Compression of Structured Big Data

Challenges and Solutions

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Talk outline

1. My personal research path to Compression of Structured Big Data (searching for new open research problems, …)

2. Some Big Data Challenges (focus on problems that motivate compression)

3. (main part) An overview of selected compression techniques
   - principles applied in well known compressors
   - grammar-based compression and its application to text, trees, and graphs
   - recompression
My Research Path to Compression of Structured Big Data (1)

My starting point:
+ 5+ years research in relational database systems
+ search for new open problems

Shift from relational databases to XML databases
- transaction synchronisation, ...
  → nearly orthogonal to data model → easy to transfer

+ non-orthogonal concepts, here: relying on data structure, e.g. queries, access control, views, ...
  → new solutions required → (potentially new) research topics

new results on
  XML access control, XML query optimization, …
Shift from XML databases to compressed XML access control, ...
→ nearly orthogonal to compression → easy to transfer

non-orthogonal concepts, here: relying on data access, e.g. queries, caching, XML schema...
→ new solutions required → (potentially new) research topics

new results on
  XML caching
  XML encoders
  schema-based XML compression
  grammar-based XML compression
  parallel multi-query optimization
  ...

My Research Path to Compression of Structured Big Data (2)
My Research Path to Compression of Structured Big Data (3)

Shift from compressed XML to compressed strings and graphs
grammar-based compression nearly orthogonal to data type
→ easy to transfer

non-orthogonal concepts, here: relying on data structure,
e.g. queries, modification, ...
→ new solutions required
→ (potentially new) research topics

new results on
IRT – an updatable BWT
parallel compression of strings
compression of commutative trees
compressing graphs
Big Data Examples

Financial transactions

Genom data

Weather forecast

Sensor data

Social networks

Big text data
Big Data Processing Examples

Pattern detection, e.g.
  crime detection in financial transactions,
  behaviour derivation from genom data, …

Prediction, e.g.
  predictive maintainance,
  market development, …

Data aggregation and data transformation, e.g.
  wheather forcast,
  big data transmission into clouds

Archiving big data, e.g.
  string data (documents, genom data, …),
  graph data (social networks, …), …
Some Big Data Processing Challenges

Too much data for efficient

- storage
- transmission
- information extraction
- search of patterns
- transformation
- data cleaning

=> most algorithms will take too long
## Some Big Data Processing Challenges

<table>
<thead>
<tr>
<th>Too much data for efficient</th>
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<td>storage</td>
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<td>data cleaning</td>
<td>- modify big data</td>
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→ most algorithms will take too long  → any way out?
Some Big Data Processing Challenges ...

“Companies are creating so much data, it has to be shipped in trucks“

e.g. DigitalGlobe‘s data transfer into the cloud → a truck, full of Amazon‘s snowball devices needs 10 days to ship the data into Amazon‘s cloud in comparison to uploading the data which currently needs 300 years

source:

Are there alternatives? → (next slides)
What is compression?

Substitute a larger data set by an “equivalent“ shorter data set

Lossless compression:
  larger data set can be reconstructed from shorter data set
  (e.g. gzip, bzip2, …)

Lossy compression:
  larger data set cannot be reconstructed from shorter data set,
  but shorter data set is sufficiently detailed for the given task
  (e.g. mp3, …)
Lossless compression principle

find repeated patterns in input data set and replace repeated patterns by pointers, shortcuts, or …

Barcelona is the capital city of Catalonia … . Barcelona is a …

Barcelona is the capital city of Catalonia … . - - a …
Lossy compression principle

find similar data or similar patterns of data in input data set and replace similar data by a unique data presentation (e.g. pointers, shortcuse, …)

Population: Barcelona (1,787,455), Hamburg (1,787,408), …

Population: Barcelona (1,787,000), Hamburg (1,787,000), …

Population: Barcelona (1,787,000), Hamburg - …
Why compression?

Goal: handle (search, …) big data efficiently

- faster memory access
  Trend towards main memory databases
    ➔ smaller memory footprint
    If relevant data fits into main memory
    ➔ faster computation possible

- faster data transmission
  ➔ shorter data transmission time

- faster algorithms (e.g. pattern search)
  ➔ search repeated patterns only once
Significant speed-up expected if …

speed of algorithms depends on size of input, e.g., search in big text data collections depends on the number of characters to be read/processed

processing repeated text (find, read, evaluate, transform, …) can be combined into a single step for all the data (find once, read once, evaluate once, transform once, …)

Very likely to be relevant for big text data
Example: Word Count

Speed of (sub-)string count in text depends on size of input, i.e., number of characters

How often is “Barcelona“ mentioned?
1. find “Barcelona“, 2. count incoming pointers to “Barcelona“

How often is “Bar“ mentioned?
1. find all words containing “Bar“, 2. count incoming pointers

faster on compressed text
Generalizing the Word Count Example to Sub-String Search

Count number of strings “Barcelona“

Find all locations of strings “Barcelona“

Navigate from all locations of strings “Barcelona“ to next word

Search for sequences of compressed words, e.g. “Barcelona is“

Search for sub-strings crossing pattern boundaries, e.g. “elona i“

… still faster on compressed text?
Generalizing to Arbitrary Algorithms on Compressed Text?

Instead of individual solutions for specialized algorithms …

What is common to all algorithms on massive compressed text?

Although text is compressed:
- access to certain text fragments (nobody can read all the text…)
- access by content or by position, e.g. relative to other content
- read access and write access to text fragments
- support of operations on massive text (multi-read, multi-write,…)

Offer efficient basic operations on massive compressed text
Basic Operations for Algorithms on Compressed Text

Algorithms on massive compressed text data rely on basic operations on sub-strings like:

- locate all positions of a given sub-string
- navigate from one huge set of positions to another
- read sub-strings at a huge number of given positions
- transform sub-strings at a huge number of given positions (including copy, insert, delete, update, … of sub-strings at a huge number of given positions)

even these elementary operations are a challenge on massive compressed text data
gzip, bzip2, … combine some of the following

**compression techniques**
- replace longer sequences of symbols (characters/words/…) by shorter sequences
- fixed replacement rules (Run Length Encoding,…)
- explicit dynamic dictionaries (Repair, Sequitur)
- implicit dynamic dictionaries (LZ77, LZ78, …)

**encoding techniques**
- use shorter codes for more frequent symbols (Huffman,…)

**additional (pre-)transformations to improve compression**
- e.g. based on rotations (MoveToFront, BWT, IRT)
Pre-Transformation by Burrows Wheeler Transform (BWT) or by Indexed Reversible Transformation (IRT)

Input = abracadabra

computed rotations:

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sorted rotations:

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BWT = rdarcaaaab

sorted rotations: BWT

E (end)

+ BWT allows to reconstruct input
+ substrings can be searched by fast LF mapping
+ BWT has more character repetitions than input ⇒ easier to compress
++ IRT additionally allows to directly access the Nth word
Compression by Run Length Encoding

\[
BWT = r \ d \ a \ r \ c \ a \ a \ a \ a \ b \ b
\]
\[
\text{run length encoding} = 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 0 \ 0 \ 0 \ 1 \ 0 \ \quad \leftarrow \text{uses bits only}
\]
\[
\text{shorter BWT} = r \ d \ a \ r \ c \ a \ b \ \quad \leftarrow \text{to save bytes}
\]

0-bit = repetition of previous character
1-bit = new character

Variants:

\[
\text{text} \quad r \ d \ a \ r \ c \ a \ a \ a \ a \ b \ b
\]
\[
\text{alternative encoding} = 1 \ 1 \ 1 \ 1 \ 4 \ 0 \ 0 \ 0 \ 2
\]
\[
\text{shorter text} = r \ d \ a \ r \ c \ a \ b
\]

\[
\text{run length encoding} = 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 0 \ 0 \ 0 \ 1 \ 0
\]
\[
\text{gamma coding} = 0 \ 6 \quad 3 \quad 1 \ 1
\]
shorter Huffman codes for more frequent letters:
2 * a → 00
1 * b → 01
1 * c → 100
1 * d → 101
2 * r → 11

Only the highlighted part is the Wavelet Tree (WT) to be stored

When using an alphabetic Huffman code, the Wavelet Tree is sorted, i.e. supports searching all symbols (letters, words, ...) <= a given constant.
Direct access on Wavelet-Tree (WT)

No direct access on Huffman codes:
e.g. letter at position 5?

11 101 00 11 100 ...

In comparison to Huffman coding, on the Wavelet Tree, only the letter at position 5 has to be decoded, but no previous symbols (letters, words, ...) need not be decoded

letter at pos 5 in WT → top-down search
Search in Wavelet-Tree (WT)

No direct access on Huffman codes: e.g. letter at position 5?

In comparison to Huffman coding, on the Wavelet Tree, only the 2nd letter has to be decoded, but no previous symbols (letters, words, ...) need not be decoded.

In comparison to Huffman coding, on the Wavelet Tree, only the 2nd letter has to be decoded, but no previous symbols (letters, words, ...) need not be decoded.

search for position of 2nd ‘r’ in input BWT → bottom-up search in the WT
Partial Solution: String Encodings

Although encodings (Huffman, Wavelet Tree, …) substitute longer words/symbols by shorter words/symbols, the number of symbols remains the same.

Also pre-transformations do not reduce the number of symbols:

→ both steps alone leave still too many symbols for big text data

→ we techniques to reduce the number of symbols/shortcuts

… and there are alternatives to Run Length Encoding
Alternative: Grammar-based string compression

Replacing (most frequent) digram occurrences uses a “look for smallest repeated pattern first“ – approach

Substitute larger frequently occurring patterns in multiple steps
Algorithms on Grammar-Compressed Strings

In the optimal case, string-grammars are exponentially smaller than a text, i.e., a text with $N$ characters/words/symbols/… can be represented by a string grammar of size $\log(N)$.

Basic operations are supported:

read
- find position(s) of given content
- determine content at position(s)
- navigate to surrounding position(s)

modify / transform
- insert text at given position(s)
- update text at given position(s)
- copy text at given position(s)
- delete text at given position(s)
(Re-)Compression by replacing a most frequent digram

\[
\begin{align*}
S & \rightarrow \ b \ \boxed{c \ \boxed{d}} \ b \ \boxed{c \ \boxed{d}} \\
S & \rightarrow \ \boxed{b \ \boxed{N}} \ b \ \boxed{N} & N & \rightarrow \ c \ d \\
S & \rightarrow \ M \ M & M & \rightarrow \ b \ N & N & \rightarrow \ c \ d \\
S & \rightarrow \ M \ M & M & \rightarrow \ b \ c \ d & & \\
\end{align*}
\]

(Re-)Compression Algorithm for strings / trees / graphs:

while at least one digram occurs more than once

choose a most frequent digram D (e.g. c d)

(if re-compression: isolate all occurrences of D by smart inlining)

replace each occurrence of digram D by a new nonterminal N,

which is thereafter treated as a terminal, i.e. not cut-off again

introduce a grammar rule (e.g. N \rightarrow c d)

inline rules called only once (e.g. N \rightarrow c d)
When can Algorithms Become Faster on Compressed Input?

Speed of many algorithms depends on size of input, e.g. for

Strings – number of characters

Trees, graphs – number of nodes and edges

Goals:
minimize number of characters / edges … by storing repeated patterns only once (=compression)

transform algorithms,
such that they need a smaller amount of data accesses
### Overview of steps towards re-compressed graphs

<table>
<thead>
<tr>
<th>String compression</th>
<th>String re-compression</th>
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<td>Graph compression</td>
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From Algorithms on Strings to Algorithms on Trees

Algorithms on massive tree-structured data rely on elementary operations on nodes and edges of trees, e.g.

- locate positions of all nodes (or edges) having a given label
- navigate from one huge set of nodes (or edges) to another
- read labels of a huge given set of nodes (or edges)
- transform sub-trees at a huge number of given positions (including copy, insert, delete, update, … of sub-trees at a huge number of given positions)

Can we extend efficient algorithms on compressed strings to efficient algorithms on compressed trees?
Each node N in a tree, can be represented by
- a path from the root node of the tree to that node N
- a path in the DAG from the DAGs root to a DAG-node corresponding to that node N
A DAG node can correspond to multiple nodes of a tree
Faster Algorithms on DAG-Compressed Trees

In the optimal case, Directed Acyclic Graphs (DAGs) are exponentially smaller, i.e., a tree with \( N \) nodes and \( N-1 \) edges can be represented by a DAG of size \( \log(N) \).

Runtime of algorithms visiting each node once (e.g. label count), may be reduced from \( N \) to \( \log(N) \) effort in the optimal case.
Updates on DAG-Compressed Trees?

Update, after isolation of the path in the DAG corresponding to the tree node to be updated.
Updates on DAG-Compressed Trees after path isolation

E.g. isolate the red path of the DAG:
Top down on the red path, copy all the nodes having incoming edges of different colors together with their outgoing edges

After path isolation, update in the DAG possible

After path isolation, update in the DAG possible
Compression by Tree Grammars (without parameters)

The tree can be represented by a Tree Grammar, i.e. a grammar where the right-hand-side of grammar rules represent (repeated) sub-trees.

Example:

\[ S \rightarrow e_1 \left( E_2, E_2 \right) \]
\[ E_2 \rightarrow e_2 \left( E_3, E_3 \right) \]
\[ E_3 \rightarrow e_3 \left( e_4, e_4 \right) \]

The last grammar rule states: the nonterminal E3 is a shortcut for an e3 node having a first child e4 and a second child e4.

S, the nonterminal of the grammar’s start rule, is a shortcut for the whole compressed tree.
Example (continued):

\[
S \rightarrow e1 \ ( E2 , E2 )
\]

\[
E2 \rightarrow e2 \ ( E3 , E3 )
\]

\[
E3 \rightarrow e3 \ ( e4 , e4 )
\]

In order to simulate operations on the compressed tree, algorithms on Tree Grammars read the grammar rules (nodes) and follow the edges calling other rules.

Less than one grammar rule (without parameters) per DAG node.
Compression by Tree Grammars with parameters

Example:

S → e1 ( E2(e4) , E2(e5) )

E2( y1 ) → e2 ( E3( e4 ) , E3( y1 ) )

E3( y1 ) → e3 ( y1 , e4 )

Each tree grammar rule with parameters ( e.g. E2(y1) → … )
is a short-cut for multiple tree grammar rules without parameters
→ in the optimal case, exponentially fewer
tree grammar rules (with parameters) than DAG nodes
→ in the optimal case, exponentially less runtime
Digrams for trees generate Tree Grammar rules

A digram is a pair of typed items (c,d) in a given relationship r

String: b c d e c d e c d

digram (c,d) with r is “d follows c“

Tree:

digram (c,d) with r is “d is the second child of c“

selecting digrams consisting of inner tree nodes results in Tree Grammar rules with parameters
Algorithms on Grammar-Compressed Trees

In the optimal case, Tree Grammars are exponentially smaller than a DAG, i.e., a DAG with N nodes and edges can be represented by a Tree Grammar of size \(\log(N)\) rules [several contributions by Markus Lohrey and Sebastian Maneth]

Basic operations are supported on Tree Grammars:

read
- find position(s) of given content
- determine content at position(s)
- navigate to surrounding position(s)

modify / transform
- insert, update, copy or delete tree at given position(s)

Goal: execute massive operations in \(O(\text{size of the grammar})\)
\(\rightarrow\) up to exponentially faster than on DAG / Tree
Can we extend efficient algorithms on compressed trees to efficient algorithms on compressed graphs?

New challenges, as graph structure is more complex, i.e.,

- graph may contain multiple paths from A to B
- graph may contain cycles
- graph may be partitioned
- graph may be difficult to partition into tractable sub-graphs
Digrams for ordered trees and for unordered trees

Intermediate step: unordered tree

Tree:

\[ \begin{array}{c}
  N \\
  c \\
  b \\
  d \\
  e \\
  \end{array} \quad \begin{array}{c}
  N \\
  c \\
  d \\
  y_1 \\
  d \\
  b \\
  e \\
  \end{array} \]

digram \((c,d)\) with \(r\) is “\(d\) is the second child of \(c\)“

Unordered Tree:

\[ \begin{array}{c}
  c \\
  b \\
  d \\
  \end{array} \quad \begin{array}{c}
  c \\
  d \\
  e \\
  \end{array} \]

digram \((c,d)\) with \(r\) is “\(d\) is a child of \(c\)“

edge order does not matter - like in graphs
A digram is a pair of typed items \((c, d)\) in a given relationship \(r\).

Graph:   \[ f \rightarrow b \rightarrow c \]
A digram is a pair of typed items \((c,d)\) in a given relationship \(r\).

**Graph:**

- **digram** \((f,b)\) with \(r\) is “nodes \(f\) and \(b\) are connected by a hyperedge from \(f\) to \(b\)”
- **digram** \((d,e)\) with \(r\) is “there is a node shared by an incoming hyperedge \(d\) and an outgoing hyperedge \(e\)”
- **digram** \((b,e)\) with \(r\) is “node \(b\) has an outgoing hyperedge \(e\)”
- **digram** \((d,b)\) with \(r\) is “node \(b\) has an incoming hyperedge \(d\)”
Digrams for a graph with labeled nodes and labeled edges

A digram is a pair of typed items \((c,d)\) in a given relationship \(r\)

Graph: \( \begin{array}{c} f \rightarrow b \rightarrow c \end{array} \)

- Digram \((f,b)\) with \(r\) is “nodes \(f\) and \(b\) are connected by a hyperedge from \(f\) to \(b\)”
- Digram \((d,e)\) with \(r\) is “there is a node shared by an incoming hyperedge \(d\) and an outgoing hyperedge \(e\)”
- Digram \((b,e)\) with \(r\) is “node \(b\) has an outgoing hyperedge \(e\)”
- Digram \((d,b)\) with \(r\) is “node \(b\) has an incoming hyperedge \(d\)”
Digrams for a graph with labeled nodes and labeled edges

A digram is a pair of typed items (c,d) in a given relationship r

Graph: \[ f \overset{d}{\rightarrow} b \overset{e}{\rightarrow} c \]

digram (f,b) with r is “nodes f and b are connected by a hyperedge from f to b“

digram (d,e) with r is “there is a node shared by an incoming hyperedge d and an outgoing hyperedge e“

digram (b,e) with r is “node b has an outgoing hyperedge e“

digram (d,b) with r is “node b has an incoming hyperedge d“
Digrams for a graph with labeled nodes and labeled edges

A digram is a pair of typed items (c,d) in a given relationship r.

Graph: \[ \begin{array}{c}
\text{d} & \rightarrow & \text{e} \\
\text{f} & \rightarrow & \text{b} & \rightarrow & \text{c}
\end{array} \]

- Digram (f,b) with r is “nodes f and b are connected by an edge from f to b”
- Digram (d,e) with r is “there is a node shared by an incoming edge d and an outgoing hyperedge e”
- Digram (b,e) with r is “node b has an outgoing edge e”
- Digram (d,b) with r is “node b has an incoming edge d”
Graph Grammars with parameters

Grammar rules with parameters,

the right-hand-side of which are graphs

which represent repeated sub-graphs

the parameters represent the connections of
a repeated sub-graphs with its environment

First results:

graph grammar is smaller than graph
(less nodes and edges)

(some) algorithms that traverse nodes and edges
are faster on (some) compressed graphs
What is re-compression of compressed data?

Transform compressed data into a more (better) compressed format without full decompression of the data.
Why Re-Compression of a Compressed Text/Tree/Graph?

Your algorithm produces an intermediate result, i.e.

- transforms big (text/tree/graph) data into big (text/tree/graph) data in a different format

or

- transforms just a sub-set of big (text/tree/graph) data into big (text/tree/graph) data in a different format

The produced data may be still too big for shipping, … but a new (better) compression fitting to the selected data sub-set may be sufficient to do next processing step (e.g. ship the data)

Use compression instead of a truck to ship the data
Why Re-Compression of a Compressed Text/Tree/Graph?

large graphs ➞ “long time“ to find a “good“ compression

idea: instead:
do any compression “fast“ and in parallel on small sub-graphs
 ➞ get compressed sub-graphs “fast“

re-compress compressed sub-graphs
 ➞ re-compression time
depends on size of compressed sub-graph
Re-compression of a compressed string / tree / graph

A string / tree / graph

\[ S \rightarrow d \text{ } [\text{red boxes}: c \text{ } d] \text{ } c \]

that has been compressed to

\[ S \rightarrow d \text{ } N \text{ } N \text{ } c \quad N \rightarrow c \text{ } d \]

can be recompressed to

\[ S \rightarrow M \text{ } M \text{ } M \text{ } M \quad M \rightarrow d \text{ } c \]

to get a better compression
Re-compress a compressed string: 1. Count digrams

\[ S \rightarrow d \ N \ N \ c \]
\[ N \rightarrow c \ d \]

- **digram generator**
  - d \( \rightarrow \) N
  - N
  - N \( \rightarrow \) c

- **generated digram**
  - d \( \rightarrow \) c
  - c \( \rightarrow \) d (occurs twice)
  - d \( \rightarrow \) c
  - d \( \rightarrow \) c

\( \rightarrow (d,c) \) with \( r = \) “d follows c“
- is the most frequent digram in decompressed graph
2. Isolate a most frequent digram by smart inlining

Task: isolate most frequent digram \((d,c)\) with \(r = "d \text{ follows } c"\)

\[
S \rightarrow d \hspace{1em} c \hspace{1em} N \hspace{1em} c \hspace{1em} N \hspace{1em} c
\]

\[
N \rightarrow \text{c e f g d}
\]

needed: partial decompression of \(N\) to isolate \(d\) from \(N\)

new rules that isolate \(d\) from the end of \(N\):

\[
N \rightarrow N_{-d} \hspace{1em} d
\]

\[
N_{-d} \rightarrow c \hspace{1em} e \hspace{1em} f \hspace{1em} g
\]

trick: inline rewritten rule \(N \rightarrow N_{-d} d\) instead of \(N \rightarrow c e f g d\)

finally, substitute digrams \((d,c)\) with new nonterminal \(M\):

\[
S \rightarrow M \hspace{1em} N_{-d} \hspace{1em} M \hspace{1em} N_{-d} \hspace{1em} M \hspace{1em} M \rightarrow d \hspace{1em} c
\]
Recompression of Grammar-Compressed Trees and of Grammar-compressed Graphs

The same two basic steps:

1. Compute the most frequent digram of the decompressed String / Tree / Graph without really decompressing the String / Tree / Graph

2. Isolate all occurrence of the selected digram from the String / Tree / Graph without really decompressing the String / Tree / Graph

First results in [ ICDE 2016 ], [ Dagstuhl 2016 ]
Overview of steps towards re-compressed graphs

- String compression → string re-compression
- Ordered tree compression → ordered tree re-compression
- Unordered tree compression → unordered tree re-compression
- Graph compression → graph re-compression

Re-compression:
- Ordered trees: SLT grammars
- Unordered trees: add commutativity
- Graphs: node-node, edge-edge & node-edge digrams

Digram counting, smart decompression