The Role of Adaptive Control in Quantum Information Systems

Mark J. Balas
Distinguished Professor
Aerospace Engineering Department
Embry-Riddle Aeronautical University
Daytona Beach, FL
It’s not theories about stars; it’s the actual stars that count.”

......... Freeman Dyson
Indirect Adaptive Control

Note: Called “Self-Organizing” System (Kalman) & “Self-Tuning” Regulator (Astrom)
Direct Adaptive Model Following Control (Wen-Balas 1989)

\[ y_e \rightarrow 0 \text{ as } t \rightarrow \infty \]

\[ (u_m, x_m, e_y) \text{ Known Signals} \]
Direct Adaptive Persistent Disturbance Rejection
(Fuentes-Balas 2000)

\[ y_m \rightarrow y \rightarrow e_y \rightarrow 0 \]

Plant

Reference model

Disturbance Generator

Adaptive Gain Laws

\( u_m \)

\( y_m \)

\( x_m \)

\( y \)

\( e_y \)

\( t \rightarrow \infty \)

\( (u_m, x_m, e_y, \phi_D) \) Known Signals

Disturbance Basis
Persistent Disturbance Example

\[
\begin{bmatrix}
\dot{\theta} \\
\ddot{\theta}
\end{bmatrix} = \begin{bmatrix}
0 & 1 \\
0 & 0
\end{bmatrix} \begin{bmatrix}
\theta \\
\dot{\theta}
\end{bmatrix} + \begin{bmatrix}
0 \\
1
\end{bmatrix} u(t) + \begin{bmatrix}
0 \\
1
\end{bmatrix} u_D
\]

\[
\begin{aligned}
u_D &\equiv A_D \sin(\omega_D t + \varphi_D) = \\
&= \begin{bmatrix}
l_1 \\
l_2
\end{bmatrix} \begin{bmatrix}
\sin \omega_D t \\
\cos \omega_D t
\end{bmatrix}
\end{aligned}
\]

\[
\begin{aligned}
u_D &\equiv a_0 + a_1 t + a_2 t^2 = \\
&= \begin{bmatrix}
a_0 \\
a_1 \\
a_2
\end{bmatrix} \begin{bmatrix}
1 \\
t \\
t^2
\end{bmatrix}
\end{aligned}
\]
Adaptive Control Is Not Complicated!

Adaptive Regulation

Nonlinear Plant

\[
\begin{aligned}
\frac{\partial x}{\partial t} &= A(x) + B(x)u \\
y &= C(x)
\end{aligned}
\]

Controller

Nonlinear Adaptive Gain Law

\[
\dot{G} = -yy^T \gamma; \gamma > 0
\]

Use ONLY Outputs & Know Almost NOTHING about the Plant
“Simplicity” via Infinite Dimensional Spaces

\[
\begin{cases}
\frac{\partial x}{\partial t} = A(x) + B(x)u \\
x(0) = x_0 \in X \quad \text{Banach or Hilbert Space} \\
y = C(x)
\end{cases}
\Rightarrow x(t, w_0) = U(t)x_0 \quad ; \forall t \geq 0
\]

“Boil Away” all the special properties of \( \mathbb{R}^N \)
Infinite-Dimensional Space

Viewpoint: **Linear** Semigroups

\[
\begin{cases}
\frac{\partial x}{\partial t} = Ax + F \\
x(0) = x_0
\end{cases}
\]

\(A : D(A) \subseteq X \rightarrow X\) closed, densely defined linear operator

\(\Rightarrow x(t, w_0) = U(t)x_0 + \int_0^t U(t - \tau)F(\tau)d\tau \in X; \forall t \geq 0\)

\(C_0 -\) Semigroup of Bounded Operators \(U(t)\):

\[
\begin{cases}
U(t + s) = U(t)U(s) \text{ (semigroup property)} \\
\frac{d}{dt}U(t) = AU(t) = U(t)A \text{ (A generates U(t))} \\
U(t)x_0 \xrightarrow{t \to 0} x_0 \text{ (continuous at } t = 0)\end{cases}
\]

J. Wen & M. Balas, “Robust Adaptive Control in Hilbert Space ”,

Example: Heat Diffusion

\[
\frac{\partial x}{\partial t} = \frac{\partial^2 x}{\partial z^2} + bu;
\]

\[b(z) \in D(A) \equiv \{ x / \text{smooth and BC: } x(t,0) = x(t,l) = 0 \}\]
\[\subset X \equiv L^2(\Omega)\]

with \((x,y) \equiv \int_\Omega x(t)y(t)dt\)
\[x(0) = x_0 \in D(A)\]
\[y = (c,x); \quad c(z) \in D(A)\]
Euler-Bernoulli Beam

\[
\begin{bmatrix}
\frac{\partial}{\partial t} w \\
\frac{\partial}{\partial t} w_t
\end{bmatrix} = \begin{bmatrix}
0 & I \\
-\frac{EI}{\rho} & 0
\end{bmatrix} \begin{bmatrix}
w \\
w_t
\end{bmatrix} + \begin{bmatrix}
0 \\
0
\end{bmatrix} b(z) u(t)
\]

where:
- \( w \) is the transverse displacement
- \( w_t \) is the transverse displacement time derivative
- \( E \) is the Young's modulus
- \( I \) is the area moment of inertia
- \( \rho \) is the material density
- \( A \) is the cross-sectional area
- \( b(z) \) is the load function
- \( u(t) \) is the time function
Symmetric Hyperbolic Systems

\[ \frac{\partial \varphi}{\partial t} = \sum_{i=1}^{n} A_i \frac{\partial \varphi}{\partial z_i} + A_0 \varphi; \ x \in D(A) \subset X \equiv L^2(\Omega; \mathbb{R}^l) \]

**Boundary Conditions**

\[ \Lambda(z)\varphi(z, t) = 0 \forall z \in \partial \Omega; t \geq 0 \]

**Theorem**: (Balas 1974, with help from Lax - Phillips 1967)

1) Symbol: \( A(\xi) = \sum_{i=1}^{n} \xi_i A_i \) is nonsingular \( \forall \xi \neq 0 \in \mathbb{R}^n \)

2) \( A_0 + A_0^* \leq 0 \)

3) \( \dim N(A) < \infty \)

4) Boundary Conditions are Coercive (\( \| \varphi \|_1 \leq \| \varphi \| + \| A\varphi \| \))

\[ \Rightarrow (A\varphi, \varphi) + (\varphi, A\varphi) = 0 \text{ and } A \text{ has compact resolvent.} \]
Examples

Wave Equation

2 - dim wave equation
\[
\frac{\partial^2 x}{\partial^2 t} = \left( \frac{\partial^2 x}{\partial^2 z_1} + \frac{\partial^2 x}{\partial^2 z_2} \right) + \gamma x
\]

\[\Rightarrow x_i = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \frac{\partial x}{\partial z_1} + \begin{bmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \frac{\partial x}{\partial z_2} + \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ -1 & -1 & 0 & 1 \\ 0 & 0 & \gamma & 0 \end{bmatrix} x \]

where \[x \equiv \begin{bmatrix} x_{z_1} \\ x_{z_2} \\ x \\ x_t \end{bmatrix}\]

Relativistic Fields (Mandl & Shaw 2010)

Dirac Equation:
\[
\frac{\partial \phi}{\partial t} = -c \left( \sum_{i=1}^{3} A_i \frac{\partial \phi}{\partial x_i} \right) + \left( i \frac{mc^2}{\hbar} \right) \phi
\]

Pauli Spin Matrices
Stability via Lyapunov-Barbalat

Nonlinear Dynamics \[ \dot{x} = f(x) \]
\[ x(0) = x_0 \in \mathbb{R}^N \]

Find Energy-like Function: \( V(x) \)
\[ V(x) > 0 \text{ when } x \neq 0 \]
\[ V(0) = 0 \]
\[ \dot{V} = \text{grad} V \cdot f(x) < 0 \implies x(t) \to 0 \text{ as } t \to \infty \text{ for all } x_0 \]

\[ \dot{V} \leq 0 \implies \text{All trajectories } x(t) \text{ are bounded} \]

From Barbalat's lemma:
\[ \dot{V}(t) \leq 0 \text{ and uniformly continuous } \implies \dot{V}(t) \to 0 \text{ as } t \to \infty \]
Infinite-Dimensional Lyapunov-Barbalat Theory: PDE & Delay Systems

**X Hilbert or Banach Space**

Let

\[
V(t, x, \Delta G) \equiv V(t, x) + \frac{1}{2} tr(\Delta G \gamma^{-1} \Delta G^T)
\]

with \( x(t) = U(t)x_0 \in X; t \geq 0 \)

Theorem: If

\[
\begin{cases}
\alpha(\|x, \Delta G\|) \leq V(t, x, \Delta G) \leq \beta(\|x, \Delta G\|)
\end{cases}
\]

and

\[
\frac{dW(x(t))}{dt} = (\frac{\partial W}{\partial x}) \frac{\partial x(t)}{\partial t}
\]

is bounded, then \( W(x(t)) \to 0 \) and \( \Delta G \) bounded.

If \( W(x) \) is coercive in the partial state \( x \), or \( W(x) \geq \gamma(\|x\|) \), then \( x(t) \to 0 \).
Adaptive Model Tracking in the Presence of Persistent Disturbances

\[ \dot{x} = Ax + Bu + \Gamma u_D \]
\[ y = Cx \]

\[ \Delta x \equiv x - x_* \xrightarrow{t \to \infty} 0 \text{ or } N(0, R_*) \]

Reference Model

\[ \begin{cases} 
\dot{x}_m = A_m x_m + B_m u_m \\
y_m = C_m x_m 
\end{cases} \]

Controller

Adaptive Gain Law

\[ \dot{G} = [\dot{G}_e \quad \dot{G}_u \quad \dot{G}_m \quad \dot{G}_D] = -h(e_y, u_m, y_m, \phi_D) \]

\[ u_D = \theta L \phi_D \]

\[ y \xrightarrow{t \to \infty} 0 \text{ or } N(0, R_e) \]
Adaptive Control Law

\[ u = G_u u_m + G_m w_m + G_D \phi_D + G_e e_y \]

where

\[
\begin{align*}
\dot{G}_u &= -e_y \cdot u_m^T \\
\dot{G}_m &= -e_y \cdot x_m^T \\
\dot{G}_D &= -e_y \cdot \phi_D^T \\
\dot{G}_e &= -e_y \cdot e_y^T
\end{align*}
\]

Gain Adaptation Laws

Using Ideal Trajectories
Long Ago and Far Away
Figure 1-1. Hubble Space Telescope
Deployable Optical Telescope DOT

Primary Mirror Supports

AFRL-Kirtland
Deployable Optical Telescope Experiment

Evolving Systems

Evolving Systems = Autonomously Assembled Active Structures

Or Self-Assembling Structures, which Aspire to a Higher Purpose; Cannot be attained by Components Alone

NASA-JPL & MSFC
Genetics of Evolving Systems: Inheritance of Component Traits

- Controllability/Observability
- Stability
- Optimality
- Robustness
- Disturbance Rejection/Signal Tracking

Stability is Necessary During the Entire Evolution Process

Ironic: Synthetic Biology???
Composability in Synthetic Biology

“It is difficult to define signal exchanges between biological units unambiguously”
F-16 Flexible Structure Model: Fluid-Structure Interaction

USAF-Edwards AFB
Flight Test Center
One Possible Solution

Aerodynamically Shaped Graduate Student
“It is surprising how quickly we replace a human operator with an algorithm and call it SMART”
Wind Energy

1979: 40 cents/kWh

2000: 4 - 6 cents/kWh

2006: 3 - 5 cents/kWh

- R & D Advances
- Increased Turbine Size
- Manufacturing Improvements
- Large Wind Farms

NREL-NWTC
Flow Control of Wind Turbine Aerodynamics
Power System
Perturbed with a Wind Farm

- When a wind farm is placed at a distance of $\alpha$, the perturbed power system becomes:

\[
\frac{\partial^2 \delta(u, t)}{\partial t^2} + \eta \frac{\partial \delta(u, t)}{\partial t} - v^2 \frac{\partial^2 \delta(u, t)}{\partial u^2} = W(u, t)
\]

with

\[ W(u, t) = P_g(t) \delta(u - \alpha) \]

- Power flow at a distance $u$ is:

\[
p(u, t) = -\frac{1}{\gamma} \frac{\partial \delta(u, t)}{\partial u}
\]
Adaptive Control in Quantum Information Systems

This might be the most fundamental application of direct adaptive control

Merde
Quantum Computing

A Quantum computer will operate differently from a Classical one. It will be involved with physical systems on an atomic scale, e.g., atoms, photons, trapped ions, or nuclear magnetic moments.

Could be improved with Adaptive Control
So Quantum Error Correction can work!!!
Erwin Schrödinger’s Cat (1935)

At “half-life of particle, cat is dead and alive!
“superposition”

\[ \Psi = | \bigcirc \rangle | \text{cat} \rangle + | \bigcirc \rangle | \text{dead cat} \rangle \]
Quantum Basics
(Dirac & Von Neumann)

Observable $A : X \xrightarrow{\text{bounded self-adjoint}} X$

Compact Resolvent $\Rightarrow Ax = \sum_{k=1}^{\infty} \lambda_k (x, \varphi_k) \varphi_k$

Pure States: $\varphi_k$ eigenfunctions of $A$

State $\varphi \in X$ complex Hilbert Space:

$(\varphi, \varphi) = 1 \text{ or } \|\varphi\| = 1 \Rightarrow \varphi = \sum_{k=1}^{\infty} c_k \varphi_k \text{ & } 1 = \|\varphi\|^2 = \sum_{k=1}^{\infty} |c_k|^2$

$:\therefore "\text{A state is a convex combination of pure states}"$
**Schrodinger Wave Equation**

\[ i\hbar \frac{\partial \varphi}{\partial t} = H_0 \varphi + H_C(u)\varphi \]

\( \varphi \in X \) complex Hilbert Space

\( \varphi \) with Diffusion with Imaginary Time:
\[ t_{\text{NEW}} = \frac{it_{\text{OLD}}}{\hbar} \]

\( \Rightarrow U_0(t) : X \rightarrow X \) Unitary Group (reversible)

\[ U_0(t)\varphi = \sum_{k=1}^{\infty} e^{i\lambda_k t} \langle \varphi, \phi_k \rangle \phi_k \text{ with } \langle \phi_k, \phi_l \rangle = \delta_{kl} \]

\( \Rightarrow \) Discrete Real Spectrum \( \sigma(H_0) = \{\lambda_k\}_{k=1}^{\infty} \)

\(-\infty \rightarrow \)

Marginally Stable

Self–Adjoint
Compact
Resolvent
Control
Hamilton–ian

Unitary Group (reversible)
Quantum Measurement

Entanglement

\[ X = X_S \otimes X_M \]

\[ \varphi = \sum_{k,l} \alpha_{kl} (\varphi_k^S \otimes \varphi_i^M) \neq h \otimes w \]

Ontology (what is) vs Epistemology (What is measured)
Uncertainty Principle

Observable $A : X \xrightarrow{\text{bounded self-adjoint}} X$ Hilbert

\[ \begin{align*}
\text{Mean } \langle A \rangle & \equiv \text{Tr}(\rho A) \\
\text{Dispersion } \Delta A & \equiv \sqrt{\text{Tr}(\rho (A - \langle A \rangle)^2)}
\end{align*} \]

Heisenberg Uncertainty Principle: Simultaneous Measurement of $A$ & $B$

\[ (\Delta A)^2 (\Delta B)^2 \geq \frac{\hbar}{2} \left| \text{Tr}(\rho [A, B]) \right|; \text{ commutator } [A, B] \equiv AB - BA \]

Recall \( (\Delta z)^2 (\Delta p)^2 \geq \frac{\hbar}{2} \left| \text{Tr}(\rho [z, p]) \right| = \left( \frac{\hbar}{2} \right)^2 \)

A Hilbert Space Property due to Cauchy - Schwarz Inequality

\[ 0 \leq \langle x, y \rangle \leq \|x\| \|y\| \]

In fact ALL of Quantum Mechanics is based on a Hilbert space of states and a (C*) algebra of bounded linear self adjoint observables.
Small Quantum Systems

- We can begin to experiment with just one electron, atom or small molecule

- Need:
  
  Precise control
  
  Isolation from the environment
  
  Simple small systems: single particles or small groups of particles

…….. David Wineland  NIST
Control of Individual Quantum Systems: Quantum Feedback Loop

Purpose:
Use information from weak QND measurements to prepare photon number (Fock) states of a cavity field and protect them against decoherence.

Method:
Quantum feedback realized by atoms as QND probes and small coherent field injections into the cavity mode as an actuator.
Adaptive Quantum Model Tracking to Reduce Decoherence

Reference Model:
Closed System

Desired Hamiltonian
\[ H^*_y \]

Open Physical System

\[ u_D = \theta L \phi_D \]

QND Measurement & Quantum Error Correction

Adaptive Quantum Controller

Adaptive Gain Law
\[ \dot{G} = \begin{bmatrix} \dot{G}_e & \dot{G}_u & \dot{G}_m & \dot{G}_D \end{bmatrix} = -h(e_y, u_m, y_m, \phi_D) \]
“Physics is like sex: sure, it may give some practical results, but that's not why we do it.”
— Richard P. Feynman

In a tile motif on the back of the Ross Dress For Less building on Lake Ave, Pasadena, CA