Passive Optical Network Configurations: Performance Analysis

TUTORIAL

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Structure

- Access Networks
- Passive Optical Networks
- Performance Analysis of EPONs
- Performance Analysis of WDM-TDMA PONs
- Performance Analysis of OCDMA PONs

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- **Access Networks**
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Access Networks

Application [1]	Bandwidth	Latency	Other requirements
Voice over IP	64 Kb/s	200 ms	Protection
Videoconferencing	2 Mb/s	200 ms	Protection
File sharing	3 Mb/s	1 s	
SDTV	4.5Mb/s/channel	10 s	Multicasting
Interactive gaming	5 Mb/s	200 ms	
Telemedicine	8 Mb/s	50 ms	Protection
Real-time video	10 Mb/s	200 ms	Content distribution
Video on demand	10 Mb/s/channel	10 s	Low packet loss
HDTV	10 Mb/s/channel	10 s	Multicasting

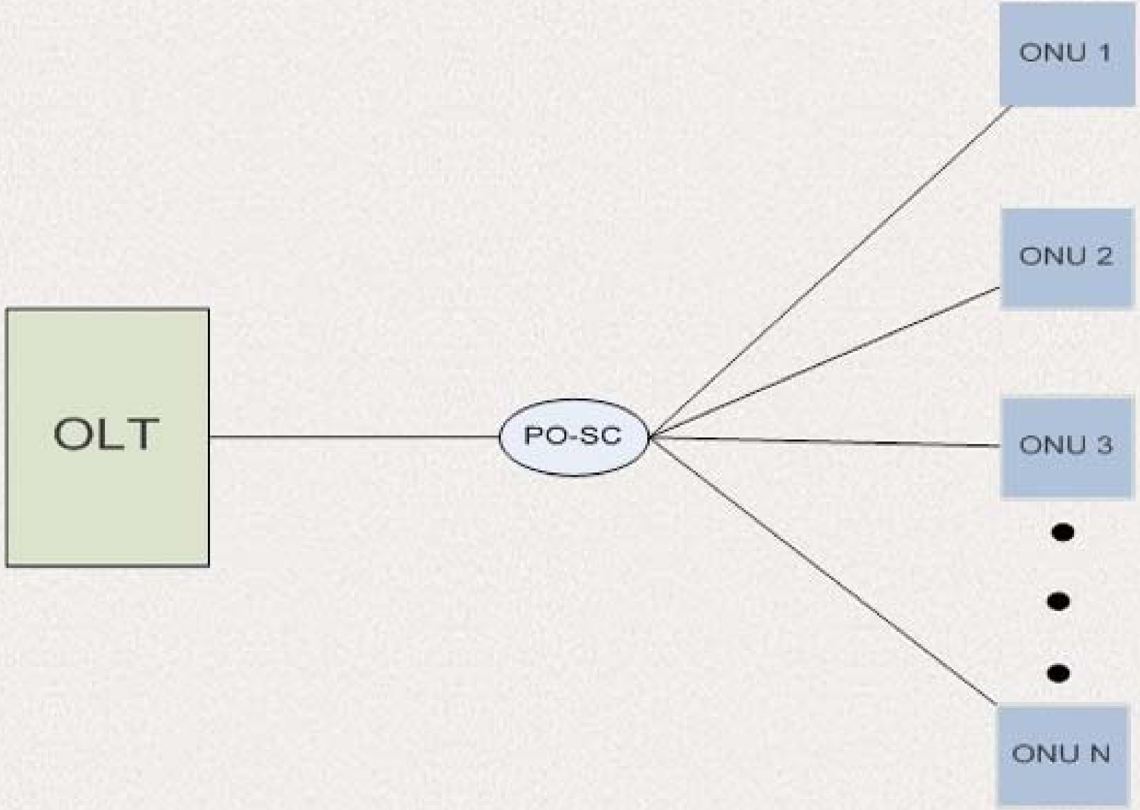
Service [1]	Medium	Downstream (Mb/s)	Upstream (Mb/s)	Max Reach (Km)
ADSL	Twisted pair	8	0.896	5.5
ADSL2	Twisted pair	15	3.8	5.5
VDSL1	Twisted pair	50	30	1.5
VDSL2	Twisted pair	100	30	0.5
HFC	Coax cable	40	9	25
BPON	Fiber	622	155	20
GPON	Fiber	2488	1244	20
EPON	Fiber	1000	1000	20
Wi-Fi	Free space	54	54	0.1
WiMAX	Free space	134	134	5

[1] L.G. Kazousky, N. Cheng, W. Shaw, D. Gutierrez, S. Wong, *Broadband Optical Access Networks*, Wiley, 2011.

Structure

- Access Networks
- **Passive Optical Networks**
- Performance Analysis of EPONs
- Performance Analysis of WDM-TDMA PONs
- Performance Analysis of OCDMA PONs

Passive Optical Network (PON) (1/3)



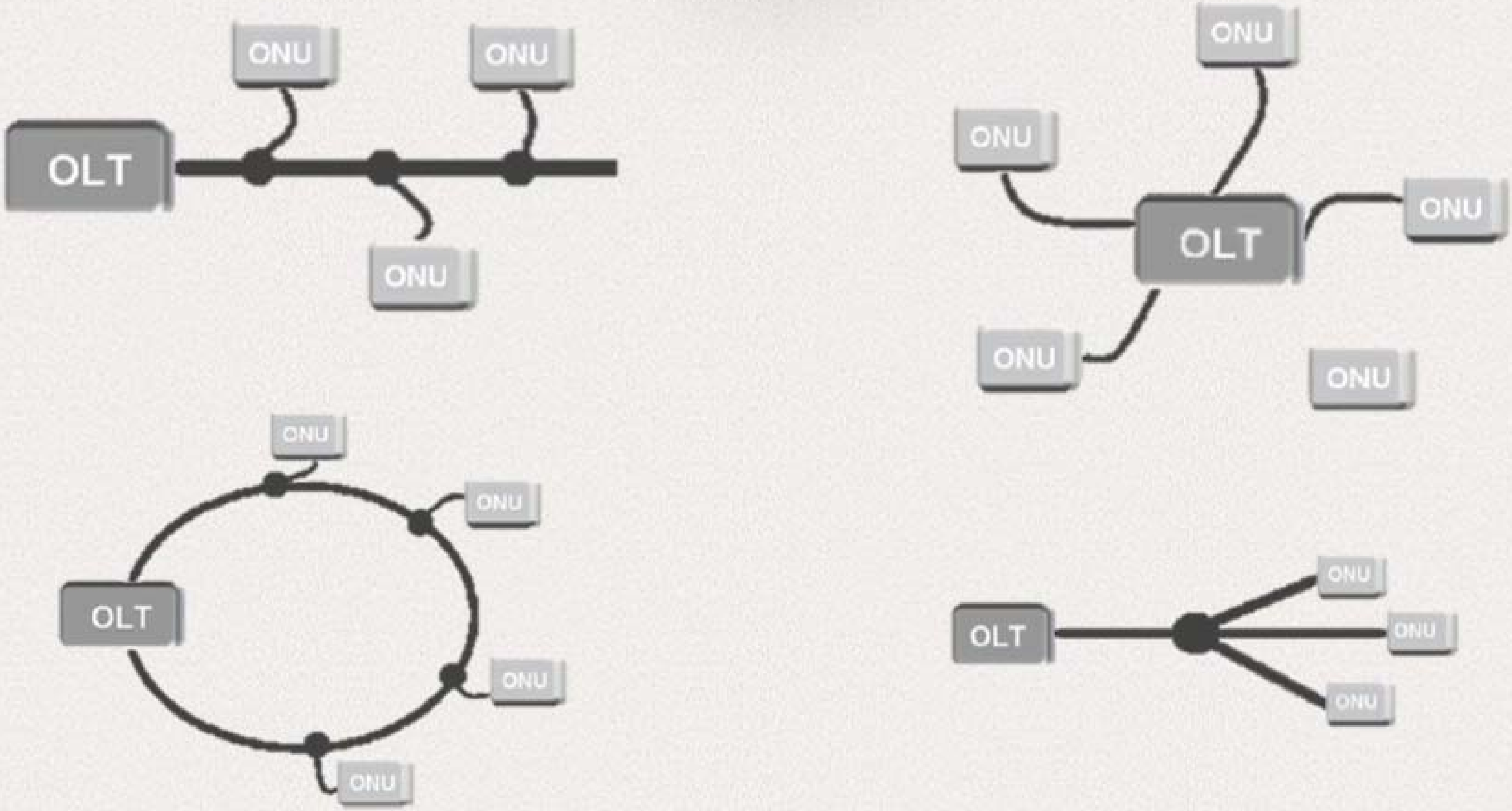
- ❑Optical Line terminal (OLT)
- ❑Optical Network Unit (ONU)
- ❑Passive Optical Splitter/Combiner (PO-SC)

Advantages

- ✓ High Bandwidth
- ✓ High transmission distance
- ✓ Less installed fiber (compared to point-to-point solutions)
- ✓ No active elements (compared to active access networks)
- ✓ Transparent→ Easy to upgrade

Passive Optical Network (PON) (2/3)

PON topologies [2]



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[2] I. Chochliouros and G. Heliotis, *Optical Access Networks and Advanced Photonics*, IGI Global, 2010.

Passive Optical Network (PON) (3/3)

Different multiple access techniques can be applied in PONs

- TDMA-PONs
- WDM PONs
- OCDMA-PONs
- OFDMA-PONs

Passive Optical Network (PON) (3/3)

TDMA-PONs

- Asynchronous Transfer Mode PON (ATM-PON)
- Broadband PON (BPON)
- Gigabit PON (GPON)
- Ethernet PON (EPON)

TDMA PONs – The ATM PON and BPON

- ✓ ITU-T G.983.x standards
- ✓ Upstream wavelength 1310 ± 50 nm
- ✓ Downstream wavelength 1530 ± 50 nm
- ✓ Different permissible data rates
- ✓ Static Bandwidth Allocation (G.983.1)
- ✓ Dynamic Bandwidth Allocation (G.983.4)

Downstream (MB/s)	Upstream (MB/s)
155.52	155.52
622.08	155.52
622.08	622.08
1244.16	155.52
1244.16	622.08

Downstream Frame Format


PLOAM 1
ATM Cell 1 to 27
PLOAM 2
ATM Cell 27 to 54
...
PLOAM
ATM
54 cells + 2 PLOAM

Upstream Frame Format


3 bytes
ATM Cell 1
3 bytes
ATM Cell 2
......
3 bytes
ATM Cell 54
53 cells

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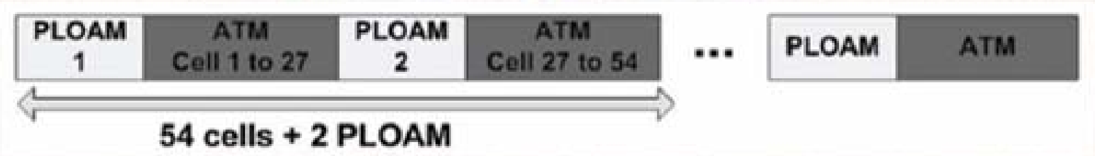
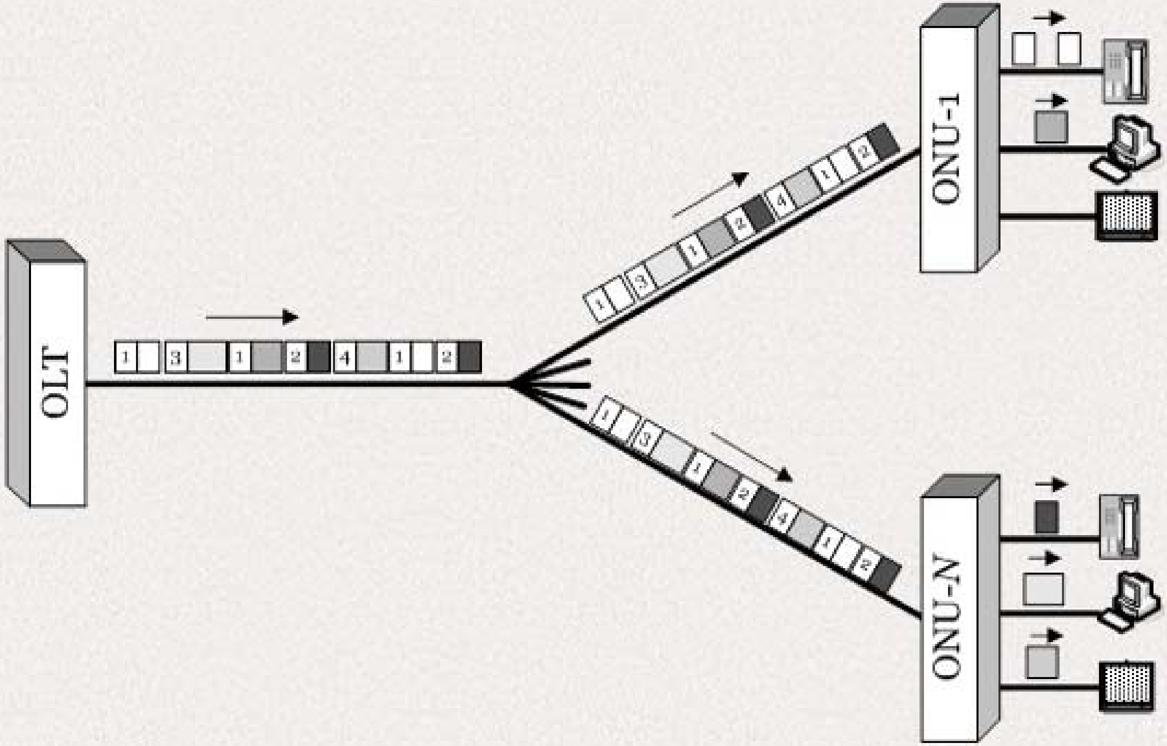
Downstream Frame Format



Upstream Frame Format

TDMA PONs – The EPON (1/2)

- ✓ IEEE 802.3ah standard
- ✓ Support of variable length packets up to 1518 bytes in fixed-length frame of 2 ms[3]
- ✓ Time-slot size 125 or 250 ms
- ✓ Upstream wavelength 1310 nm
- ✓ Downstream wavelength 1500±10 nm
- ✓ Dynamic Bandwidth Allocation
- ✓ Support of different QoS requirements



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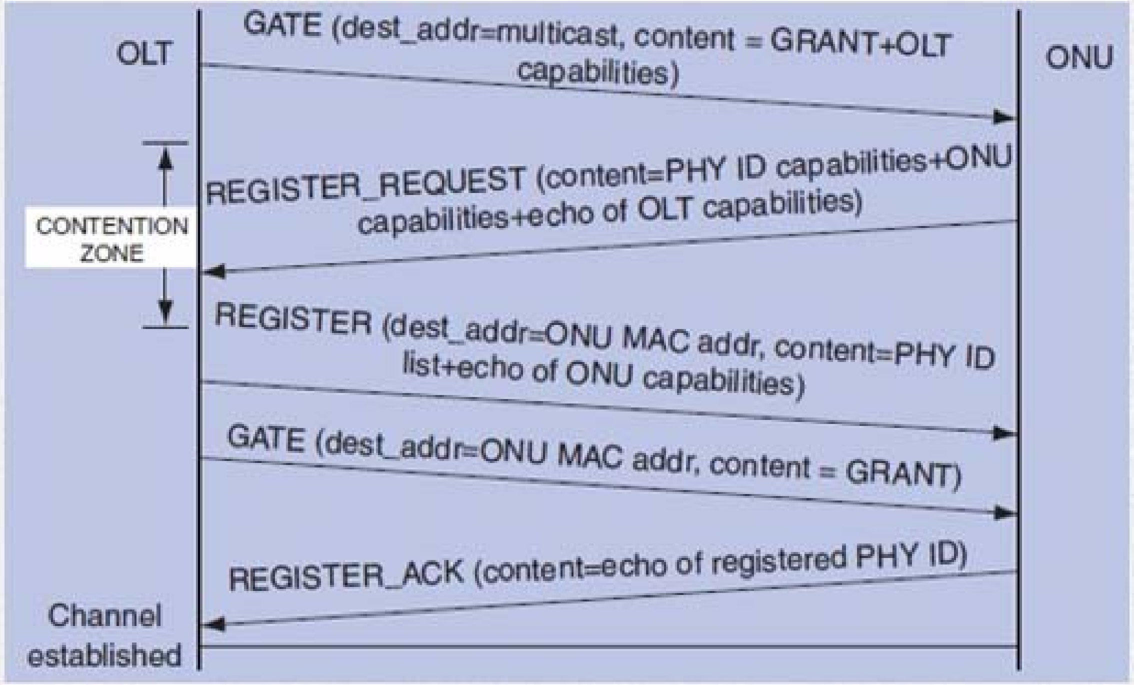
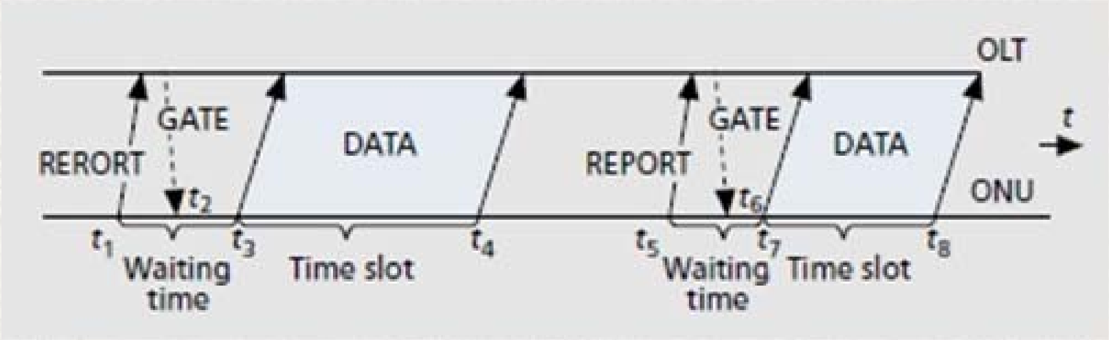
[3] T. Koonen, "Fiber To The Home/ Fiber To The Premises: What, Where and When", *Proceedings of IEEE,* Vol. 94, No. 5, May 2006, pp.911-934.

TDMA PONs – The EPON (2/2)

Multi-Point Control Protocol (MPCP)

- ✓ Performs ranging, bandwidth arbitration and discovery functions
- ✓ For different bandwidth allocation algorithms [4]
- ✓ Use of 64-byte MAC control messages
- ✓ Bandwidth allocation is performed by using the GATE and REPORT control messages [5]





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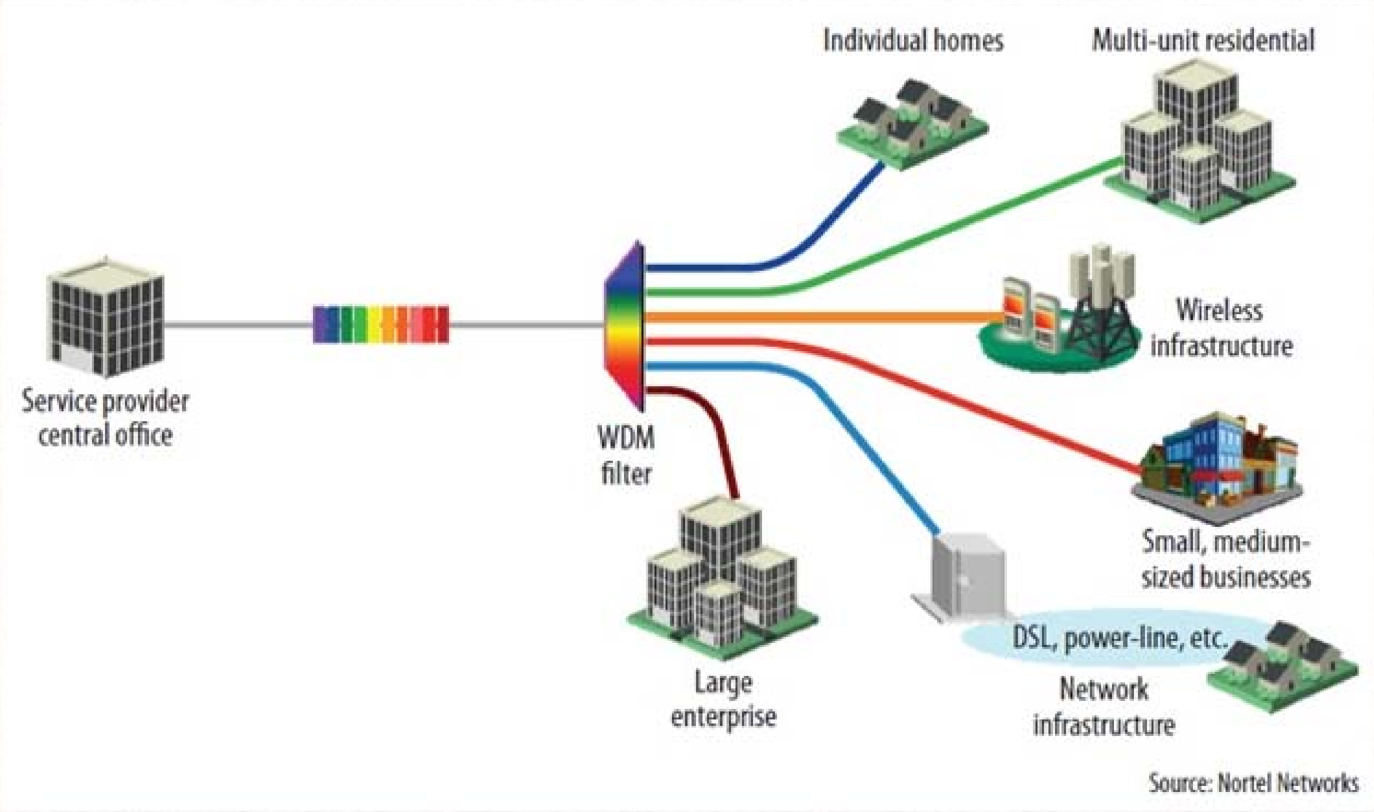
[4] M. Ma, *Current Research progress of Optical Networks*, Springer, 2009.

[5] Y. Luo and N. Ansari, “Bandwidth Allocation For Multiservice Access on EPONs”, *IEEE Optical Communications*, February 2005, pp. S16-S21

WDM PONs

Application of Wavelength Division Multiplexing in PONs

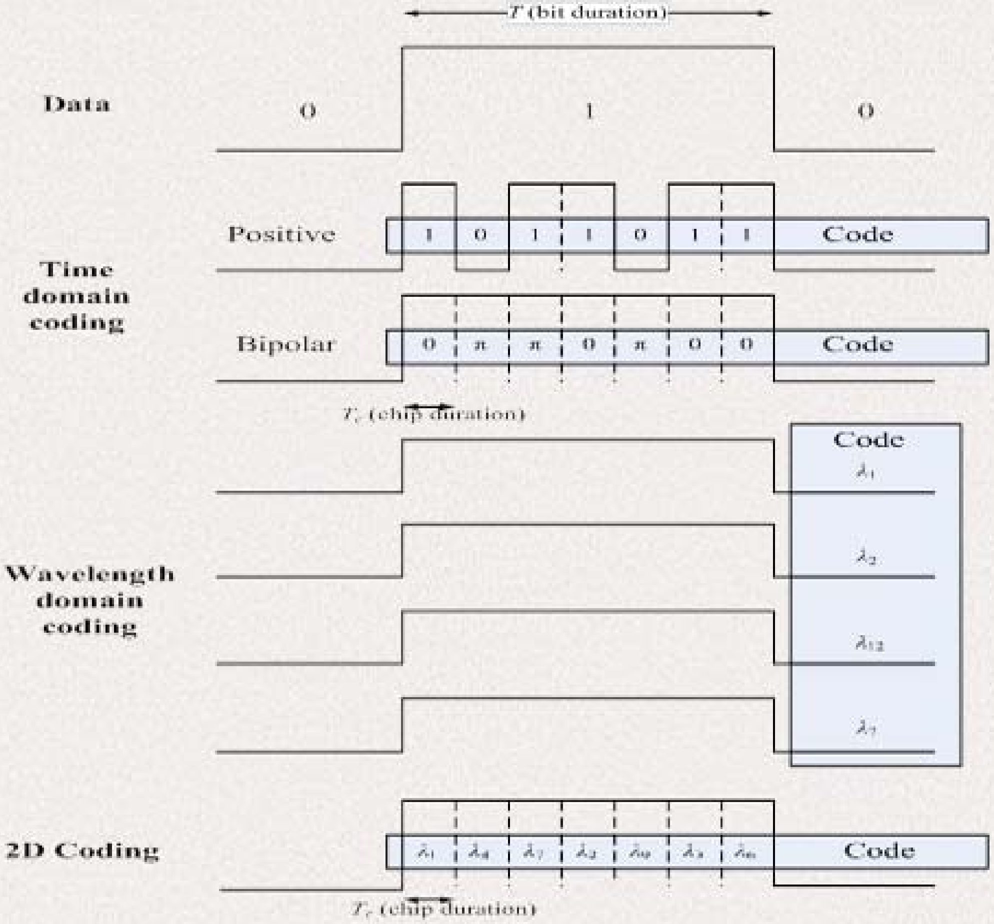
- ✓ Combination of PON's efficiency with WDM's high bandwidth
- ✓ Use of 1 wavelength per ONU -> increased security
- ✓ The optical power is not divided → longer transmission range (40-100 Km)
- ✓ 2.5-10 Gb/s per wavelength
- ✓ Coarse WDM → up to 16 wavelengths
- ✓ Dense WDM → 16, 32, 64 128 wavelengths

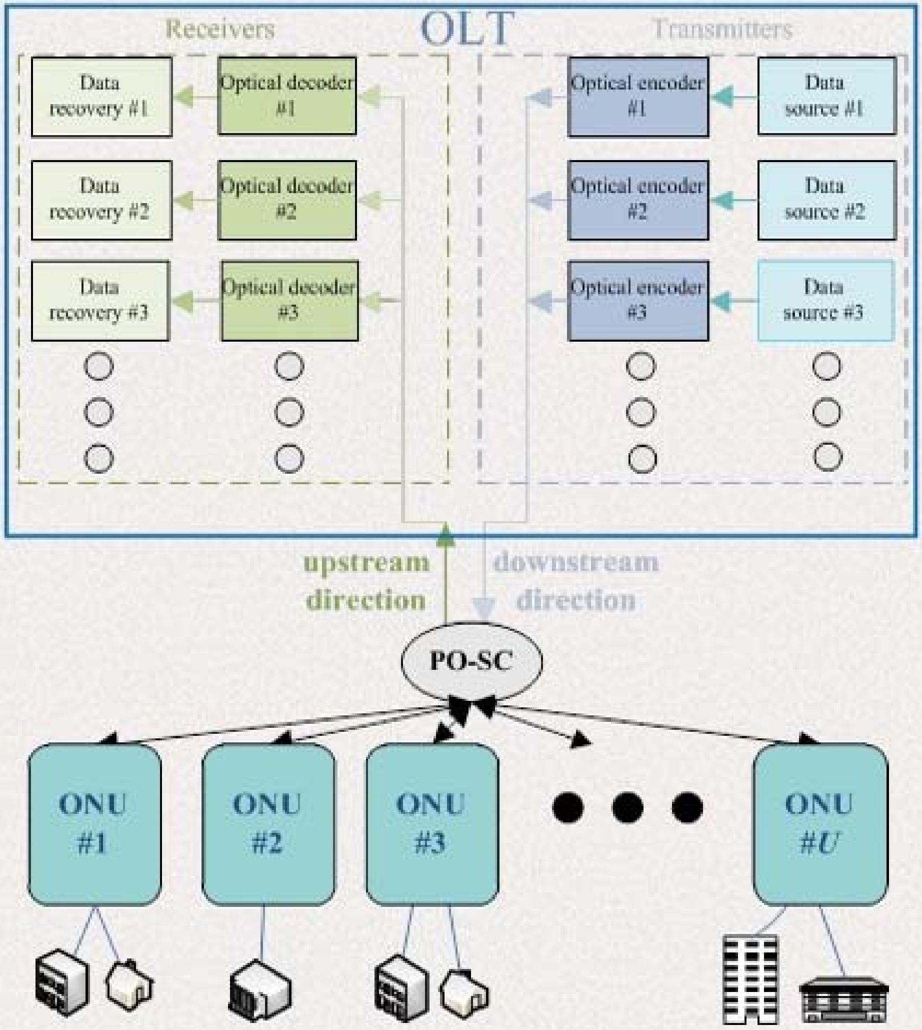


Source: Nortel Networks

OCDMA PONs

- ✓ Each communication channel is distinguished by a specific optical code
- ✓ Encoding involves multiplying the data bit by a code either in the time domain, the wavelength domain or a combination.
- ✓ Unwanted signals appear as noise → MAI





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IPACT – A DBA Algorithm for EPONs

Interleaved Polling with Adaptive Cycle Time

- ✓ Dynamic Bandwidth Allocation algorithm
- ✓ Use of GATE and REPORT messages of MPCP
- ✓ ONUs send REPORT to inform about the status of their queues
- ✓ OLT calculates the time-slots and responses with GATE messages

Max Time-slot $W_i^{\max}$

Max polling cycle time

Bandwidth of ONU$_i$

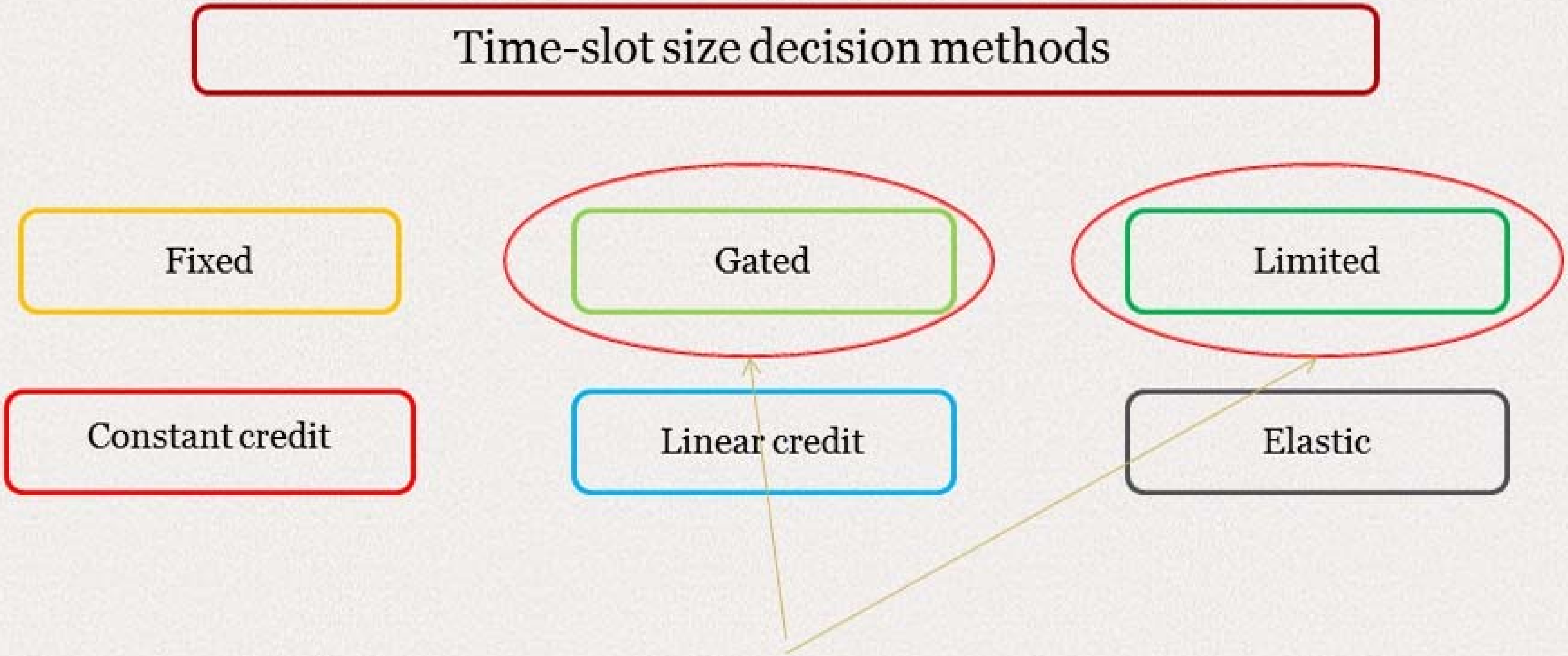
$$T^{\max} = \sum_{i=1}^{N}(G + \frac{W_i^{\max}}{R_N})$$

$$B_i^{\min} = \frac{W_i^{\max} - W^{REPORT}}{T^{\max}}$$

G → Guard time

R_N → Data rate of upstream channel

IPACT – A DBA Algorithm for EPONs



Time-slot size decision methods

Fixed

Gated

Limited

Constant credit

Linear credit

Elastic

Lannoo B., Verslegers L., Colle D., Pickavet M., Gagnaire M., Demeester P., "Analytical model for the IPACT dynamic bandwidth allocation algorithm for EPONs," *Journal of Optical Networking*. Vol. 6, No. 6, pp. 677-688 June 2007.

Analysis of the Gated service in EPONs

Cycle time has no maximum value
There is a minimum value:

$$T_{cycle}^{\min} = T_{fiber} + T_{proc}$$

$T_{fiber} \rightarrow$ Transmission time (ONU ⇆ OLT)
$T_{proc} \rightarrow$ Processing time

The probability of having more packet arrivals than can be sent in a minimum cycle time

$$1 - \sum_{k=0}^{P} \left(\exp\left(-\frac{\Lambda}{B} T_{cycle}^{\min} \right) \frac{\left(\frac{\Lambda}{B} T_{cycle}^{\min} \right)^{k}}{k!} \right) \Longleftarrow P = \left\lfloor \left(T_{cycle}^{\min} - N\left(T_{guard} + \frac{B_{req}}{R_U} \right) \right) \frac{R_U}{B + B_{eth}} \right\rfloor$$

```
if probability < 0.05
    Low traffic-load analysis
else
    High traffic-load analysis
end
```

Low traffic-load analysis of the Gated service in EPONs

Average cycle time

$$\bar{T}_{cycle} = T_{cycle}^{\min} + (1+C)\frac{\lambda}{B}T_{cycle}^{\min}\frac{B+B_{eth}}{R_U}$$

$C \rightarrow$ clustering factor, $C=(N-1)/(P+1)$

Average waiting time

$$\bar{W}_{low} = \frac{3}{2}\bar{T}_{cycle}$$

Symbol	Explanation	Value
N	Number of ONUs	16
λ	ONU arrival rate (Poisson traffic)	from 5 to 57.5 Mbits/s
T_{fiber}	Two-way delay on the EPON	200 μs
T_{proc}	Processing time	35 μs
T_{guard}	Guard time	1.5 μs
B	Packet size (network layer)	12,000 bits (=1500 bytes)
B_{eth}	Ethernet overhead	304 bits (=38 bytes)
B_{req}	REPORT or request message size	576 bits (=64 bytes+8 bytes preamble)
R_U	Upstream bandwidth on the EPON	1 Gbits/s
$P_{\max}$	Maximum transmission window (fixed or limited)	10 packets

High traffic-load analysis of the Gated service in EPONs

The cycle time only takes discrete values:

$$T_{cycle}^{m} = \frac{m(B + B_{eth}) + NB_{req}}{R_U} + NT_{guard} \quad \text{for } m \geq 0$$

The minimum value of the cycle time:

$$T_{cycle}^{\min'} = T_{cycle}^{k} \ \text{ with } k = \min\left\{ m : \frac{m(B + B_{eth}) + NB_{req}}{R_U} + NT_{guard} > \bar{T}_{cycle} \right\}$$

Queuing model with Poisson arrivals

$$p_{i,j} = \Pr[T_{cycle}(n+1) = T_{cycle}^{j+k} \mid T_{cycle}(n) = T_{cycle}^{i+k}] = \exp\left(-\frac{\Lambda}{B} T_{cycle}^{i+k}\right) \frac{\left(\frac{\Lambda}{B} T_{cycle}^{i+k}\right)^{j}}{j!} \qquad i \geq 0, j > 0$$

Steady state equations

$$\pi P = \pi \qquad \tilde{\pi} = \frac{\pi_i T_{cycle}^{i+k}}{\sum_{j=0}^{M} \pi_j T_{cycle}^{i+k}}$$

Average waiting time

$$\bar{W} = \frac{3}{2} \sum_{j=0}^{M} \pi_j T_{cycle}^{i+k}$$

Analysis of the Limited service in EPONs

Similar analysis as in the Gated service

Low traffic-load

High traffic load

$$\overline{W}_{low} = \frac{3}{2}\overline{T}_{cycle}$$

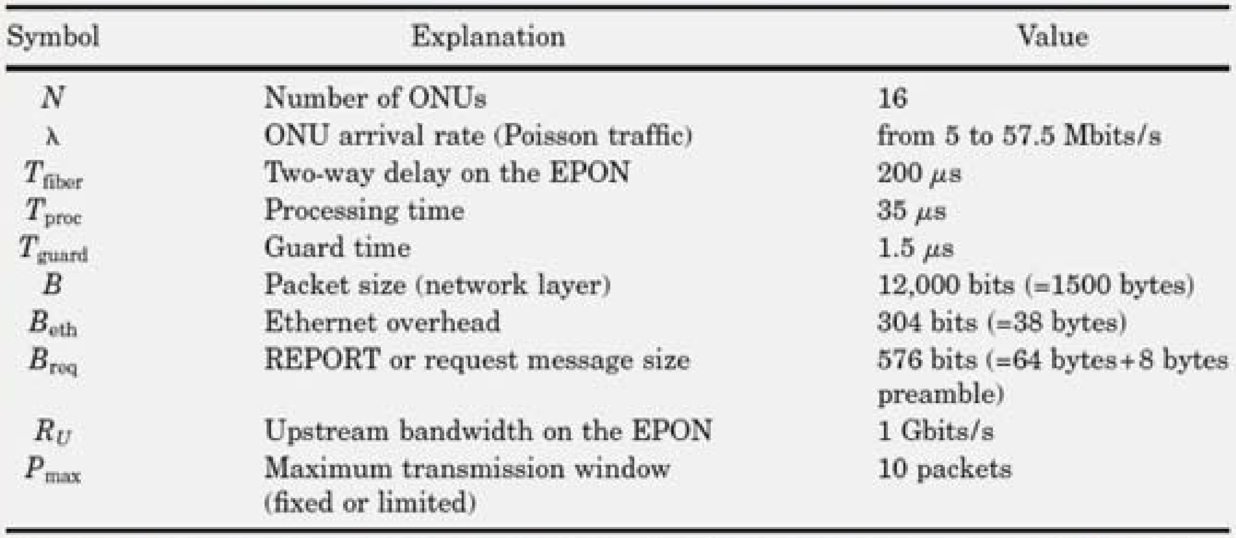
$$\overline{W} = \frac{3}{2}\sum_{j=0}^{M}\pi_j T_{cycle}^{i+k}$$

The cycle time is limited by a maximum value of packets $P_{\max}$

$$T_{cycle}^{\max} = \frac{NP_{\max}(B + B_{eth}) + NB_{req}}{R_U} + NT_{guard}$$

Results (1/2)

Comparison of analytical and simulation results



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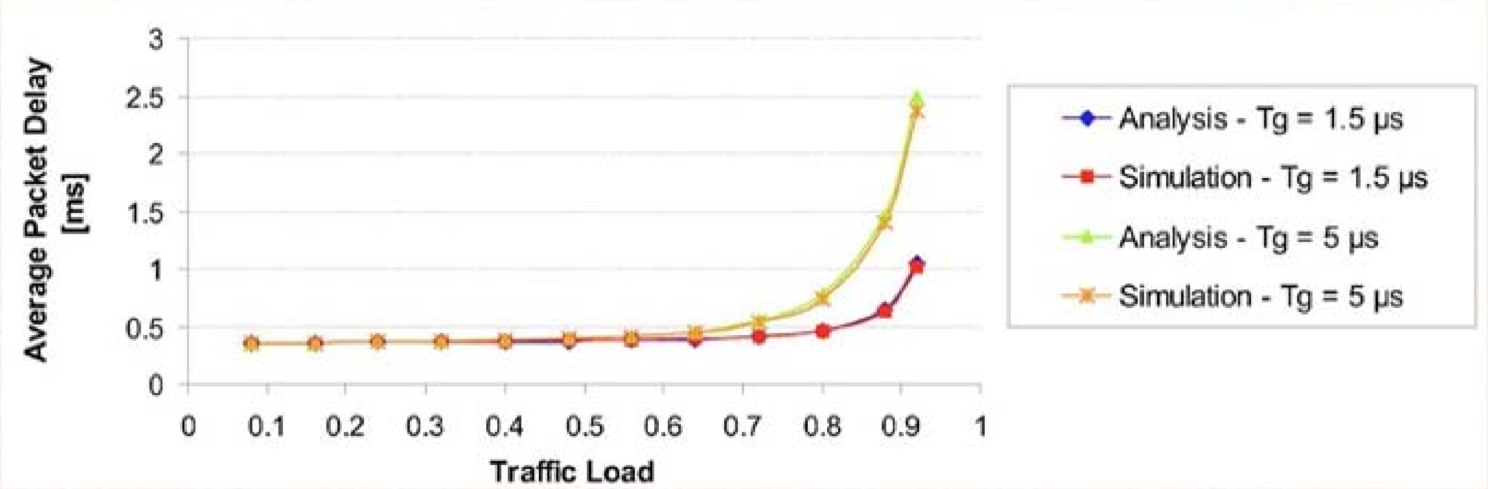
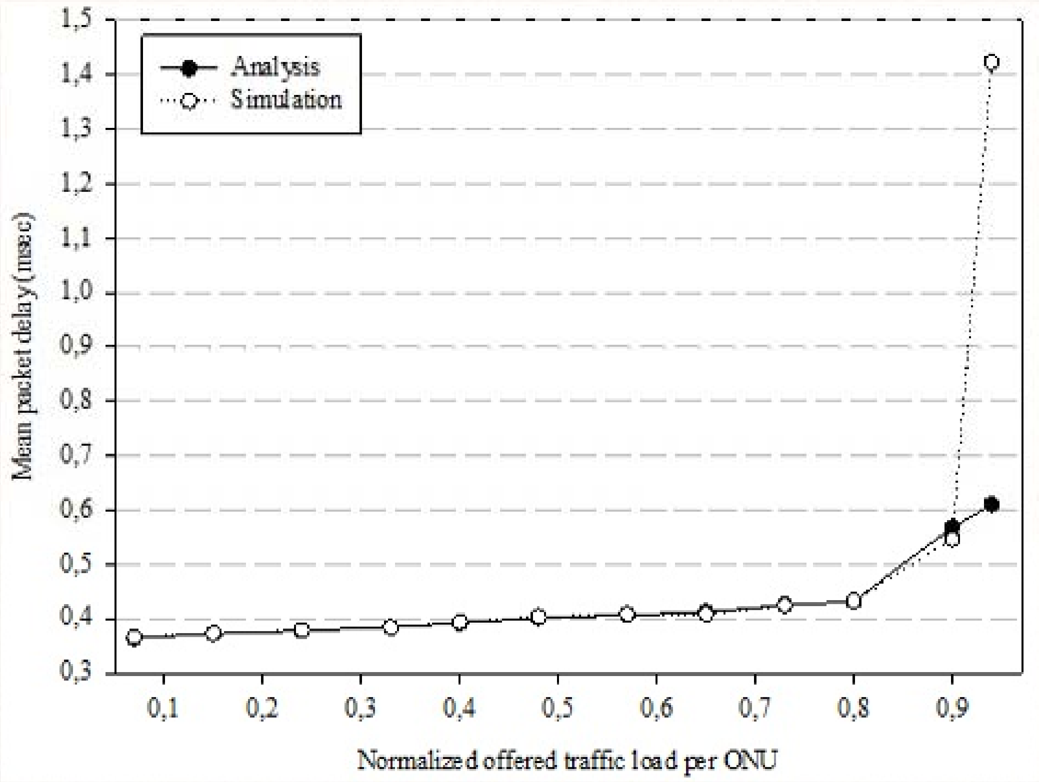
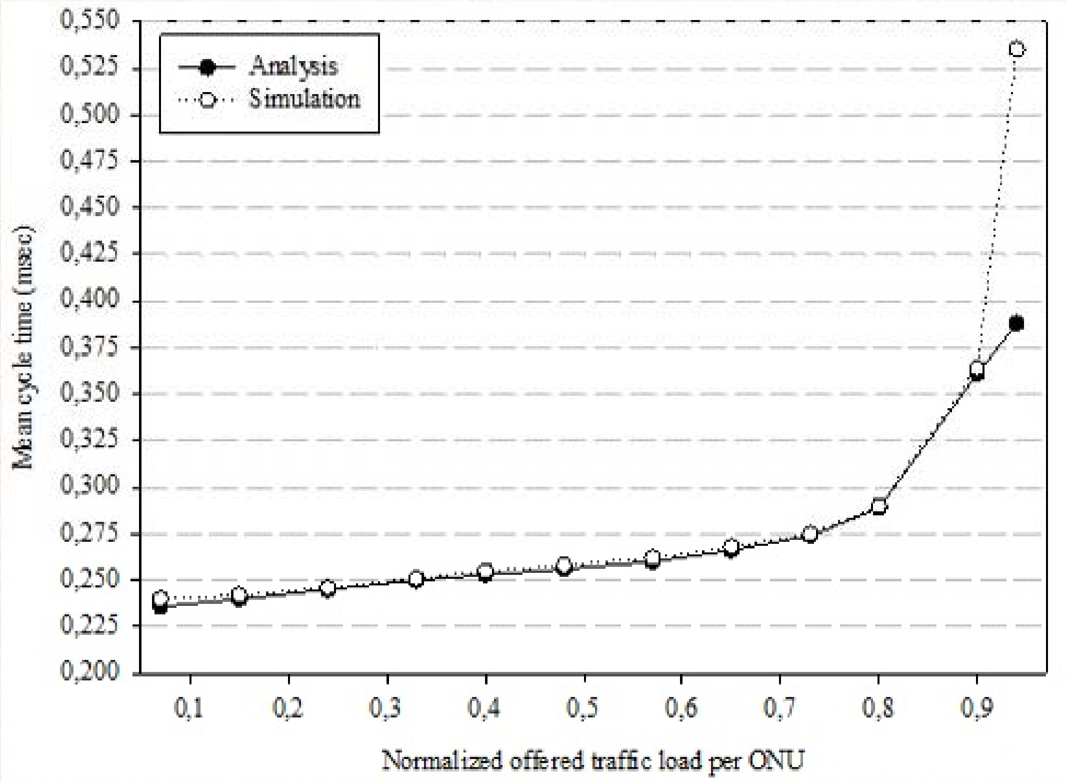
Results for the Gated service



Results (2/2)

Results for the Limited service





Analysis of the Fixed service in EPONs with multiple service-classes

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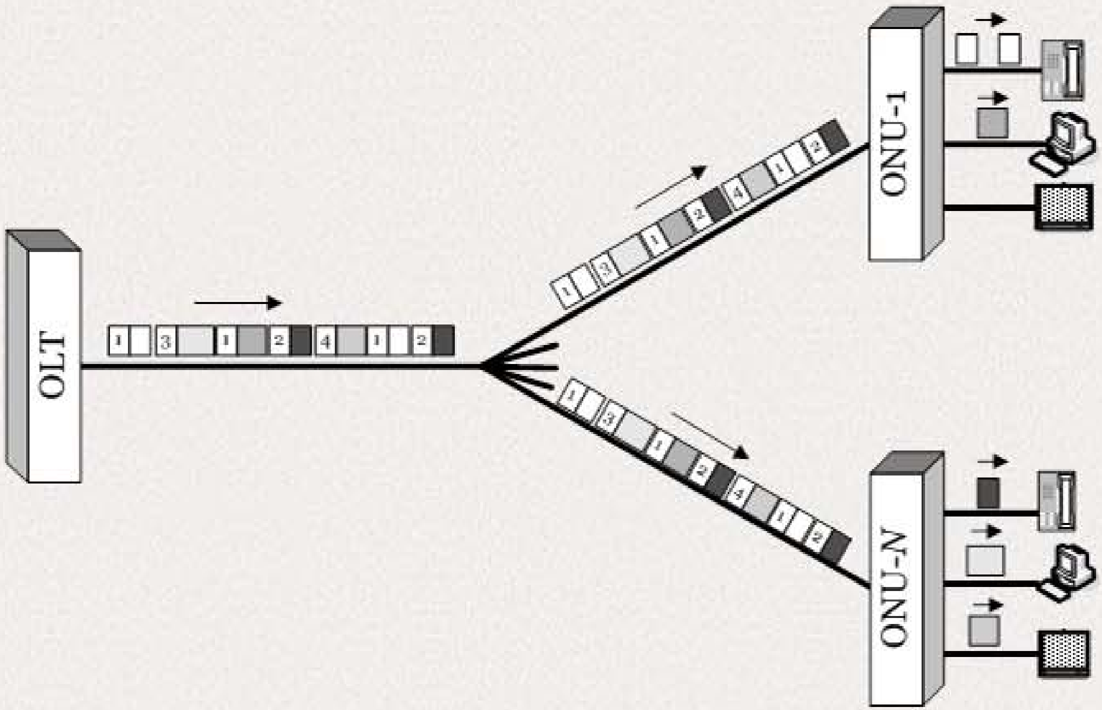

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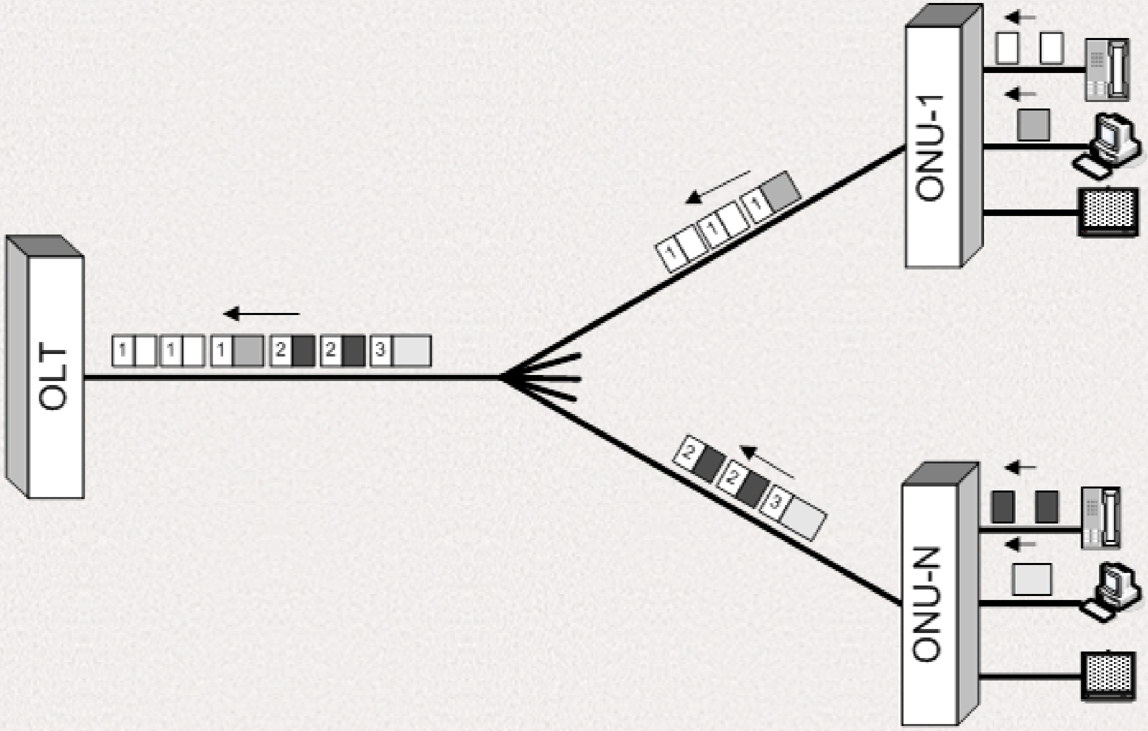

1. B. Lannoo, L. Verslegers, D. Colle, M. Pickavet, M. Gagnaire, and P. Demeester, "Analytical model for the IPACT dynamic bandwidth allocation algorithm for EPONs," J. Opt. Netw. 6, 677-688 (2007).
2. F. Aurzada, M. Scheutzow, M. Herzog, M. Maier, and M. Reisslein, "Delay analysis of Ethernet passive optical networks with gated service," J. Opt. Netw. 7, 25-41 (2008).
3. Bhatia, S.; Garbuzov, D.; Bartos, R.; , "Analysis of the Gated IPACT Scheme for EPONs," *Communications, 2006. ICC '06. IEEE International Conference on* , vol.6, no., pp.2693-2698, June 2006

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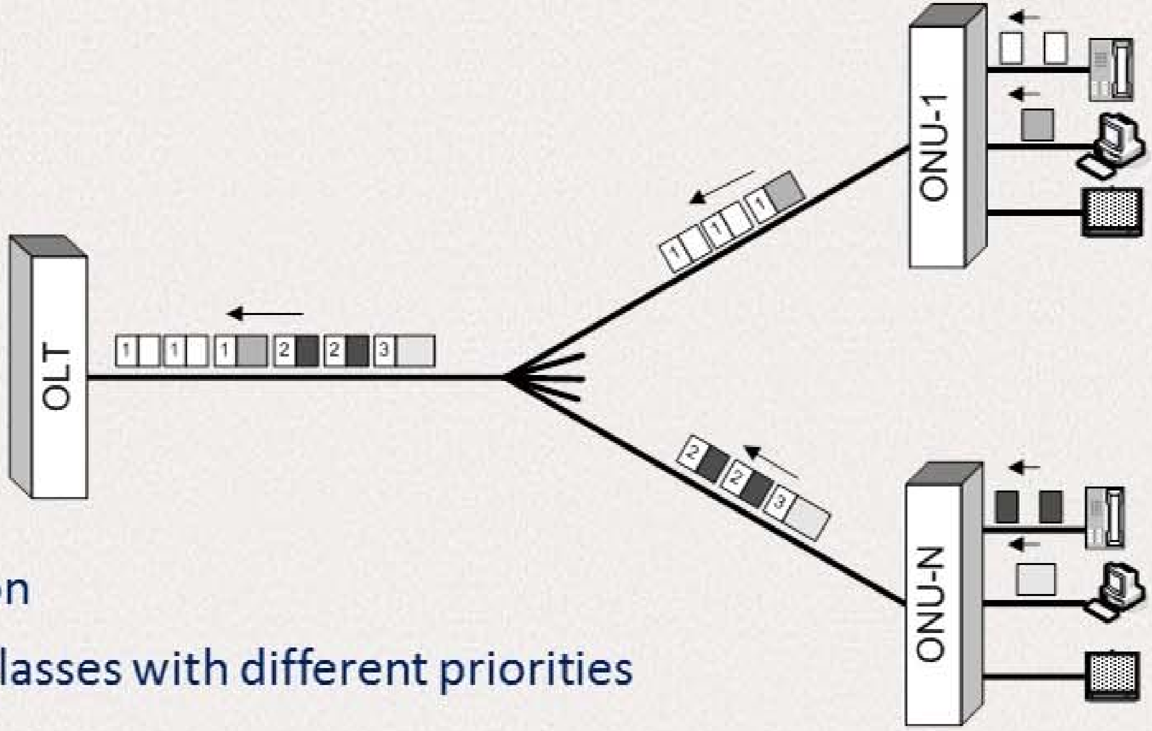


✓Upstream direction

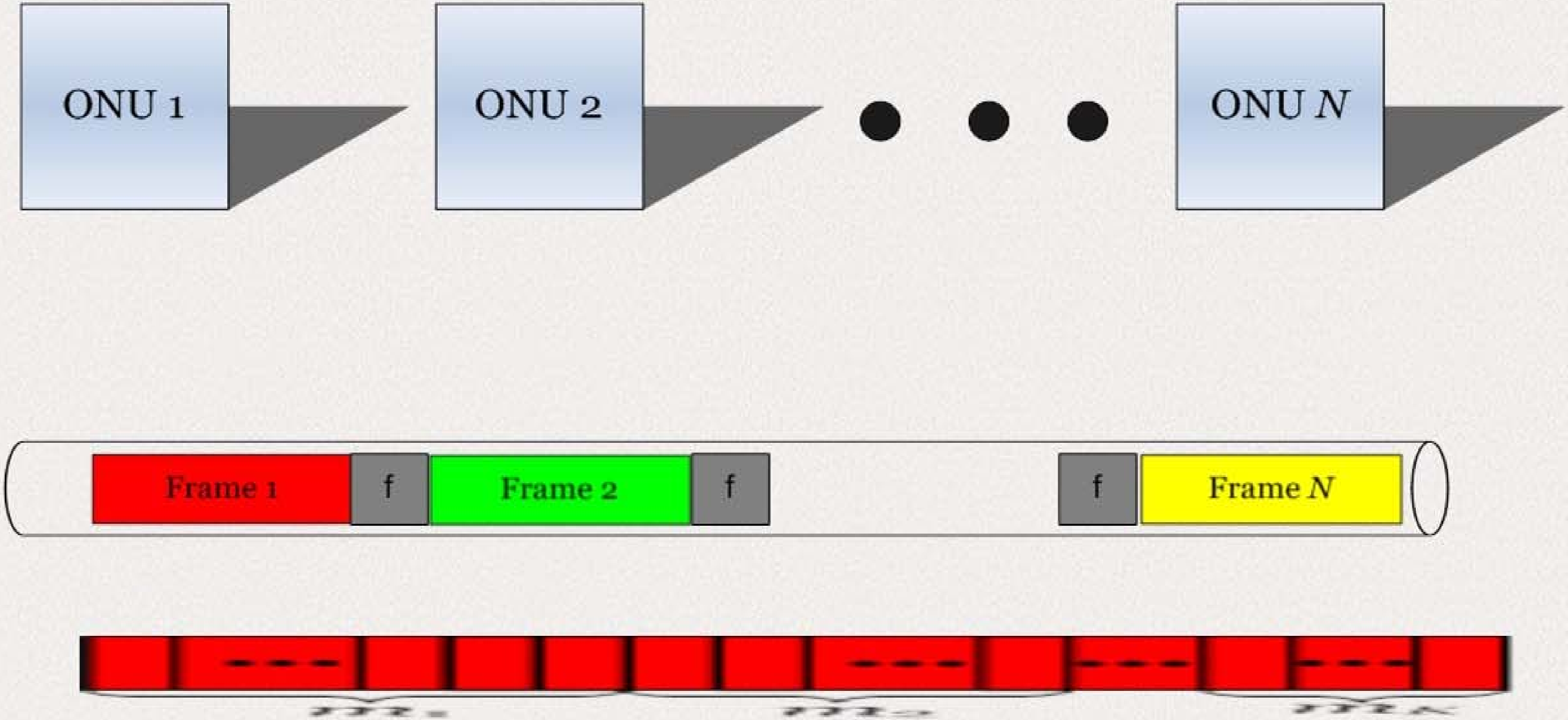
✓Multiple service-classes with different priorities

Target of the analysis

Queuing delay

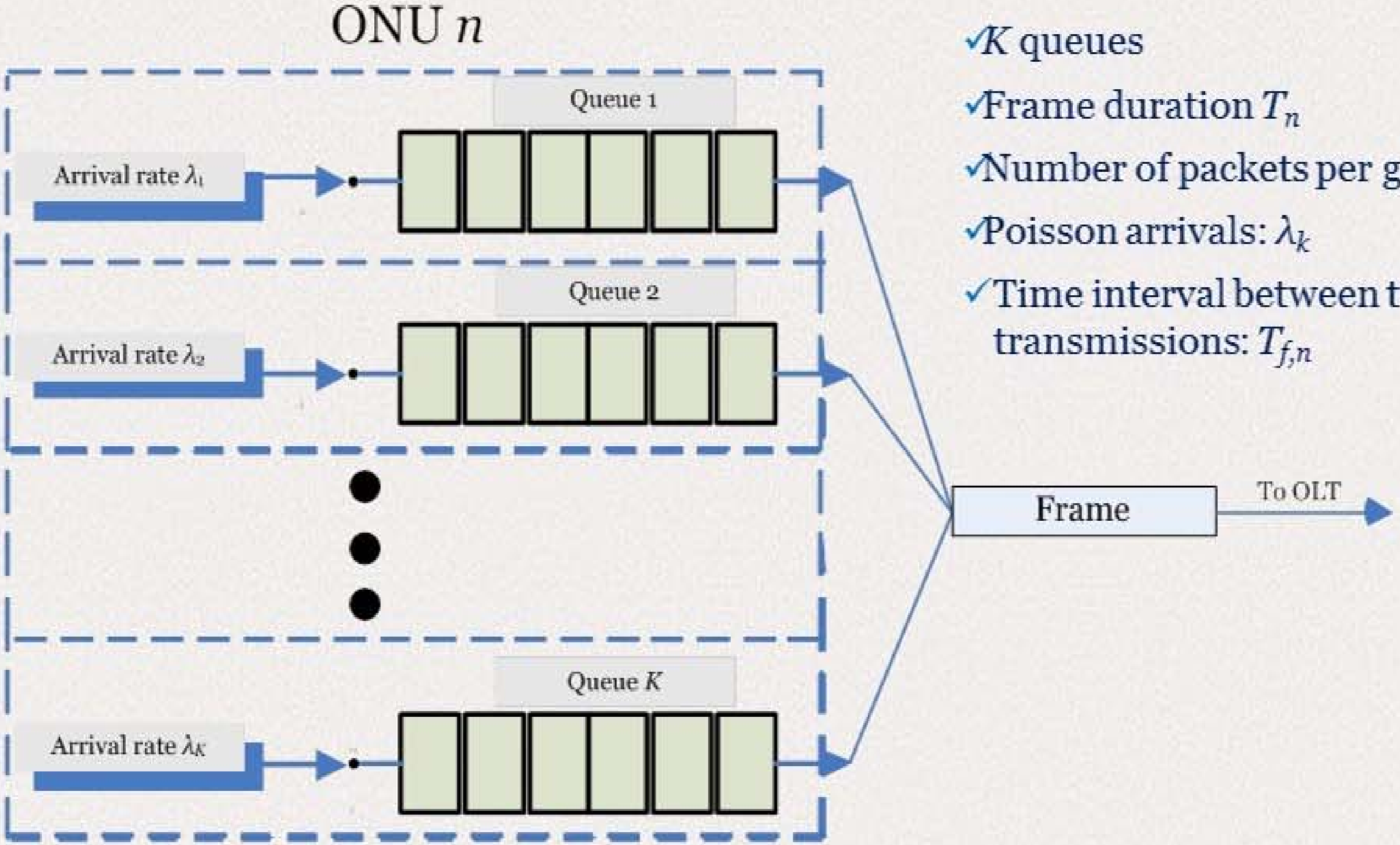
End-to-end delay

System model



- ✓ N ONUs
- ✓ K service-classes
- ✓ Time-slot duration: σ
- ✓ Frame duration: T_n
- ✓ Safety interval: f
- ✓ $m_{n,1} > m_{n,2} > ... > m_{n,K}$

Queuing model (1/4)



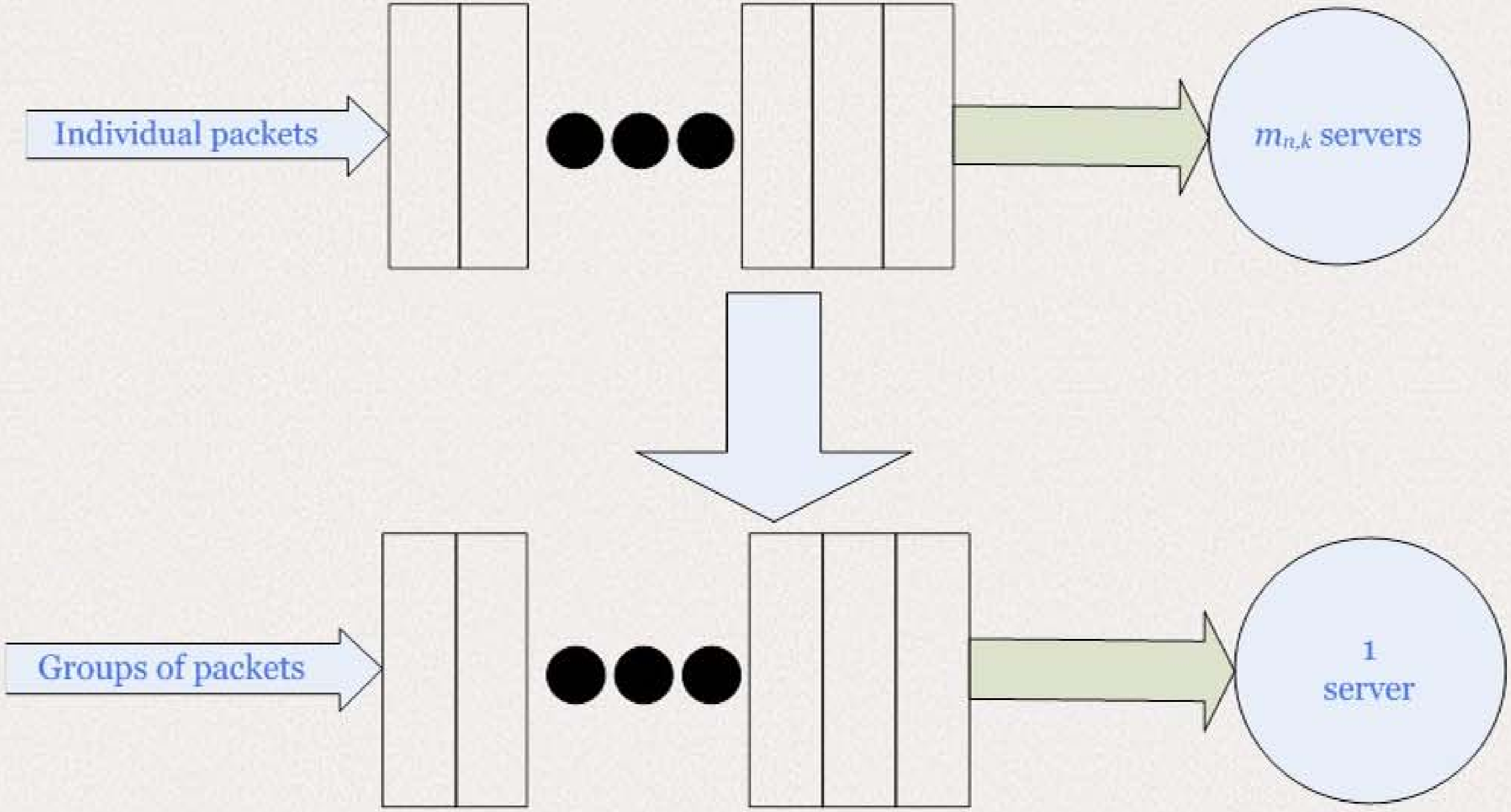
- ✓K queues
- ✓Frame duration T_n
- ✓Number of packets per group $[w_{n,k}, m_{n,k}]$
- ✓Poisson arrivals: λ_k
- ✓Time interval between two consecutive transmissions: $T_{f,n}$

Time interval between two consecutive transmissions

$$T_{f,n} = \sum_{i=1}^{N} T_i - T_n + (N-1) \cdot f$$

Queuing model (2/4)

Each service-class follows the queuing model M/D $^{[wn,k,\ mn,k]}$/ $m_{n,k}$



The groups of packets follow the queuing model M/D/1

Queuing model (3/4)

Equivalent arrival rate of the groups of packets and equivalent offered traffic load:

$$\lambda'_{n,k} = \frac{\lambda_{n,k}}{m_{n,k}}$$

$$A'_{n,k} = \lambda'_{n,k} \cdot T_{f,n} \cdot \sigma$$

Mean waiting time in the M/D/1 system:

$$W'_{n,k} = \frac{T_{f,n} \cdot \sigma}{2 \cdot (1 - A_{n,k})}$$

By using Little's theorem,
the mean queue length of the M/D/1 system is:

$$L'_{n,k} = \lambda'_{n,k} \cdot W'_{n,k} = \frac{\lambda'_{n,k} \cdot T_{f,n} \cdot \sigma}{2 \cdot (1 - A_{n,k})}$$

Queuing model (4/4)

Mean length of queue of the individual packets:

$$L_{n,k} \approx m_{n,k} \cdot L'_{n,k} + P^{w}_{n,k} \frac{m_{n,k}-1}{2} + (1-P^{w}_{n,k}) \cdot \frac{w_{n,k}-1}{2}$$

Probability of waiting in the M/D/$m_{n,k}$ system:

$$P^{w}_{n,k} = P(j \geq m_{n,k}) = \sum_{j=m_{n,k}}^{\infty} \pi_j^{n,k}$$

Steady state distribution of the M/D/$m_{n,k}$ system:

$$\pi_i^{n,k} = (1-A_{n,k})\sum_{j=1}^{i} \{(-1)^{i-j} e^{jA_{n,k}} [\frac{(jA_{n,k})^{i-j}}{(i-j)!} + \frac{(jA_{n,k})^{i-j-1}}{(i-j-1)!}]\}$$

By using Little's theorem, we obtain the mean waiting time of the individual packets:

$$W_{n,k} = \frac{L_{n,k}}{\lambda_{n,k}}$$

End-to-end delay

End-to-end delay

- Queuing delay
- Transmission delay
- Propagation delay

$$W_{n,k} = \frac{L_{n,k}}{\lambda_{n,k}}$$

$$T_{tr} = \frac{l}{C}$$

$$T_{p,n} = \frac{d_n}{\tilde{c}}$$

Mean end-to-end delay

$$\mathrm{E}[B_{n,k}] = W_{n,k} + T_{tr} + T_{p,n}$$

Results (1/2)

Comparison of analytical and simulation results

EPON topology:

N = 30 ONUs, C=1 Gbps
K = 3 service-classes

Parameter	**Value**
Distance d_n	20 Km
Frame size	50 time-slots
Time-slot σ	10 μsec
Safety interval f	2 time-slots
(m_1, m_2, m_3)	(25, 15, 10) time-slots
Upstream channel data-rate C	1 Gbps
Refractive index	1.45
Packet length l	1000 bits

Results (1/2)

Parameter	Value
Distance d_n	20 Km
Frame size	50 time-slots
Time-slot σ	10 μsec
Safety interval f	2 time-slots
(m_1, m_2, m_3)	(25, 15, 10) time-slots
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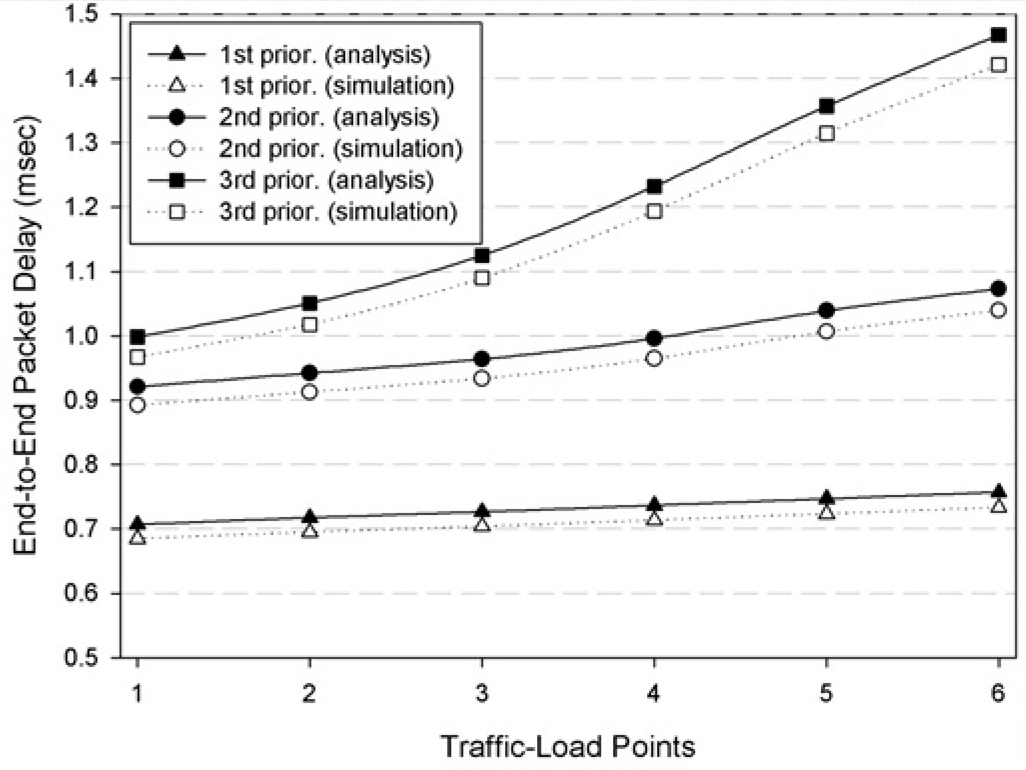
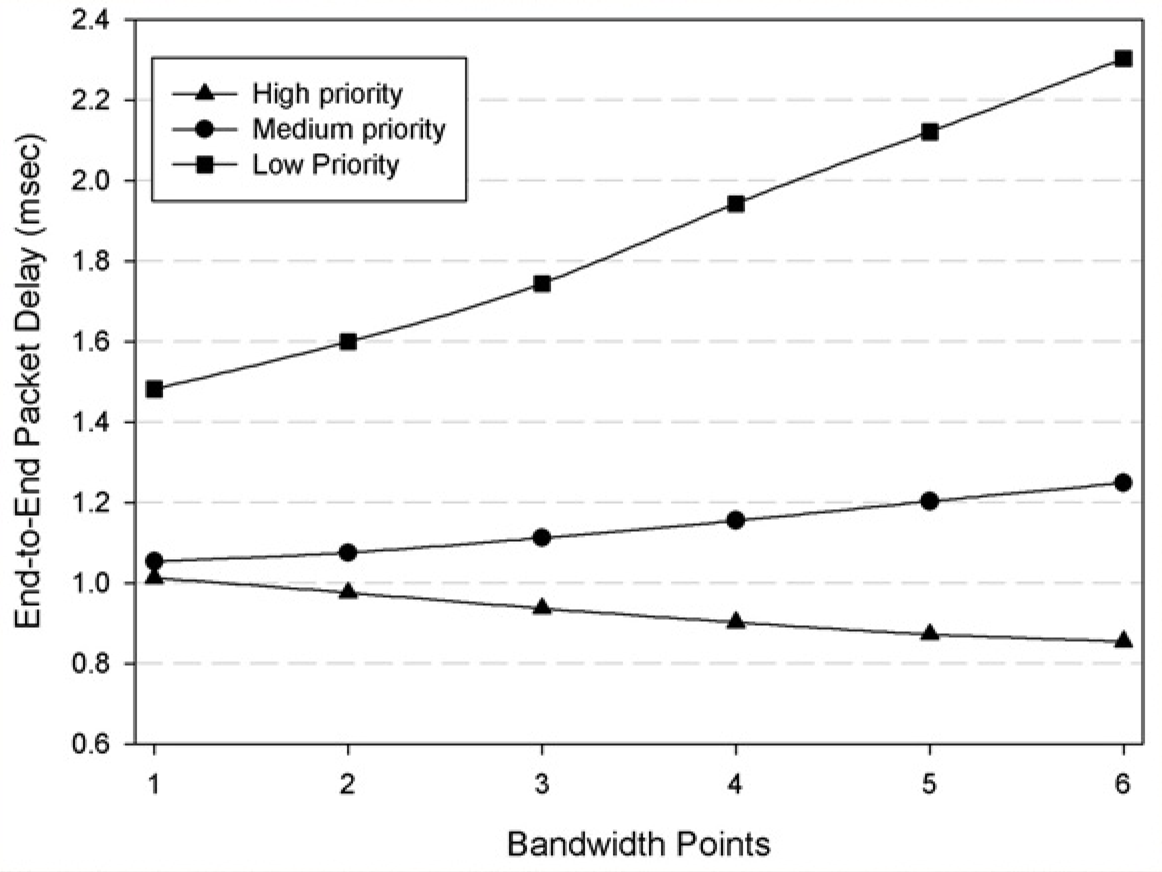


Results (2/2)



	x-axis points of the diagram					
$(m_{n,1}, m_{n,2}, m_{n,3})$ (time-slots)	**1**	**2**	**3**	**4**	**5**	**6**
$m_{n,1}$	22	24	26	28	30	32
$m_{n,2}$	16	15	14	13	12	11
$m_{n,3}$	12	11	10	9	8	7

Structure

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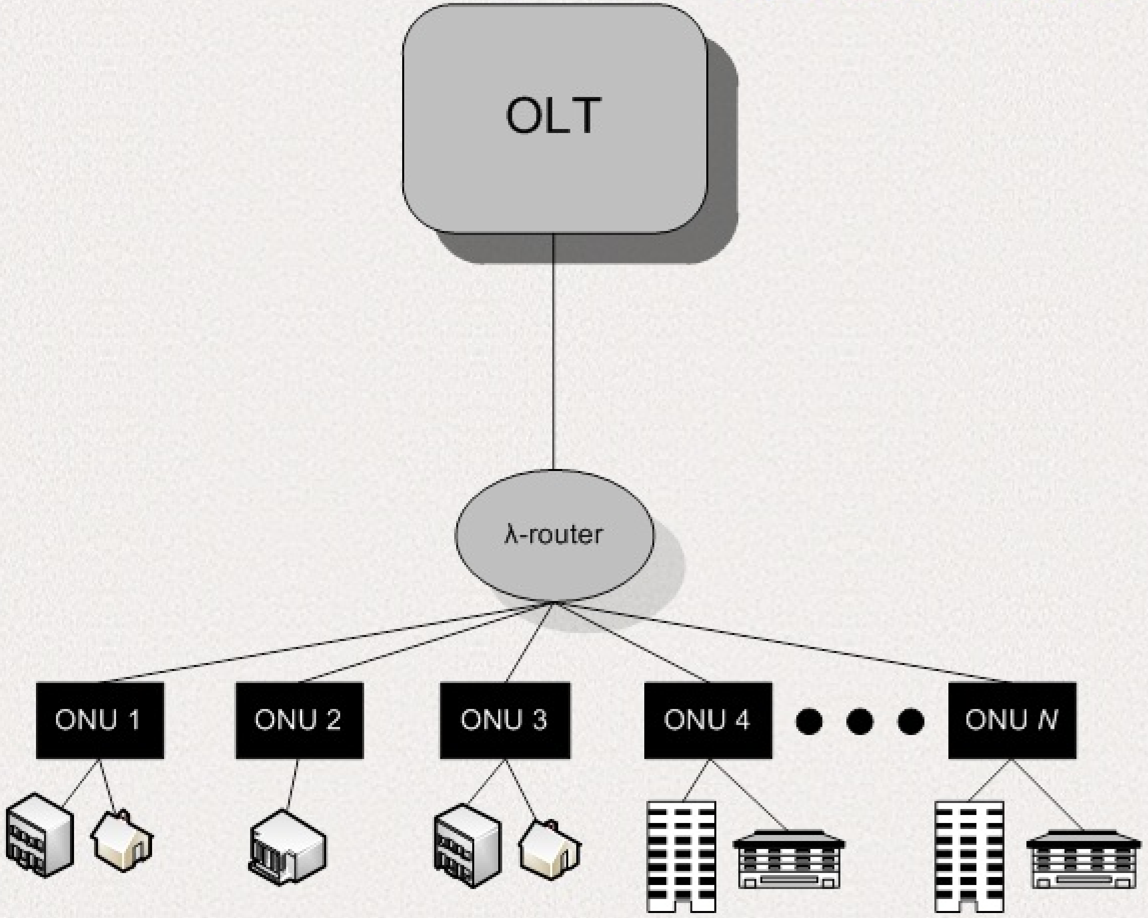
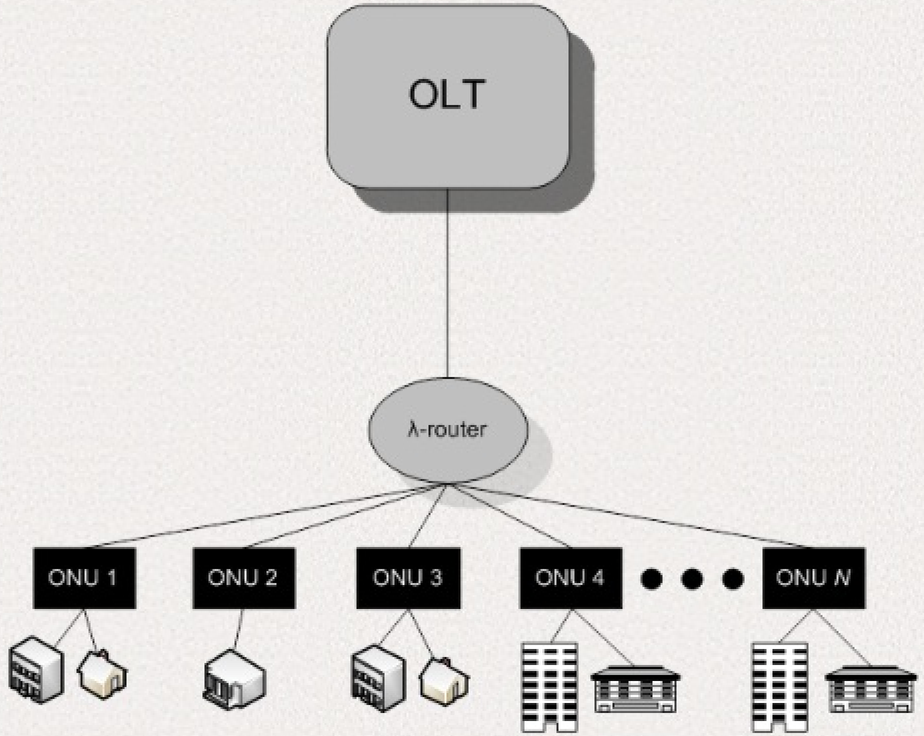
Hybrid WDM-TDMA PONs



1. Talli, G.; Townsend, P.D.; , "Hybrid DWDM-TDM long-reach PON for next-generation optical access," *Lightwave Technology, Journal of* , vol.24, no.7, pp. 2827- 2834, July 2006
2. Wong, E.; Chan, S.; , "Dynamic Wavelength Allocation Schemes in WDM-PON," *PhotonicsGlobal@Singapore, 2008. IPGC 2008. IEEE* , vol., no., pp.1-4, 8-11 Dec. 2008
3. Dixit, A.; Lannoo, B.; Das, G.; Colle, D.; Pickavet, M.; Demeester, P.; , "Flexibility evaluation of hybrid WDM/TDM PONs," *Advanced Networks and Telecommunication Systems (ANTS), 2011 IEEE 5th International Conference on* , vol., no., pp.1-6, 18-21 Dec. 2011

Hybrid WDM-TDMA PONs



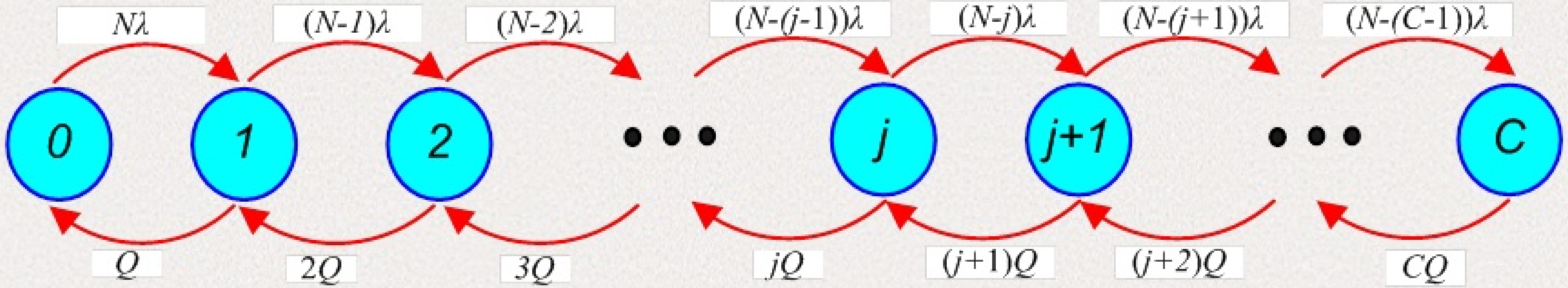
- ✓Upstream direction
- ✓Dynamic wavelength allocation
- ✓Multiple service-classes of stream, elastic or ON-OFF traffic
- ✓Infinite of finite traffic sources

Target of the analysis

Calculation of
connection failure probabilities
and call blocking probabilities

Analytical model for stream traffic (1/3)

Distribution of occupied wavelengths in the PON



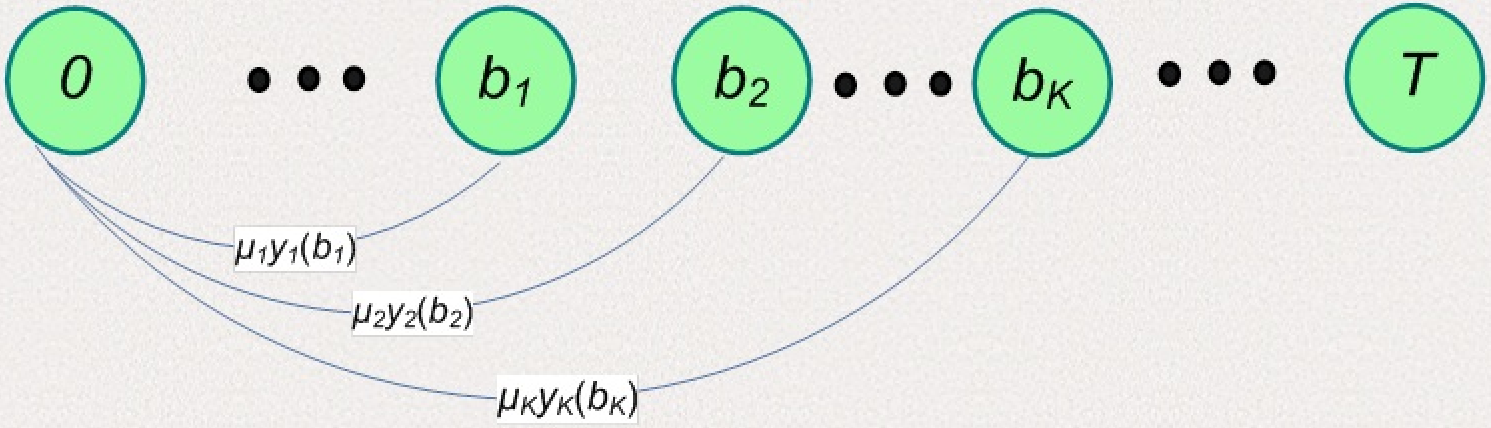
- C: number of wavelengths
- T: capacity of a wavelength
- N: total number of ONUs
- K: number of service-classes

- λ_k: arrival rate
- b_k: bandwidth requirements
- μ_k: service rate
- the total arrival rate of calls

from an ONU $\lambda = \sum_{k=1}^{K} \lambda_k$

Analytical model for stream traffic (2/3)

Occupancy distribution of b.u. inside the wavelength



Wavelength service rate

$$Q = \sum_{k=1}^{K} \mu_k y_k\left(b_k\right) \frac{q\left(b_k\right)}{\sum_{i=1}^{T} q\left(i\right)}$$

Occupancy distribution of b.u.

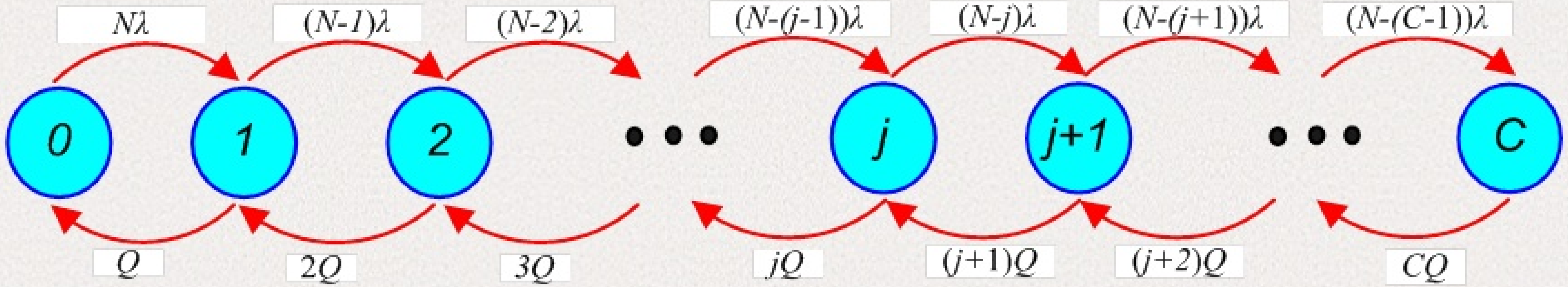
$$iq(i) = \sum_{k=1}^{K} a_k b_k q\left(i - b_k\right)$$

Mean number of service-class *k* calls when *i* b.u. are occupied in the wavelength

$$y_k\left(i\right) = \frac{a_k q\left(i - b_k\right)}{q\left(i\right)}$$

Analytical model for stream traffic (3/3)

Distribution of occupied wavelengths



$$P(j)=\left(\frac{\lambda}{Q}\right)^{j}\cdot\frac{\prod_{i=1}^{j}\left[N-(j-1)S\right]}{j!}\cdot\left[\sum_{l=0}^{C}\left(\frac{\lambda}{Q}\right)^{l}\frac{\prod_{j=1}^{l}\left[N-(j-1)S\right]}{l!}\right]^{-1}$$

Connection Failure Probability (CFP)

$P(C\ )$

Call Blocking Probability (CBP)

$$B_k=\sum_{j=T-b_k+1}^{T}q(j)$$

Calculation of the Total CBP

A call is accepted for the service when:

(the ONU has already established a connection)
AND
(enough free b.u. are available in the wavelength)

OR

(is the first call that arrives at an ONU)
AND
(there is a free wavelength in the PON)

$$P_{accept}^{k} = P_s \cdot \frac{\sum_{i=1}^{T-b_k} q(i)}{\sum_{i=1}^{T} q(i)} \quad + \quad (1-P_S)\cdot\left(1-P(C)\right)$$

Probability that an ONU has already established a connection

$$P_s = \sum_{j=1}^{C} P(j) \frac{\binom{N-1}{j-1}}{\binom{N}{j}} = \sum_{j=1}^{C} P(j) \frac{j}{N}$$

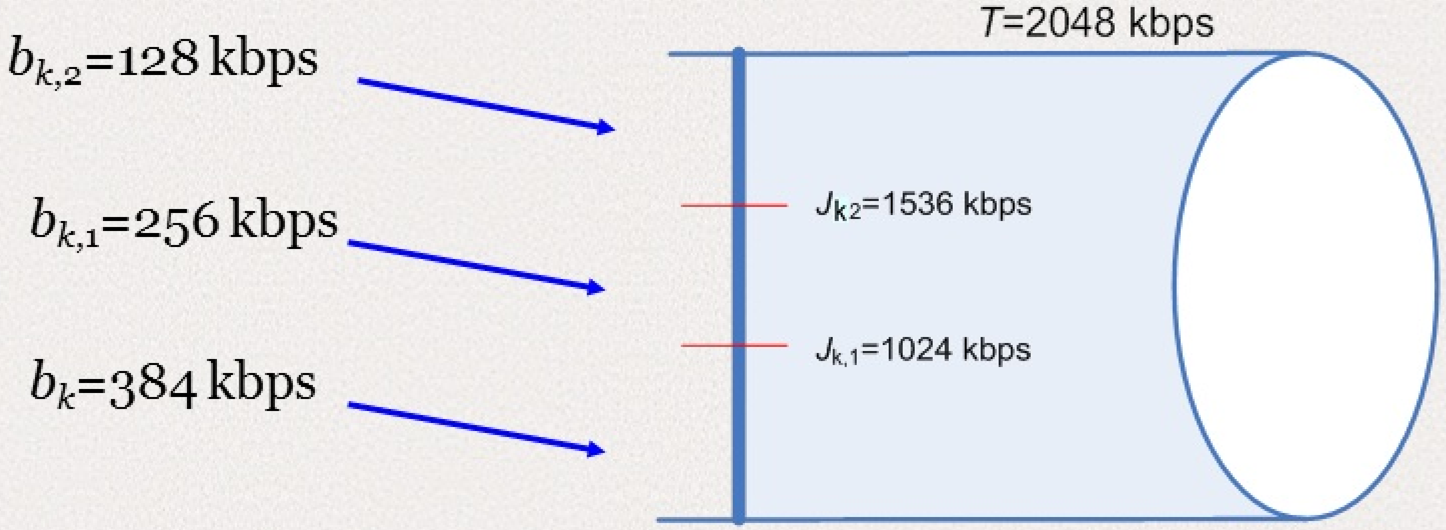
Total CBP

$$TB_k = 1 - P_{accept}^{k} = 1 - \left(P_s \cdot \frac{\sum_{i=1}^{T-b_k} q(i)}{\sum_{i=1}^{T} q(i)} + (1-P_s)\cdot\left(1-P(C)\right) \right)$$

Analytical model for stream and elastic traffic (1/3)

Example
An arriving video-call
has 3 different
bandwidth requests
upon call arrival:

J_{k1}, J_{k2} thresholds of k_{th} service
$b_k > b_{k,1} > b_{k,2}$ bandwidth requirements
$\mu_k > \mu_{k,1} > \mu_{k,2}$ service rates



$b_{k,2}$=128 kbps

$b_{k,1}$=256 kbps

b_k=384 kbps

Analytical model for stream and elastic traffic (2/3)

- We use the Markov chain of the stream traffic
- New service rate of the wavelength

Last call

(b_k, μ_k)

$(b_{k,s}, \mu_{k,s})$

$$Q = \sum_{k=1}^{K} \left(\mu_k y_k(b_k) \frac{q(b_k)}{\sum_{i=1}^{T} q(i)} \right) + \sum_{k=1}^{K} \left(\sum_{s=1}^{S(k)} \mu_{k,s} y_{k,s}(b_{k,s}) \frac{q(b_{k,s})}{\sum_{i=1}^{T} q(i)} \right)$$

Analytical model for stream and elastic traffic (3/3)

$q(i)$: occupancy distribution of the wavelength

$$iq(i)=\sum_{k=1}^{K}a_{k,0}b_{k,0}\delta_{k}(i)q(i-b_{k,0})+\sum_{k=1}^{K}\sum_{s=1}^{S(k)}a_{k,s}b_{k,s}\delta_{k,s}(i)q(i-b_{k,s})$$

Connection Failure Probability (CFP)

$$P(C)$$

Total Call Blocking Probability (TCBP)

$$TB_{k}=1-\left(P_{s}\cdot\frac{\sum_{i=1}^{T-b_{k,S(k)}}q(i)}{\sum_{i=1}^{T}q(i)}+(1-P_{s})\cdot\left(1-P(C)\right)\right)$$

Call Blocking Probability (CBP)

$$B_{k}=\sum_{i=T-b_{k,S(k)}+1}^{T}q(i)$$

Results (1/2)

PON topology:

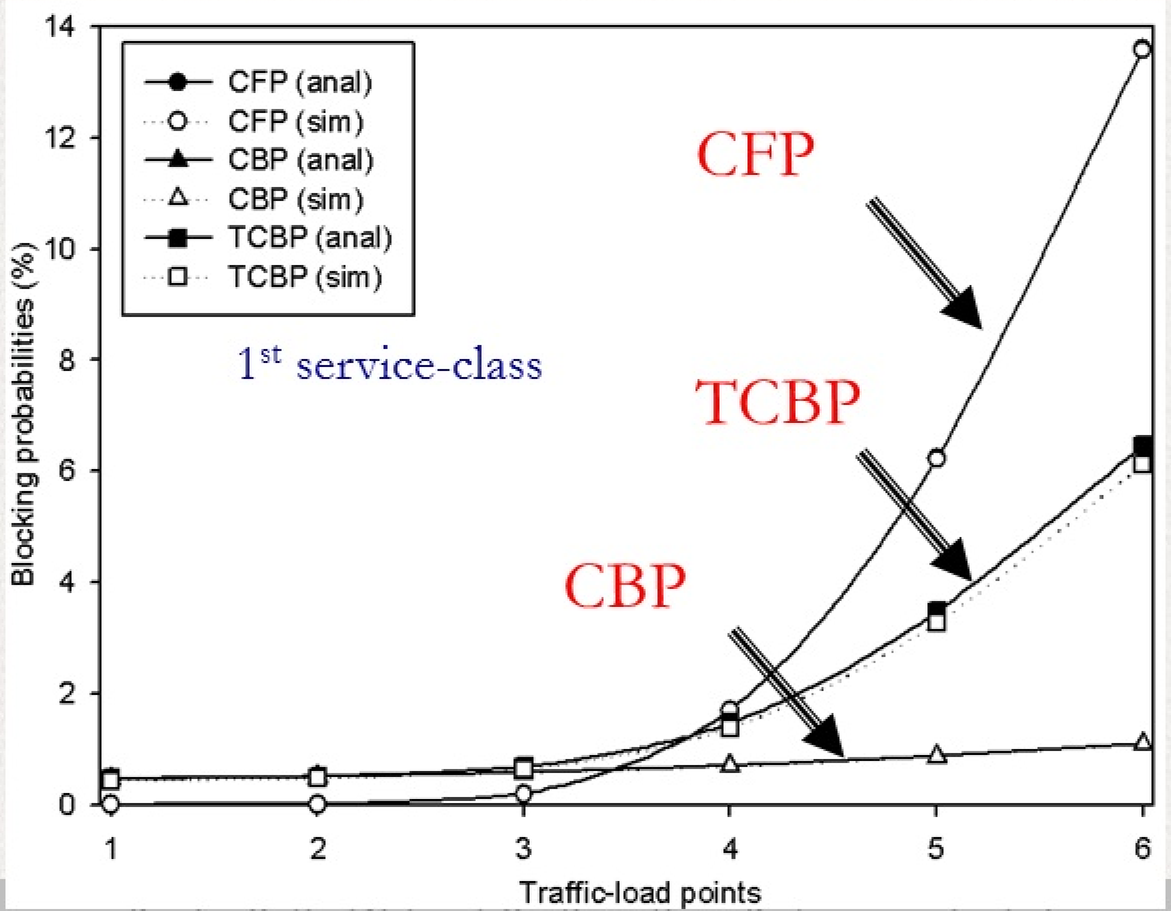
C = 64 wavelengths

N = 100 ONUs, T = 155 b.u.

Stream traffic, two service classes:

s_1: b_1 = 24 b.u.

s_2: b_2 = 36 b.u



Results (1/2)

PON topology:

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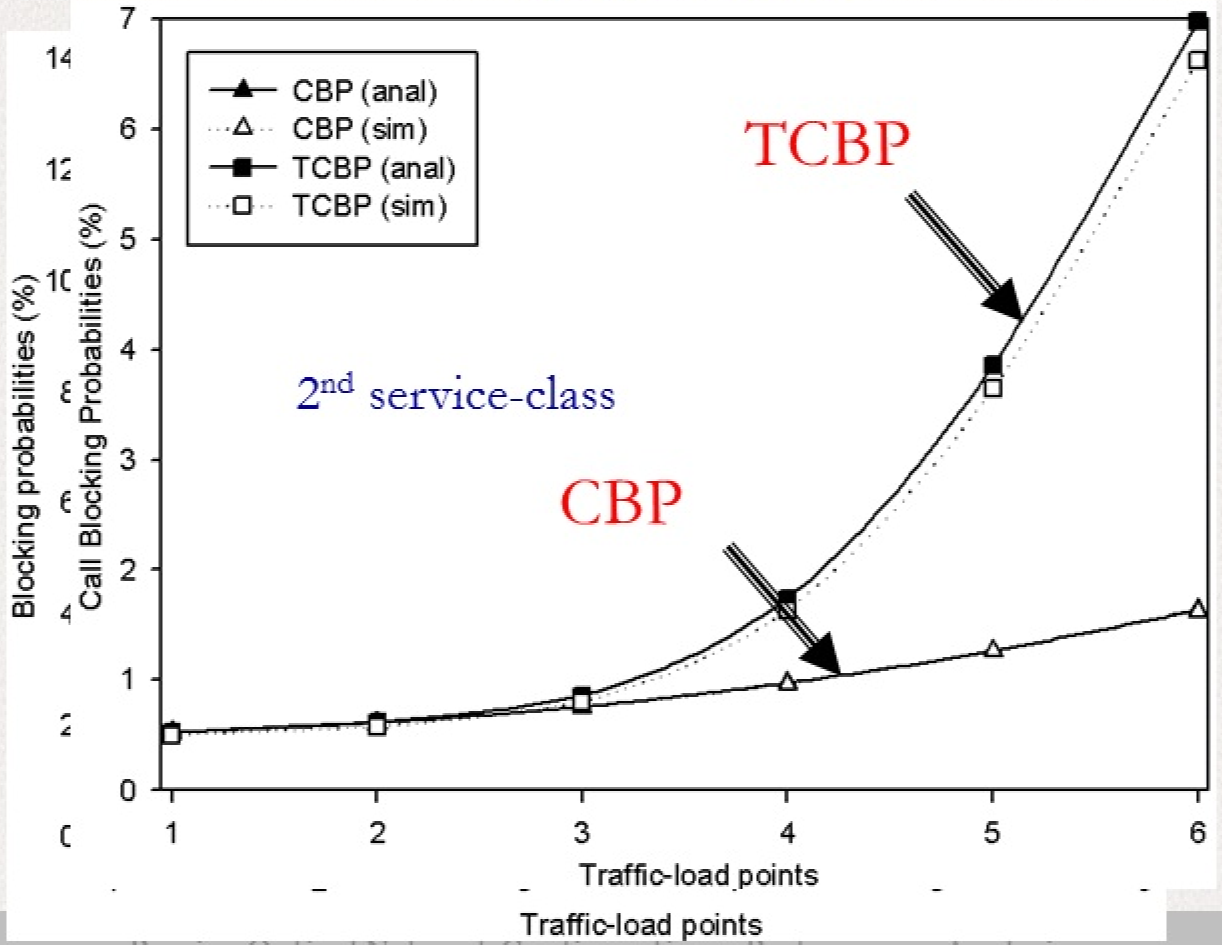
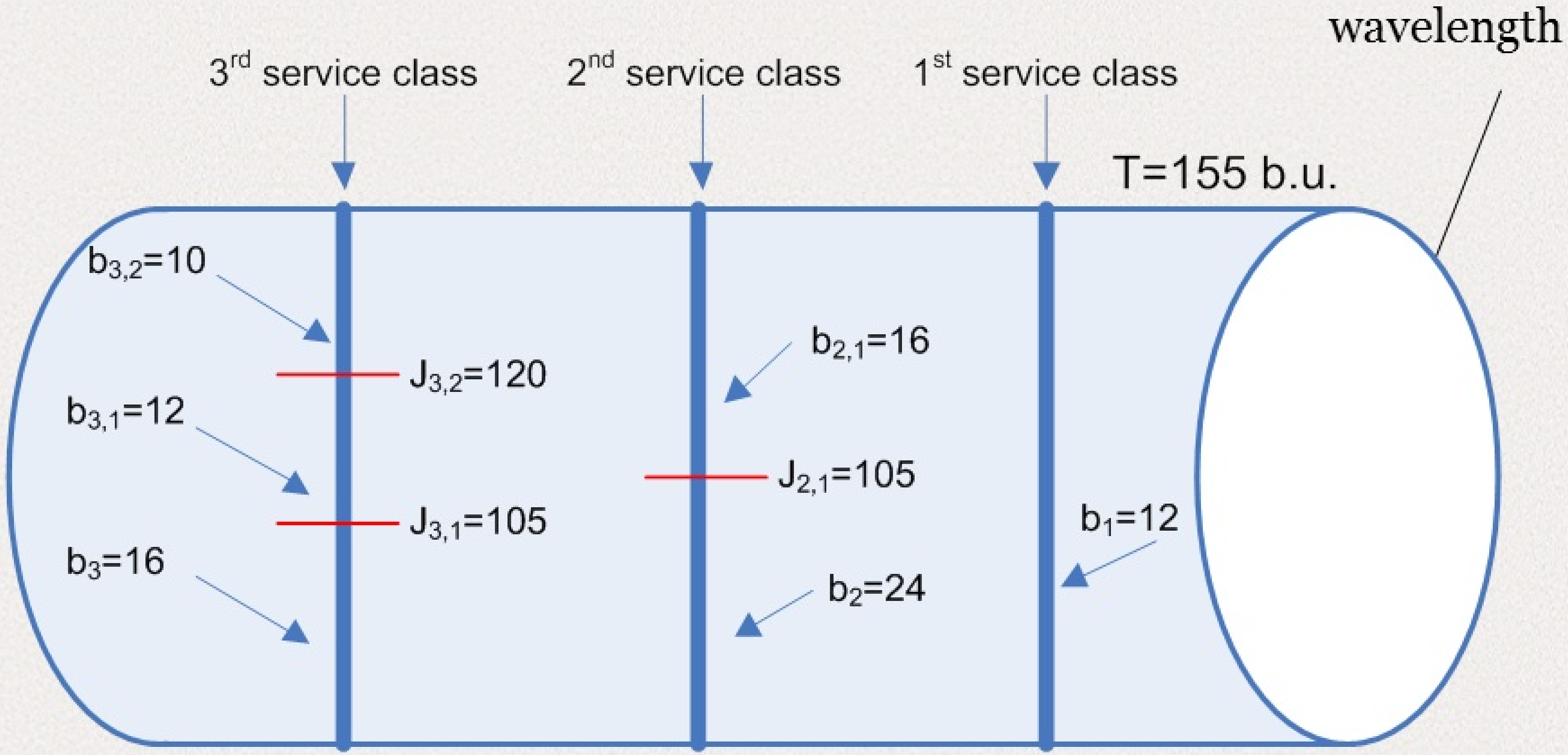


Results (2/2)



PON topology:

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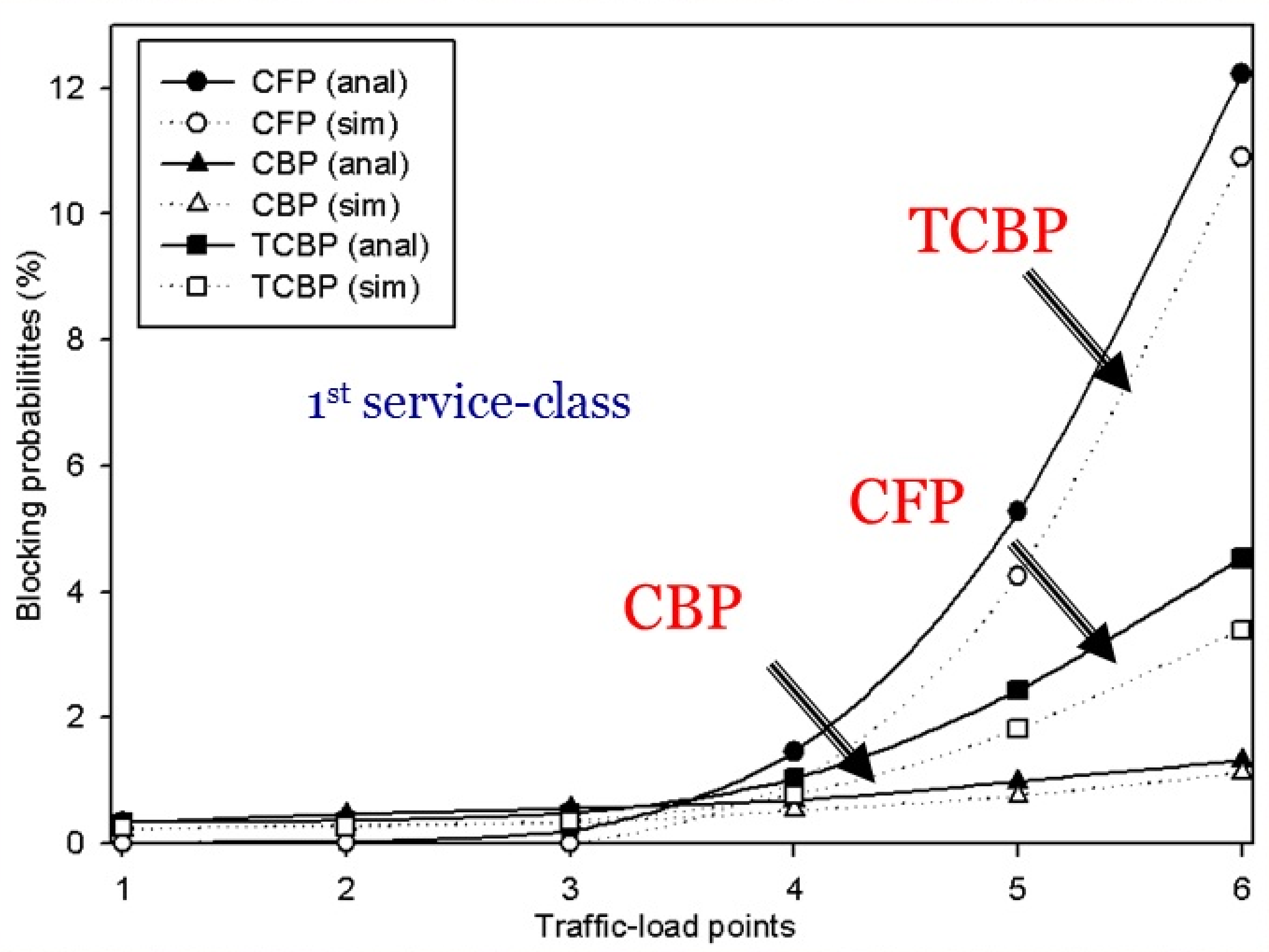
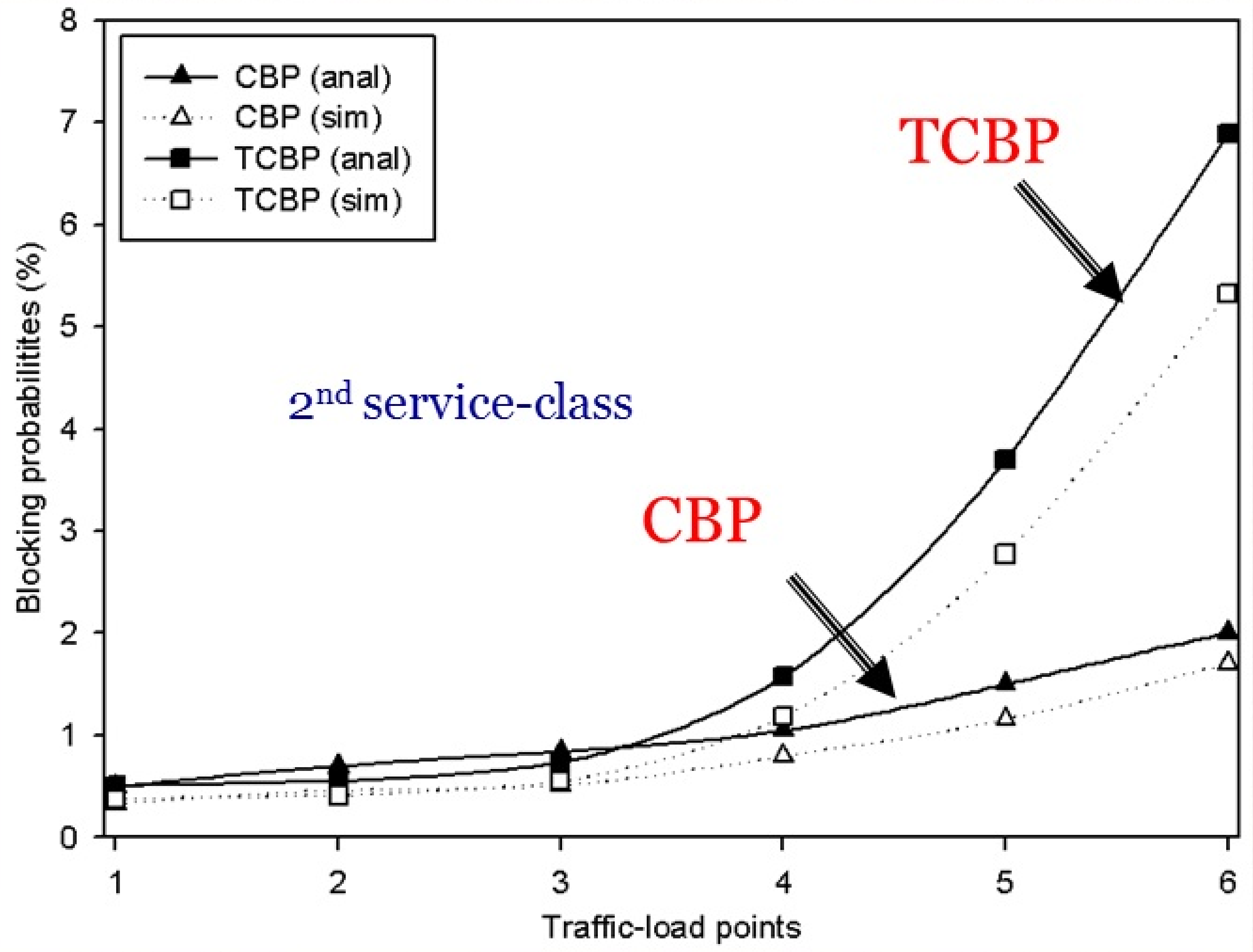
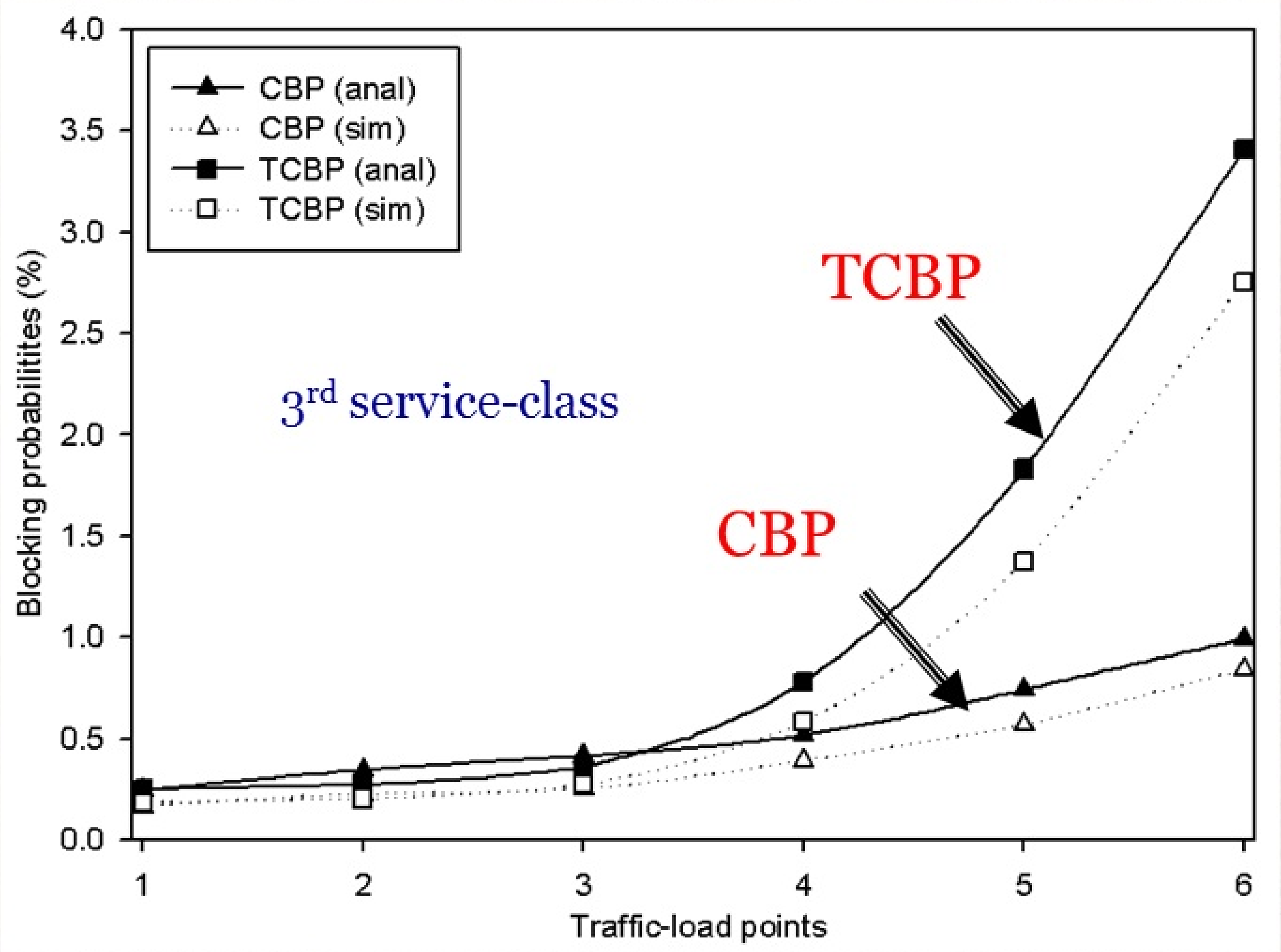
Results (2/2)


CFP (anal)
CFP (sim)
CBP (anal)
CBP (sim)
TCBP (anal)
TCBP (sim)
1st service-class
TCBP
CFP
CBP
Blocking probabilitites (%)
Traffic-load points
0
2
4
6
8
10
12
1
2
3
4
5
6

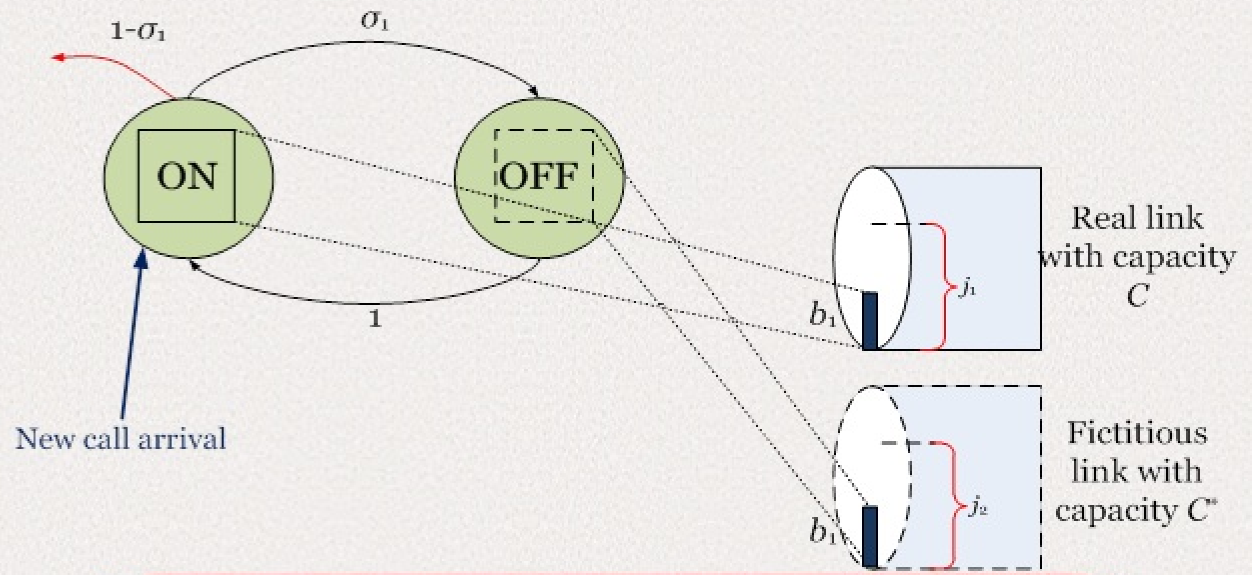
Results (2/2)


8
7
6
5
4
3
2
1
0
Blocking probabilitites (%)
1
2
3
4
5
6
Traffic-load points
CBP (anal)
CBP (sim)
TCBP (anal)
TCBP (sim)
TCBP
CBP
2nd service-class

Results (2/2)


4.0
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
Blocking probabilitites (%)
1
2
3
4
5
6
Traffic-load points
CBP (anal)
CBP (sim)
TCBP (anal)
TCBP (sim)
TCBP
CBP
3rd service-class

Analytical model for ON-OFF traffic (1/2)

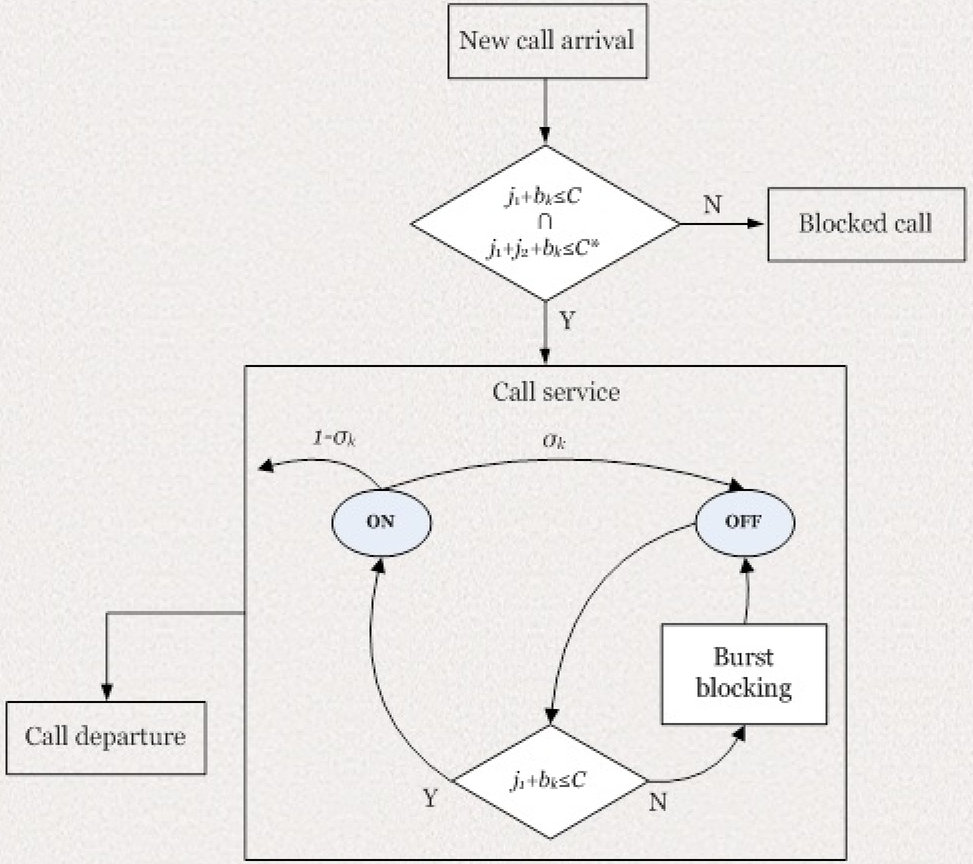


Link of capacity *C*

- ✓ K service-classes
- ✓ λ_k: arrival rate
- ✓ b_k: bandwidth requirements
- ✓ μ_{ik}: service-rate in state i
- ✓ C^*: capacity of fictitious link
- ✓ σ_k: probability of transition to state OFF
- ✓ p_{ik}: utilization of link i

1. Asrin M. A., “Call-burst blocking and call admission control in a broadband network with bursty sources”, *Performance Evaluation*, Vol. 38, pp. 1-19, 1999.
2. Moscholios I., Logothetis M., Kokkinakis G., “Call-burst blocking of ON-OFF traffic sources with retrials under the complete sharing policy”, *Performance Evaluation*, Vol. 59, Issue 4, pp. 279-312, 2005.

Analytical model for ON-OFF traffic (1/2)



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- ✓ μ_{ik}: service-rate in state i
- ✓ C^*: capacity of fictitious link
- ✓ σ_k: probability of transition to state OFF
- ✓ p_{ik}: utilization of link i

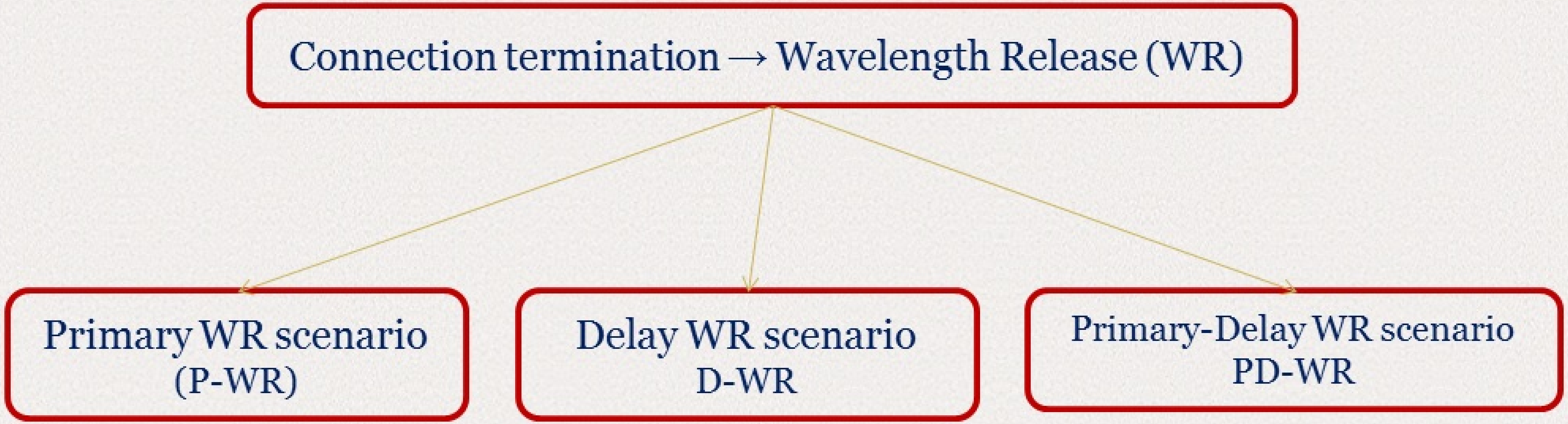
$$q_{INF}(\vec{j}) = \begin{cases} 1 & \text{for } \vec{j} = 0 \\ \sum_{i=1}^{2}\sum_{k=1}^{K} b_{i,k,s}\, p_{INF_{ik}}\, q_{INF}(\vec{j} - B_{i,k}) = j_s & \text{for } j_1 = 1,\dots,C \ (\text{if } s = 1) \ \text{ or for } j_2 = 1,\dots,C^* - j_1 \ (\text{if } s = 2) \\ 0 & \text{otherwise} \end{cases}$$

$$s = \begin{cases} 1 & \Rightarrow \text{real link} \\ 0 & \Rightarrow \text{fictitious link} \end{cases}$$

$$\vec{j} = (j_1, j_2),\ B_{i,k} = (b_{i,k,1}, b_{i,k,2}) \qquad b_{i,k,s} = \begin{cases} b_k, & \text{if } s = i \\ 0, & \text{if } s \neq i \end{cases}$$

$$i = \begin{cases} 1 & \Rightarrow \text{state ON} \\ 0 & \Rightarrow \text{state OFF} \end{cases}$$

Analytical model for ON-OFF traffic (2/2)



Connection termination → Wavelength Release (WR)

Primary WR scenario
(P-WR)

Delay WR scenario
D-WR

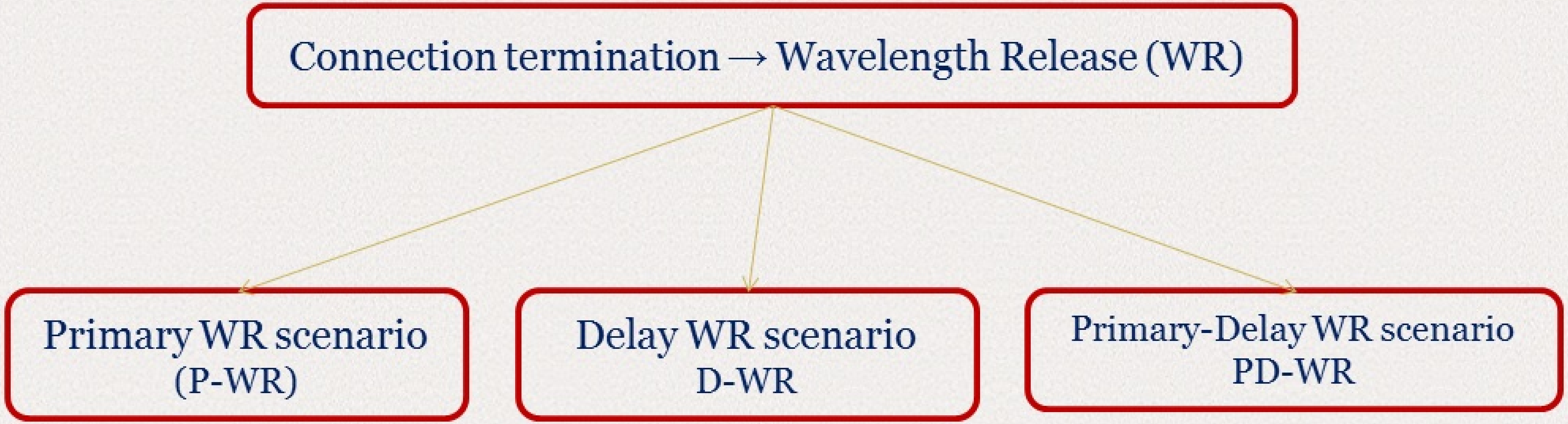
Primary-Delay WR scenario
PD-WR

For each scenario the following metrics are calculated:

- ✓ Connection Failure Probability (CFP)
- ✓ Call Blocking Probability (CBP)
- ✓ Burst Blocking Probability (BBP)
- ✓ Delay*
- ✓ Average number of delays calls*

(* if delay exists)

Analytical model for ON-OFF traffic (2/2)



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For each scenario the following metrics are calculated:

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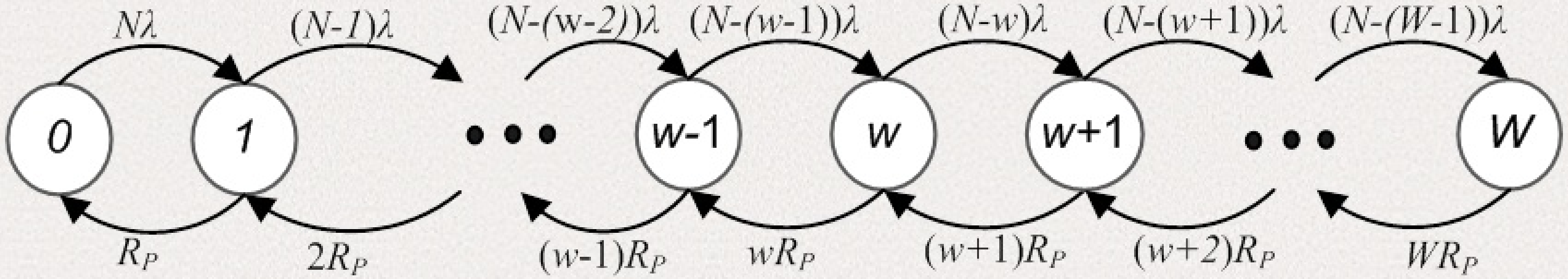
The Primary WR scenario (2/2)



Distribution of occupied wavelengths in the PON

$$P(w)=\left(\frac{\lambda}{R_p}\right)^w\frac{\prod_{z=1}^{w}\left[N-(z-1)\right]}{w!}\left[\sum_{l=0}^{W}\left(\frac{\lambda}{R_p}\right)^l\frac{\prod_{m=1}^{l}\left[N-(m-1)\right]}{l!}\right]^{-1}$$

CFP

$$P(W)$$

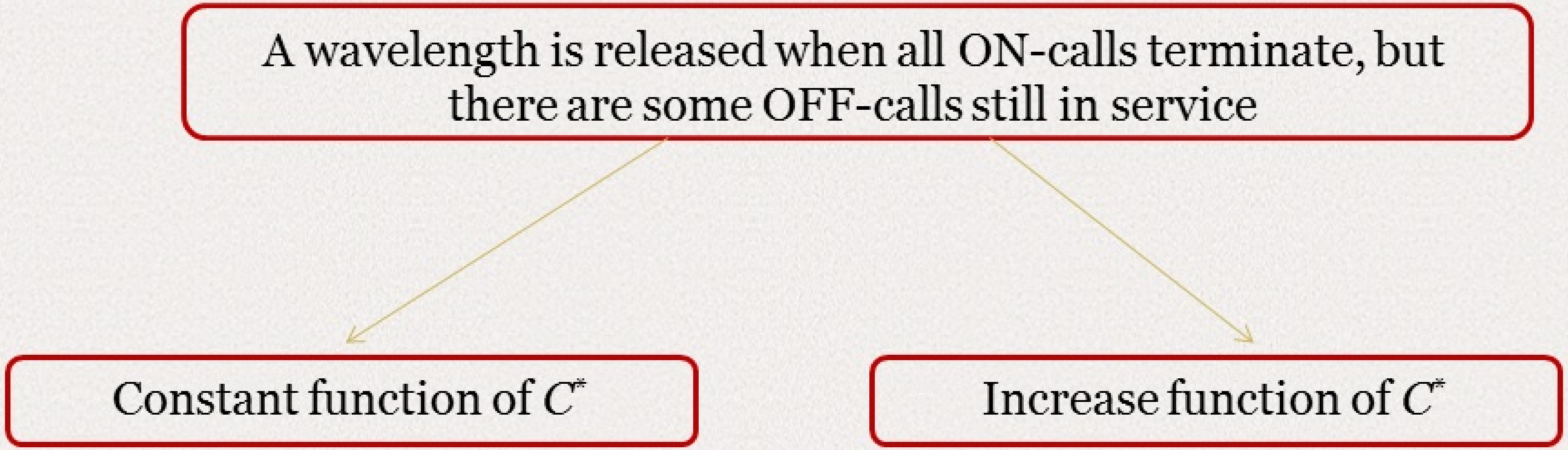
CBP

$$P_{b_k}=\sum_{\left\{\vec{j}\,|\left[(b_{1,k,1}+j_1)>C\right]\cup\left[(b_{1,k,2}+j_1+j_2)>C^*\right]\right\}}G^{-1}q_{INF}(\vec{j})$$

BBP

$$P_{b_k}^*=\frac{\sum_{\vec{j}\in\Omega^*}y_{2k,INF}(\vec{j})q_{INF}(\vec{j})\mu_{2k}}{\sum_{\vec{j}\in\Omega}y_{2k,INF}(\vec{j})q_{INF}(\vec{j})\mu_{2k}}$$

The Delay WR scenario (1/3)



A wavelength is released when all ON-calls terminate, but there are some OFF-calls still in service

Constant function of C^*

Increase function of C^*

The Delay WR scenario (1/3)

A wavelength is released when all ON-calls terminate, but there are some OFF-calls still in service

Constant function of C^*

$$mC^*,\ m : mC^* \in \mathbb{N}$$

Service rate of a wavelength

$$R_{D_1} = \sum_{k=1}^{K} \sum_{j_2=0}^{mC^*} \mu_{1k} \cdot y_{INF_{1k}}(b_k, j_2)(1-\sigma_k)\hat{q}_{INF}(b_k, j_2) =$$

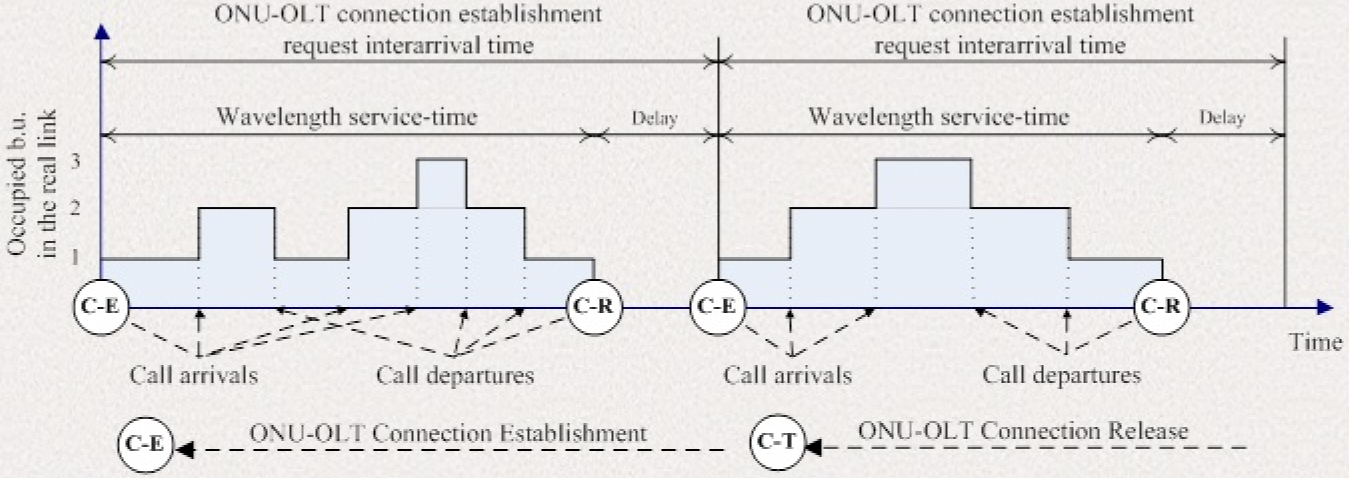
$$\sum_{k=1}^{K} \sum_{j_2=0}^{mC^*} \mu_{1k} \cdot y_{INF_{1k}}(b_k, j_2)(1-\sigma_k)\frac{q_{INF}(b_k, j_2)}{G - q_{INF}(0,0)}.$$

Distribution of occupied wavelengths

$$P(w) = \left(\frac{\lambda}{R_{D_1}}\right)^{w} \frac{\prod_{z=1}^{w}\left[N-(z-1)\right]}{w!}\left[\sum_{l=0}^{W}\left(\frac{\lambda}{R_{D_1}}\right)^{l}\frac{\prod_{m=1}^{l}\left[N-(m-1)\right]}{l!}\right]^{-1}$$

The Delay WR scenario (2/3)

Constant function of *C*:
Calculation of the delay of calls that remain in state OFF



ONU-OLT connection establishment request interarrival time

$$\frac{1}{\Lambda}=\left(\left\lfloor\frac{\frac{1}{R_{D1}}}{\frac{1}{\lambda}}\right\rfloor+1\right)\frac{1}{\lambda}$$

Arrival rate of connection establishment requests

$$\Lambda_s=\Lambda(1-P(W))$$

Delay

$$T_D=\frac{1}{\Lambda_s}-\frac{1}{R_{D1}}$$

Number of calls that suffer the delay

$$N_{D,k}=\sum_{j_2=1}^{m\cdot C^*} y_{INF_{2k}}(b_k,j_2)$$

The Delay WR scenario (3/3)

Increase function of C^*

$\left\lfloor (mC^*/W)w \right\rfloor$ w: number of occupied wavelengths

Wavelength service rate

$$R_{D_2}(w) = \sum_{k=1}^{K} \sum_{j_2=0}^{\lfloor (m \cdot C^*/W) \cdot w \rfloor} \mu_{1k} \cdot y_{INF_{1k}}(b_k, j_2) \cdot (1-\sigma_k) \cdot \hat{q}_{INF}(b_k, j_2) =$$

$$= \sum_{k=1}^{K} \sum_{j_2=0}^{\lfloor (m \cdot C^*/W) \cdot w \rfloor} \mu_{1k} \cdot y_{INF_{1k}}(b_k, j_2) \cdot (1-\sigma_k) \cdot \frac{q_{INF}(b_k, j_2)}{G - q_{INF}(0,0)}$$

Distribution of occupied wavelengths

$$P(w) = \frac{(\lambda)^w}{w!} \prod_{z=1}^{w} \frac{[N-(z-1)]}{R_{D_2}(z)} \cdot \left[\sum_{l=0}^{W} \frac{(\lambda)^l}{l!} \prod_{m=1}^{l} \frac{[N-(m-1)]}{R_{D_2}(m)} \right]^{-1}$$

Delay function

$$T_{D_2}(w) = \frac{1}{\Lambda_s} - \frac{1}{R_{D2}(w)}$$

Average delay

$$\bar{T}_{D_2} = \frac{\sum_{w=1}^{W} T_{D_2}(w)}{W} = \frac{1}{\Lambda_s} - \frac{\sum_{w=1}^{W} \frac{1}{R_{D_2}(w)}}{W}$$

Number of calls that suffer delay

$$N_{D,k} = \frac{\sum_{w=1}^{W} \sum_{j_2=1}^{\lfloor (mC^*/W)w \rfloor} y_{INF_{2k}}[b_k][j_2]}{W}$$

The Primary Delay WR scenario

if $w \leq W\text{-}W_T$
The P-WR scenario is applied
else
The D-WR scenario is applied

Wavelength service rate

$$R_{PD}(w) = \begin{cases} \sum_{k=1}^{K} \mu_{1k} \cdot y_{INF_{1k}}(b_k, 0) \cdot (1-\sigma_k) \cdot \hat{q}_{INF}(b_k, 0), & w \leq W - W_T \\ \sum_{k=1}^{K} \sum_{j_2=0}^{m \cdot C^*} \mu_{1k} \cdot y_{INF_{1k}}(b_k, j_2) \cdot (1-\sigma_k) \cdot \hat{q}_{INF}(b_k, j_2) & w > W - W_T \end{cases}$$

Distribution of occupied wavelengths

$$P(w) = \frac{(\lambda)^w}{w!} \prod_{z=1}^{w} \frac{\left[N-(z-1)\right]}{R_{PD}(z)} \cdot \left[\sum_{l=0}^{W} \frac{(\lambda)^l}{l!} \prod_{m=1}^{l} \frac{\left[N-(m-1)\right]}{R_{PD}(m)} \right]^{-1}$$

Delay function

$$T_{PD}(w) = \frac{1}{\Lambda_s} - \frac{1}{R_{PD}(w)}$$

Average delay

$$\bar{T}_{PD} = \frac{\sum_{w=1}^{W} T_{PD}(w)}{W} = \frac{1}{\Lambda_s} - \frac{\sum_{w=1}^{W} \frac{1}{R_{PD}(w)}}{W}$$

Number of calls that suffer delay

$$N_{\Pi D,k} = \frac{\sum_{w=W-W_T}^{W} \sum_{j_2=1}^{m \cdot C^*} y_{INF_{2k}}[b_k][j_2]}{W}$$

Results (1/7)

Comparison of analytical and simulation results

Parameter	**Value**
Number of ONUs N	24
Number of wavelengths W	16
Service-classes K	2
Real capacity C	90 b.u.
Fictitious capacity C^*	100 b.u.
Bandwidth requirements	(b_1, b_2)=(20,16)
Service-time in state ON	$(\mu_{11}{}^{-1}, \mu_{12}{}^{-1})$=(0.009, 0.008)
Service-time in state OFF	$(\mu_{11}{}^{-1}, \mu_{12}{}^{-1})$=(0.01, 0.01)
Probability of transition to state ON	(σ_1, σ_2)=(0.95, 0.9)

Results (1/7)

Results for the P-WR scenario

Parameter	Value
Number of ONUs N	24
Number of wavelengths W	16
Service-classes K	2
Real capacity C	90 b.u.
Fictitious capacity C^*	100 b.u.
Bandwidth requirements	(b_1, b_2)=(20,16)
Service-time in state ON	$(\mu_{11}^{-1}, \mu_{12}^{-1})$=(0.009, 0.008)
Service-time in state OFF	$(\mu_{11}^{-1}, \mu_{12}^{-1})$=(0.01, 0.01)
Probability of transition to state ON	(σ_1, σ_2)=(0.95, 0.9)

Arrival rate (calls/sec)	**CFP**	
	Analysis (%)	***Simulation***
0.6	0.00652	0.00656 ± 2.52e-4
0.8	0.12221	0.12129 ± 2.55e-3
1.0	0.82844	0.82394 ± 0.0122
1.2	3.00679	3.00790 ± 0.0145
1.4	7.25745	7.24689 ± 0.0129
1.6	13.37831	13.3706 ± 0.0557

Arrival rate (calls/sec)	**CBP of 1st service-class**		**CBP of 2nd service-class**	
	Analysis (%)	***Simulation***	***Analysis (%)***	***Simulation***
0.6	3.725e-3	3.87e-3 ± 3.12e-4	0.0032	0.003 ± 3.12e-4
0.8	1.255e-2	1.21e-2 ± 8.92e-4	0.0109	0.011 ± 5.81e-4
1.0	3.189e-2	3.21e-2 ± 1.27e-3	0.0281	0.028 ± 1.29e-3
1.2	6.748e-2	6.75e-2 ± 1.29e-3	0.0602	0.061 ± 2.11e-3
1.4	12.57e-2	12.47e-2 ± 2.59e-3	0.1132	0.013 ± 2.74e-3
1.6	21.33e-2	21.16e-2 ± 3.61e-3	0.1934	0.192 ± 1.99e-3

Results (1/7)

Results for the P-WR scenario

Parameter	**Value**
Number of ONUs N	24
Number of wavelengths W	16
Service-classes K	2
Real capacity C	90 b.u.
Fictitious capacity C^*	100 b.u.
Bandwidth requirements	(b_1, b_2)=(20,16)
Service-time in state ON	$(\mu_{11}^{-1}, \mu_{12}^{-1})$=(0.009, 0.008)
Service-time in state OFF	$(\mu_{11}^{-1}, \mu_{12}^{-1})$=(0.01, 0.01)
Probability of transition to state ON	(σ_1, σ_2)=(0.95, 0.9)

Arrival rate (calls/sec)	**CFP**	
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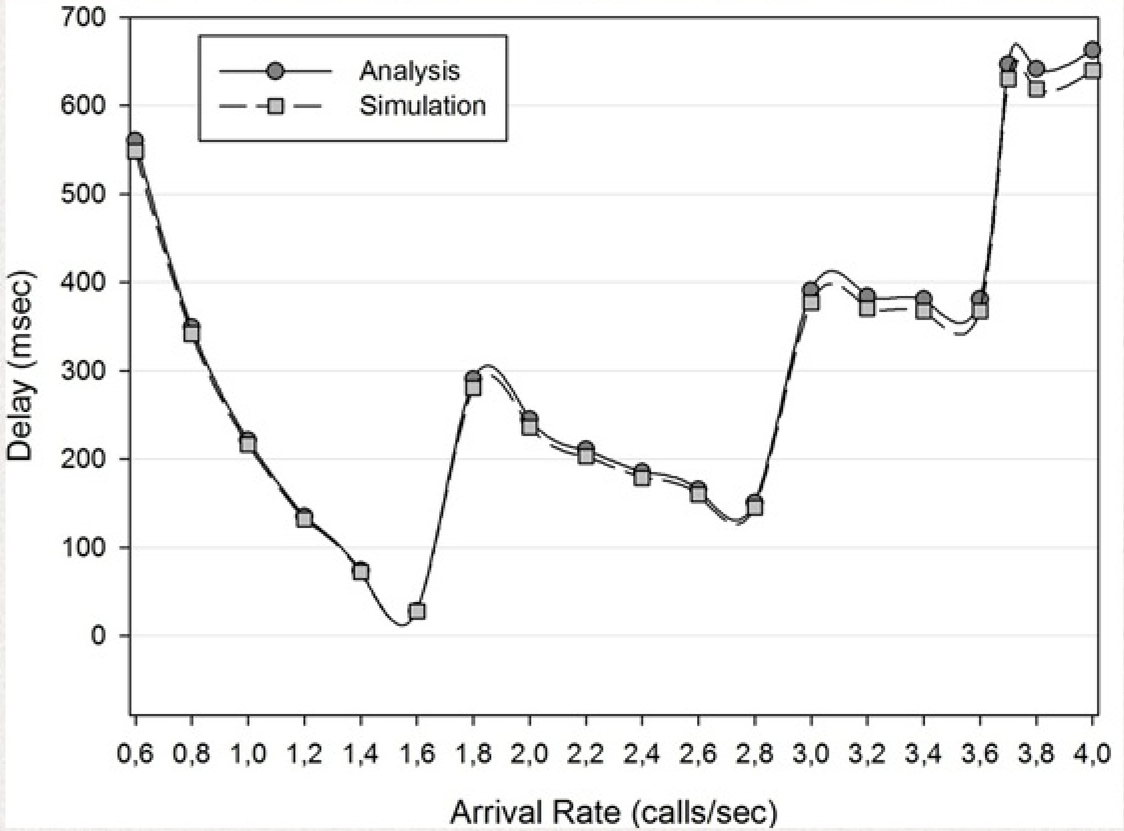
1.6	13.37831	13.3706 ± 0.0557

Arrival rate (calls/sec)	**BBP of 1st service-class**		**BBP of 2nd service-class**	
	Analysis(%)	***Simulation***	***Analysis(%)***	***Simulation***
0.6	1.62e-3	1.59e-3 ± 1.24e-4	1.14e-3	1.12e-2 ± 4.86e-5
0.8	4.61e-3	4.58e-3 ± 1.28e-4	3.24e-2	3.27e-3 ± 2.67e-4
1.0	1.01e-2	1.01e-2 ± 1.63e-4	7.10e-2	7.29e-3 ± 1.68e-4
1.2	1.88e-2	1.91e-2 ± 2.10e-4	13.2e-2	1.40e-2 ± 1.29e-4
1.4	3.13e-2	3.21e-2 ± 6.11e-4	2.20e-2	2.20e-2 ± 5.02e-4
1.6	4.80e-2	4.72e-2 ± 1.02e-3	3.38e-2	3.40e-2 ± 4.56e-4

Results (2/7)

Results for the D-WR scenario with the constant function (m=0.2)

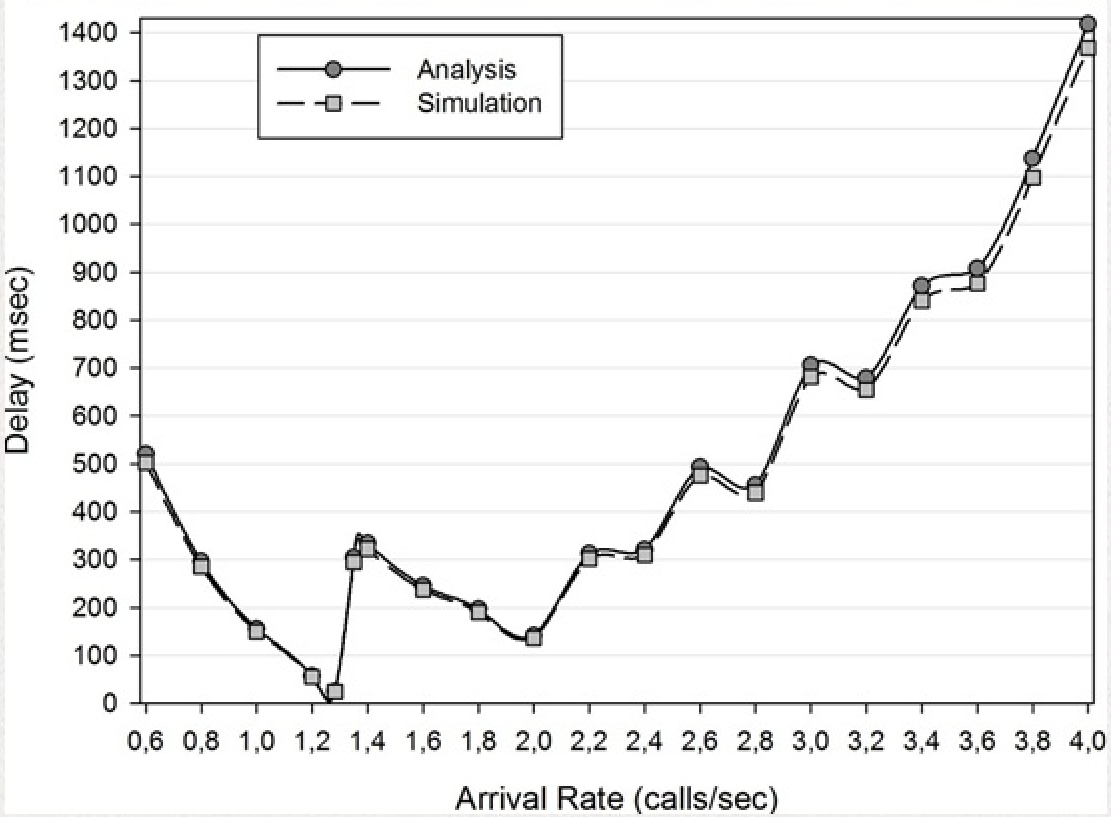
Arrival rate (calls/sec)	**CFP**	
	Analysis (%)	***Simulation***
0.6	0.00652	0.00656 ± 2.52e-4
0.8	0.12221	0.12129 ± 2.55e-3
1.0	0.82844	0.82394 ± 0.0122
1.2	3.00679	3.00790 ± 0.0145
1.4	7.25745	7.24689 ± 0.0129
1.6	13.37831	13.3706 ± 0.0557



Results (3/7)

Results for the D-WR scenario with the increase function(m=0.2)

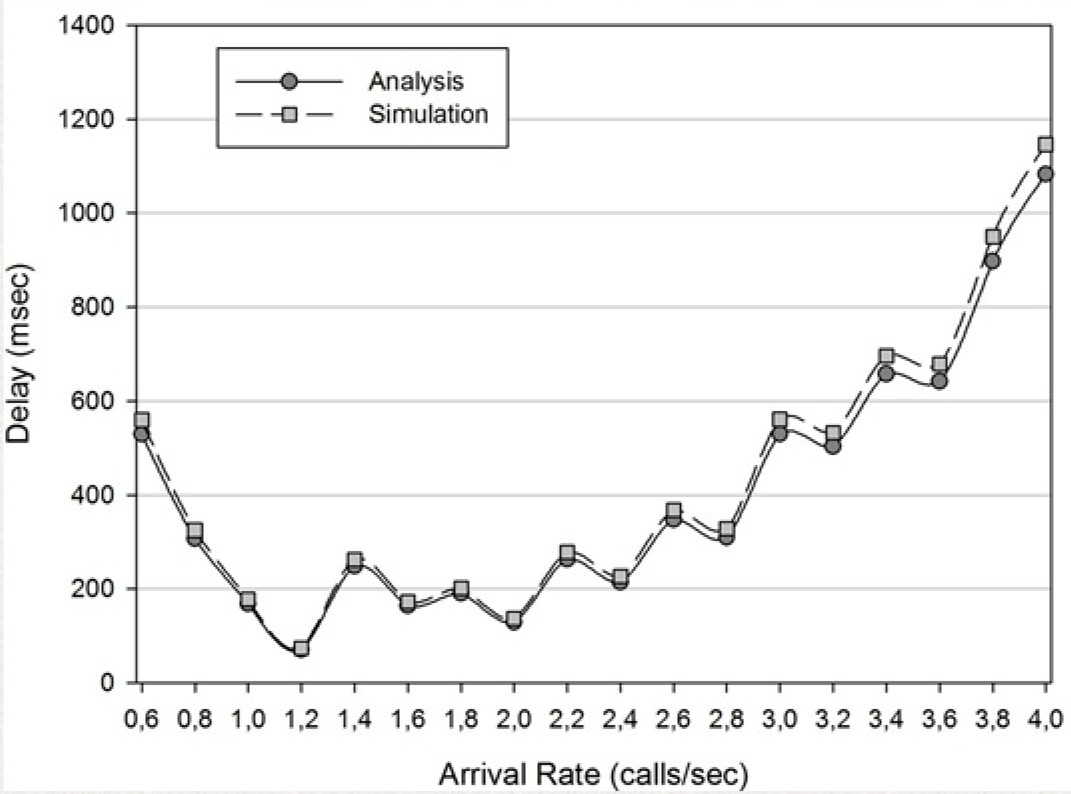
Arrival rate (calls/sec)	**CFP**	
	Analysis (%)	***Simulation***
0.6	0.0048	0.0049 ± 1.84e-4
0.8	0.0813	0.0880 ±2.47e-3
1.0	0.5106	0.5315 ± 1.42e-2
1.2	1.7603	1.7721 ± 1.85e-2
1.4	4.1569	4.0845 ± 1.48e-2
1.6	7.7801	7.3075 ± 2.64e-2



Results (4/7)

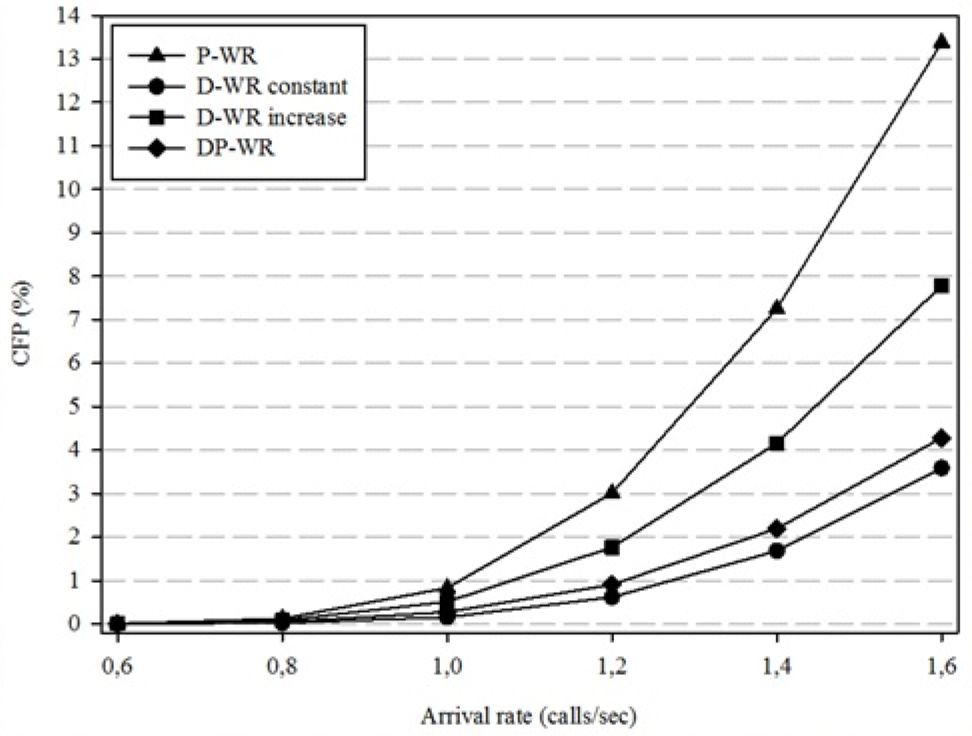
Results for the PD-WR scenario (W_T=4)

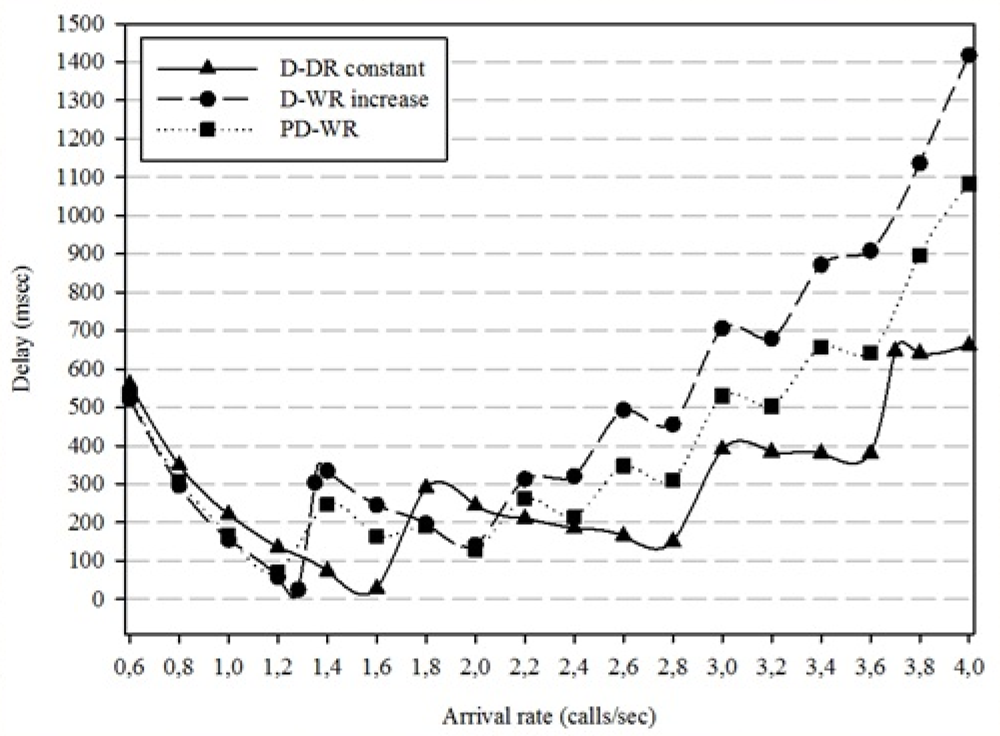
Arrival rate (calls/sec)	**CFP**	
	Analysis (%)	***Simulation***
0.6	0.0039	0.0041 ± 3.41e-4
0.8	0.0456	0.0530 ± 1.92e-3
1.0	0.2769	0.2990 ± 6.62e-3
1.2	0.9042	0.9590 ± 1.60e-2
1.4	2.1933	2.3475 ± 1.45e-2
1.6	4.2779	4.4255 ± 4.11e-2



Results (5/7)

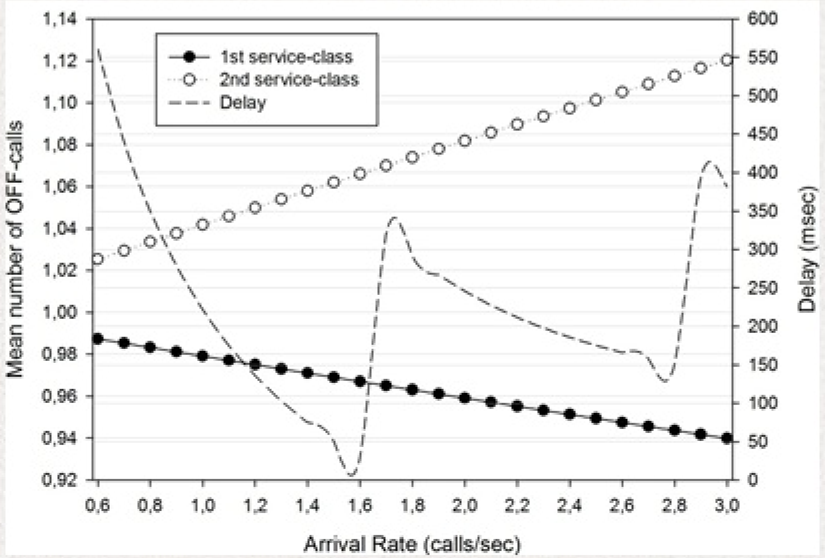
Comparison of the scenarios

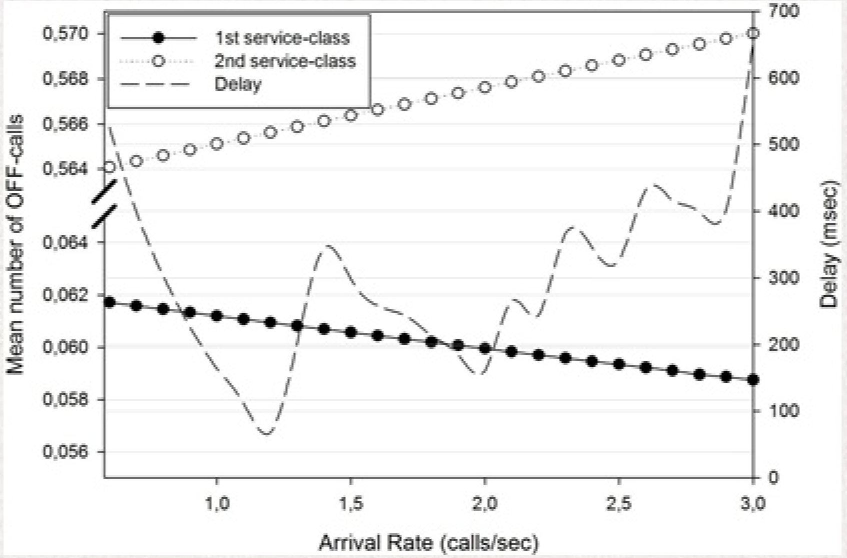


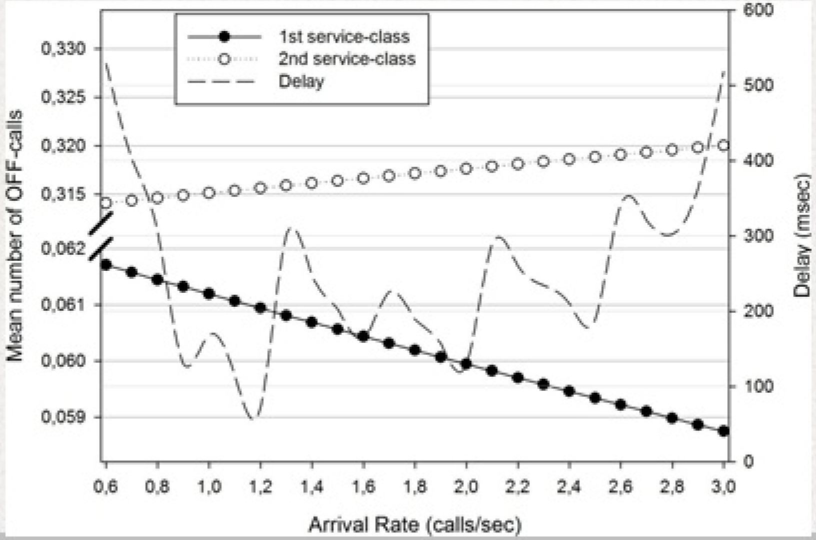


Results (6/7)

Average number of calls that suffer delay

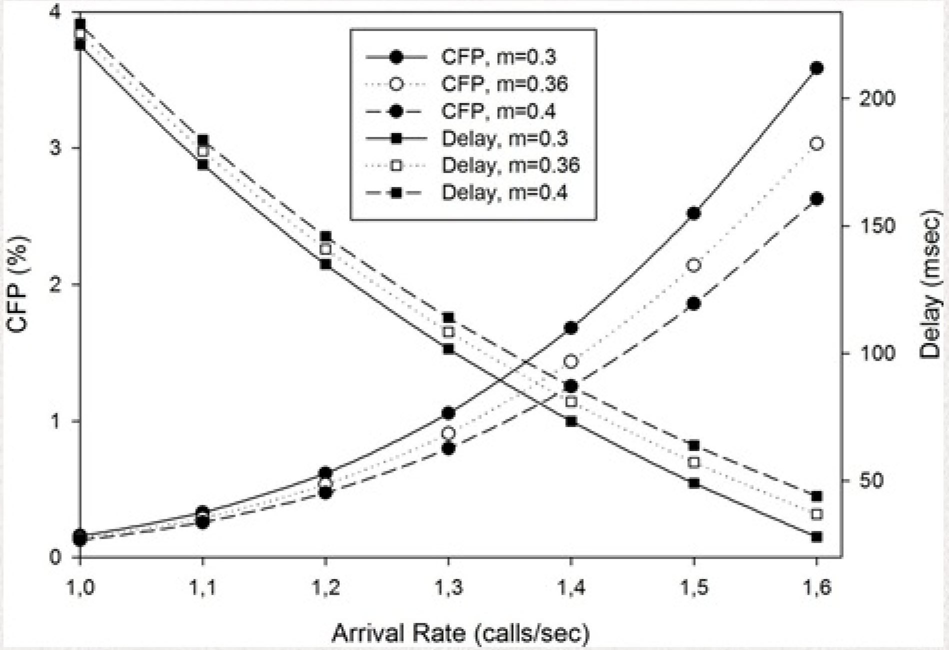


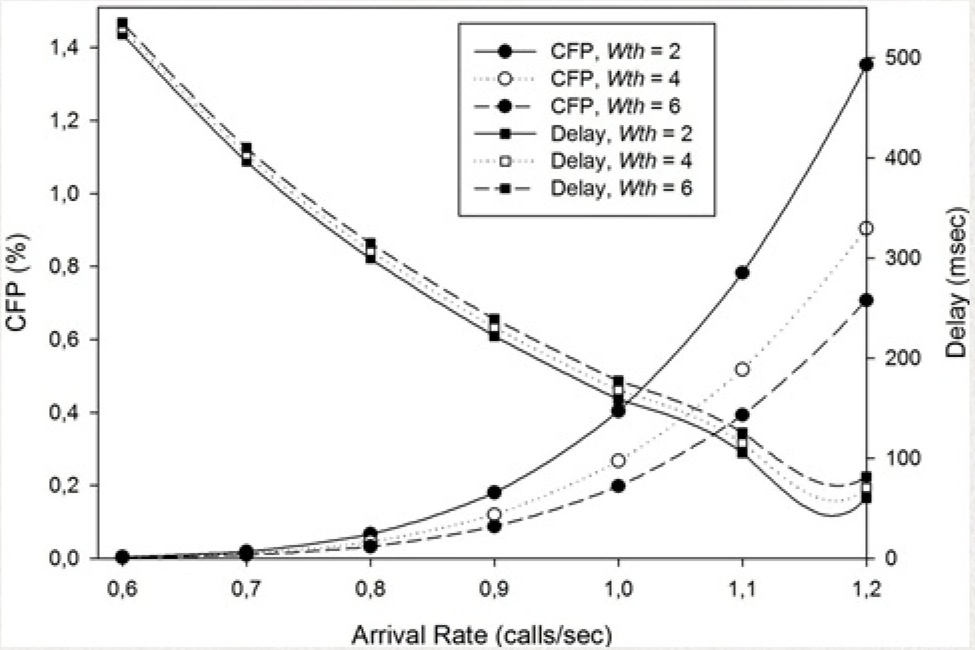




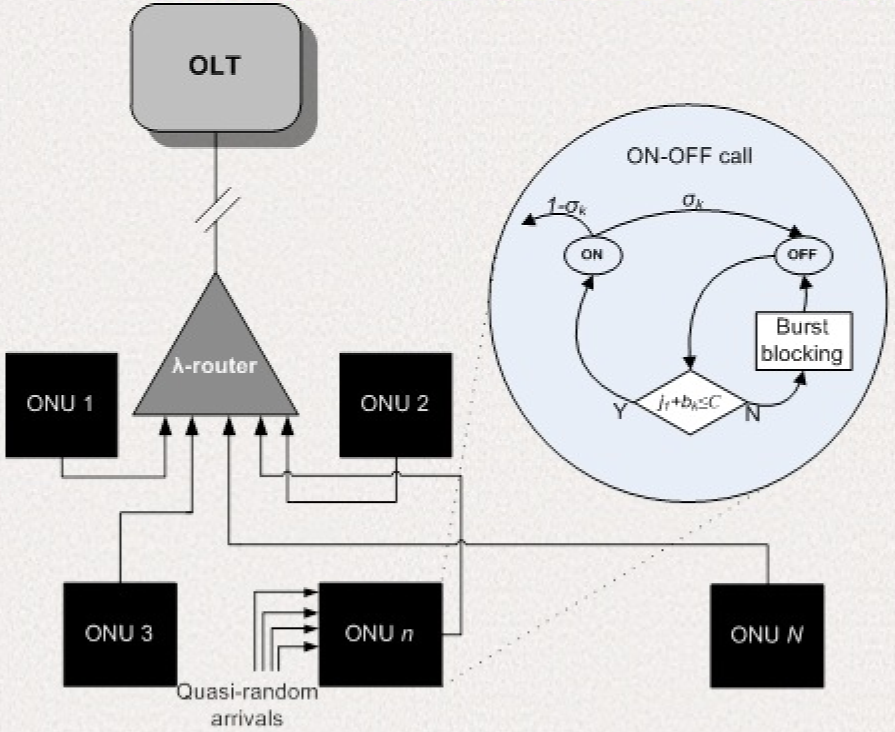
Results (7/7)

Effect of the parameters m and W_T to CFP and to the delay





Analytical model for ON-OFF traffic an finite traffic sources



- Traffic is generated from a finite number of traffic sources
- Number of traffic sources of service-class k: M_k
- Arrival rate per traffic source: v_k
- Application of the multi-rate ON-OFF model with finite traffic sources¹

$$n_k^i(\vec{j}) \approx y_{INF_{ik}}(\vec{j}) = \frac{M(i)\cdot p_{F_{ik}} \cdot q_{INF}(\vec{j}-B_{i,k})}{q_{INF}(\vec{j})}$$

$$q_F(\vec{j}) = \begin{cases} 1 & \text{for } \vec{j}=0 \\ \sum_{i=1}^{2}\sum_{k=1}^{K}\left(M_k - n_k^1 - n_k^2 + 1\right) b_{i,k,s} p_{F_{ik}} q_F(\vec{j}-B_{i,k}) = j_s & \text{for } j_1 = 1,\dots,C \text{ (if } s=1) \text{ or for } j_2 = 1,\dots,C^* \text{ - } j_1 \text{ (if } s=2) \\ 0 & \text{otherwise} \end{cases}$$

$$s = \begin{cases} 1 & \Rightarrow \text{real link} \\ 0 & \Rightarrow \text{fictitious link} \end{cases}$$

$$p_{F_{ik}} = \frac{e_{F_{i,k}}}{\mu_{i,k}} = \begin{cases} \dfrac{v_k}{(1-\sigma_k)\mu_{1k}} & \text{for } i=1 \\ \dfrac{v_k\sigma_k}{(1-\sigma_k)\mu_{2k}} & \text{for } i=2 \end{cases}$$

$$\vec{j} = (j_1, j_2),\ B_{i,k} = (b_{i,k,1}, b_{i,k,2}) \qquad b_{i,k,s} = \begin{cases} b_k, & \text{if } s=i \\ 0, & \text{if } s \ne i \end{cases}$$

$$i = \begin{cases} 1 & \Rightarrow \text{state ON} \\ 0 & \Rightarrow \text{state OFF} \end{cases}$$

1. Moscholios I., Logothetis M., Koukias M. N., "An ON-OFF Multi-Rate Loss Model of Finite Sources", *IEICE Transactions on Communications*, Vol. E90-B, No. 7, pp. 1608-1619, July 2007.

Wavelength Release scenarios

✓Primary WR scenario

✓Delay WR scenario

✓Primary-Delay WR scenario

✓ Priority WR scenario

Wavelength Release scenarios

Priority WR scenario

K service-classes

K_1 high priority service-classes

K_2 low priority service-classes

Connection release

- When all ON-calls of both priorities and all OFF-calls of the high priority terminate
- Only a number of low priority OFF-calls suffer delay
- For these calls the D-WR scenario with the constant function is applied

Wavelength Release scenarios

Priority WR scenario

Connection release

- When all ON-calls of both priorities and all OFF-calls of the high priority terminate
- Only a number of low priority OFF-calls suffer delay
- For these calls the D-WR scenario with the constant function is applied

Wavelength service-rate

$$R_{\mathrm{Pr}} = \sum_{k=1}^{K} \sum_{j_2=0}^{J} \mu_{1k} \cdot y_{F_{1k}}(b_k, j_2) \cdot (1-\sigma_k) \cdot \hat{q}_F(b_k, j_2) \qquad J = \begin{cases} j-1 & \text{if } y_{F_{2k_1}}(0,j) > 0 \text{ and } y_{F_{2k_1}}(0,j_2) = 0,\ \forall j_2 < j-1 \\ mC^* & \text{otherwise} \end{cases}$$

$$P(w) = \left(\frac{\lambda}{R_{\mathrm{Pr}}}\right)^{w} \frac{\prod_{z=1}^{w}[N-(z-1)]}{w!} \left[\sum_{l=0}^{W} \left(\frac{\lambda}{R_{\mathrm{Pr}}}\right)^{l} \frac{\prod_{m=1}^{l}[N-(m-1)]}{l!} \right]^{-1} \Rightarrow \text{CFP: } P(W)$$

Delay ⇒ $T_D = \frac{1}{\Lambda_s} - \frac{1}{R_{\mathrm{Pr}}}$

Average number of calls that suffer delay ⇒ $N_{Pr,k} = \sum_{j_2=1}^{J} y_{F_{2k}}(b_k, j_2)$

Results (1/5)

Comparison of analytical and simulation results

Parameter	**Value**
Number of ONUs N	24
Number of wavelengths W	16
Service-classes K	2
Number of service-classes M_k	20
Real capacity C	90 b.u.
Fictitious capacity C^*	100 b.u.
Bandwidth requirements	(b_1, b_2)=(20,16)
Service-time is state ON	$(\mu_{11}^{-1}, \mu_{12}^{-1})$=(0.009, 0.008)
Service-time is state OFF	$(\mu_{11}^{-1}, \mu_{12}^{-1})$=(0.01, 0.01)
Probability of transition to state ON	(σ_1, σ_2)=(0.95, 0.9)

Results (1/5)

Results for the Primary WR scenario

Parameter	**Value**
Number of ONUs N	24
Number of wavelengths W	16
Service-classes K	2
Number of service-classes M_k	20
Real capacity C	90 b.u.
Fictitious capacity C^*	100 b.u.
Bandwidth requirements	(b_1, b_2)=(20,16)
Service-time is state ON	$(\mu_{11}^{-1}, \mu_{12}^{-1})$=(0.009, 0.008)
Service-time is state OFF	$(\mu_{11}^{-1}, \mu_{12}^{-1})$=(0.01, 0.01)
Probability of transition to state ON	(σ_1, σ_2)=(0.95, 0.9)

Arrival rate per idle source (calls/sec)	**CFP**	
	Analysis(%)	***Simulation***
0.03	0.002437	0.00233 ± 2.1e-4
0.04	0.051031	0.04882 ± 4.2e-4
0.05	0.387576	0.37078 ± 5.9e-4
0.06	1.568473	1.50049 ± 1.0e-3
0.07	4.181966	4.00071 ± 7.1e-3
0.08	8.395249	8.03137 ± 2.1e-2

Arrival rate per idle source (calls/sec)	**CBP of 1st service-class**		**CBP of 2nd service-class**	
	Analysis (%)	***Simulation***	***Analysis (%)***	***Simulation***
0.03	0.00311	0.00224 ± 2.7e-4	0.00234	0.00224 ± 1.4e-4
0.04	0.01009	0.00966 ± 3.3e-4	0.00778	0.00745 ± 3.3e-4
0.05	0.02493	0.02385 ± 6.3e-4	0.01982	0.01896 ± 5.5e-4
0.06	0.05117	0.04895 ± 8.1e-4	0.04118	0.03939 ± 6.1e-4
0.07	0.09470	0.09060 ± 1.8e-3	0.07772	0.07435 ± 1.6e-3
0.08	0.15854	0.15167 ± 5.5e-3	0.13162	0.12591 ± 5.2e-3

Results (1/5)

Results for the Primary WR scenario

Parameter	Value
Number of ONUs N	24
Number of wavelengths W	16
Service-classes K	2
Number of service-classes M_k	20
Real capacity C	90 b.u.
Fictitious capacity C^*	100 b.u.
Bandwidth requirements	(b_1, b_2)=(20,16)
Service-time is state ON	$(\mu_{11}^{-1}, \mu_{12}^{-1})$=(0.009, 0.008)
Service-time is state OFF	$(\mu_{11}^{-1}, \mu_{12}^{-1})$=(0.01, 0.01)
Probability of transition to state ON	(σ_1, σ_2)=(0.95, 0.9)

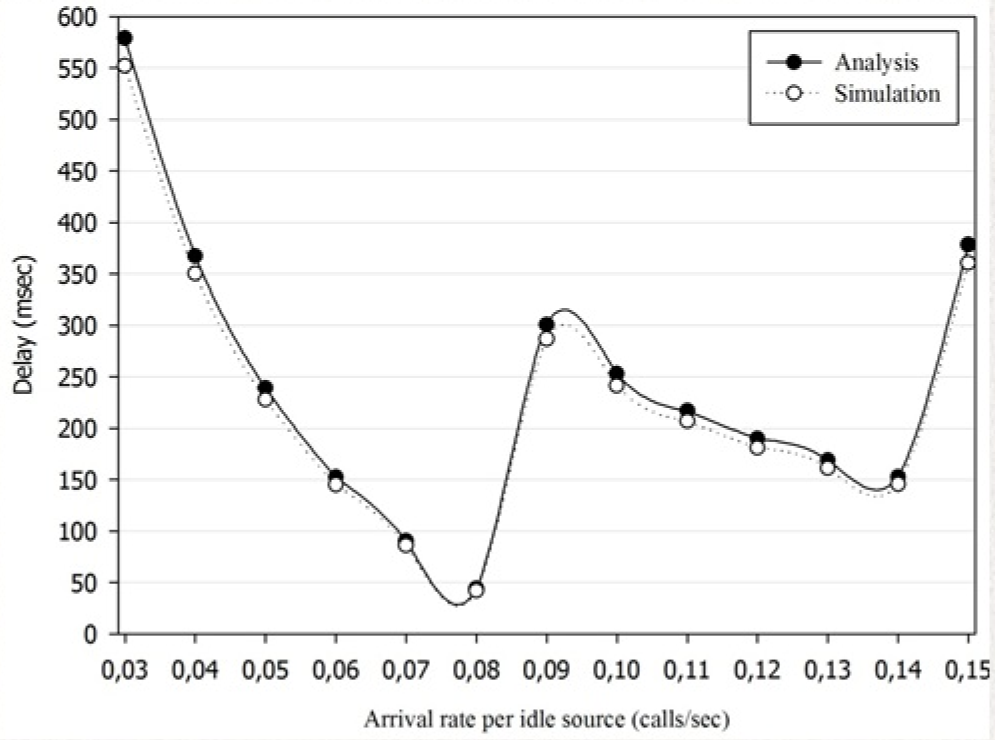
Arrival rate per idle source (calls/sec)	CFP	
	Analysis(%)	*Simulation*
0.03	0.002437	0.00233 ± 2.1e-4
0.04	0.051031	0.04882 ± 4.2e-4
0.05	0.387576	0.37078 ± 5.9e-4
0.06	1.568473	1.50049 ± 1.0e-3
0.07	4.181966	4.00071 ± 7.1e-3
0.08	8.395249	8.03137 ± 2.1e-2

Arrival rate per idle source (calls/sec)	BBP of 1st service-class		BBP of 2nd service-class	
	Analysis (%)	*Simulation*	*Analysis (%)*	*Simulation*
0.03	0.00122	0.00107 ± 1.5e-4	0.00083	0.00073 ± 5.5e-5
0.04	0.00329	0.00288 ± 1.7e-4	0.00225	0.00197 ± 1.0e-4
0.05	0.00693	0.00607 ± 2.2e-4	0.00474	0.00415 ± 2.5e-4
0.06	0.01246	0.01091 ± 4.5e-4	0.00852	0.00746 ± 2.9e-4
0.07	0.02013	0.01763 ± 4.9e-4	0.01376	0.01205 ± 6.5e-4
0.08	0.03009	0.02635 ± 8.9e-4	0.00206	0.00180 ± 8.8e-4

Results (2/5)

Results for the Delay WR scenario (m=0.4)

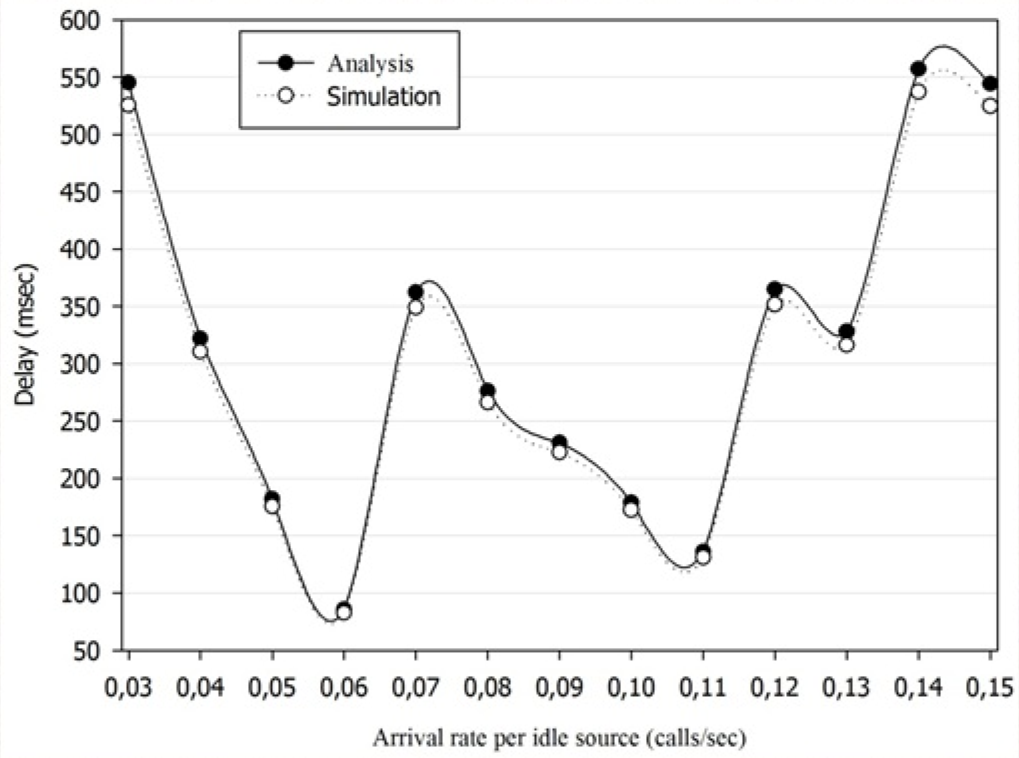
Arrival rate per idle source (calls/sec)	**CFP**	
	Analysis (%)	***Simulation***
0.03	0.00070	0.00066 ± 2.5e-4
0.04	0.01295	0.01200 ± 3.0e-4
0.05	0.09547	0.08997 ± 5.0e-4
0.06	0.40280	0.37957 ± 1.1e-3
0.07	1.17427	1.10655 ± 1.9e-3
0.08	2.64426	2.49174 ± 3.1e-3



Results (3/5)

Results for the Primary Delay WR scenario(W_T=4)

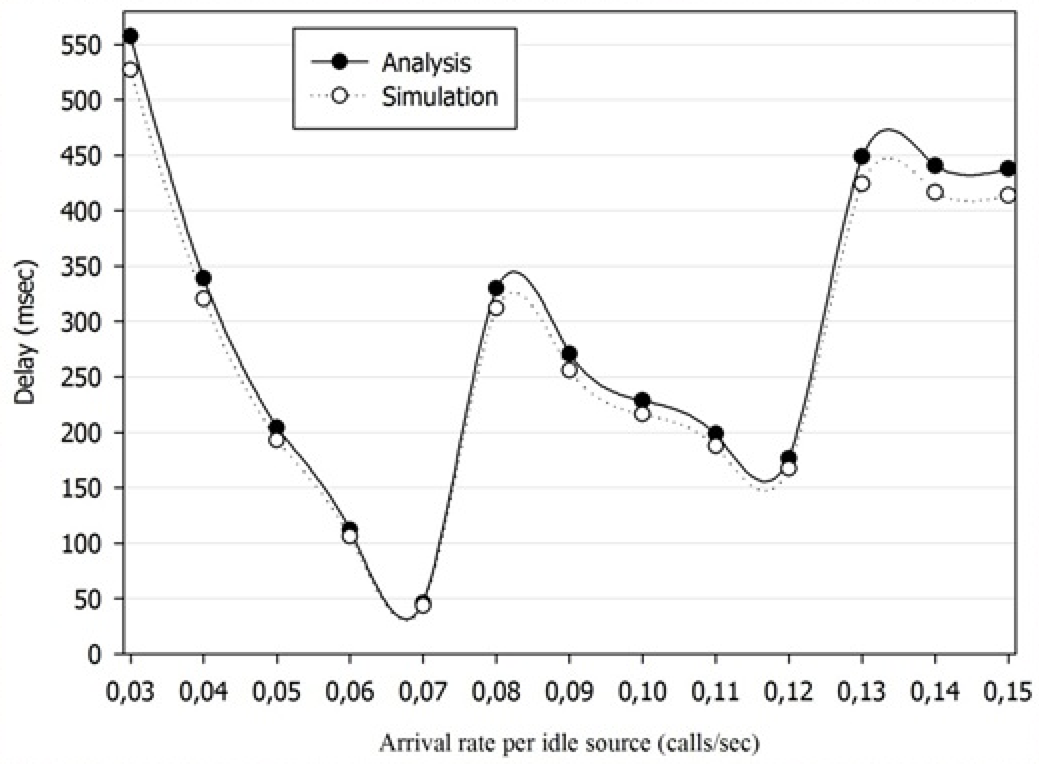
Arrival rate per idle source (calls/sec)	**CFP**	
	Analysis (%)	***Simulation***
0.03	0.00234	0.00225 ± 2.0e-4
0.04	0.04787	0.04616 ± 4.1e-4
0.05	0.35407	0.34136 ± 7.2e-4
0.06	1.39187	1.34193 ± 1.3e-3
0.07	3.60877	3.47930 ± 2.5e-3
0.08	7.07757	6.82363 ± 7.5e-3



Results (4/5)

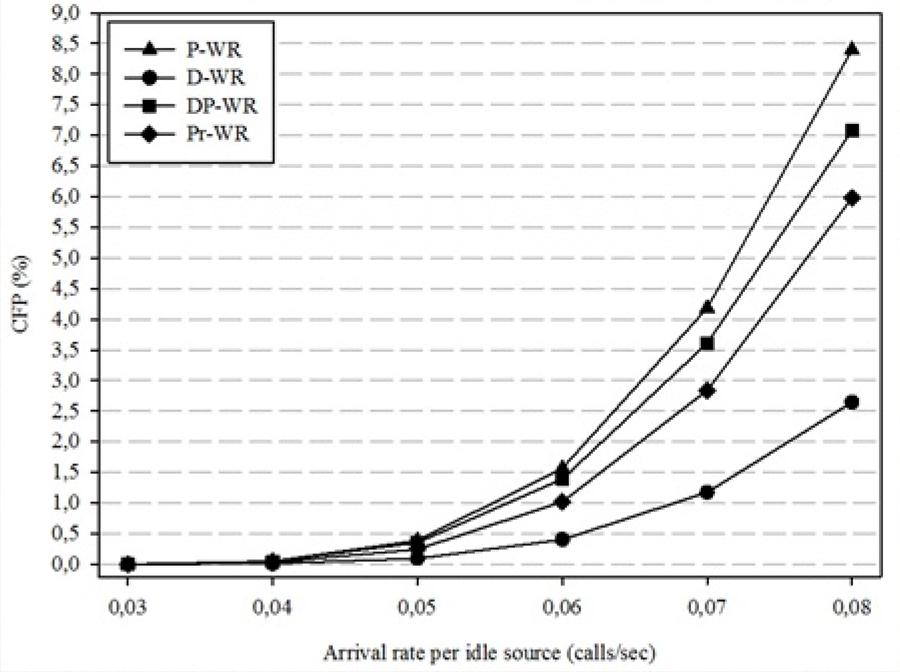
Results for the Priority WR scenario

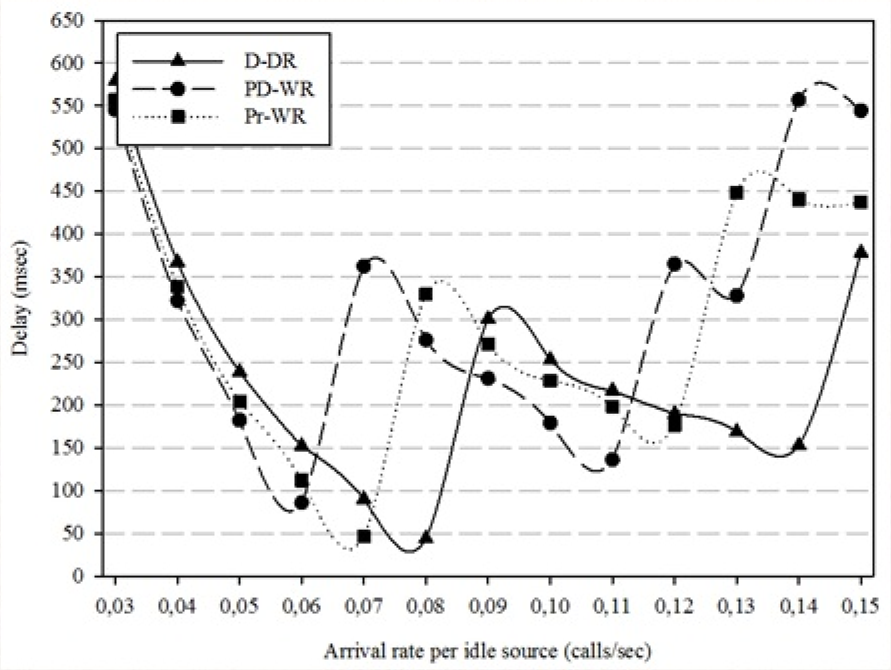
Arrival rate per idle source (calls/sec)	CFP	
	Analysis (%)	***Simulation***
0.03	0.001604	0.00106 ± 1.3e-4
0.04	0.032423	0.02521 ± 2.1e-4
0.05	0.246216	0.21897 ± 3.4e-4
0.06	1.022117	0.95857 ± 8.9e-4
0.07	2.839859	2.70545 ± 1.1e-3
0.08	5.980038	5.59144 ± 2.7e-3



Results (5/5)

Comparison of the 4 scenarios



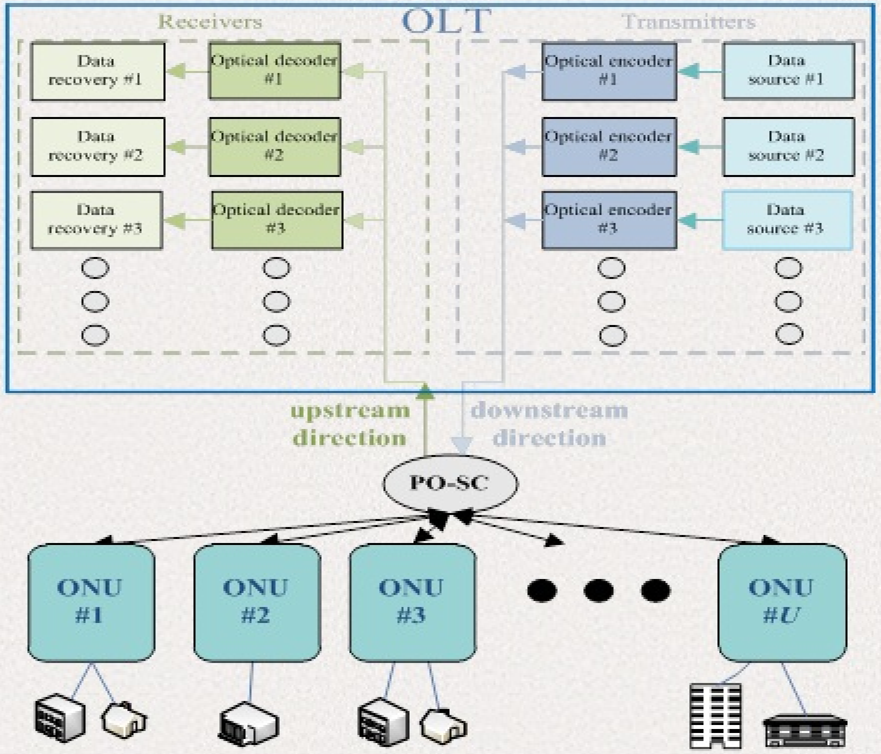


Number of calls that suffer delay				
	1st service-class		2nd service-class	
	Analysis	*Simulation*	*Analysis*	*Simulation*
D-WR	1.05263	0.992	1.05263	0.989
PD-DR	0.06579	0.062	0.06579	0.061
Pr-WR	0	0	1.05263	0.957

Structure

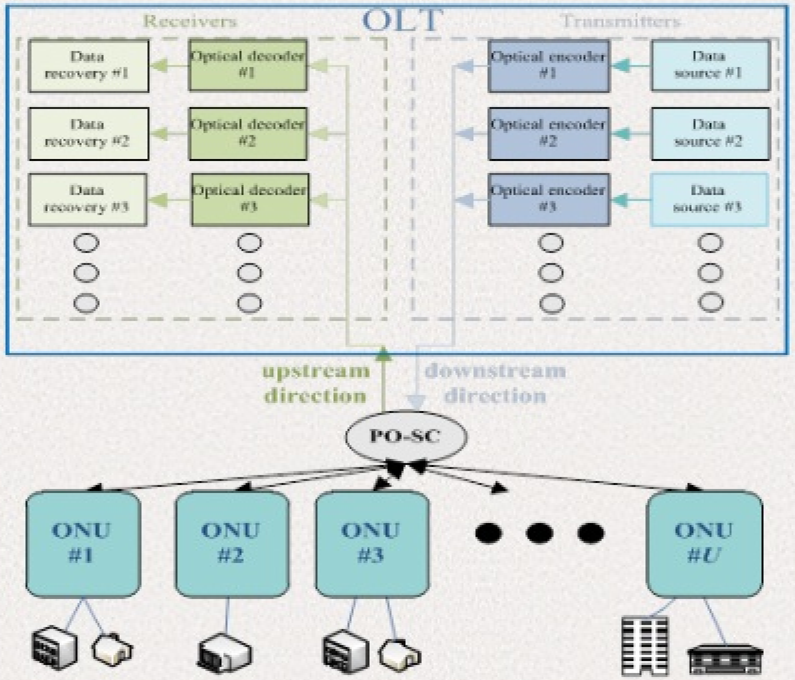
- Access Networks
- Passive Optical Networks
- Performance Analysis of EPONs
- Performance Analysis of WDM-TDMA PONs
- **Performance Analysis of OCDMA PONs**

Analytical model for OCDMA PON



1. S. Goldberg and P. R. Prucnal, "On the Teletraffic Capacity of Optical CDMA", IEEE Trans. Commun., Vol. 55, No. 7, Jul. 2007, pp. 1334-1343.
2. E. Mutafungwa and S. J. Halme, "Analysis of the Blocking Performance of Hybrid OCDM-WDM Transport Networks", Microwave and Optical Technol. Lett., Vol. 34, No. 1, Jul. 2002, pp. 61-67.
3. M. Gharaei, C. Lepers, O. Affes, and P. Gallion, "Teletraffic Capacity Performance of WDM/DS-OCDMA Passive Optical Network", NEW2AN/ruSMART 2009, LNCS 5764, Springer-Verlag Berlin Heidelberg, 2009, pp. 132-142.
4. Y. Deng and P. R. Prucnal, "Performance Analysis of Heterogeneous Optical CDMA Networks with Bursty Traffic and Variable Power Control", IEEE/OSA J. Optical Commun. and Netw., Vol. 4, No. 4, Jul. 2011, pp. 313-316.

Analytical model for OCDMA PON



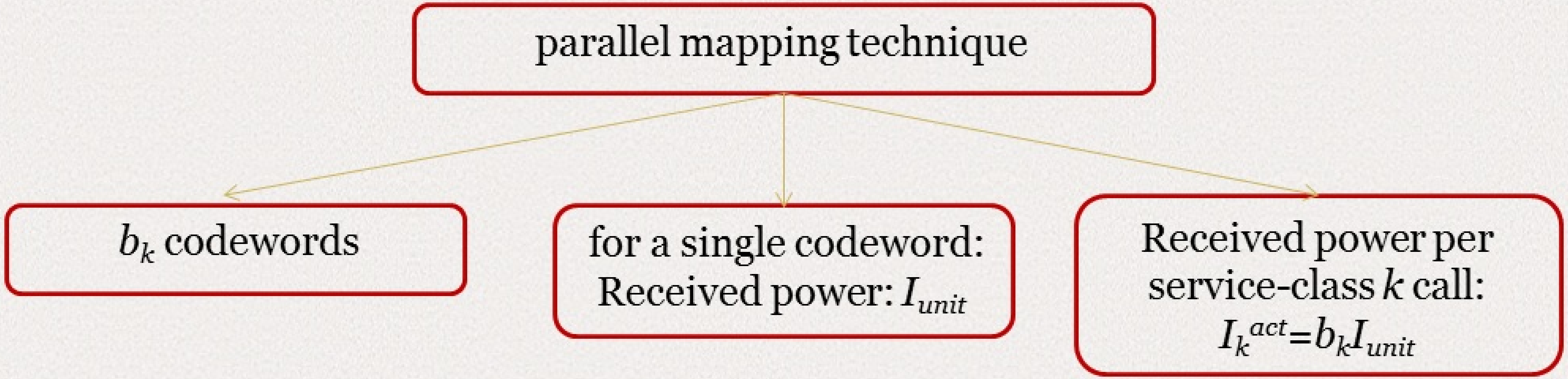
- ✓Upstream direction
- ✓*K* service-classes
- ✓Parallel mapping technique
- ✓Active/Passive users
- ✓Additive noise
- ✓Poisson arrivals

Target of the analysis

An analytical model for the calculation
of the distribution of active and passive users

System model (1/3)

Use of (F, W, l_a, l_c) codes with constant length F, constant weight W, while the auto-correlation parameter l_a and cross-correlation parameter l_c are defined by the desirable BER



parallel mapping technique

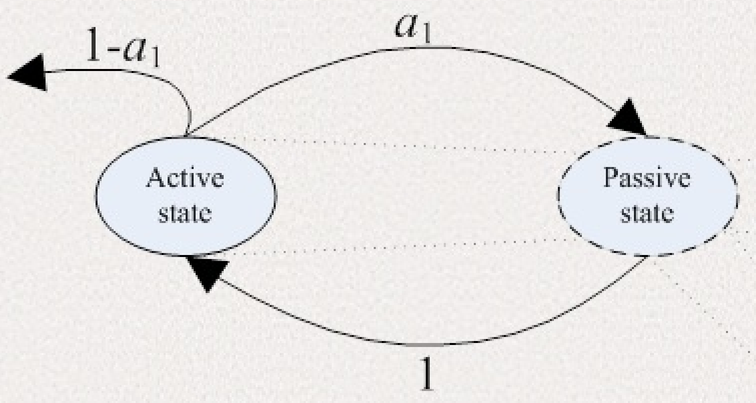
b_k codewords

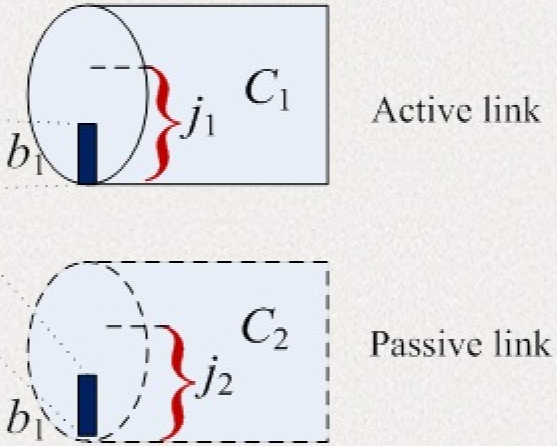
for a single codeword:
Received power: I_{unit}

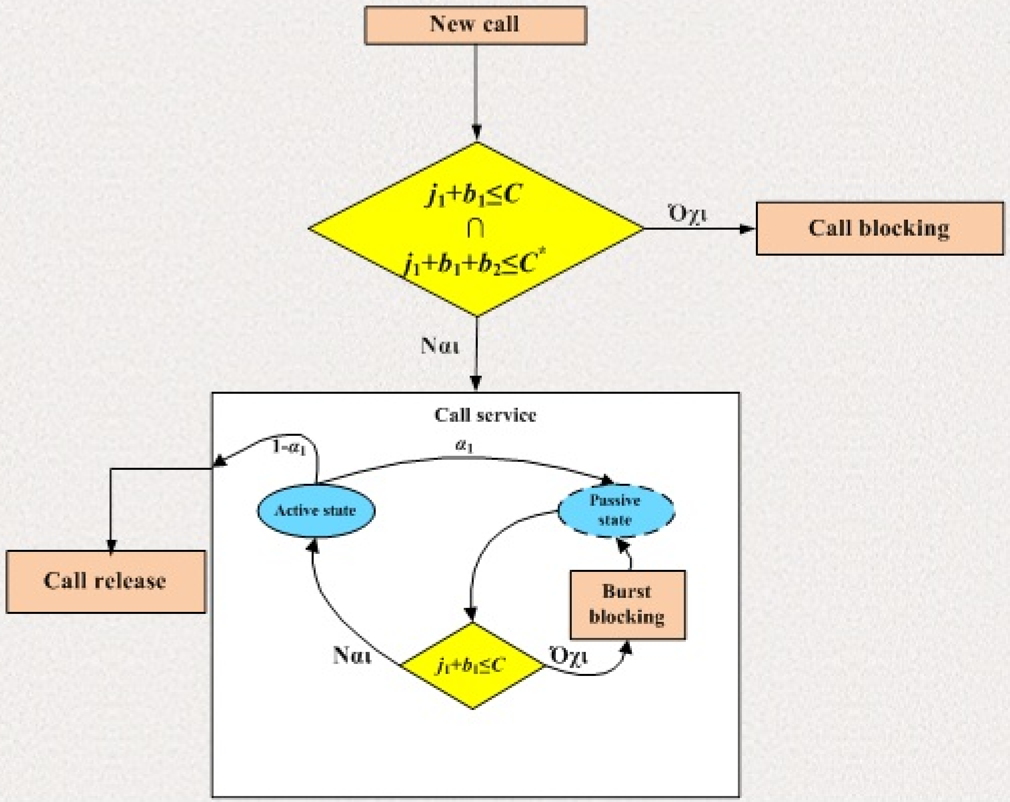
Received power per service-class k call:
$I_k^{act}=b_kI_{unit}$

Total received power at the receiver: $I_{\max}$
Total number of codewords: C

System model (2/3)

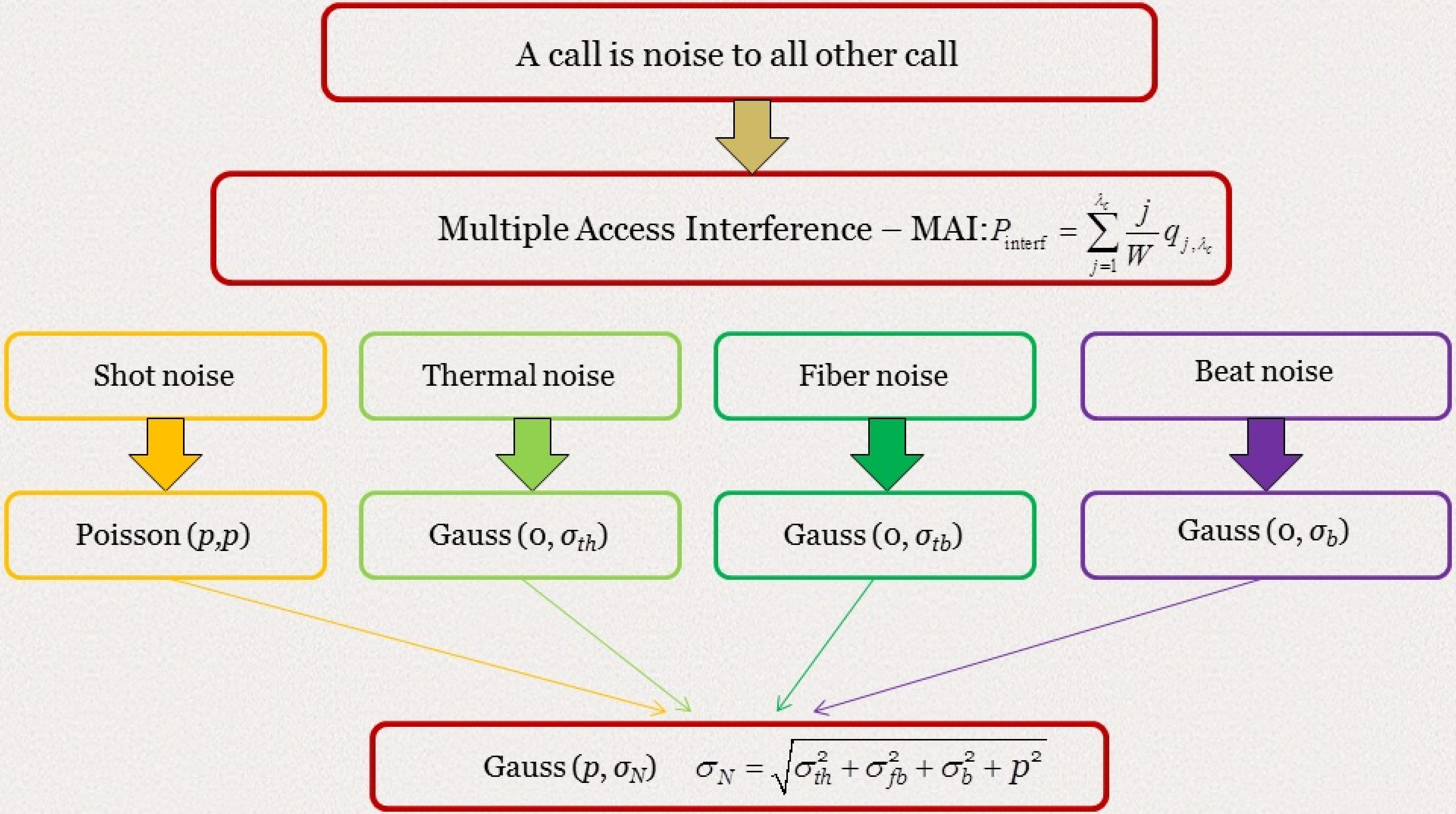






- C: Number of the supported codewords
- C*: Number of fictitious codewords
- K service-classes
- λ_k: arrival rate
- b_k: απαιτήσεις σε κωδικές λέξεις
- μ_{ik}: χρόνος εξυπηρέτησης στην κατάσταση i
- α_k: πιθανότητα μετάβασης στην ενεργή κατάσταση

System model (3/3)



A call is noise to all other call

Multiple Access Interference – MAI: $P_{\text{interf}} = \sum_{j=1}^{\lambda_c} \frac{j}{W} q_{j,\lambda_c}$

Shot noise

Poisson (p,p)

Thermal noise

Gauss $(0, \sigma_{th})$

Fiber noise

Gauss $(0, \sigma_{tb})$

Beat noise

Gauss $(0, \sigma_b)$

Gauss (p, σ_N) $\sigma_N = \sqrt{\sigma_{th}^2 + \sigma_{fb}^2 + \sigma_b^2 + p^2}$

Local Blocking Probability (LBP)

Call Admission Control

$$\sum_{k=1}^{K}(n_k^1 I_k^{act} P_{\text{interf}}) + I_k^{act} + I_N > I_{\max} \Leftrightarrow \frac{I_N}{I_{\max}} > 1 - \sum_{k=1}^{K}(n_k^1 \frac{I_k^{act}}{I_{\max}} P_{\text{interf}}) - \frac{I_k^{act}}{I_{\max}}$$

LBP

$$L_k(n_k^1) = P\left(\frac{I_N}{I_{\max}} > 1 - \sum_{k=1}^{K}\left(n_k^1 \frac{I_k^{act}}{I_{\max}} P_{\text{interf}}\right) - \frac{I_k^{act}}{I_{\max}}\right)$$

or

$$1 - L_k(n_k^1) = P\left(\frac{I_N}{I_{\max}} \leq 1 - \sum_{k=1}^{K}\left(n_k^1 \frac{I_k^{act}}{I_{\max}} P_{\text{interf}}\right) - \frac{I_k^{act}}{I_{\max}}\right)$$

$I_N/I_{\max} \rightarrow$ Gauss $(\mu_N/I_{\max}, \sigma_N/I_{\max})$

CDF of $I_N/I_{\max} \rightarrow$ $$F_n(x) = \frac{1}{2}\left(1 + erf\left(\frac{x - \mu_N / I_{\max}}{(\sigma_N / I_{\max})\sqrt{2}}\right)\right)$$ $$x = 1 - \sum_{k=1}^{K}(n_k^1 \frac{I_k}{I_{\max}}) - \frac{I_k}{I_{\max}}$$

Distribution of active and passive calls (1/3)

Recursive formula

$$\sum_{i=1}^{2}\sum_{k=1}^{K} b_{i,k,s} p_{ik,_{INF}}(\vec{j}) q_{INF}(\vec{j}-B_{i,k}) = j_s q_{INF}(\vec{j}) \qquad \vec{j}=(j_1,j_2), \quad j_1=\sum_{k=1}^{K} n_k^1 b_k \text{ and } j_2=\sum_{k=1}^{K} n_k^2 b_k$$

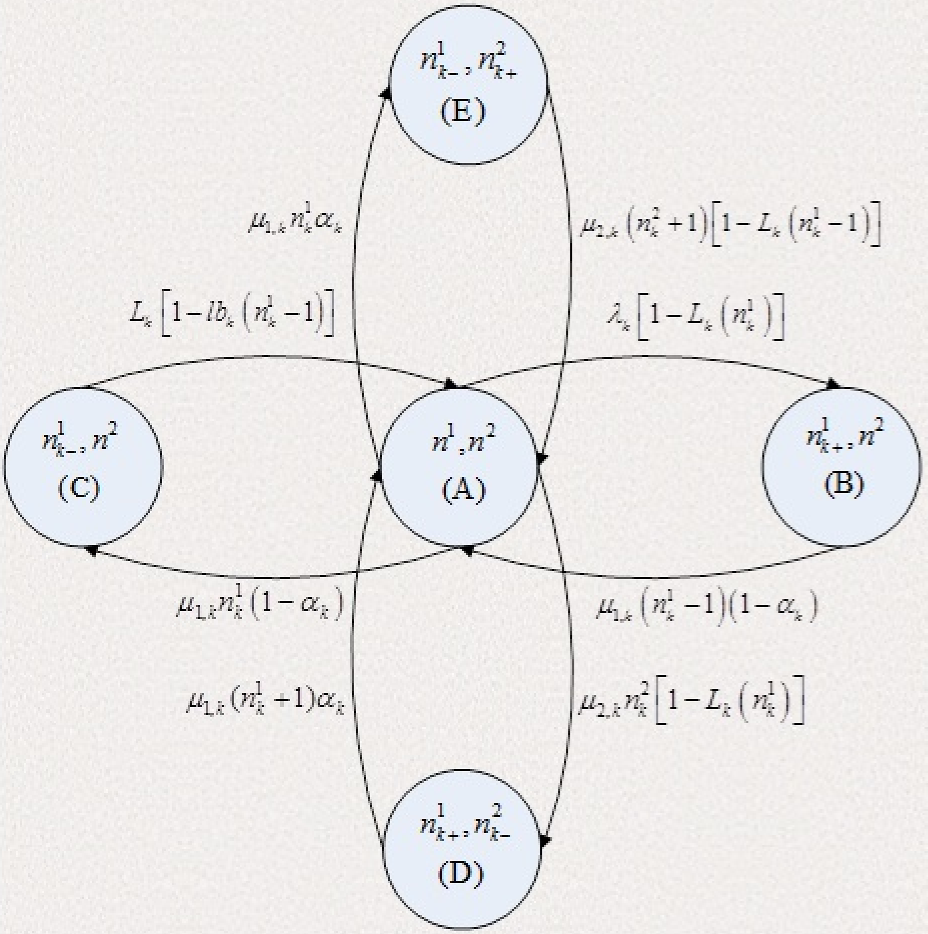
$$\vec{j}\in\Omega \Leftrightarrow \left\{\left(j_1 \le C \cap \left(\sum_{s=1}^{2} j_s \le C^*\right)\right)\right\} \qquad j_s=\sum_{i=1}^{2}\sum_{k=1}^{K} n_k^i b_{i,k,s}$$

Utilization of system *i* from service-class *k*

$$p_{i,k,INF}\left(\vec{j}\right)=\begin{cases}\dfrac{\lambda_k\left[1-L_k\left(j_1-b_k\right)\right]}{(1-\alpha_k)\mu_{1k}} & \text{for } i=1\\[2ex] \dfrac{\lambda_k\sigma_k}{(1-\alpha_k)\mu_{2k}} & \text{for } i=2\end{cases}$$

Distribution of active and passive calls (2/3)

Proof of the recursive formula



$$P(\vec{n})\mu_{ik}n_k^1 v_k = P(\vec{n}_{k-+})\mu_{2k}(n_k^2+1)[1-L_k(n_k^1-1)]$$
$$P(\vec{n})\lambda_k[1-L_k(n_k^1)] = P(\vec{n}_{k+}^1)\mu_{1k}(n_k^1+1)(1-v_k)$$
$$P(\vec{n})\mu_{2k}n_k^2[1-L_k(n_k^1)] = P(\vec{n}_{k+-})\mu_{1k}(n_k^1+1)v_k$$
$$P(\vec{n})\mu_{1k}n_k^1(1-v_k) = P(\vec{n}_{k-}^1)\lambda_k[1-L_k(n_k^1-1)]$$

The system has a product form solution

$$P(\bar{n}) = \frac{1}{G}\prod_{i=1}^{2}\prod_{k=1}^{K}\frac{p_{ik}^{n_k^i}(n_k)}{n_k^1!}$$

$$p_{i,k,INF}\left(\vec{j}\right) = \begin{cases} \dfrac{\lambda_k\left[1-L_k\left(j_1-b_k\right)\right]}{(1-\alpha_k)\mu_{1k}} & \text{για } i=1 \\ \dfrac{\lambda_k\sigma_k}{(1-\alpha_k)\mu_{2k}} & \text{για } i=2 \end{cases}$$

Assumption

$$1-L_k(n_k^1) \approx 1-L_k(n_k^1-1)$$

Distribution of active and passive calls (3/3)

Call Blocking Probability

$$Pb_k = \underbrace{\sum_{\vec{j}\in\Omega-\Omega_h} G^{-1} L_k(j_1) q_{INF}(\vec{j})}_{\text{LBP}} + \underbrace{\sum_{j\in\Omega_h} G^{-1} q_{INF}(\vec{j})}_{\text{HBP}}$$

$$\Omega_h = \left\{\vec{j} \middle| \left[(b_{i,k,1}+j_1)>C_1\right] \cup \left[(b_{i,k,2}+j_1+j_2)>C_2\right]\right\}$$

Burst Blocking Probability

$$P_{b_k} = \frac{\sum_{\vec{j}\in\Omega^*} n_k^2 q_{INF}(\vec{j})\mu_{2k} + \sum_{\vec{j}\in\{\Omega-\Omega^*\}} n_k^2 L_k(\vec{j}) q_{INF}(\vec{j})\mu_{2k}}{\sum_{\vec{j}\in\Omega} n_k^2 q_{INF}(\vec{j})\mu_{2k}}$$

$$\vec{j} \in \Omega^* \Leftrightarrow \left\{\left(C_1 - b_k + 1 \le j_1 \le C_1 \cap \left(\sum_{s=1}^{2} j_s \le C_2\right)\right)\right\}$$

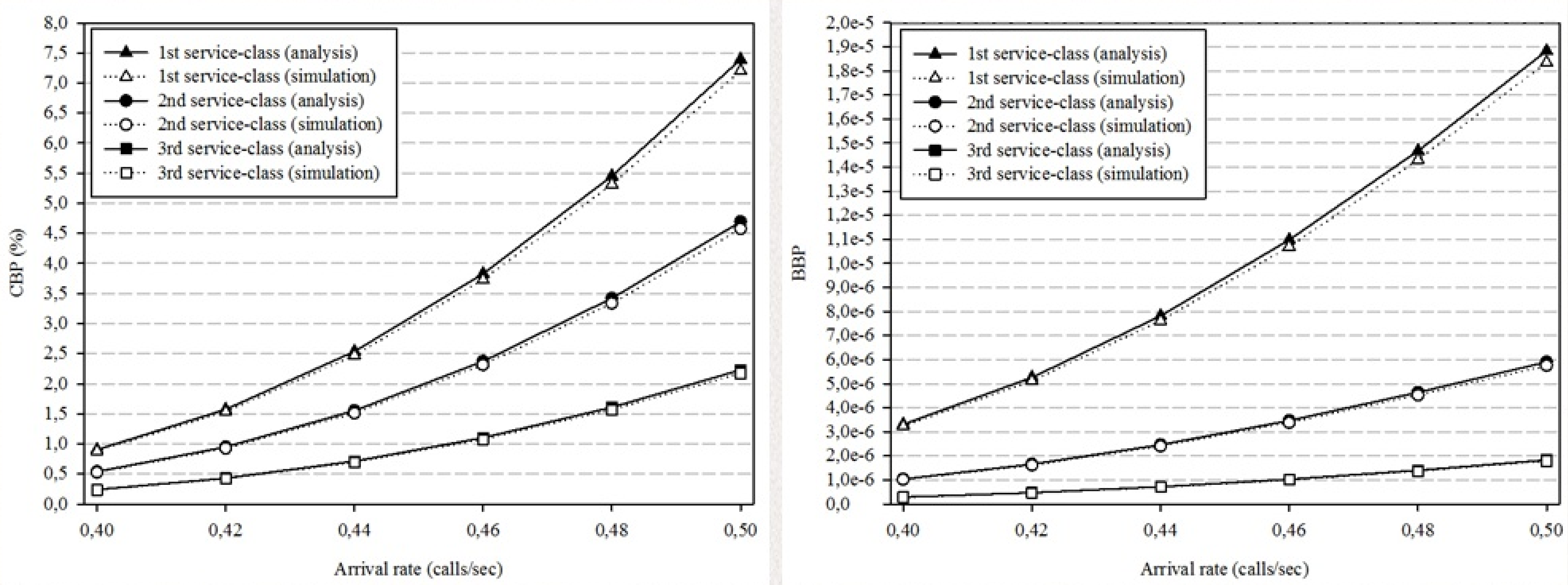
Results

Comparison of analytical and simulation results

- K=3 service-class
- C=350 codewords (BER=10^{-6})
- I_{unit}= 0.5 μW (BER=10^{-6}), $I_{\max}$=12 μW
- Total additive noise (1, 0.2) μW

$(b_k, a_k, \mu_{1k}, \mu_{2k})$

- (18, 0.85, 0.5, 0.8)
- (12, 0.9, 1.0, 1.9)
- (6, 0.95, 1.2, 0.9)



Analytical model for OCDMA PON with code reservation (1/3)

A fraction of the total number of codewords are reserved to benefit service-classes, in order to imbalance the CBP

CBP equalization can be achieved when the number of reserved codewords c_k are chosen so that:

$$c_1 + b_1 = c_2 + b_2 = \ldots = c_K + b_K$$

If the call is not blocked due to the additive noise, the CAC checks both the conditions:

$$j_1 + b_k = C_1 - c_k \text{ and } j_1 + j_2 + b_k = C_2 - c_k$$

Analytical model for OCDMA PON with code reservation (2/3)

Recursive formula

$$\sum_{i=1}^{2}\sum_{k=1}^{K} D(\vec{j}-B_{i,k})p_{ik}(\vec{j})q_{INF}(\vec{j}-B_{i,k}) = j_s q_{INF}(\vec{j}) \quad \vec{j}=(j_1, j_2), \quad j_1=\sum_{k=1}^{K} n_k^1 b_k \text{ and } j_2=\sum_{k=1}^{K} n_k^2 b_k$$

$$\vec{j}\in\Omega \Leftrightarrow \left\{\left(j_1 \le C \cap \left(\sum_{s=1}^{2} j_s \le C^*\right)\right)\right\} \quad j_s=\sum_{i=1}^{2}\sum_{k=1}^{K} n_k^i b_{i,k,s} \quad D_{i,k,s}(\vec{j}-B_{i,k})=\begin{cases} b_{i,k,s} & \text{if } j_1 \le C_1 - c_k \text{ and } j_1 + j_2 \le C_2 - c_k \\ 0 & \text{otherwise} \end{cases}$$

Utilization of system *i* from service-class *k*

$$p_{i,k,INF}\left(\vec{j}\right)=\begin{cases} \dfrac{\lambda_k\left[1-L_k\left(j_1-b_k\right)\right]}{(1-\alpha_k)\mu_{1k}} & \text{for } i=1 \\ \dfrac{\lambda_k\sigma_k}{(1-\alpha_k)\mu_{2k}} & \text{for } i=2 \end{cases}$$

Analytical model for OCDMA PON with code reservation (3/3)

Call Blocking Probability

$$Pb_k = \underbrace{\sum_{\vec{j}\in\Omega-\Omega_h} G^{-1} L_k(j_1) q(\vec{j})}_{\text{LBP}} + \underbrace{\sum_{j\in\Omega_h} G^{-1} q(\vec{j})}_{\text{HBP}}$$

$$\Omega_h = \left\{ \vec{j} \middle| \left[(b_{i,k,1}+j_1) > C_1 - c_k \right] \cup \left[(b_{i,k,2}+j_1+j_2) > C_2 - c_k \right] \right\}$$

Burst Blocking Probability

$$P_{b_k} = \frac{\sum_{\vec{j}\in\Omega^*} n_k^2 q_{INF}(\vec{j})\mu_{2k} + \sum_{\vec{j}\in\{\Omega-\Omega^*\}} n_k^2 L_k(\vec{j}) q_{INF}(\vec{j})\mu_{2k}}{\sum_{\vec{j}\in\Omega} n_k^2 q_{INF}(\vec{j})\mu_{2k}}$$

$$\vec{j} \in \Omega^* \Leftrightarrow \left\{ \left(C_1 - b_k + 1 \le j_1 \le C_1 \cap \left(\sum_{s=1}^{2} j_s \le C_2 \right) \right) \right\}$$

Results

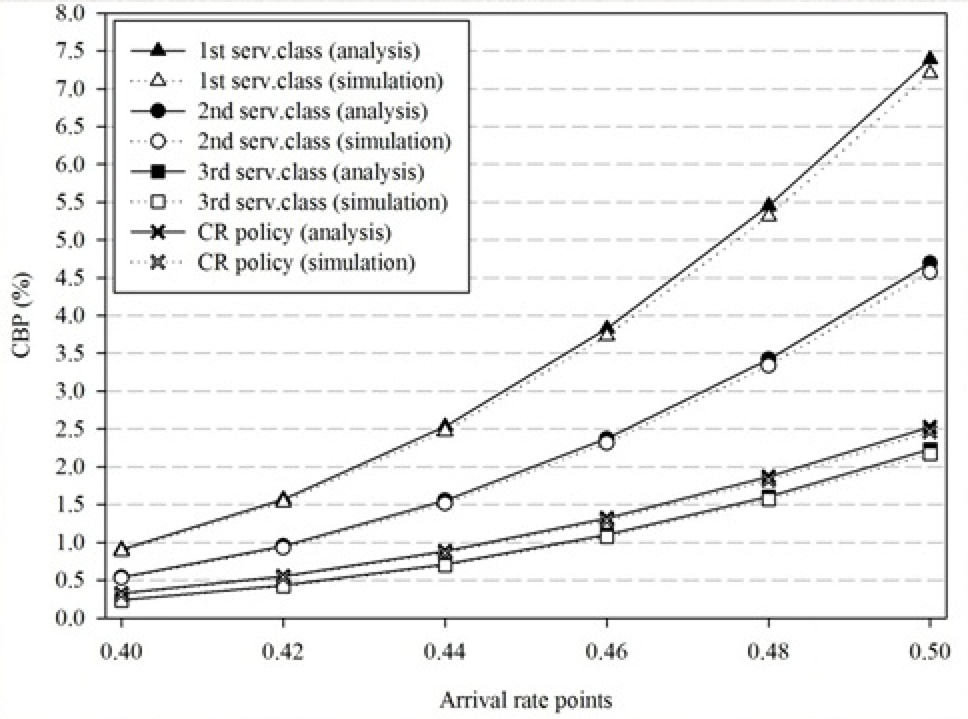
Comparison of analytical and simulation results

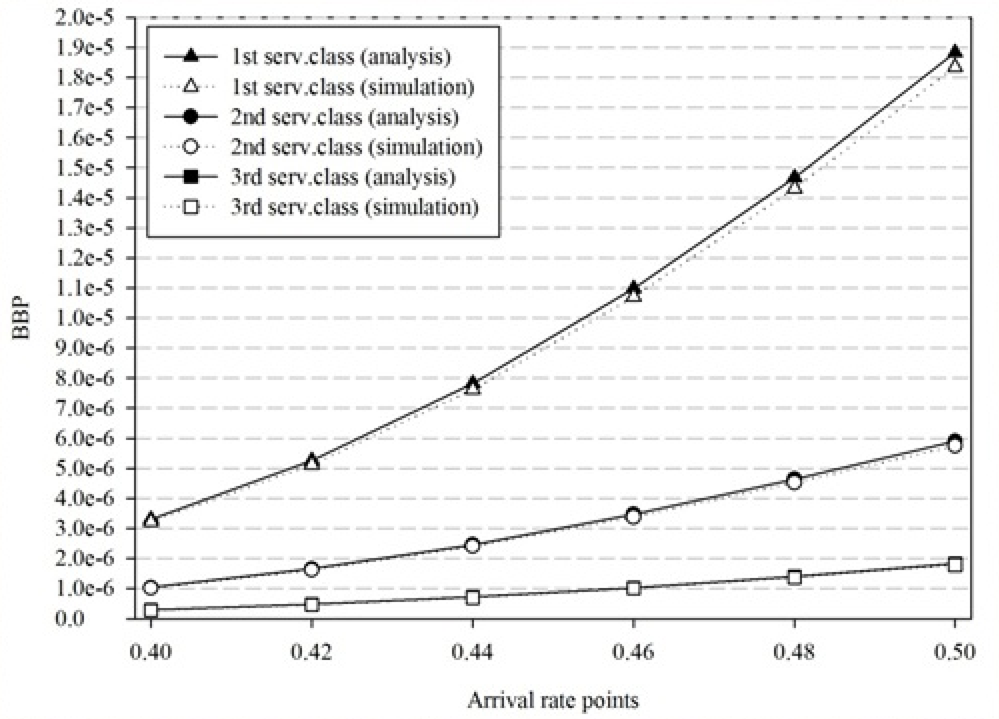
- K=3 service-class
- C=350 codewords (BER=10^{-6})
- I_{unit}= 0.5 μW (BER=10^{-6}), $I_{\max}$=12 μW
- Total additive noise (1, 0.2) μW

(b_k, a_k, μ_{1k}, μ_{2k})

- (18, 0.85, 0.5, 0.8)
- (12, 0.9, 1.0, 1.9)
- (6, 0.95, 1.2, 0.9)

(c_1, c_2, c_3)=(0, 6, 12)





Analytical model for OCDMA PON with finite traffic sources(1/2)

Recursive formula

$$\sum_{i=1}^{2}\sum_{k=1}^{K}(N_k - n_k^1 - n_k^2 + 1)b_{i,k,s}p_{ik,F}(\vec{j})q_F(\vec{j}-B_{ik}) = j_s q_F(\vec{j}) \qquad \vec{j}=(j_1,j_2),\ \ j_1=\sum_{k=1}^{K}n_k^1 b_k \text{ and } j_2=\sum_{k=1}^{K}n_k^2 b_k$$

$$\vec{j}\in\Omega \Leftrightarrow \left\{\left(j_1 \le C \cap \left(\sum_{s=1}^{2} j_s \le C^*\right)\right)\right\} \qquad j_s=\sum_{i=1}^{2}\sum_{k=1}^{K} n_k^i b_{i,k,s}$$

Utilization of system i from service-class k

$$p_{ik,F}\left(\vec{j}\right)=\begin{cases}\dfrac{v_k\left[1-L_\kappa\left(j_1-b_k\right)\right]}{(1-a_k)\mu_{1k}} & \text{for } i=1\\[2ex] \dfrac{v_k a_k}{(1-a_k)\mu_{2k}} & \text{for } i=2\end{cases}$$

Number of traffic sources $n_k{}^i$

$$n_k^i(\vec{j}) \approx y_{INF_{ik}}(\vec{j}) = \frac{N_k \cdot p_{F_{ik}} \cdot q_{INF}(\vec{j}-B_{i,k})}{q_{INF}(\vec{j})}$$

Analytical model for OCDMA PON with finite traffic sources(2/2)

Call blocking probability

$$Pb_k = \sum_{\vec{j}\in\Omega-\Omega_h} L_k(j_1)q_F(\vec{j}) + \sum_{j\in\Omega_h} G^{-1}q_F(\vec{j})$$

LBP

HBP

$$\Omega_h = \left\{\vec{j} \middle| \left[(b_{i,k,1}+j_1)>C\right] \cup \left[(b_{i,k,2}+j_1+j_2)>C^*\right]\right\}$$

Burst Blocking Probability

$$P_{b_k} = \frac{\sum_{\vec{j}\in\Omega^*} n_k^2 q_F(\vec{j})\mu_{2k} + \sum_{\vec{j}\in\{\Omega-\Omega^*\}} n_k^2 L_k(\vec{j}) q_F(\vec{j})\mu_{2k}}{\sum_{\vec{j}\in\Omega} n_k^2 q_F(\vec{j})\mu_{2k}}$$

$$\vec{j}\in\Omega^* \Leftrightarrow \left\{\left(C_1 - b_k + 1 \le j_1 \le C_1 \cap \left(\sum_{s=1}^{2} j_s \le C_2\right)\right)\right\}$$

User activity

$$w_k = \frac{\bar{T}_k^{act}}{\bar{T}_k^{act} + \bar{T}_k^{pas}} = \frac{\mu_{1k}^{-1}}{\mu_{1k}^{-1} + \sum_{l=0}^{\infty} \mu_{2k}^{-1} B_{b_k}^{l}} = \frac{\mu_{1k}^{-1}}{\mu_{1k}^{-1} + \mu_{2k}^{-1}/(1 - B_{b_k}^{l})}$$

Results (1/4)

Comparison of analytical and simulation results

- K=3 service-classes
- C=350 codewords (BER=10^{-6})
- I_{unit}= 0.5 μW (BER=10^{-6}), $I_{\max}$=13 μW
- Total additive noise (1, 0.2) μW

(b_k, a_k, μ_{1k}, μ_{2k})

- (20, 0.85, 0.5, 0.8)
- (16, 0.9, 1.0, 1.9)
- (10, 0.95, 1.2, 0.9)

Arrival rate per idle source (calls/sec)	CBP of 1st service-class		BBP of 1st service-class	
	Analysis (%)	*Simulation*	*Analysis (%)*	*Simulation*
1	0.0099	0.0126±3.76e-03	6.903e-04	9.929e-04±8.20e-04
2	0.0249	0.0242±6.41e-03	1.363e-03	1.585e-03±5.59e-04
3	0.0564	0.0631±7.68e-03	2.511e-03	3.543e-03±1.91e-03
4	0.1166	0.1197±1.48e-02	4.340e-03	4.183e-03±1.49e-03
5	0.2226	0.2169±1.67e-02	7.077e-03	7.596e-03±1.92e-03
6	0.3958	0.4056±3.52e-02	1.095e-02	1.332e-02±2.00e-03
7	0.6606	0.6631±6.11e-02	1.614e-02	1.835e-02±3.20e-03
8	1.0421	1.0778±5.97e-02	2.279e-02	2.868e-02±3.44e-03
9	1.5630	1.6045±8.87e-02	3.096e-02	3.665e-02±4.17e-03
10	2.2414	2.3012±1.06e-01	4.062e-02	4.634e-02±4.57e-03
11	3.0883	3.1050±9.13e-02	5.169e-02	5.807e-02±4.82e-03

Results (1/4)

Comparison of analytical and simulation results

- K=3 service-classes
- C=350 codewords (BER=10^{-6})
- I_{unit}= 0.5 μW (BER=10^{-6}), $I_{\max}$=13 μW
- Total additive noise (1, 0.2) μW

$(b_k, a_k, \mu_{1k}, \mu_{2k})$

- (20, 0.85, 0.5, 0.8)
- (16, 0.9, 1.0, 1.9)
- (10, 0.95, 1.2, 0.9)

Arrival rate per idle source (calls/sec)	CBP of 2nd service-class		BBP of 2nd service-class	
	Analysis (%)	*Simulation*	*Analysis (%)*	*Simulation*
1	0.0065	0.0062±3.14e-03	1.190e-04	1.729e-04±1.22e-04
2	0.0167	0.0162±3.92e-03	2.707e-04	3.128e-04±8.28e-05
3	0.0386	0.0371±5.58e-03	5.600e-04	6.405e-04±2.86e-04
4	0.0812	0.0806±1.65e-02	1.066e-03	1.349e-03±2.04e-04
5	0.1574	0.1579±1.88e-02	1.885e-03	2.319e-03±4.57e-04
6	0.2836	0.2915±2.19e-02	3.124e-03	3.404e-03±6.21e-04
7	0.4790	0.4654±3.53e-02	4.888e-03	5.557e-03±7.80e-04
8	0.7634	0.7569±3.58e-02	7.269e-03	7.904e-03±5.74e-04
9	1.1557	1.1522±5.06e-02	1.033e-02	1.333e-02±1.08e-03
10	1.6715	1.6514±4.58e-02	1.411e-02	1.452e-02±4.61e-03
11	2.3211	2.2947±5.18e-02	1.860e-02	2.051e-02±1.55e-03

Results (1/4)

Comparison of analytical and simulation results

- K=3 service-classes
- C=350 codewords (BER=10^{-6})
- I_{unit}= 0.5 μW (BER=10^{-6}), $I_{\max}$=13 μW
- Total additive noise (1, 0.2) μW

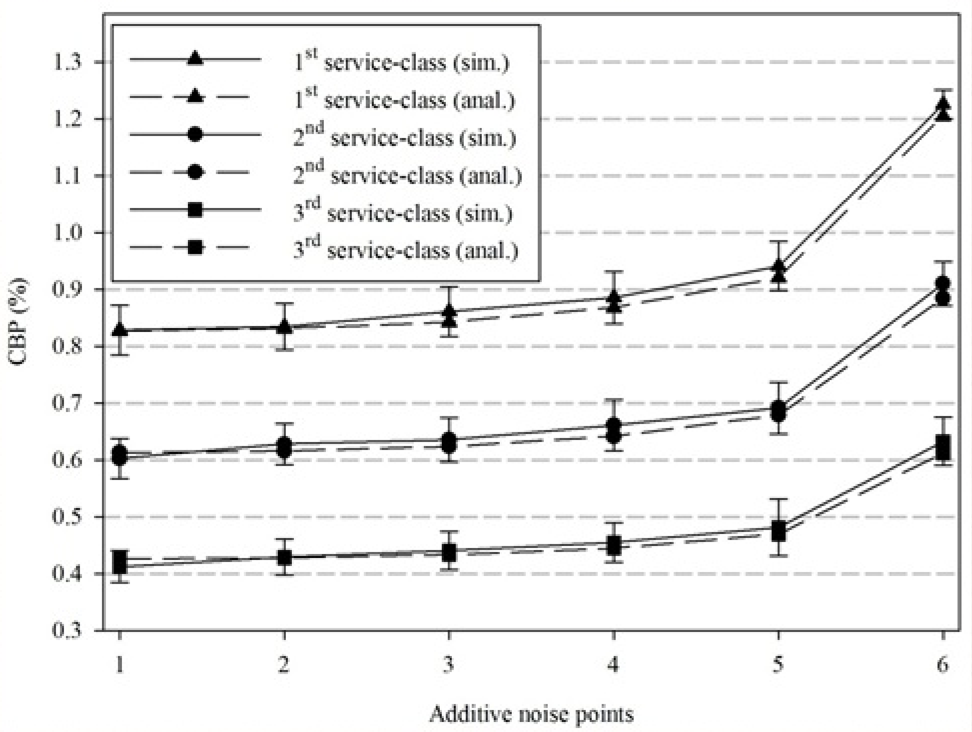
$(b_k, a_k, \mu_{1k}, \mu_{2k})$

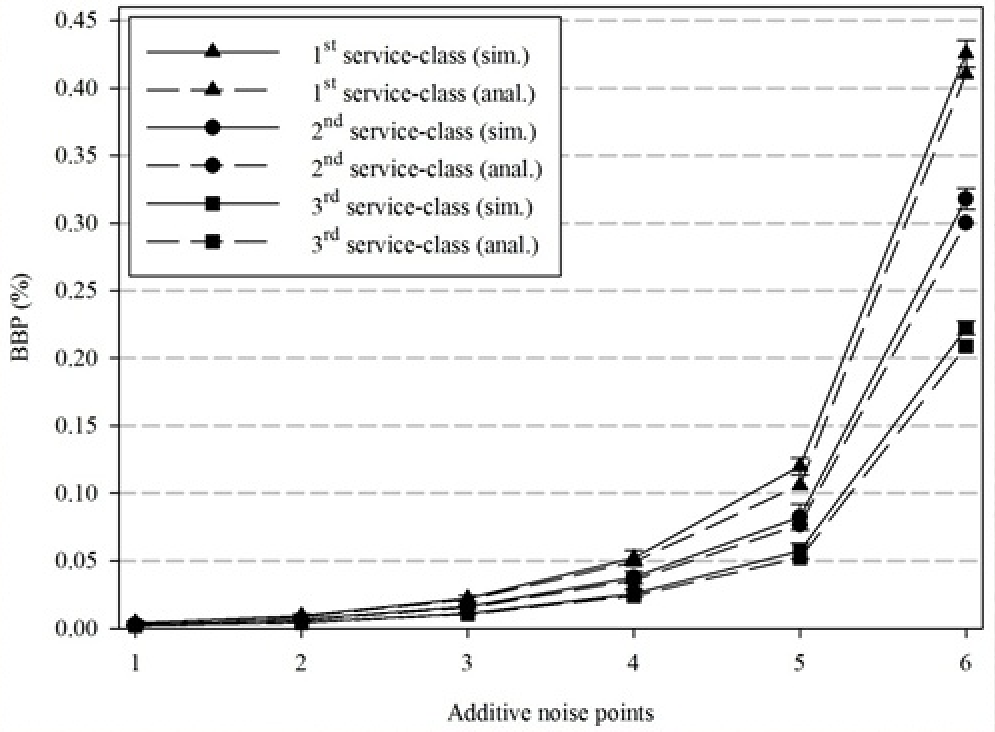
- (20, 0.85, 0.5, 0.8)
- (16, 0.9, 1.0, 1.9)
- (10, 0.95, 1.2, 0.9)

Arrival rate per idle source (calls/sec)	CBP of 3rd service-class		BBP of 3rd service-class	
	Analysis (%)	*Simulation*	*Analysis (%)*	*Simulation*
1	0.0033	0.0027±1.51E-03	5.488E-05	5.226E-04±6.49E-04
2	0.0086	0.0098±3.46E-03	1.279E-04	1.404E-04±6.73E-05
3	0.0202	0.0190±6.00E-03	2.705E-04	3.281E-04±1.82E-04
4	0.0432	0.0413±8.19E-03	5.253E-04	6.195E-04±7.44E-05
5	0.0848	0.0906±7.64E-03	9.463E-04	1.141E-03±2.17E-04
6	0.1548	0.1560±1.10E-02	1.595E-03	1.858E-03±3.62E-04
7	0.2645	0.2584±1.34E-02	2.536E-03	3.022E-03±2.83E-04
8	0.4262	0.4210±1.56E-02	3.825E-03	4.170E-03±5.11E-04
9	0.6520	0.6385±2.96E-02	5.508E-03	6.205E-03±4.10E-04
10	0.9524	0.9371±3.41E-02	7.613E-03	8.378E-03±6.72E-04
11	1.3349	1.2808±4.91E-02	1.014E-02	1.240E-02±9.83E-04

Results (2/4)

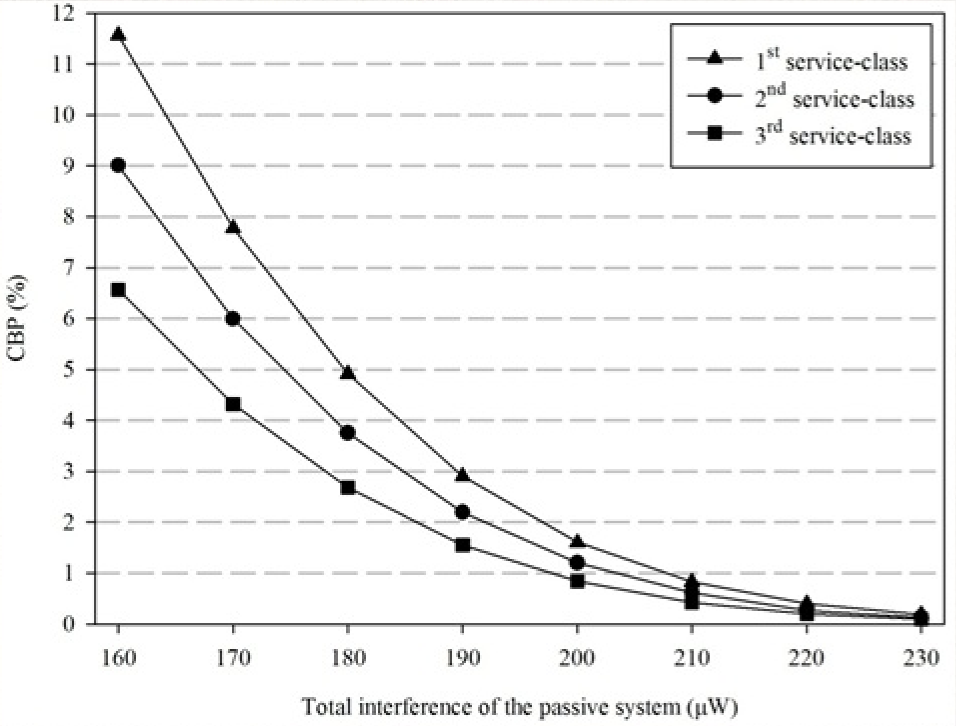
Effect of the additive noise on CBP and BBP

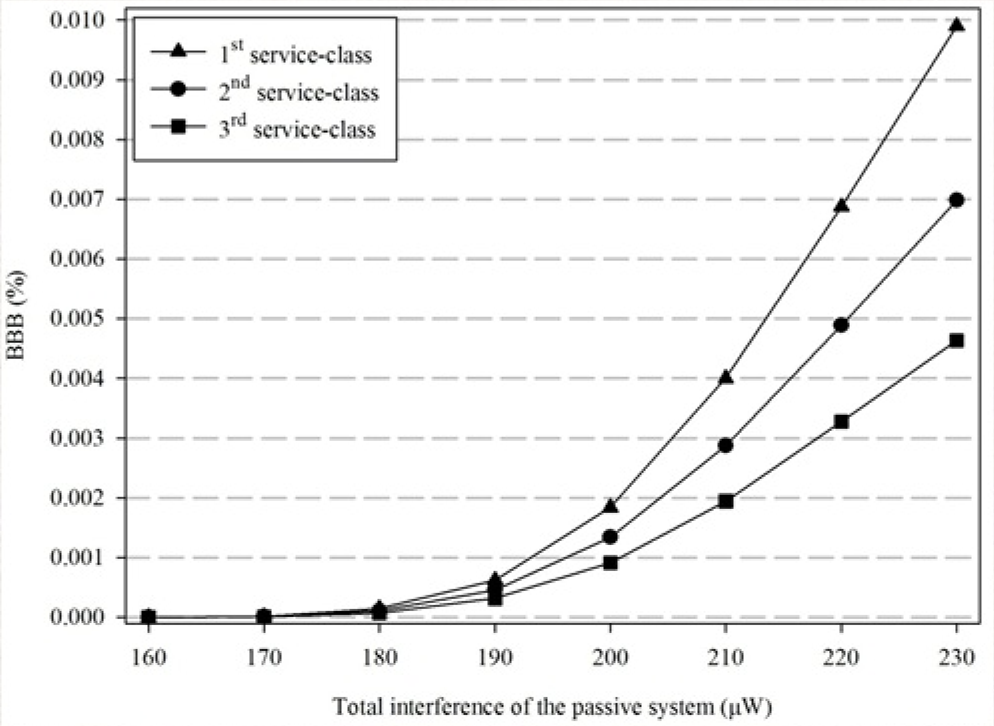




Results (3/4)

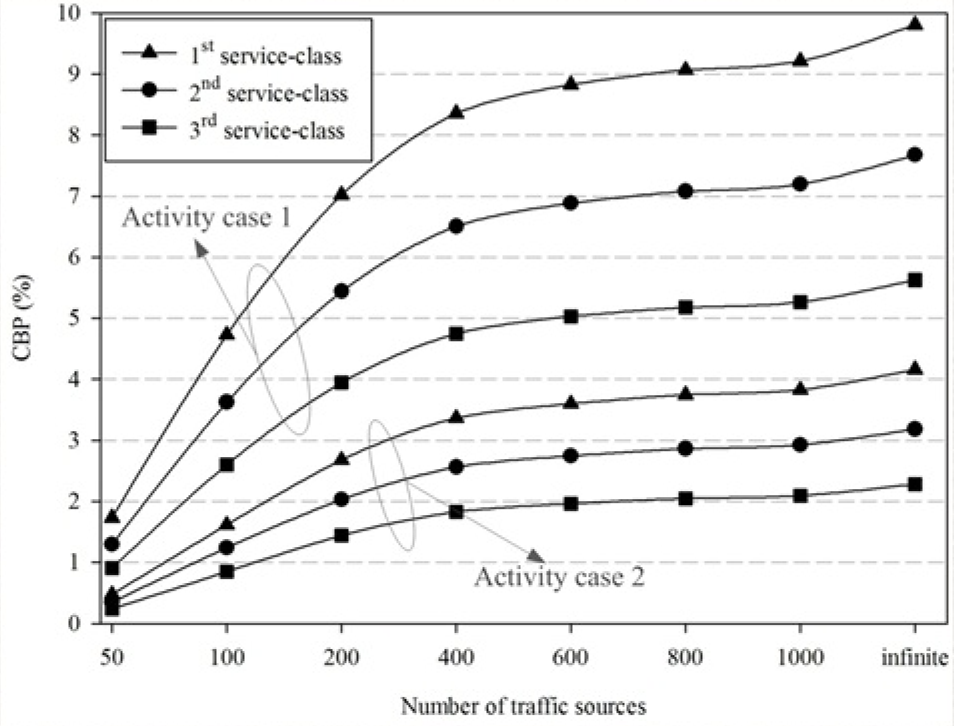
Effect of the total number of fictitious codewords on CBP and BBP

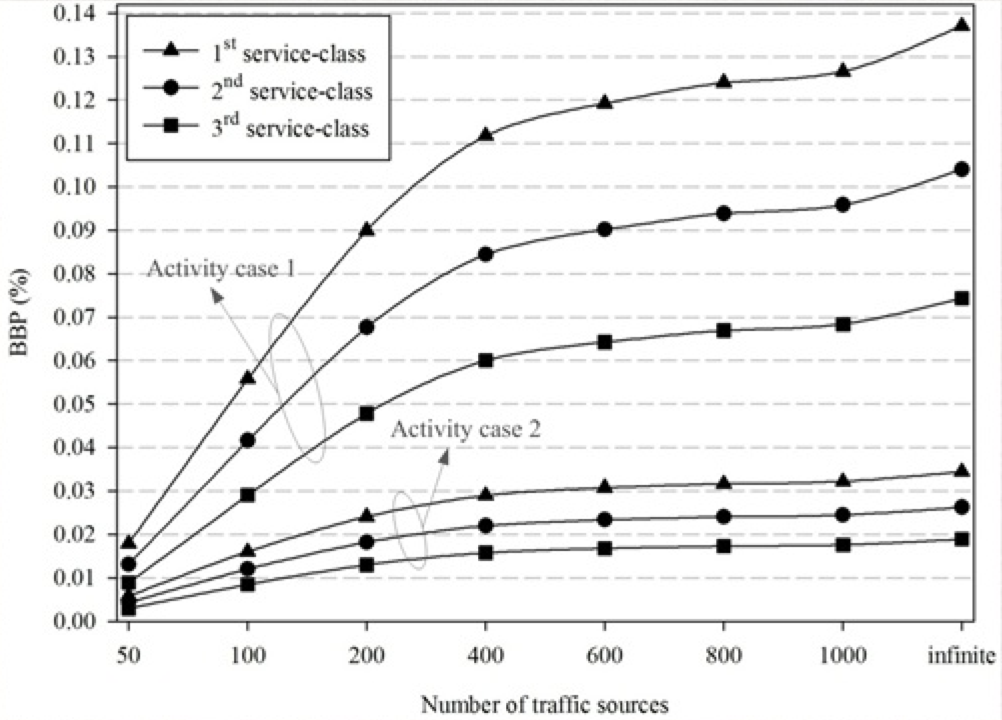




Results (4/4)

Effect of number of traffic sources on CBP and BBP
for 2 different user activity cases





More can be done!!

EPONs

Limited service
Elastic service
and
constant-linear credit
for the case of
multiple service-classes

OCDMA PONs

Retry models
Batch arrivals

Different codes

OFDMA PONs

Multiple service-classes
Finite traffic sources
Retry models
Batch arrivals

Thank you !!!