GRUPPO BANCA CARIGE

DATA ANALYTICS 2020 FinTech Risk Management

Seasonality Modeling through LSTM Network in Inflation-Indexed Swaps

Looking for a bridge between traditional standard pricing approach and the new FinTech techniques

Università di Genova

IARIA CONFERENCE
International Academy, Research and Industry Association

Pier Giuseppe Giribone

NICE 26th October 2020

TABLE OF CONTENTS

THE PRICING FRAMEWORK

An Inflation-Indexed Swap (IIS) is a swap deal in which, for each payment date, $T_1, \dots, T_M$, counterparty A pays to counterparty B the inflation rate in the considered period, while counterparty B pays to counterparty A the fixed rate. The inflation rate is calculated as the percentage return of the Consumer Price Index (CPI) over the reference time interval.

There are two main types of IIS traded on the market:

Zero-Coupon Inflation-Indexed Swap (ZCIIS)

In a ZCIIS, at maturity date T_M, assuming $T_M = M$ years, counterparty B pays to counterparty A the fixed quantity: $N\left[(1+K)^M - 1\right]$ where K and N are the fixed interest rate and the principal, respectively.

In return, at the maturity date T_M, counterparty A pays to counterparty B the floating amount: $N\left[\frac{I(T_M)}{I_0} - 1\right]$

Year-on-Year Inflation-Indexed Swap (YYIIS)

In a YYIIS, for each payment date T_i, counterparty B pays to counterparty A the fixed amount: $N\varphi_i K$ where φ_i is the year fraction of the fixed swap leg in the range $[T_{i-1}, T_i]$, $T_0 \coloneqq 0$ and N is the principal of the deal.

Counterparty A pays to counterparty B the floating amount equals to: $N\varphi_i\left[\frac{I(T_i)}{I(T_{i-1})} - 1\right]$

THE PRICING FRAMEWORK

ZCIIS and YYIIS are typically quoted in terms of the corresponding equivalent fixed rate K

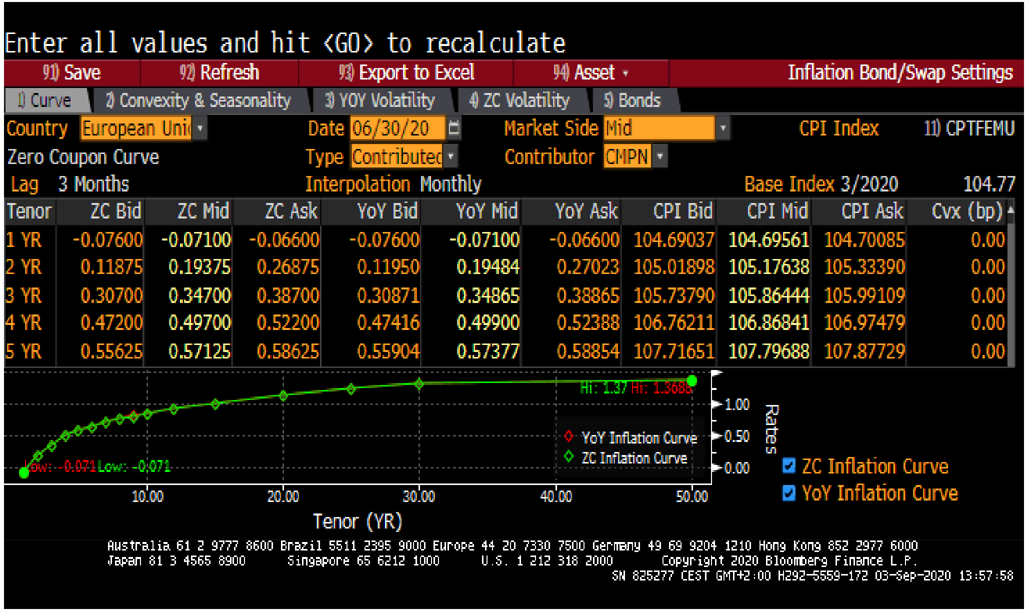
Based on these quotes and using stochastic calculus, pricing formulas can be derived for both classes of derivatives.

Kazziha derived the CPI forward values, $\Im_i$:

$$\Im_M(0) = \Im_{REF}(0) \cdot [1 + K(T_M)]^M$$

$\Im_{REF}(0)$ is the CPI reference value. It corresponds to the one set n months back in relation to the settlement date. Typically, the standard time lag is 3 months.

$K(T_M)$ is the Inflation Zero Swap Rate quoted on the market in correspondence to T_M.



Tenor	ZC Bid	ZC Mid	ZC Ask	YoY Bid	YoY Mid	YoY Ask	CPI Bid	CPI Mid	CPI Ask	Cvx (bp)
1 YR	-0.07600	-0.07100	-0.06600	-0.07600	-0.07100	-0.06600	104.69037	104.69561	104.70085	0.00
2 YR	0.11875	0.19375	0.26875	0.11950	0.19484	0.27023	105.01898	105.17638	105.33390	0.00
3 YR	0.30700	0.34700	0.38700	0.30871	0.34865	0.38865	105.73790	105.86444	105.99109	0.00
4 YR	0.47200	0.49700	0.52200	0.47416	0.49900	0.52388	106.76211	106.86841	106.97479	0.00
5 YR	0.55625	0.57125	0.58625	0.55904	0.57377	0.58854	107.71651	107.79688	107.87729	0.00

Source: Bloomberg® Market Data: 30th June 2020

THE PRICING FRAMEWORK

We are able to project the index values in the future according to the swap rates listed on the market.

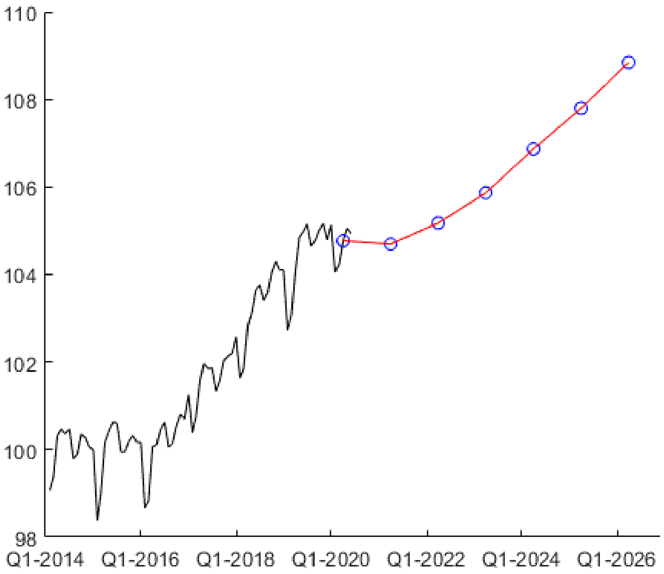
Since the frequency with which the index is published is monthly, it is necessary to provide a simulation of the CPI with such periodicity.

The missing curve points are therefore estimated by adding the logarithm of the monthly increase between a calculated value $\Im_M(0)$ and its subsequent value $\Im_{M+1}(0)$: $\Delta\Im_M = \dfrac{\ln\left(\frac{\Im_{M+1}(0)}{\Im_M(0)}\right)}{12\cdot\tau}$

where τ is the time interval expressed in year fraction between $\Im_M(0)$ and $\Im_{M+1}(0)$.

The points making up the simulated curve of the consumer price index are defined by the formula:

$$\Im_{i+1} = \Im_i \exp(\Delta\Im_M + \Re_M), \quad \Im_M(0) \leq \Im_i \leq \Im_{M+1}(0)$$



CPI Projection without Seasonality

THE PRICING FRAMEWORK

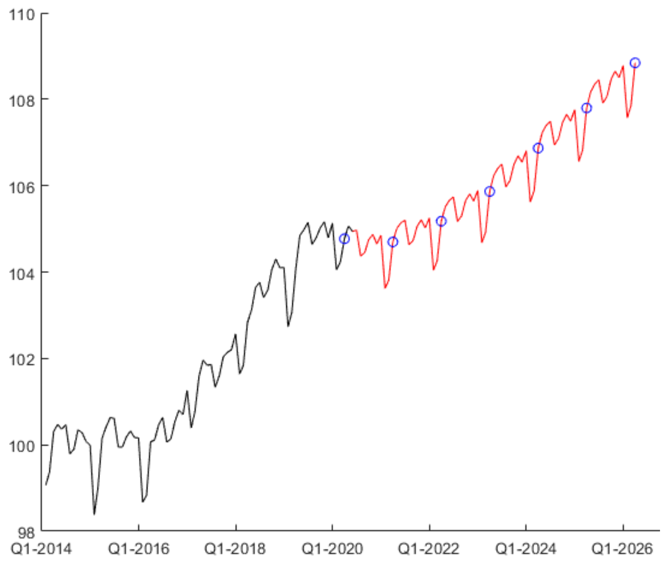
The standard methodology, suggested by the main benchmark info provider pricing modules, takes into account the index seasonality algebraically adding the normalized residuals $\Re_M$ obtained from the historical values of the CPI, in accordance with the expression:

$$\Re_M = \frac{\sum_{i=1}^{seasyear} \ln\left[\frac{\Im_{i+1}^{Monthly}}{\Im_{i}^{Monthly}}\right]}{seasyear} - \frac{\sum_{i=1}^{12 \cdot seasyear} \ln\left[\frac{\Im_{i+1}^{Monthly}}{\Im_{i}^{Monthly}}\right]}{12 \cdot seasyear}$$

where $\Re_M$ are the standardized residuals obtained from the effect of seasonality over $seasyear$ years.

The first contribution is the logarithmic variation of the CPI values on the considered month, the second one represents the overall logarithmic variation recorded in the time period considered for seasonality.

The idea is to use a Long Short-Term Memory network with the aim of providing a better model for the seasonality.

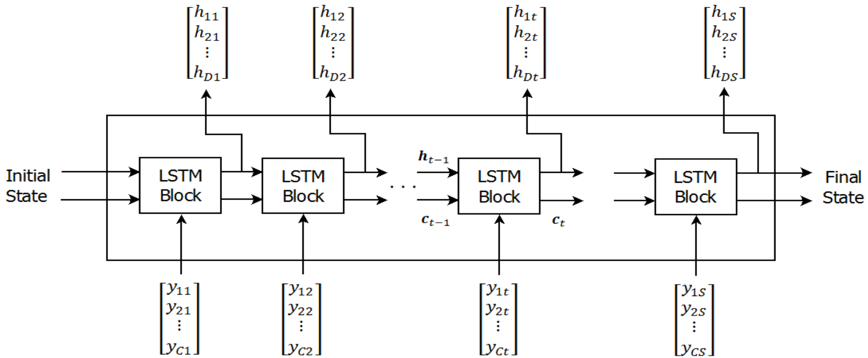


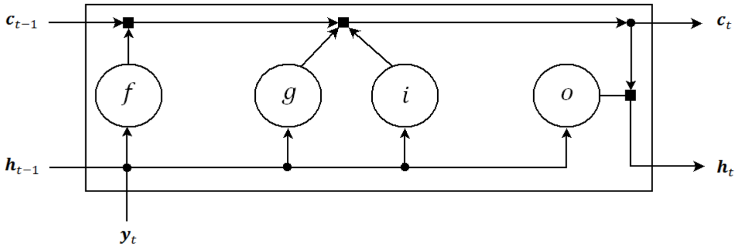
CPI Projection with Seasonality

SEASONALITY MODELING LSTM network

LSTM networks are able to learn long-term relationships between the time intervals of a time series, therefore without the need to pre-set the number of time lags.

A common LSTM unit is composed of a cell, an input gate and a forget gate. The cell remembers values over arbitrary time intervals and the three gates regulate the flow of information into and out of the cell. Intuitively, the cell is responsible for keeping track of the dependencies between the elements in the input sequence. The input gate controls the extent to which a new value flows into the cell, the forget gate controls the extent to which a value remains in the cell and the output gate controls the extent to which the value in the cell is used to compute the output activation of the LSTM unit.





LSTM network architecture and LSTM unit

LSTM TRAINING

LSTMs are supervised networks, as a result, after the design of the model, it is essential to implement a robust procedure for the training phase.

This is the part in which the designer decides how many neurons must be implemented in order to make reliable predictions.

Statistical tests

The objective of this kind of test is to tune the LSTM in order to have a good fitting of the training dataset. Special measures for avoiding data overfitting are taken into consideration:

- Random-splitting dataset method:
- Adding a term to the traditional loss function (RMSE) which put in a penalty (the λ coefficient) if a further weight (ω) associated to an arch has been activated: $J = RMSE + \frac{1}{2}\lambda\|\omega\|^2$.
- Dropout, which is a technique consisting of training only a group of randomly selected neurons rather than the entire network.

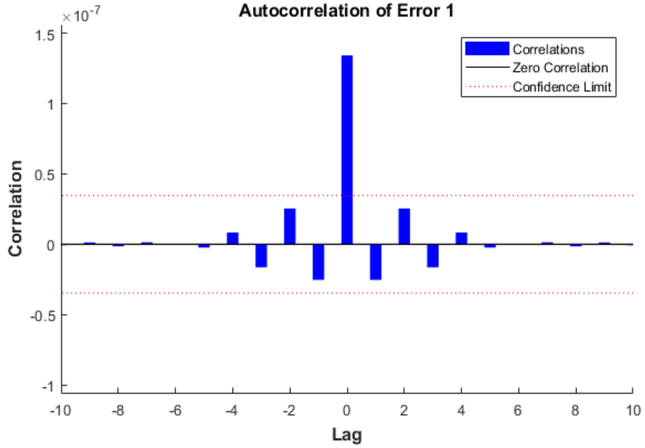
LSTM TRAINING

Econometric test

Given that the objective is to perform a prediction of the most reasonable CPI projections, the second test has an econometric nature.

It is based on the verification of the autocorrelation error absence so that the model error is unstructured and the predicted values can be econometrically reliable.

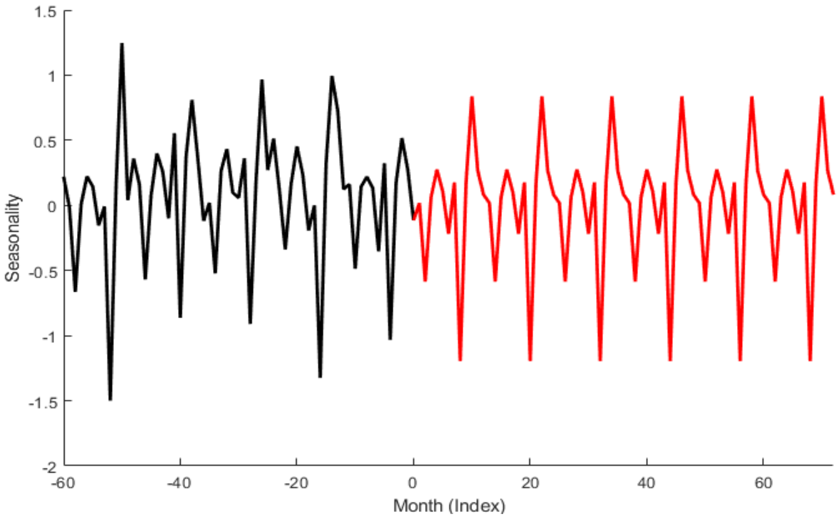
Figure shows that the auto-correlation error for the tuned model has been kept, with a confidence interval equal to 95%, under an acceptable threshold (represented in red dotted lines) for the non-zero lags.



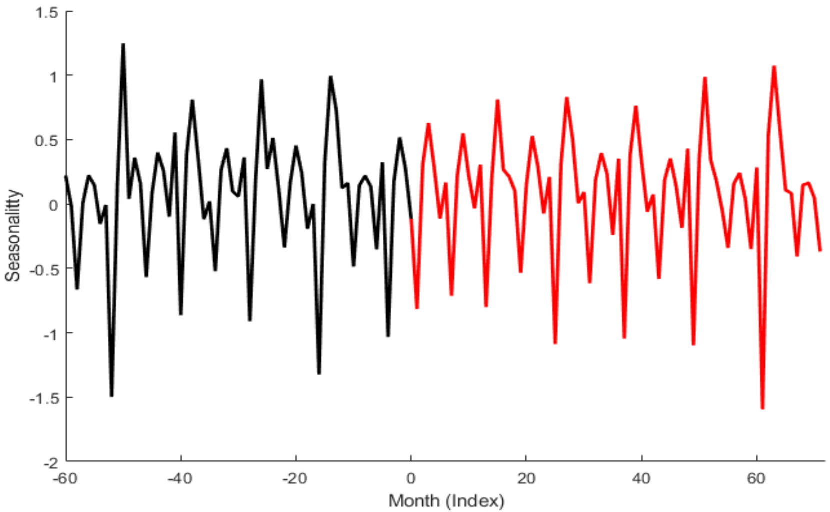
For the training set, we use the monthly return of the index computed in the last 5 years, according to the market standard convention. Adopting an ADAM optimizer and implementing all the described techniques in order to avoid overfitting, we can achieve excellent results in the training phase.

COMPARISON between TECHNIQUES

Having checked the forecasting reliability of the LSTM network, we proceed to compute the following 6 years returns (72 values). Figures show the difference between the two approaches: the black line represents the realized past returns of the last five years and the red line represents the forecasted returns.



Traditional approach



FinTech approach

COMPARISON between TECHNIQUES

It is sufficient to look at the figures to realize that the red line (i.e. the projected time-series) obtained from the traditional method has a behavior which is too simplified.

In fact, it is based on the estimation of the twelve normalized residuals of the previous months which are repeated equal for the future values.

Implementing a properly trained LSTM allows to use a model able to capture highly nonlinear relationship among the time-series in accordance with the rigorous statistical and econometric tests.

As a result, facing the forecasting problem with the FinTech approach, the red line has a more realistic forward-looking behavior thanks to both the advanced technology (deep learning) and the careful tuning.

As we will see in the market case, which regards the pricing of a YYIIS, these differences in the simulation of the seasonality cause an impact on the derivative fair-value that is not always negligible.

We proceed with the valorization of a YYIIS using the two approaches previously described.

YYIIS PRICING

The valuation date of the "In Arrears" swap is 30th June 2020, as a result we use the historical and prospective inflation data already computed.

Regarding the discount curve we use, according to the new benchmark standard for collateralized derivatives, the EUR OIS ESTR term structure.

As a result, zero rates and discount factors used for pricing are those implied from the new market benchmark curve.

Using the pricing framework described, we proceed with the estimation of the future cash-flows for the swap and then we go through the discounting process for obtaining the NPVs for the two legs.

The difference between the two NPVs gives the price of the swap.

	Receiving Leg	Paying Leg
Leg Type	Y-o-Y Inflation	Fixed
Notional	10 MM	10 MM
Currency	Euro	Euro
Index	CPTFEMU Index	Fixed Coupon: 0.5%
Effective Date	30th June 2020	30th June 2020
Maturity Date	30th June 2026	30th June 2026
Lag	3 Month	-
Interpolation	Monthly	-
Spread	0	-
Reset Frequency	Semi-Annual	-
Payment Freq.	Semi-Annual	Annual
Day Count	ACT/ACT	ACT/ACT
Discount Curve	EUR-OIS-ESTR	EUR-OIS-ESTR

YYIIS financial characteristics

YYIIS CASH FLOWS

Date	Reset CPI	Payment	Discount	PV
12/31/2020	104.74729	-2,185.66	1.002913	-2192.02
06/30/2021	104.69561	-4,894.44	1.006019	-4923.89
12/31/2021	105.06038	35,066.31	1.009331	35393.52
06/30/2022	105.17638	10,944.48	1.012638	11082.8
12/30/2022	105.6452	44,597.45	1.015884	45305.83
06/30/2023	105.86444	20,674.18	1.019137	21069.82
12/29/2023	106.49159	58,904.25	1.022122	60207.33
06/28/2024	106.86841	35,225.72	1.02504	36107.77
12/31/2024	107.45914	56,181.35	1.027729	57739.21
06/30/2025	107.79688	31,122.41	1.030223	32063.02
12/31/2025	108.44679	60,603.27	1.032507	62573.3
06/30/2026	108.84187	360,65.68	1.034607	37313.81

YYIIS Receiving Leg (standard approach)

Reset CPI	Payment	Discount	PV
104.80403	3274,34	1.002913	3283.87
104.69561	-10265.09	1.006019	-10326.88
104.75465	5683.47	1.009331	5736.51
105,17638	39847.66	1.012638	40351.25
105.33777	15375.28	1.015884	15619.50
105.86444	49737.38	1.019137	50689.20
106.18170	29841.55	1.022122	30501.70
106.86841	64287.66	1.02504	65897.43
107.14643	26480.24	1.027729	27214.51
107.79688	60025.26	1.030223	61839.40
108.13120	31220.98	1.032507	32235.88
108.63801	46376.07	1.034607	47981.01

YYIIS Receiving Leg (LSTM approach)

Payment Date	Cash Flow	Discount Rate	Present Value
06/30/2021	-49,930.76	1.006019	-50,231.30
06/30/2022	-50,000.00	1.012638	-50,631.89
06/30/2023	-50,000.00	1.019137	-50,956.87
06/30/2024	-49,796.02	1.025040	-51,042.90
06/30/2025	-50,203.98	1.030223	-51,721.29
06/30/2026	-50,000.00	1.034607	-51,730.37

YYIIS Paying Leg

YYIIS Rec. Leg: +391,740.5 € (standard)
YYIIS Rec. Leg: +371,023.4 € (FinTech)
YYIIS fixed Paying Leg: -306,314.62

It is interesting to highlight that both methodologies are consistent with the market. The gap between the values from the two pricing methodologies is equal to 85,425.87 – 64,708.77 = 20,717.1. The percentage error is higher than 20% compared to the Mark to Market of the analyzed derivative.

CONCLUSION & FUTURE DEVELOPMENTS

This study shows how a Deep Learning methodology can be usefully implemented in a pricing framework aimed at determining the fair value of derivatives linked to the inflation index.

The Long Short-Term Memory has allowed to identify the effect of seasonality more reliably than the traditional standard methodology. In fact, the proposed technique is able to simulate the future values of the time series by applying the described rigorous statistical and econometric tests, reasonably guaranteeing the reliability of the forecast.

On the contrary, the traditional approach, based on the estimation of the historical normalized residuals, does not consider these important tests and it is not able to capture highly nonlinear relationships as a LSTM network does.

It is particularly interesting considering how artificial intelligence paradigms can be integrated with traditional pricing methodologies in the quantitative finance field.

For the continuation of the study, it is interesting to apply the suggested technology to derivatives written on an underlying which differs from inflation, where the seasonality modeling is of fundamental importance, such as commodity and energy derivatives.

MAIN REFERENCES

A. Agosto, P. G. Giribone, "Artificial Intelligence in Risk Management: Machine Learning methods in application to market risk" XIV Convention AIFIRM (Italian Association of Financial Industry Risk Managers), Milan, 11th April 2019.

D. Brigo and F. Mercurio, "Interest Rate Models: Theory and Practice with smile, inflation and credit" Springer Finance, 2006.

O. Caligaris and P. G. Giribone, "Modeling seasonality in inflation indexed swap through machine learning techniques: analysis and comparison between traditional methods and neural networks" Risk Management Magazine, vol. 13, N. 3, pp. 37–53, December 2018.

S. Hochreiter and J. Schmidhuber, "Long Short-Term Memory" Neural Computing, vol. 9, N. 8, pp. 1735–1780, 1997.

S. Kazziha, "Interest rate models, inflation-based derivatives, trigger notes and cross-currency swaptions", PhD Thesis, Imperial Collage of Science, Technology and Medicine, London, 1999.

R. S. Tsay, "Analysis of Financial Time series" Third Edition, Wiley, 2010.

THANKS

Does anyone have any questions?

Pier Giuseppe Giribone

PhD, CIIA®, CESGA®, CIWM®
pier.giuseppe.giribone@economia.unige.it
piergiuseppe.giribone@carige.it
http://www.diptem.unige.it/piergiribone/