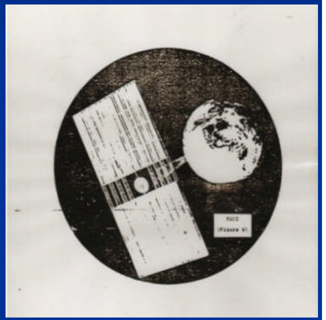
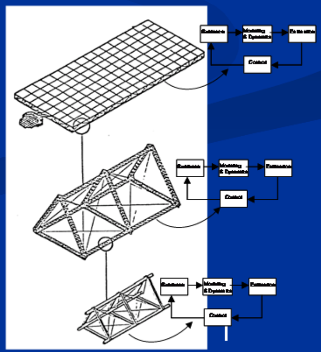
Reduction of Decoherence in Quantum Information Systems Using Direct Adaptive Control of Infinite Dimensional Systems

Mark's Autonomous Control Laboratory

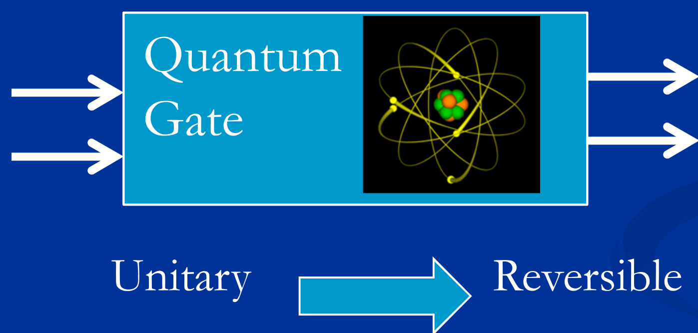
Mark J. Balas
Professor
of
Dynamics and Control Systems
Mechanical Engineering Department
Texas A&M University
College Station, Texas, USA
mbalas@tamu.edu





Motivation: Quantum Computing

"A Quantum computer will operate differently from a Classical one. It will be involved w physical systems on an atomic scale, eg atoms, photons, trapped ions, or nuclear magnetic moments"
… R. Feynman 40 years ago

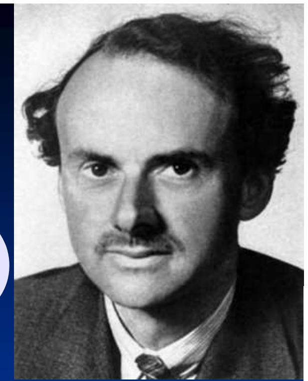


Decoherence is the loss of information from a system into the environment. Entanglements are generated between the system and environment, which have the effect of sharing quantum information with—or transferring it to—the surroundings

Reduced with Infinite Dimensional Direct Adaptive Control
(And Quantum Error Correction)

What really happens

Quantum Basics (Dirac & Von Neumann)



Observable $A: X \xrightarrow[\text{compact}]{\substack{\text{bounded / unbounded} \\ \text{self-adjoint}}} X$

Orthonormal Eigen –Basis for X

$$Ax = \sum_{k=1}^{\infty} \lambda_k \underbrace{(x, \phi_k)\phi_k}_{P_k x} = \sum_{k=1}^{\infty} \lambda_k P_k x \;\&\; \sigma(A) \equiv \left\{ \underbrace{\lambda_1, \lambda_2, \lambda_3, \ldots}_{\text{Observed Values of } A} \right\}$$

Pure States: ϕ_k eigenfunctions of A

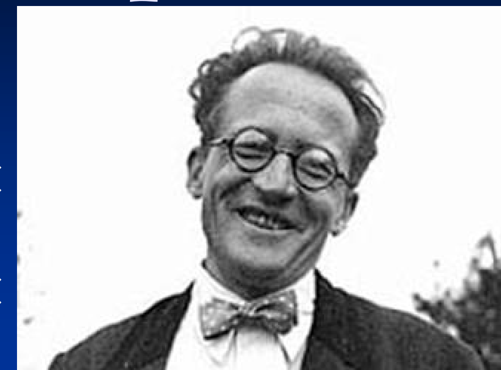
State $\phi \in X$ complex infinite-dimensional separable Hilbert Space:

$$(\phi, \phi) = 1 \text{ or } \|\phi\| = 1 \Rightarrow \phi = \sum_{k=1}^{\infty} c_k \phi_k \;\&\; \|\phi\|^2 = \sum_{k=1}^{\infty} |c_k|^2 = 1$$

$\therefore$ "A (mixed) state is a linear combination of pure states"

Special Case: Quantum SPIN Systems are FINITE Dimensional

Dynamics: Schrodinger Wave Equation



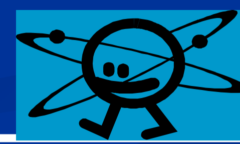
$\phi \in X$ complex Hilbert Space

$$i\hbar \frac{\partial \phi}{\partial t} = \underbrace{H_0}_{\text{Hamiltonian Energy Operator}} \phi \qquad \text{Discrete Spectrum } \sigma(H_0) = \{\lambda_k\}_{k=1}^{\infty}$$

$$\Rightarrow \phi(t) = \underbrace{U_0(t)}_{\text{Unitary Group}} \phi(0) = e^{-\frac{i}{\hbar}H_0 t}\phi(0) = \sum_{k=1}^{\infty} e^{-\frac{i\lambda_k}{\hbar}t}(\phi(0),\phi_k)\phi_k \text{ with } (\phi_k,\phi_l) = \delta_{kl}$$

$\therefore \|\phi(t)\|^2$ = Probability Distribution for the Energy

in the Quantum State ϕ(t) $\Rightarrow \|\phi(t)\| = \|\phi(0)\|$



$-\infty$

Marginally Stable

$\Rightarrow \therefore \|\phi(t)\|^2$ = Probability Distribution for the Energy

in the Quantum State ϕ(t)

$\Rightarrow \|\phi(t)\| = \|\phi(0)\|$

Quantum Measurement

The interpretation of Quantum Measurement is still a controversial part of Quantum Theory

Entanglement

$$X = X_S \otimes X_M$$

$$\phi = \sum_{k,l} \alpha_{kl} (\phi_k^S \otimes \phi_l^M) \neq h \otimes w$$

A quantum measurement is an entanglement with the environment (measuring device)

S

M

Back Action

"Simplicity" via Infinite Dimensional Spaces

$$\begin{cases} \dfrac{\partial x}{\partial t} = Ax + Bu + \Gamma u_D ; A \text{ generates a } C_0 - \text{semigroup } U(t) \\ Bu = \sum_{i=1}^{M} b_i u_i \\ x(0) = x_0 \in D(A) \subset X \\ y = Cx = \left[(c_1, x) \quad (c_2, x) \quad \ldots \quad (c_m, x) \right]^* ; b_i, c_j \in D(A) \end{cases}$$

$$\Rightarrow x(t, w_0) = \underbrace{U(t) x_0}_{\substack{Evolution \\ in\ X}} ; \forall t \geq 0$$

C_0 – Semigroup of Bounded Operators $U(t)$:

$$\begin{cases} U(t+s) = U(t)U(s) \ \text{(semigroup property)} \\ \dfrac{d}{dt} U(t) = AU(t) = U(t)A \ (A \text{ generates } U(t)) \\ U(t)x_0 \xrightarrow[t \to 0]{} x_0 \ (\text{continuous at t} = 0) \end{cases}$$

J. Wen & M.Balas, "Robust Adaptive Control in Hilbert Space", J. Mathematical. Analysis and Applications, Vol 143, pp 1-26,1989.

J. Wen & M.Balas ,"Direct Model Reference Adaptive Control in Infinite-Dimensional Hilbert Space," Chapter in Applications of Adaptive Control Theory, Vol.11, K. S. Narendra, Ed., Academic Press, 1987

Semigroups

Closed Linear Operator

$$\text{Solve}\begin{cases}\frac{\partial x}{\partial t} = Ax \\ x(0) = x_0 \in D(A)\end{cases} \Rightarrow x(t) = U(t)x_0$$

$$\dim X < \infty \Rightarrow U(t) = e^{At} \equiv \sum_{k=0}^{\infty} A^k \frac{t^k}{k!}$$

$\underline{C_0 - \text{Semigroup}}$

$U(t): X \to X$ bounded operators $t \geq 0$

Generator: $Ax = \lim_{t\to 0+} \frac{U(t)x - x}{t}$ with $D(A) \equiv \{x / \lim_{t\to 0+} \text{ exists }\}$ dense in X

$$\text{LaPlace Transform}\begin{cases} L(U(t)) = (\lambda I - A)^{-1} \equiv R(\lambda, A) \ \underline{\text{Resolvent Operator}} \\ L^{-1}(R(\lambda, A)) = U(t)\end{cases}$$

Spectrum of A

Resolvent Set $\rho(A) \equiv \left\{ \lambda \,/\, R(\lambda, A): X \to X \text{ bounded linear op on } X \right\}$

Spectrum $\sigma(A) \equiv \rho(A)^{\mathrm{C}} = \sigma_{\text{point}}(A) \cup \sigma_{cont}(A) \cup \sigma_{residual}(A)$

$\sigma_{point}(A) \equiv \left\{ \lambda \,/\, \lambda I - A \text{ is NOT 1-1} \right\} = \left\{ \lambda \,/\, \exists \phi \neq 0 \ni \lambda\phi = A\phi \right\}$

$\sigma_{cont}(A) \equiv \left\{ \lambda \,/\, \lambda I - A \text{ is 1-1, but its range is only dense in } X \right\}$

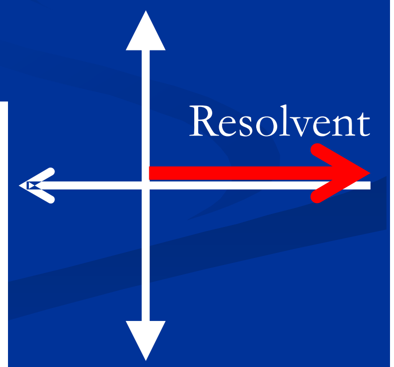
$\sigma_{residual}(A) \equiv \left\{ \lambda \,/\, \lambda I - A \text{ is 1-1, but range is a proper subspace of } X \right\}$

Theorem (Gearhart,Pruss,&Greiner):

Assume A generates a C_0-semigp $U(t)$ on a <u>Hilbert</u> space X.

$U(t)$ is exponentially stable $\Leftrightarrow$ $\mathrm{Re}\lambda > 0 \Rightarrow \lambda \in \rho(A)$ and

$\|R(\lambda, A)\| \le M < \infty$, for <u>all</u> such complex λ



When is a Semigroup Exponentially Stable ?

Lumer–Phillips (Renardy & Rogers 1993):

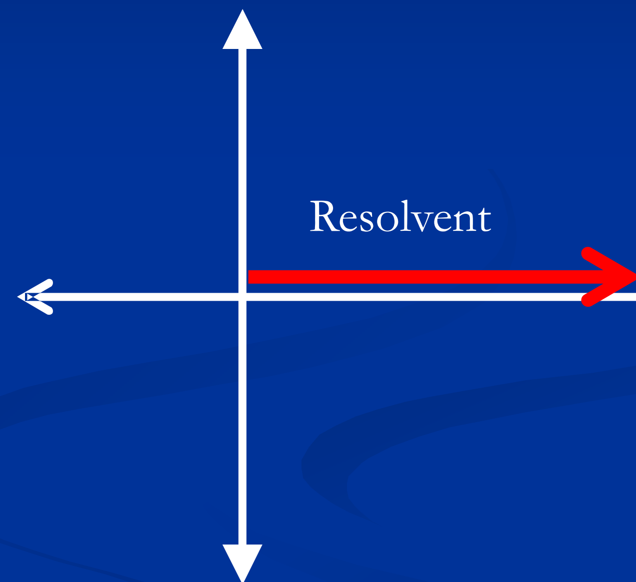
$D(A)$ dense in complex Hilbert space X,

$\mathrm{Re}(Ax, x) \le -\alpha \|x\|^2 \; \forall x \in D(A), \alpha > 0$

and

$\exists$ real $\lambda_* > -\alpha \ni A - \lambda_* I$ is onto

$\Rightarrow \|U(t)\| \le e^{-\alpha t}; t \ge 0$ (exponentially stable semigroup)



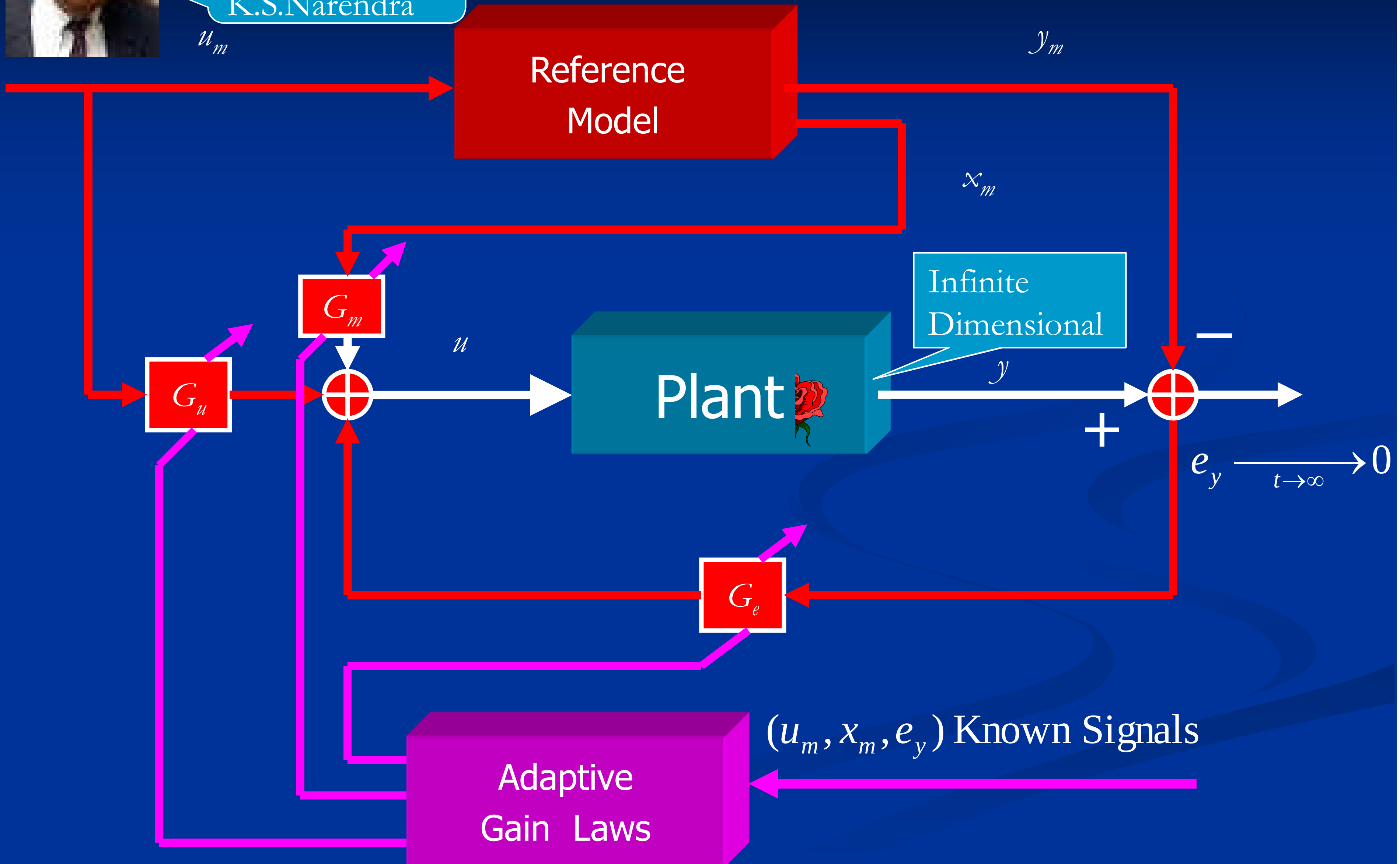
Theorem (Gearhart, Pruss, & Greiner) :

Assume A generates a C_0 - semigp $U(t)$ on a <u>Hilbert</u> space X.

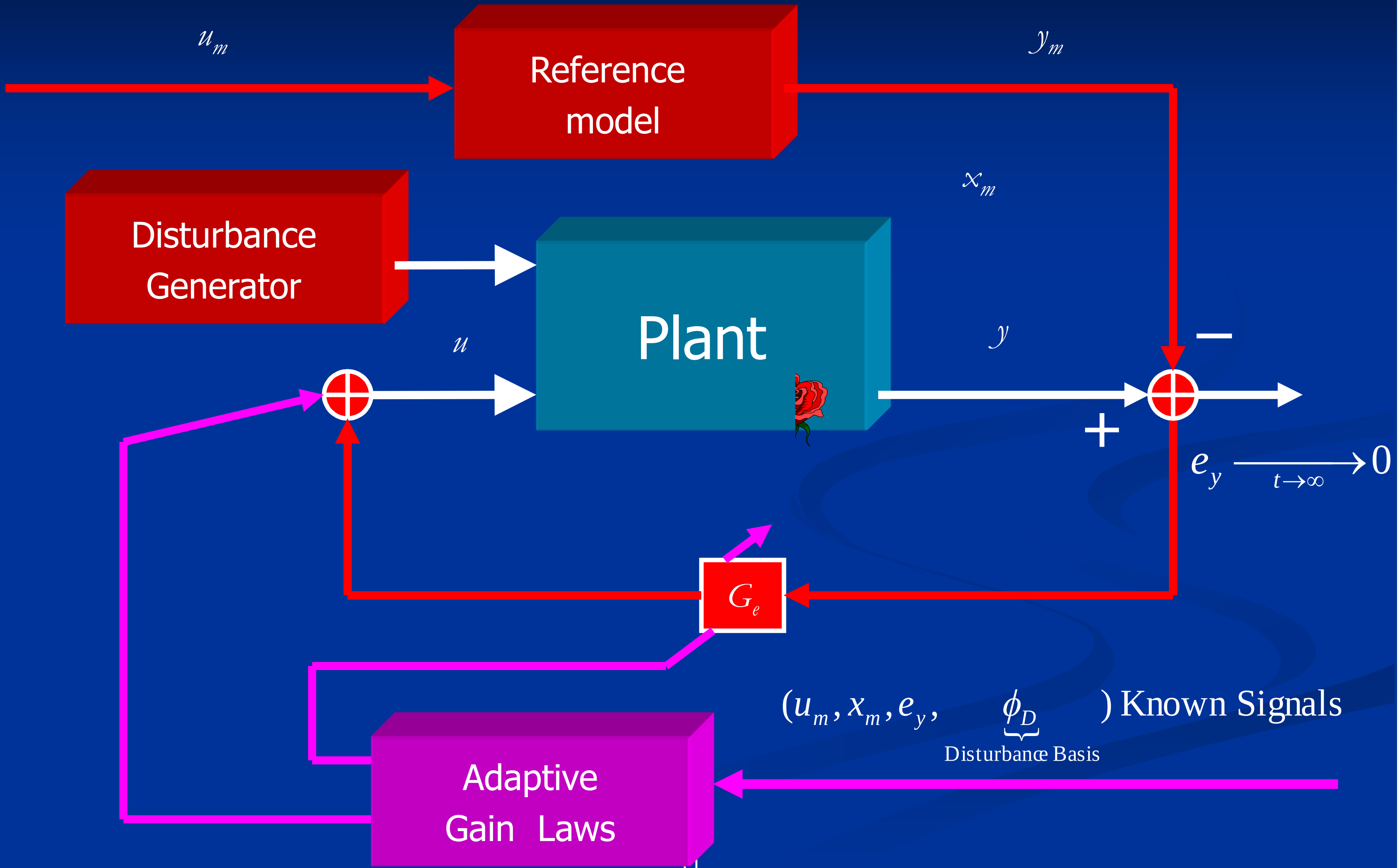
$U(t)$ is exponentially stable $\Leftrightarrow \mathrm{Re}\lambda > 0 \Rightarrow \lambda \in \rho(A)$ and

$\|R(\lambda, A)\| \le M < \infty$, for <u>all</u> such complex λ

Direct Adaptive Model Following Control (Wen-Balas 1989)

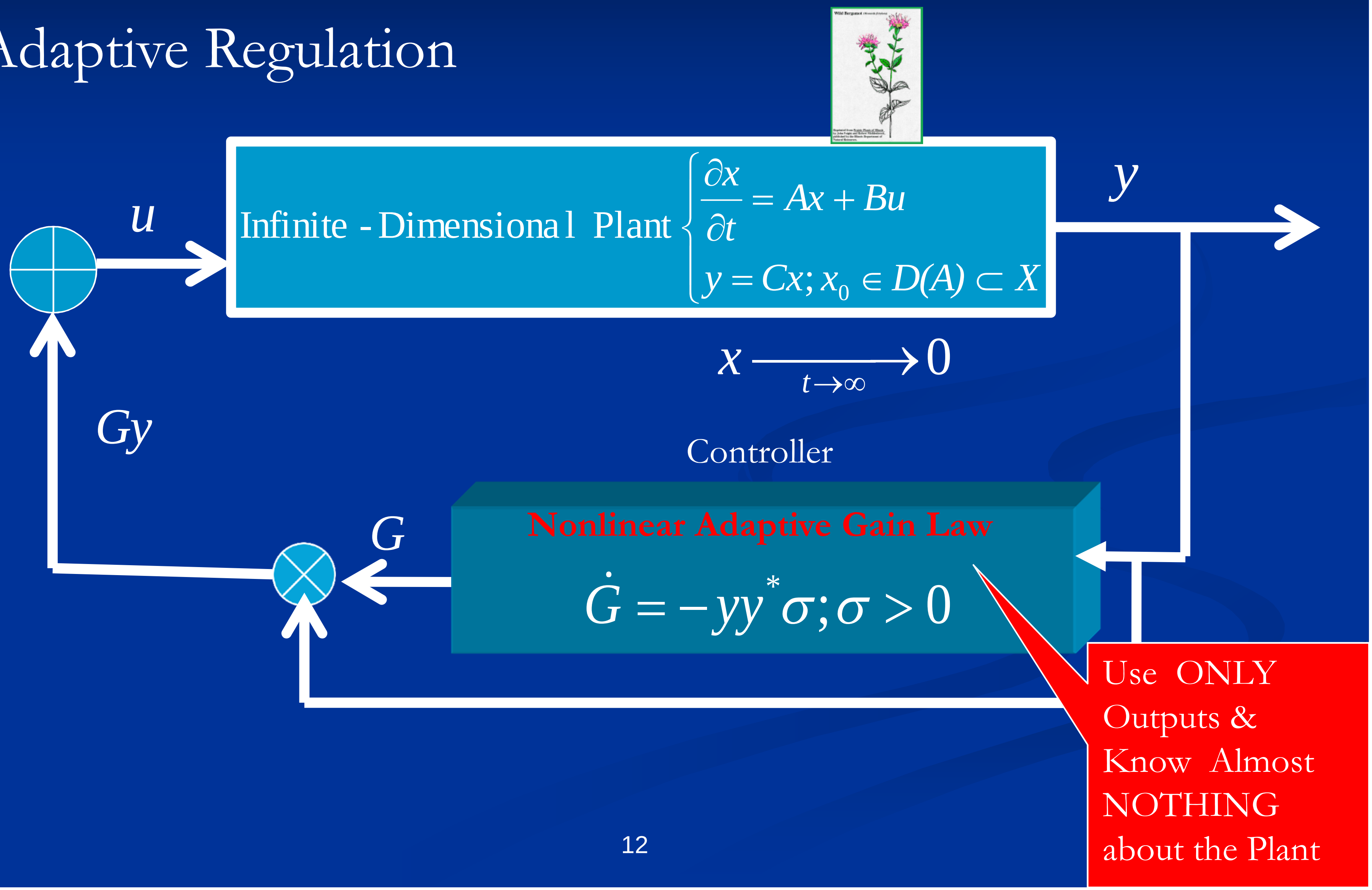


Direct Adaptive Persistent Disturbance Rejection (Fuentes-Balas 2000)



Direct Adaptive Control Theory Is Not Complicated !

Adaptive Regulation



For Finite & Infinite Dimensions
All Roads Lead To Rome

$$\begin{cases} \dfrac{\partial x}{\partial t} = Ax + Bu = Ax + \sum_{i=1}^{m} b_i u_i \\ x(0) = x_0 \in D(A) \subset X \\ y = Cx = \left[(c_1, x) \quad (c_2, x) \quad \ldots \quad (c_m, x) \right]^T \end{cases}$$

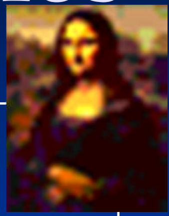
with (A, B, C) Almost Strictly Dissipative (ASD)

$$\Rightarrow \text{Adaptive Controller} \begin{cases} u = G(t)y \\ \dot{G}(t) = -yy^{*}\sigma; \sigma > 0 \end{cases}$$

produces $x(t) \xrightarrow[t \to \infty]{} 0$

with bounded adaptive gains $G(t)$

Finite- Dimensional LINEAR ASD: Two Simple Open-Loop Properties



High Frequency Gain is Sign-Definite (CB>0)

Open-Loop Transfer Function is Minimum Phase (i.e. Transmission Zeros are all stable)

⟺ Almost Strictly Dissipative

⟹ Adaptive Regulation $\begin{cases} u = Gy \\ \dot{G} = -yy^{*}\sigma; \sigma > 0 \end{cases}$

produces $x(t) \xrightarrow[t\to\infty]{} 0$

with bounded adaptive gains $G(t)$

Our Infinite-Dimensional Version of the "Two Simple Open Loop Properties" Theorem

$$
\begin{cases}
\dfrac{\partial x}{\partial t} = Ax + Bu = Ax + \sum_{i=1}^{m} b_i u_i ; A \text{ generates a } C_0 \text{ semigroup} \\
x(0) = x_0 \in D(A) \subset X \\
y = Cx = \left[(c_1, x) \quad (c_2, x) \quad \ldots \quad (c_m, x) \right]^* ; b_i, c_j \in D(A)
\end{cases}
$$

<u>*Theorem*</u>: Def : $\lambda_* \in C$ is a <u>transmission zero</u> of (A, B, C) when $N(H(\lambda_*)) \neq \{0\}$

where $H(\lambda) \equiv \begin{bmatrix} A - \lambda I & B \\ C & 0 \end{bmatrix} : D(A) x \Re^M \rightarrow X x \Re^M$ closed linear operator

(A, B, C) is Almost Strictly Dissipative <u>if and only if</u>

$CB = \left[(c_j, b_i) \right]_{\text{mxm}} > 0$ and Transmission Zeros$(A,B,C) \equiv \{ \lambda / N(H(\lambda)) \neq \{0\} \} = \sigma_p(\overline{A}_{22})$ "*stable*"

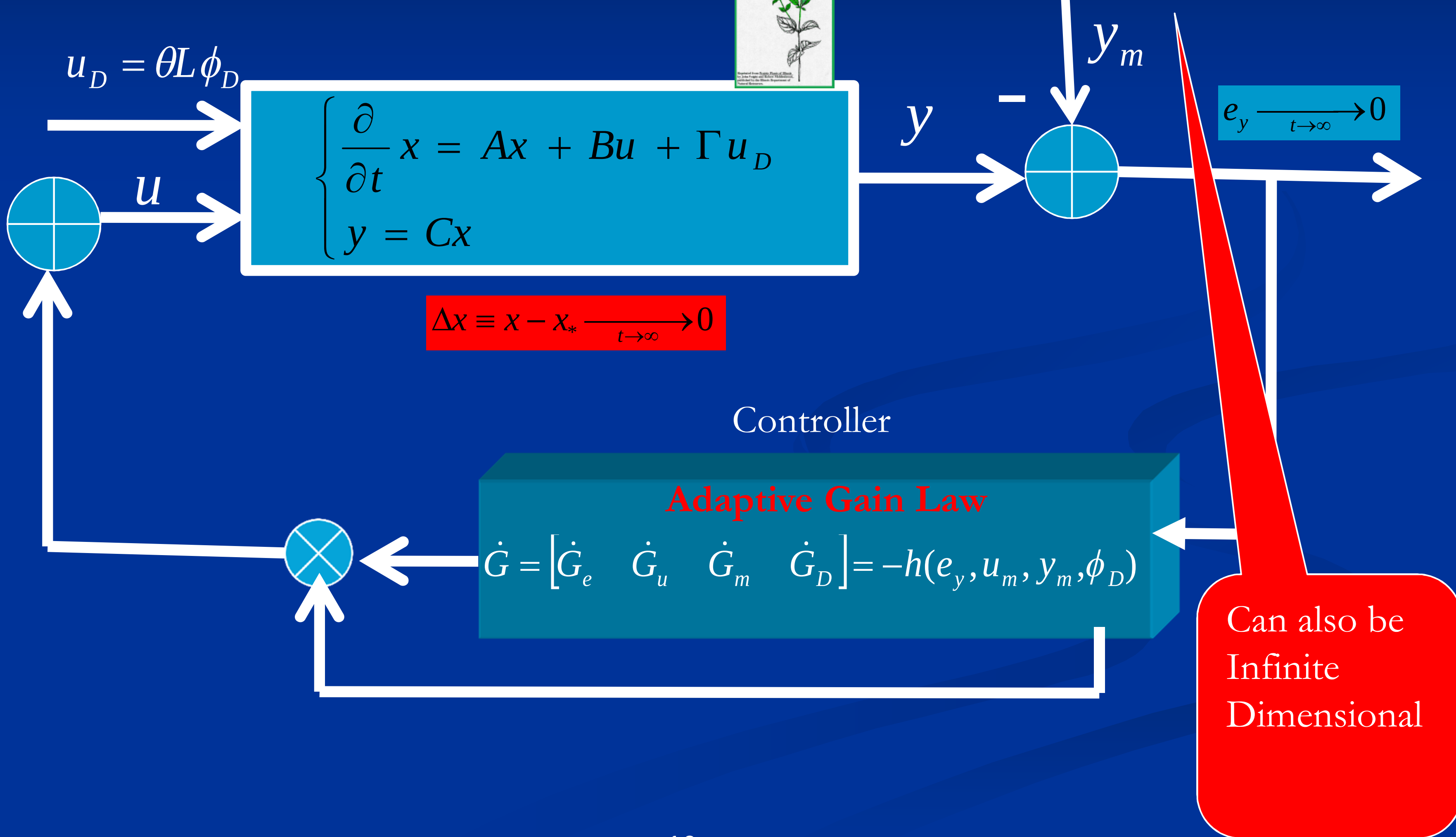
(i.e., $\overline{A}_{22}$ generates exponentially stable semigroup)

Mark Balas and Susan Frost, "Robust Adaptive Model Tracking for Distributed Parameter Control of Linear Infinite-dimensional Systems in Hilbert Space",IEEE/CAA JOURNAL OF AUTOMATICA SINICA, VOL. 1, NO. 3, JULY 2014.

Adaptive Model Tracking in the Presence of Persistent Disturbances

Reference Model

$$\begin{cases} \dot{x}_m = A_m x_m + B_m u_m \\ y_m = C_m x_m \end{cases}$$



Adaptive Control Law

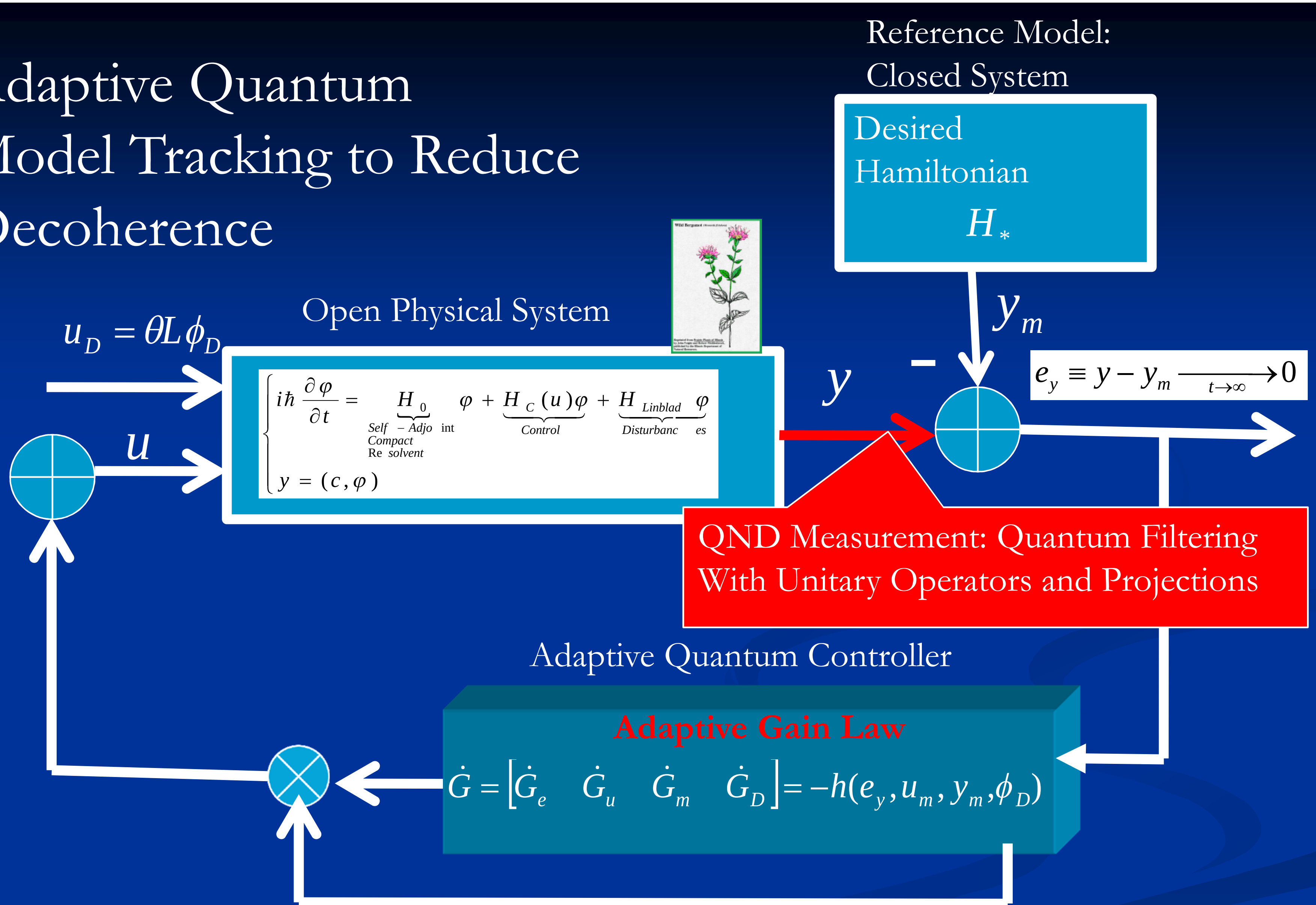
$$u = \underbrace{G_u u_m + G_m x_m}_{\text{Model Tracking}} + \underbrace{G_D \phi_D}_{\text{Disturbance Rejection}} + \underbrace{G_e e_y}_{\text{Stabilization}}$$

where

$$\begin{cases} \dot{G}_u = -e_y u_m^* \sigma_u; \sigma_u > 0 \\ \dot{G}_m = -e_y x_m^* \sigma_m; \sigma_m > 0 \\ \dot{G}_D = -e_y \phi_D^* \sigma_D; \sigma_D > 0 \\ \dot{G}_e = -e_y e_y^* \sigma_e; \sigma_e > 0 \end{cases}$$

Gain Adaptation Laws

Adaptive Quantum Model Tracking to Reduce Decoherence



Schrodinger Equation Control

$$i\hbar \frac{\partial \phi}{\partial t} = \underbrace{H_0}_{\substack{\text{Self-Adjoint} \\ \text{Energy Hamiltonian}}} \phi + \underbrace{H_C}_{\substack{\text{Feedback Control} \\ \text{Hamiltonian}}} \phi$$

Time-Varying Hamiltonian

$$\begin{cases} \dfrac{\partial \varphi}{\partial t} = A\varphi + Bu = A\varphi + \sum_{i=1}^{m} b_i u_i ;\ A \equiv -\dfrac{i}{\hbar} H_0 \\ y = C\varphi = \left[(c_1,\varphi) \quad (c_2,\varphi) \quad \ldots \quad (c_m,\varphi) \right]^* \end{cases}$$

Adaptive Controller: $u = G(t)y \;\&\; u_i = \sum_{j=1}^{m} g_{ij}(t)(c_j, x)$

$$\Rightarrow \frac{\partial}{\partial t}\varphi = A\varphi + \sum_{i=1}^{m} b_i u_i = A\varphi + \underbrace{\sum_{i=1}^{m}\sum_{j=1}^{m} b_i g_{ij}(t)(c_j,\varphi)}_{BG(t)Cx}$$

$$\Rightarrow \varphi(t,x_0) = \underbrace{U(t)\varphi_0}_{\substack{\text{Evolution} \\ \text{in } X}} ;\forall t \geq 0 \text{ (Not Unitary)}$$

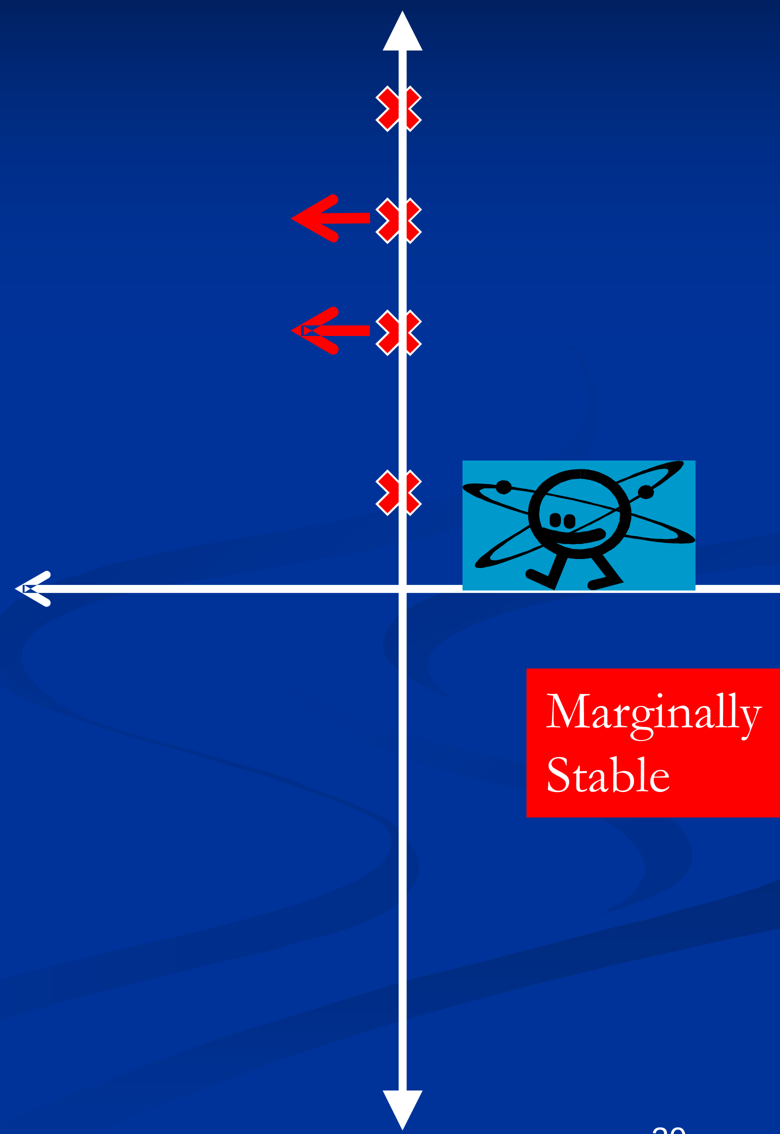
Quantum Adaptive Control Approach

$$\begin{cases} x_N \equiv P_N x \xrightarrow[t\to\infty]{} 0 & \text{exponentially} \\ \|x_R\| \equiv \|P_R x\| = \|P_R x_0\| & \text{constant} \end{cases}$$

$$\text{with} \begin{cases} P_N x \equiv \sum_{k=1}^{N} (\phi_k, x)\phi_k \\ P_R x \equiv \sum_{k=N+1}^{\infty} (\phi_k, x)\phi_k \end{cases}$$

via robust adaptive control with bounded gains:

$$\begin{cases} H_C x \equiv i\ [b(G_e(b,x) + G_D\varphi_D)]; b \in P_N(X) \\ \dot{G}_e = -yy^* \gamma_e; \gamma_e > 0 \\ \dot{G}_D = -y\varphi_D{}^* \gamma_D; \gamma_D > 0 \end{cases}$$



Adaptive Control: Convergence to a Decoherence-Free Subspace 1

Let S be an N-dimensional A-invariant subspace

P_N orthogonal projection onto S & $P_R = I - P_N$ orthogonal projection onto $S^{\perp}$

$$\begin{cases} \dfrac{\partial P_N x}{\partial t} = P_N \dfrac{\partial x}{\partial t} = \underbrace{(P_N A P_N)}_{A_N} P_N x \\ \dfrac{\partial P_R x}{\partial t} = P_R \dfrac{\partial x}{\partial t} = \underbrace{(P_R A P_R)}_{A_R} P_R x + \underbrace{(P_R B)}_{B_R} u \\ y = \underbrace{(C P_R)}_{C_R} P_R x \end{cases} \Rightarrow \begin{cases} \dfrac{\partial x_N}{\partial t} = A_N x_N \\ \dfrac{\partial x_R}{\partial t} = A_R x_R + B_R u \\ y = C_R x_R \end{cases}$$

Adaptive Control: Convergence to a Decoherence-Free Subspace 2

Choosing actuators b_i & sensors $c_j \Rightarrow B_N = P_N B = 0 \,\&\, C_N = P_N C = 0$

$$\begin{cases} \dfrac{\partial P_N x}{\partial t} = P_N \dfrac{\partial x}{\partial t} = \underbrace{(P_N A P_N)}_{A_N} P_N x + \underbrace{(P_N A P_R)}_{A_{NR}=0} P_R x + \underbrace{(P_N B)}_{B_N} u \\ \dfrac{\partial P_R x}{\partial t} = P_R \dfrac{\partial x}{\partial t} = \underbrace{(P_R A P_N)}_{A_{RN}=0} P_N x + \underbrace{(P_R A P_R)}_{A_R} P_R x + \underbrace{(P_R B)}_{B_R} u \\ y = \underbrace{(C P_N)}_{C_N} P_N x + \underbrace{(C P_R)}_{C_R} P_R x \end{cases}$$

Adaptive Control: Convergence to a Decoherence-Free Subspace 3

Theorem: If (A_R, B_R, C_R) is ASD,i.e. $C_R B_R > 0$ & $C_R(sI - A_R)B_R$ minimum phase ,

then the Adaptive Controller:

$$\begin{cases} u = G_e y + G_D \varphi_D \\ \dot{G}_e = -yy^* \gamma_e \\ \dot{G}_D = -y\varphi_D{}^* \gamma_D \end{cases}$$

will produce $\|x_R\| = \|P_R x\| \xrightarrow[t\to\infty]{} 0$ (Convergence of the state x to the subspace S)

Choose S as a "Decoherence-Free Subspace" (see references [20]-[22])

which are finite dimensional Hamiltonian -Invariant (A-Invariant)

subspaces of the Schrodinger PDE

where all decoherence effects are removed,

i.e. the Schrodinger Evolution Operator is unitary

Direct Adaptive Control of Infinite Dimensional Linear Systems

- Can be used on a Quantum System to cause it to converge to a Decoherence-Free subspace
- This requires careful Selection of actuators and Sensors
- So decoherence in Quantum Computing Gates can potentially be reduced by direct adaptive control
- Implementation in simple quantum systems is not trivial and certainly remains to be developed.

Famous
Lisbon Poet

"No intelligent idea can gain general acceptance unless some stupidity is mixed in with it" …..

Fernando Pessoa, The Book of Disquiet