gamma)\left(x^T(0)\bar{P}x(0) + \frac{\|\tilde{\theta}(0)\|^2}{\lambda_{min}(\Gamma)}\right)}$$

- And for $t \geq t_s$:

$$\|x(t)\| \leq \sqrt{\frac{\exp^{-\rho_\omega(t-t_s)}}{\lambda_{min}(P)}\left(x^T(t_s)\bar{P}x(t_s) + \frac{\|\tilde{\theta}(t_s)\|^2}{\lambda_{min}(\Gamma)}\right)}$$

$$\|\tilde{\theta}(t)\| \leq \sqrt{\lambda_{max}(\Gamma)\exp^{-\rho_\omega(t-t_s)}\left(x^T(t_s)\bar{P}x(t_s) + \frac{\|\tilde{\theta}(t_s)\|^2}{\lambda_{min}(\Gamma)}\right)}$$

(detailed proof in the paper)

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Summary

- We have proposed a mixed graph, optimal, and adaptive control theoretic formulation.
- We build a two-layer control protocol for the distributed stabilization and data-assisted adaptive decoupling of interconnected multiagent systems.
- The proposed approach captures the architectural aspect of cyber-physical systems with separate agent (physical) and control (cyber) layers.
- Compared to our previous publications, the cooperation layer topology can be chosen nonsymmetric and completely independent of the agent- and data-assisted decoupling layers.
- A new collective finite excitation condition relaxes the PE condition.
- Exponential stability of all state trajectories and exponential convergence of all interconnection matrices estimates are established.

Thank you