Robust Supply Chain Costs Minimization Considering Operational Risks

Invited Talk: NexTech 2020
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Invited Talk

- 2001: Diploma Degree at Saarland University
- 2001-2003: Scientific Assistant at the German Research Center for Artificial Intelligence (DFKI)
- 2003-2006: Scientist at the German Meteorological Service
- 2006-2010: PhD Student at the Distance University of Hagen
- 2011-2015: Postdoc at Goethe University Frankfurt am Main
- 2015-now: Research Associate at Lucerne University of Applied Science and Arts
- 2019-now: Lecturer at FFHS (Fernfachhochschule der Schweiz)
Supply Chain

- Consider the production of a notebook
- To produce a notebook one needs
  - Hard drive
  - LCD
  - Keyboard
  - Touchpad …

To produce an LCD one need:
  - Rare Earth Materials
  - …. 
A supply chain can be represented by a graph. Nodes: locations / items. Edges: transportation between locations.
Costs minimization can be accomplished using an optimization model: Minimize $\sum_{iz} c_{iz} P_{iz} + \sum_{ijz} t_{ijz} Q_{ijz} + \sum_{itz} d_{itz}$

- $D_{iz} \leq \sum_{j,i,z} Q_{j,i,z}$
- $Q_{j,i,z} \leq P_{jz} + I_{ji}$

$i,j$ : locations, $z$: item, $c_{iz}$ : production/inventory/transportation costs, $d_{itz}$ : production/inventory/transportation costs
- $D_{iz}$ : demand of item $z$ at location $I$
- $P_{jz}$ : Number of items $z$ produced at location $j$
- $I_{ji}$ : number of items $z$ contained in inventory at $j$
- $Q_{j,i,z}$ : number of items $z$ transported from $i$ to $j$
A supply chain can be affected by several risks
• Disruption risks
• Price escalation risks
• Inventory and scheduling risks
• Technology access risk
• Quality risks
Possible disruption risks:
- Natural disaster
  - Fire, earthquake, lightning, strike, volcano eruption
- Political risks
  - Labor strike, Brexit
- Sabotage
  - Cyber-attack, burglars
A certain risk event can cause a simultaneous breakdown of several suppliers.

- **Examples**
  - Strike: can affect several countries in the same country and industry branch
  - Natural disaster: can cause a breakdown of all suppliers in a certain region
Principle:

Get rough overview over potential supply chain threats by looking at the supply chain structure or geographical map [2,3]
Critical notes[2,3]: Nodes that are essential for the proper functioning of the total supply chain
Density: Supply chain containing a geographical area with high number of suppliers might be vulnerable to group risks.
Topological risk measures only provide a rough overview over potential risks. Is there a more precise method?

Yes → using stochastic optimization

- Two possibilities to extend optimization model to consider disruption risks:
  - Iterate first over risks and then over locations
  - Iterate first over locations and then over risks
What are the advantages and drawbacks of the two methods?
What are advantages and drawbacks of automatic vs manual scenario generation?
How large is the conducted error by assuming the scenarios are disjoint, while in reality they are stochastically independent?
Possibility 1: Iterate first over risks and then over locations

Stochastic optimization
Stochastic optimization

Risk costs

Risk costs

Risk costs

Risk costs
Stochastic optimization

Minimal risk costs obtained by optimizer

Risk costs

Risk costs

Risk costs
A risk scenario is always associated to exactly one node in the supply chain
A local estimation of risk costs is required
- Fast, only one optimization run with only few decision variables
- As result: one single optimal supply chain flow

- Unclear, how to assess single and multiple source risks
- Unclear, how to deal with dependencies between risks and group risks
So far:
• Aggregate costs
  • Iterate over scenarios

Alternative model (Babazadeh and Razmi):
• Iterate over scenarios
• Aggregate costs
Robust optimization [1]

Risk scenario 1

Risk scenario 2

Risk scenario 3
Robust optimization

Risk scenario 1
Costs $C_1$

Risk scenario 2
$C_2$

Risk scenario 3
$C_3$
Robust optimization

Risk scenario 1
\[ p_1 C_1 \]

Risk scenario 2
\[ p_2 C_2 \]

Risk scenario 3
\[ p_3 C_3 \]
Robust optimization

Risk scenario 1: \( p_1 C_1 \)

Risk scenario 2: \( p_2 C_2 \)

Risk scenario 3: \( p_3 C_3 \)
Robust optimization

Risk scenario 1
\[ E(C) = p_1 C_1 \]

Risk scenario 2
\[ + \quad p_2 C_2 \]

Risk scenario 3
\[ + \quad p_3 C_3 \]
Robust optimization

Risk scenario 1

\[ \mathbb{E}(C) = p_1 C_1 \]

Risk scenario 2

\[ + p_2 C_2 \]

Risk scenario 3

\[ + p_3 C_3 \]

\[ \mathbb{V}(C) = \sum_s (p_s (\mathbb{E}(C) - C_s)^2) \]
Risk scenario 1: 
\[ E(C) = p_1 C_1 + \max(0, E(C) - C_1) \]

Risk scenario 2: 
\[ E(C) = p_2 C_2 + \max(0, E(C) - C_2) \]

Risk scenario 3: 
\[ E(C) = p_3 C_3 + \max(0, E(C) - C_3) \]

Robust optimization

\[ V_{\text{abs}}(C) = \sum_s p_s \max(0, E(C) - C_s) \]
\[ \mathbb{E}(C) = p_1 C_1 + p_2 C_2 + p_3 C_3 \]

\[ \mathbb{V}_{\text{abs}}(C) = \sum_s p_s |(\mathbb{E}(C) - C_s)| \]
Robust optimization

Objective function:
\[
\text{obj: Min } E(C) + \gamma \int_{\text{abs}} (C) + \omega \sum_s p_s \alpha_k \delta_{k_s} \\
\delta_{k_s} \text{ not satisfied demand} \\
\sum_j \sum_n Q_{jkns} + \delta_{k_s} \geq d_{k_s} \\
d_{k_s}: \text{ demand of customer zone } k \text{ in scenario } s \\
Q_{jkns}: \text{ quantity shipped from } j \text{ to } k \text{ by mode } n \text{ in scenario } s
Robust optimization

- For each risk scenario, a separate minimal flow is determined by the optimizer
- Optimizer is only applied once

What is not provided by this method:
- An optimal overall flow
• Without considering risks: Deterministic optimization
• Considering risks but no variance: Stochastic optimization
  Considering risk and costs variance: Robust optimization
Do single source situations have a higher expected costs than dual source situations?

Yes, since single source situations lead more often to unsatisfied demands, which are penalized.
Pro / Cons

Pro:
- Unsatisfied demand / group risks / single and multiple sourcing are treated properly
- Determines optimal flow for each risk scenario

Con:
- Rather slow caused by large amount of decision variables
- Determines no global overall optimal flow
Manager-defined scenarios

Supply chain expert specifies list of risk scenarios where each scenario contains:
- probability of occurrence
- impact in form of supplier or transport disruption (example: production capacity is reduced by 80% due to this risk)
Supply chain expert specifies list of risk scenarios templates where each scenario template contains:
- probability of occurrence
- impact in form of supplier or transport disruption degree distribution (here Weibull) typically given by variance and expected value
- Scenarios are then sampled using this distribution
Research questions

- How many automatically generated scenarios are needed for a precise estimate?
- What is the critical point (point of convergence) using automatically generated scenarios?
- What is the convergence rate?
- (Future) What is the gain to use the full distribution in contrast to just use the expected value? For that we need a non-linear model.
We are using a real-world supply chain with:
- 40 Customer
- 100 suppliers
- 230 Items
- 200 transport links
So far we employed only stochastic and not robust optimization for the automated scenario generation approach.
Findings:
- Manager defined approach can be seen as special case of automated scenario generation approach with scenarios → ∞ and variance → 0
- so, costs obtained for manager-defined scenarios be seen as approximately the convergence point for the automated approach with small variance (here 0.05)
- with 100 scenarios, we still have an estimated error of 0.05 %
- converge rate: only sublinear, i.e. rather slow convergence
Error Estimation

• For the robust/stochastic optimization approach it is assumed that the risk scenarios are all disjoint:
• However in practice they are more likely to be independent.
• This can cause the sum of the probabilities to be too large, in extreme cases even exceed one.
• How much can the sum of probabilities deviate from the correct value?
• We can estimate this error using the formula of Sylvester.
Error = \left( \sum_{i=1}^{n} P(S_i) - \sum_{k=1}^{n} (-1)^{k-1} \sum_{1 \leq i_1 < \ldots < i_k \leq n} P(S_{i_1} \cap \ldots \cap S_{i_k}) \right) \\
= \left( \sum_{i=1}^{n} P(S_i) - \sum_{k=1}^{n} (-1)^{k} \sum_{1 \leq i_1 < \ldots < i_k \leq n} P(S_{i_1} \ldots S_{i_k}) \right)

S_i: risk scenario \ i
P(S_i): probability of risk scenario \ i

In our setup:
Absolute error: 0.0001
Relative error: 0.0067
Conclusion

- Robust optimization is a powerful approach for dealing with supply chain risks
- It allows for considering partial breakdowns as well as group risks
- The number of automatically generated scenarios required for reliable cost estimates in our example setup is $>100$ and the costs converges only sublinear
- Performance of single period robust optimization model is high for our example setup with 8 scenarios (only several seconds)
Future Work

- Use automatic scenario generation together with nonlinear models (for instance: models containing integer constraints) and robust optimization (currently only stochastic optimization)
- Use of a multiperiod robust optimization model
- Combining topological risk models with a robust optimization model
Bibliography


