



Anatomy of Generalized Wireless Fading Channels

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ORGANIZATION OF THE PRESENTATION

ABSTRACT

1. INTRODUCTION

2. DIFFERENT GENERAL FADING DISTRIBUTION

3. PRESENCE OF CO-CHANNEL INTERFERENCE

4. MITIGATION OF FADING AND CCI

5. PERFORMANCE ANALYSIS BASED ON THE SIR AT THE OUTPUT OF THE RECEIVER - FEW EXAMPLES

6. CONCLUSION

1. INTRODUCTION

- One of the most critical influence to signal propagation in wireless channels is fading
- The macroscopic fading, also called large scale or slow fading, and microscopic, called small scale or fast fading, can be considered as basic effects of wireless propagation.
- Macroscopic fading results from the shadowing effect by buildings, foliage and other objects
- Microscopic fading results from multipath propagation, which occurs in indoor environments, but also in both macrocellular and microcellular outdoor environments.

- Multiple rays arrive at the receiving antennas with different delays and make complex constructive or destructive fading.
- Also, we have to take into account reflections from buildings, towers, trees, and so on, as well as diffraction from edges and different obstacles as hill tops.
- Since the same signal arrives over several paths, their phases will be different, resulting in constructive or destructive fading amplitude.

- Fading in the mobile communications is different than this phenomenon in the situation with line-of-sight (LOS) where radio paths can be optimized
- The mobile terminals may be fixed during the process of the telecommunication connection or data transfer, but they will be in motion more probably
- When terminals are in motion, the propagation path characteristics are constantly changing and there is multipath propagation.

- Fading channel modeling is generally defined as the variation of the attenuation of a signal with various variables: time, geographical position, and radio frequency
- Multipath is the appearance of non-line-of-sight (NLOS) signals at the receiver with a sufficiently short delay that leads to an undesired distortion of useful signal
- Furthermore, it is important to find as accurate a way as possible to describe this phenomenon in wireless systems

- Fading is modeled as a random process, i.e. random variable with a certain statistical distribution
- It is very important that this distribution describes the conditions in the wireless channel as closely as possible
- To exceed the gap between theoretical considerations and measurements performed in a fading environment, intensive investigations of new general fading distributions are being carried out
- There was a need to improve known, traditional models

- In recent years, significant new results have been obtained in the study of the physical basis of fading, which are not taken into account in traditional models
- New models take into account such phenomena as
 - the nonlinearity of the radio wave propagation medium,
 - the effects of radio wave scattering,
 - the presence of dominant components,
 - the cluster nature of multipath propagation, and
 - the imbalance of quadrature signal components
- This lecture presents a description of non-traditional generalized fading models, shows the relationships between the models, and gives some calculated examples

- Known distribution as are the Ricean and Nakagami- m have benefits as fading models, but also some lacks
- For example, lacks are because of reflections and shadowing in the case of Ricean model
- Nakagami- m is a more general statistical distribution with greater flexibility and accuracy in matching certain experimental data than Rayleigh, Rician and Nakagami- q
- For a long time, Nakagami- m distribution was considered the most faithful representation of fading, but it cannot describe the LOS channels, but only scatter channels since it consists of both, line-of-sight (LOS) and NLOS components

- To overcome this drawback, Norman Beaulieu and Xie Jiandong defined new distribution, called based on their names Beaulieu-Xie (BX), for modeling wireless systems containing multiple dominant specular components
- The new distribution is suitable for modeling the fading channels with more LOS components and diffuse scattering components
- Because of that is necessary to develop new, powerful models in order to limit the errors induced by fading

- In recent years, a significant number of distributions that present new generalized models of the communication channel with fading have also appeared
- These include such models as κ - μ , η - μ , η - κ , α - μ , λ - μ , η - λ - μ , α - κ - μ , α - η - μ , α - η - κ - μ , α - λ - μ - η ,...
- There are the relationship between new and traditional models.
- This considerations is tied to small-scale fading

Beaulieu-Xie fading model

- The new fading model was introduced by Beaulieu and Xie to overcome these requirements and simultaneously includes both these models (Rician and Nakagami- m)
- BX fading model contains advantage of Nakagami- m model thanks to its flexible fading parameter and advantage of the Rician model since its use of a non-central chi-distribution what allows description of both, LOS and NLOS fading channels
- So, this fading distribution model describes fading channels with multiple dominant specular components and is given in closed-form

- The probability density function modelled by Beaulieu-Xie distribution:

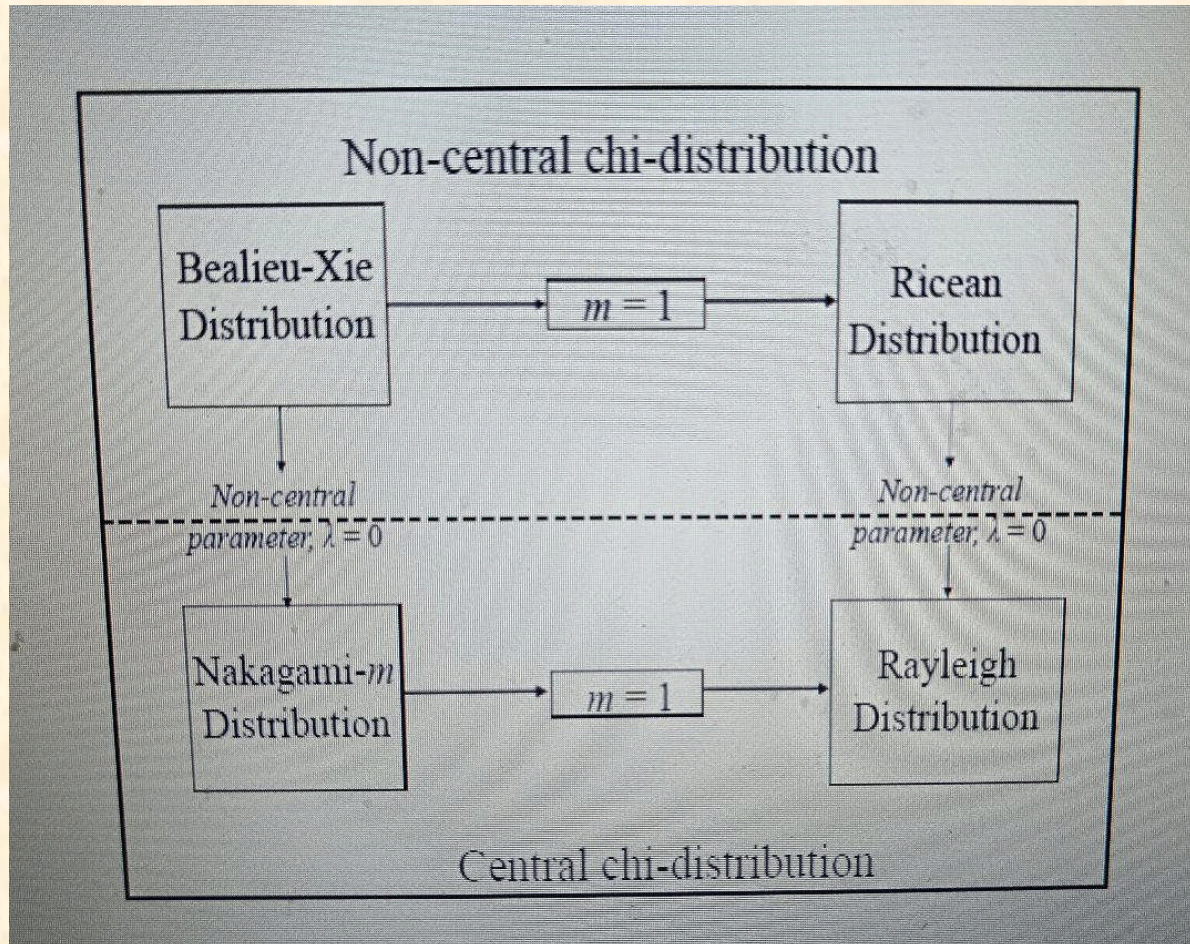
$$p_X(x) = 2e^{-\frac{m}{\Omega}(x^2 + \lambda^2)} \sum_{i=0}^{\infty} \frac{\lambda^{2i} x^{2i+2m-1}}{i! \Gamma(i+m)} \left(\frac{m}{\Omega}\right)^{2i+m}$$

- m presents the fading severity parameter and controls the shape;
- λ presents the non-centrality parameter and controls the location and the height of the mode of the PDF;
- Ω reflects the power and controls the spread;
- λ^2 represents the power of the LOS components, and
- λ^2/Ω is the Rician K -factor representing the ratio between the total average power of the LOS components and the total average power of the scattered components

- BX distribution is useful in presenting practical fading for the femtocells, millimeter (mmWave) and terahertz (THz) wireless communication systems, and also for 6G short distance random access channels, in the presence of more components obtained due to the reflection of the signal

- Thanks to the various values of its three parameters which characterize it, the BX fading model is very flexible.
- This is general distribution because it includes some others known distributions as special cases for different values of m and λ :
 - $\kappa - \mu$ distribution,
 - Nakagami- m ,
 - generalized Rician,
 - non-central chi,
 - and a few others included in them

The relationships between the Rayleigh, Ricean, Nakagami- m , and BX models.



The parameter m and represent the fading severity parameter and λ is non-centrality parameter

The relationship between the BX distribution and other unimodal distributions

TRANSFORMATION OF DISTRIBUTIONS

Distribution	Operation
Non-central chi-distribution	normalize by $\sqrt{\frac{2m}{\Omega}}$ and set k to $2m$
generalized Ricean distribution	normalize by $\sigma\sqrt{\frac{2m}{\Omega}}$ and set n to $2m$
$\kappa - \mu$ distribution	Substitute the parameters, $\mu = m$, $\hat{r} = \sqrt{2k\sigma^2 + d^2}$ and $d^2 = s^2$, $k = m$ [19, Appendix A] into the $\kappa - \mu$ distribution, then normalize the random variable by $\sigma\sqrt{\frac{2m}{\Omega}}$

Co-channel interference (CCI)

- The co-channel interference (CCI) exists when two or more devices are operating on the same frequency channel
 - This is actually the crosstalk from two different radio transmitters using the same frequency channel
 - The most common reasons causing CCI are: bad weather condition and bad frequency planning
 - The CCI can have the same distribution as fading in observed environment, but also it can be different
- CCI also contribute to a significant reduction in data rates and system performance degradation

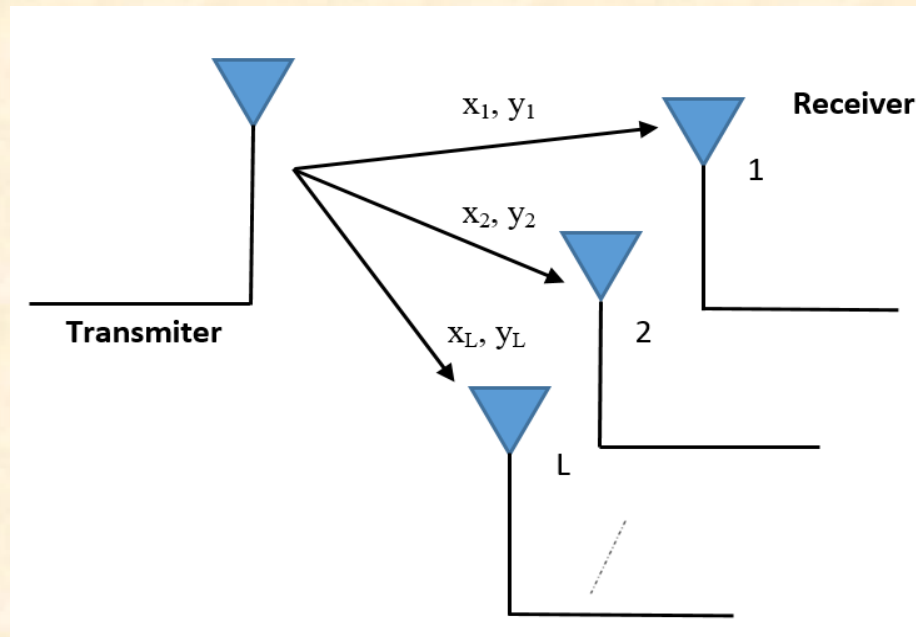
Mitigation of fading – using of diversity combining systems

- In wireless communication systems, diversity methods play a main role in mitigating fading influence
- One of the most important techniques is using a large number of receiving antennas.
- When diversity techniques are used, the probability that all signals on branches are seriously obstructed by fading is considerably reduced.
- The number of such approximately independent branches is called the diversity order.

- There are several well-known techniques of receiver diversity combining:
 - maximum-ratio combining (MRC),
 - equal-gain combining (EGC),
 - switch and stay combining (SSC),
 - selection combining (SC)
-
- MRC is optimal method in the sense of quality, but with highest price, whereas at other methods some information are lost in order to reduce the implementation price of the MRC.

- Selection combining (SC) is often used in spatial diversity systems
- SC combiner sample signals from L antennas, sending the largest one to the demodulator
- SC diversity combining is not optimal technique since it does not use all the received signals (or signal to interference ratios), but is quite easy to implement
- The importance of SC receivers in wireless systems was shown through many papers in the literature dealing with this type of diversity combining.

- The model of the SC diversity receiver with L branches is shown:



- The SC receiver transmits to the user the signal from the input antenna whose value is the highest

- The input signal are x_i and the output signal is x , $i=1, 2, \dots, L$
- The CCI envelopes at the input are y_i , the output value is y .
- The input ratios of the useful signals and the CCIs are $z_i = x_i / y_i$, and output SIR is denoted with z :

$$z = \max (z_1, z_2, \dots, z_L)$$

Derivation of the Level Crossing Rate in Wireless Channel Limited by Beaulieu-Xie Fading and Co-Channel Interference

- In this example, we analyze wireless system with selection combining (SC) in the presence of Beaulieu-Xie fading and co-channel interference (CCI)
- For such scenario, the formula for level crossing rate (LCR) will be derived and some graphs plotted
- Then, we examined the influence of the fading parameters having real physical meaning and allow setting up of the fading severities

- Each signal at the input of the receiver has the probability density function modelled by BX distribution:

$$p_{X_i}(x_i) = 2e^{-\frac{m_1}{\Omega_i}(x_i^2 + \lambda_1^2)} \sum_{i_1=0}^{\infty} \frac{\lambda_1^{2i_1} x_i^{2i_1+2m_1-1}}{i_1! \Gamma(i_1 + m_1)} \left(\frac{m_1}{\Omega_i} \right)^{2i_1+m_1}$$

- The CCI envelope y_i is also under BX distribution:

$$p_{Y_i}(y_i) = 2e^{-\frac{m_2}{s_i}(y_i^2 + \lambda_2^2)} \sum_{i_2=0}^{\infty} \frac{\lambda_2^{2i_2} y_i^{2i_2+2m_2-1}}{i_2! \Gamma(i_2 + m_2)} \left(\frac{m_2}{s_i} \right)^{2i_2+m_2}$$

where s_i stands for CCI power.

The ratio z_i of the useful signal and the CCI interference (SIR) at the i^{th} input branch in the SC receiver is: $z_i = x_i/y_i$.

- The PDF of SIR z_i is:

$$p_{z_i}(z_i) = \int_0^\infty dy_i y_i p_{x_i}(z_i y_i) p_{y_i}(y_i) =$$

$$= 2e^{-\left(\frac{m_1}{\Omega_i} \lambda_1^2 + \frac{m_2}{s_i} \lambda_2^2\right)} \sum_{i_1=0}^{\infty} \sum_{i_2=0}^{\infty} \frac{\lambda_1^{2i_1} \lambda_2^{2i_2}}{i_1! i_2!}.$$

$$\cdot \frac{z_i^{2i_1+2m_1-1} m_1^{2i_1+m_1} m_2^{2i_2+m_2} \Omega_i^{i_2-i_1+m_2} s_i^{i_1-i_2+m_1} \Gamma(i_1+i_2+m_1+m_2)}{\Gamma(i_1+m_1) \Gamma(i_2+m_2) (m_2 \Omega_i + m_1 s_i z_i^2)^{i_1+i_2+m_1+m_2}}$$

- The cumulative distribution function (CDF) of the SIR z_i is:

$$\begin{aligned}
 F_{z_i}(z_i) &= \int_0^{z_i} p_{z_i}(t) dt = \\
 &= e^{-\left(\frac{m_1}{\Omega_i} \lambda_1^2 + \frac{m_2}{s_i} \lambda_2^2\right)} \sum_{i_1=0}^{\infty} \sum_{i_2=0}^{\infty} \left(\frac{m_1}{\Omega_i}\right)^{i_1} \cdot \\
 &\quad \cdot \frac{\lambda_1^{2i_1} \lambda_2^{2i_2} \Gamma(i_1 + i_2 + m_1 + m_2)}{i_1! i_2! \Gamma(i_1 + m_1) \Gamma(i_2 + m_2)} \left(\frac{m_2}{s_i}\right)^{i_2} B_{\frac{m_1 s_i \lambda^n}{m_2 \Omega_i + m_1 s_i \lambda^n}}(i_1 + m_1, i_2 + m_2)
 \end{aligned}$$

where with $B_z(a, b)$ is marked the incomplete Beta function.

the conditional probability density functions of z_i and $\dot{z}_i$ are

The variance of $\dot{z}_i$ is:

$$\sigma_{\dot{z}_i}^2 = \frac{\pi^2 f_m^2}{y_i^2} \left(\frac{m_2 \Omega_i + m_1 s_i z_i^2}{m_1 m_2} \right)$$

The conditional probability density functions of z_i and $\dot{z}_i$ are:

$$p_{\dot{z}_i}(\dot{z}_i | z_i y_i) = \frac{1}{\sqrt{2\pi} \sigma_{\dot{z}_i}} e^{-\frac{\dot{z}_i^2}{2\sigma_{\dot{z}_i}^2}}$$

$$p_{z_i}(z_i | y_i) = \left| \frac{dx_i}{dz_i} \right| p_{x_i}(z_i y_i) = y_i p_{x_i}(z_i y_i)$$

conditional joint PDF of y_i , z_i , and $\dot{z}_i$ is:

$$p_{\dot{z}_i z_i y_i}(\dot{z}_i z_i y_i) = p_{\dot{z}_i}(\dot{z}_i | z_i y_i) p_{x_i}(z_i y_i) p_{y_i}(y_i) y_i$$

The joint JPDF of z_i and $\dot{z}_i$ is:

$$p_{\dot{z}_i z_i}(\dot{z}_i z_i) = \int_0^{\infty} p_{\dot{z}_i z_i y_i}(\dot{z}_i z_i y_i) dy_i$$

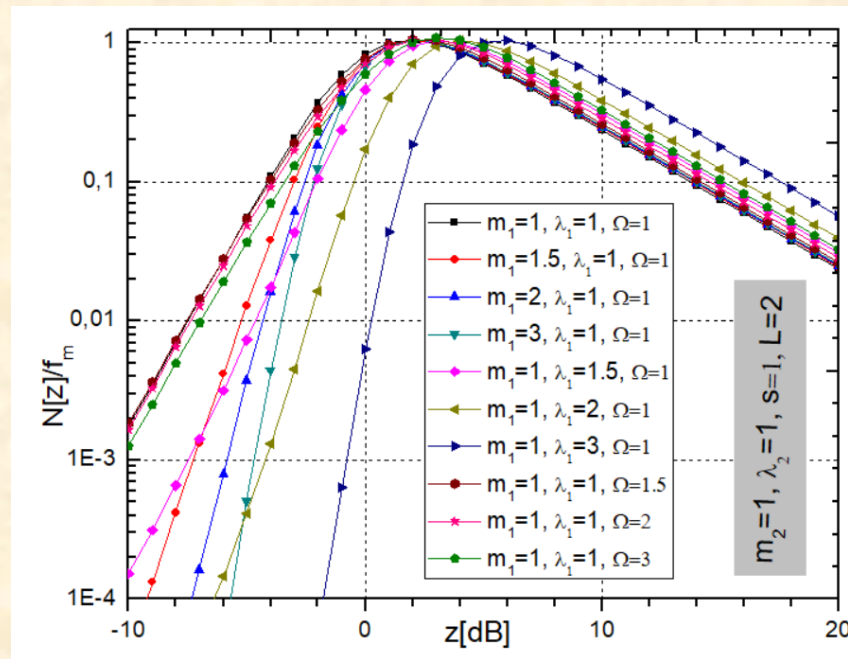
- LCR of SIR z_i is:

$$\begin{aligned}
 N_{z_i}(z_i) &= \int_0^\infty \dot{z}_i p_{\dot{z}_i z_i}(\dot{z}_i z_i) d\dot{z}_i = \\
 &= \frac{\sqrt{2\pi} f_m}{e^{\frac{m_1}{\Omega_i} \lambda_1^2 + \frac{m_2}{s_i} \lambda_2^2}} \sum_{i_1=0}^\infty \sum_{i_2=0}^\infty \frac{\lambda_1^{2i_1} \lambda_2^{2i_2} z_i^{2i_1+2m_1-1}}{i_1! i_2!} \cdot \\
 &\cdot \frac{\Omega_i^{i_2-i_1+m_2+\frac{1}{2}-1} s_i^{i_1-i_2+m_1+\frac{1}{2}-1} m_1^{2i_1+m_1-\frac{1}{2}} m_2^{2i_2+m_2-\frac{1}{2}} \Gamma(i_1+i_2+m_1+m_2-1/2)}{\Gamma(i_1+m_1)\Gamma(i_2+m_2)(m_2\Omega_i+m_1s_i z_i^2)^{i_1+i_2+m_1+m_2-1}}
 \end{aligned}$$

- The LCR for the output SIR z is given by:

$$\begin{aligned}
 N_{z|\Omega_i s_i}(z) / f_m &= L\left(F_{z_i}(z_i)\right)^{L-1} N_{z_i}(z_i) = \\
 &= \frac{L\sqrt{2\pi} f_m}{e^{\frac{m_1}{\Omega_i} \lambda_1^2 + \frac{m_2}{s_i} \lambda_2^2}} \sum_{i_1=0}^{\infty} \sum_{i_2=0}^{\infty} \frac{\lambda_1^{2i_1} \lambda_2^{2i_2}}{i_1! i_2!} \cdot \\
 &\cdot \frac{z_i^{2i_1+2m_1-1} \Omega_i^{i_2-i_1+m_2-\frac{1}{2}} s_i^{i_1-i_2+m_1-\frac{1}{2}} m_1^{2i_1+m_1-\frac{1}{2}} m_2^{2i_2+m_2-\frac{1}{2}} \Gamma(i_1+i_2+m_1+m_2-1/2)}{\Gamma(i_1+m_1)\Gamma(i_2+m_2)(m_2\Omega_i+m_1s_iz_i^2)^{i_1+i_2+m_1+m_2-1}} \\
 &\left(e^{-\left(\frac{m_1}{\Omega_i} \lambda_1^2 + \frac{m_2}{s_i} \lambda_2^2\right)} \sum_{i_3=0}^{\infty} \sum_{i_4=0}^{\infty} \left(\frac{m_1}{\Omega_i}\right)^{i_3} \left(\frac{m_2}{s_i}\right)^{i_4} \frac{\lambda_1^{2i_3} \lambda_2^{2i_4} \Gamma(i_3+i_4+m_1+m_2)}{i_3! i_4! \Gamma(i_3+m_1)\Gamma(i_4+m_2)} \cdot B_{\frac{m_1 s_i z^2}{m_2 \Omega_i + m_1 s_i z^2}}(i_3+m_1, i_4+m_2) \right)^{L-1}
 \end{aligned}$$

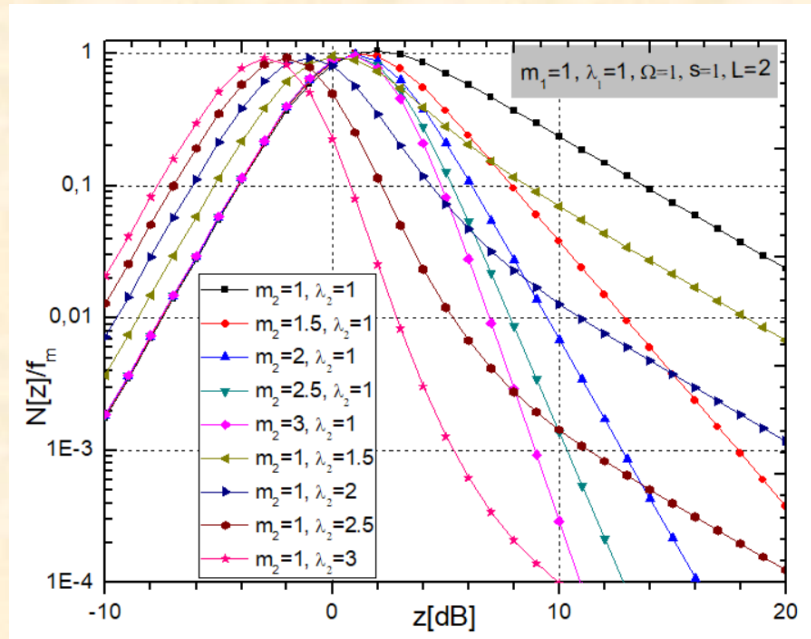
- We plotted a few graphs based on formula for LCR, normalized by f_m , versus output SIR in order to analyze the impact of BX fading and CCI parameters on the concerned characteristic. We assumed that the correlation between the input branches in the SC receiver is minimal.



Normalized LCR of SC receiver versus output SIR for different sets of fading parameters m_1 , λ_1 , and Ω .

- From this figure, one can see that due to the increase of the BX fading parameter m_1 , the LCR decreases and the system has better characteristics, according theory.

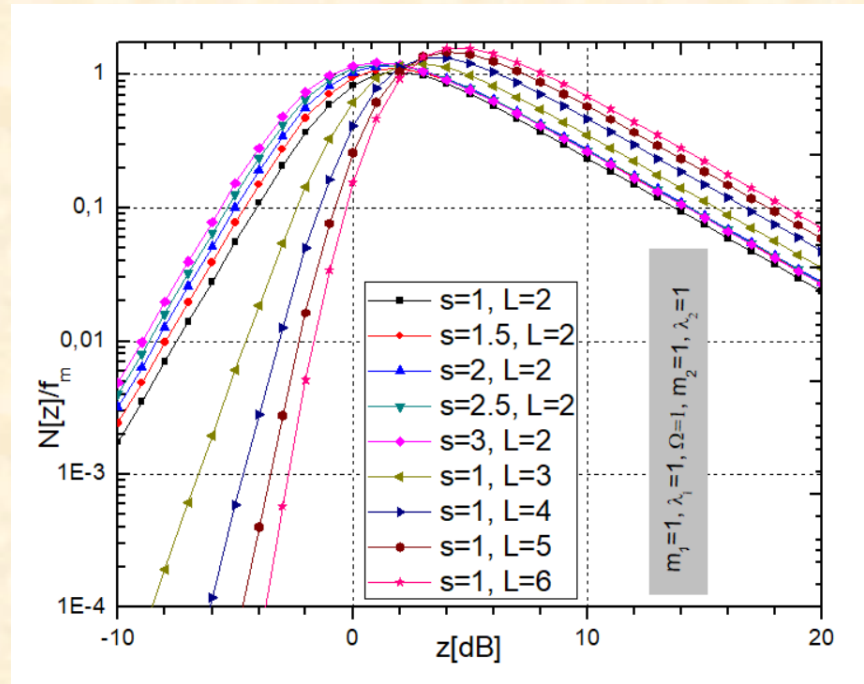
- It is also possible to see that LCR decreases when BX fading parameter λ_1 increases, for small SIR values, $z < 0$. This increasing of λ_1 improves the system performance. For big z , the situation is the opposite. Finally, with an increase of signal power Ω , LCR increases for all values of z .



Normalized LCR of SC receiver versus output SIR for different values of CCI parameters m_2 and λ_2 .

- From graphs in Fig. 2, the impact of m_2 is noticeable for $z > 0$. In this situation, the LCR decreases with increasing m_2 . From this figure is also visible that the LCR decreases with increasing λ_2 for $z > 0$.

- Opposite situation is for $z < 0$, when LCR increases with increasing m_2 . So, increasing in m_2 and λ_2 improve the system performance for bigger SIRs z .



Normalized LCR of SC receiver versus output SIR for different power of CCI s , and number of branches L .

- It is possible to notice that the LCR increases with an increase of CCI power s for all z . Also, from this figure is visible that LCR decreases with the increasing of the number of branches L , but with the increase in the power of the useful signal, the further increase in the number of branches has no sense or economic justification.

The papers with Beaulieu-Xie fading

- 1. **Dragana Krstić**, Suad Suljović, Devendra S. Gurjar, Suneel Yadav, “Improving the outage probability using SC diversity for GNSS signals limited by Beaulieu-Xie fading and Rician co-channel interference”, The 16th Royal Institute of Navigation Annual Baška GNSS Conference: Technologies, Techniques and Applications across PNT, Baška, Krk Island, Croatia 14 – 18 May 2023.
- 2. **Dragana Krstić**, Suad Suljović, Devendra S. Gurjar, Suneel Yadav, “Outage Probability Determining for Wireless Systems in the Presence of Beaulieu-Xie Fading and Co-channel Interference Rayleigh Modeled”, 2023, 46th MIPRO ICT and Electronics Convention (MIPRO), Opatija, Croatia, May 22 - 26, 2023, ICT for a Smart and Green Present and Future, pp. 523-526, doi:10.23919/MIPRO57284.2023.10159907.
- 3. **Dragana Krstić**, Suad Suljović, Nenad Petrovic, Goran Nestorovic, Dejan Milic, „Machine Learning Approach to QoS Adjustment Leveraging LCR in Mobile Networks Limited by Beaulieu-Xie Fading and Co-Channel Interference”, 10th Jubilee International Conference on Electrical, Electronic and Computing Engineering, IcETRAN 2023, East Sarajevo, Bosnia and Herzegovina, June 5 - 8, 2023. DOI: [10.1109/IcETRAN59631.2023.10192132](https://doi.org/10.1109/IcETRAN59631.2023.10192132)

- 4. Suad Suljović, **Dragana S. Krstić**, Goran Nestorovic, Nenad N. Petrović, Siniša Minić, Devendra S. Gurjar, “Using Level Crossing Rate of Selection Combining Receiver Damaged by Beaulieu-Xie Fading and Rician Co-Channel Interference with a Purpose of Machine Learning QoS Level Prediction”, Elektronika ir Elektrotechnika, Issue Vol. 29, No. 3, 2023, pp. 68-73.
DOI: <https://doi.org/10.5755/j02.eie.34018>
- 5. **Dragana S. Krstić**, Suad Suljović, Dejan Milic, Nenad Petrovic, “Approach to QoS Prediction Leveraging Impact of Beaulieu-Xie Fading and κ - μ Co-Channel Interference on SC Diversity Receiver Outage Probability”, 17th International Conference on Telecommunications - ConTEL 2023, July 11-13, 2023, Graz, Austria
- 6. **Dragana S. Krstić**, Suad Suljović, Nenad Petrovic, Sinisa Minic, Zoran Popovic, “Utilizing LCR of Wireless System with SC Receiver Weakened by Beaulieu-Xie Fading and κ - μ Interference for Machine Learning-Based QoS Prediction”, The IEEE 21st International Symposium on Intelligent Systems and Informatics (SISY 2023), September 21-23, 2023 in Pula, Croatia,

LEVEL CROSSING RATE OF SIGNAL TO INTERFERENCE RATIO AT THE OUTPUT OF THE RECEIVER WHERE CCI IS WITH RICIAN DISTRIBUTION

- The CCI envelope y_i is under Rician distribution:

$$p_{Y_i}(y_i) = 2e^{-\frac{(1+K_i)y_i^2}{s_i} - K_i} \sum_{i_2=0}^{\infty} \frac{K_i^{i_2} y_i^{2i_2+1}}{i_2! \Gamma(i_2 + 1)} \left(\frac{1 + K_i}{s_i} \right)^{i_2+1}$$

s_i is CCI power, and

K_i is Rician factor equal to the ratio of direct and scattered components.

The PDF is:

$$\begin{aligned}
 p_{z_i}(z_i) &= \int_0^\infty p_{x_i}(z_i y_i) p_{y_i}(y_i) dy_i = \\
 &= 2e^{-\frac{m}{\Omega} \lambda^2 - K_i} \sum_{i_1=0}^{\infty} \sum_{i_2=0}^{\infty} \frac{K_i^{i_2}}{i_1! i_2!} \\
 &\times \frac{\lambda^{2i_1} z_i^{2i_1+2m-1} m^{2i_1+m} s_i^{i_1+m} (1+K_i)^{i_2+1} \Gamma(i_1+i_2+m+1)}{\Gamma(i_1+m) \Gamma(i_2+1) \Omega_i^{i_1-i_2-1} (\Omega(1+K_i) + m s_i z_i^2)^{i_1+i_2+m+1}}
 \end{aligned}$$

LEVEL CROSSING RATE OF SIGNAL TO INTERFERENCE RATIO AT THE OUTPUT OF THE RECEIVER FOR BX FADING AND RICIAN CCI

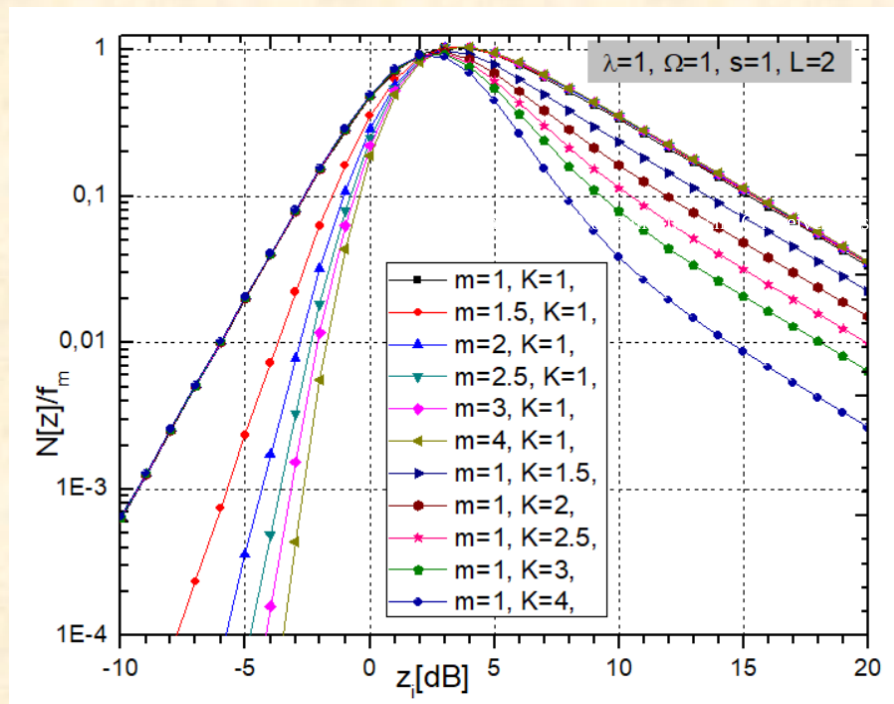
- The formula for LCR of SIRs z_i is:

$$\begin{aligned}
 N_{z_i}(z_i) &= \int_0^\infty \dot{z}_i d\dot{z}_i \int_0^\infty p_{\dot{z}_i}(\dot{z}_i | z_i y_i) p_{y_i}(y_i) y_i p_{x_i}(z_i y_i) dy_i = \\
 &= \frac{f_m \sqrt{2\pi}}{e^{K_i + (m\lambda^2/\Omega)}} \\
 &\times \sum_{i_1=0}^{\infty} \sum_{i_2=0}^{\infty} \frac{m^{2i_1+m-1/2}}{i_1! i_2!} \frac{K_i^{i_2} \lambda^{2i_1} z_i^{2i_1+2m-1} s_i^{i_1+m-1/2} (K_i + 1)^{i_2+1/2} \Gamma(i_1 + i_2 + m + 1/2)}{\Gamma(i_1 + m) \Gamma(i_2 + 1) \Omega^{i_1-i_2-1/2} ((1 + K_i)\Omega + m s_i z_i^2)^{i_1+i_2+m}}
 \end{aligned}$$

- The formula for LCR for the output SIR z is given by:

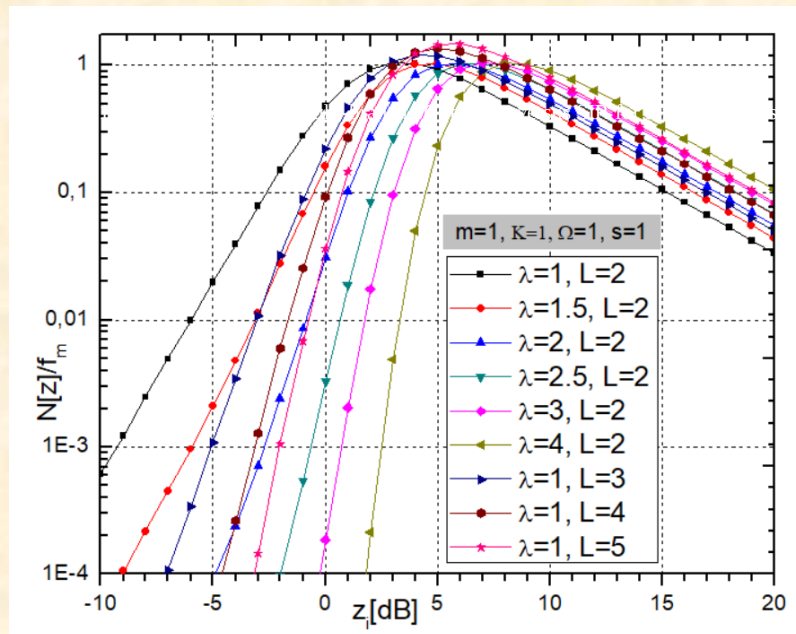
$$\begin{aligned}
N_{z|\Omega_i s_i}(z) &= L \left(F_{z_i}(z_i) \right)^{L-1} N_{z_i}(z_i) = \\
&= L \frac{f_m \sqrt{2\pi}}{e^{K_i + (m\lambda^2/\Omega)}} \sum_{i_1=0}^{\infty} \sum_{i_2=0}^{\infty} \frac{K_i^{i_2}}{i_1! i_2!} \\
&\frac{m^{2i_1+m-1/2} \lambda^{2i_1} z_i^{2i_1+2m-1} s_i^{i_1+m-1/2} (K_i+1)^{i_2+1/2} \Gamma(i_1+i_2+m+1/2)}{\Gamma(i_1+m) \Gamma(i_2+1) \Omega^{i_1-i_2-1/2} \left((1+K_i)\Omega + m s_i z_i^2 \right)^{i_1+i_2+m}} \\
&\left(e^{-\frac{m}{\Omega} \lambda^2} e^{-K_i} \sum_{i_3=0}^{\infty} \sum_{i_4=0}^{\infty} \frac{K_i^{i_4} \lambda^{2i_3} \Gamma(i_3+i_4+m+1)}{i_3! i_4! \Gamma(i_3+m) \Gamma(i_4+1)} \left(\frac{m}{\Omega} \right)^{i_3} B \frac{m s_i z_i^2}{\Omega(1+K_i) + m s_i z_i^2} (i_3+m, i_4+1) \right)^{L-1}
\end{aligned}$$

Normalized LCR of multi-branch SC receiver versus SIR for different values of BX fading parameter m and Rician factor K



•Fig.2. Normalized LCR of multi-branch SC receiver versus SIR for different values of BX fading parameter λ and number of branches L

Normalized LCR of multi-branch SC receiver versus SIR for different values of BX fading parameter λ and number of branches L



*Fig.2. Normalized LCR of multi-branch SC receiver versus SIR for different values of BX fading parameter λ and number of branches L

DERIVATION OF SIR BASED LCR AT THE SC RECEIVER OUTPUT FOR BX FADING AND κ - μ CCI

- The CCI envelope y_i is under κ - μ distribution:

$$p_{y_i}(y_i) = \frac{2e^{-\frac{\mu(1+\kappa)}{s_i}y_i^2}}{e^{\mu\kappa}} \sum_{i_2=0}^{+\infty} \frac{\mu^{2i_2+\mu} \kappa^{i_2} y_i^{2i_2+2\mu-1}}{\Gamma(i_2+\mu) i_2!} \left(\frac{1+\kappa}{s_i} \right)^{i_2+\mu}$$

- The κ - μ distribution is characterized by two parameters:
- κ is called Rician factor, $\kappa=K$, and
- μ presents the number of clusters in the propagation channel.
- The mean square values of the CCI envelopes are marked by s_i :

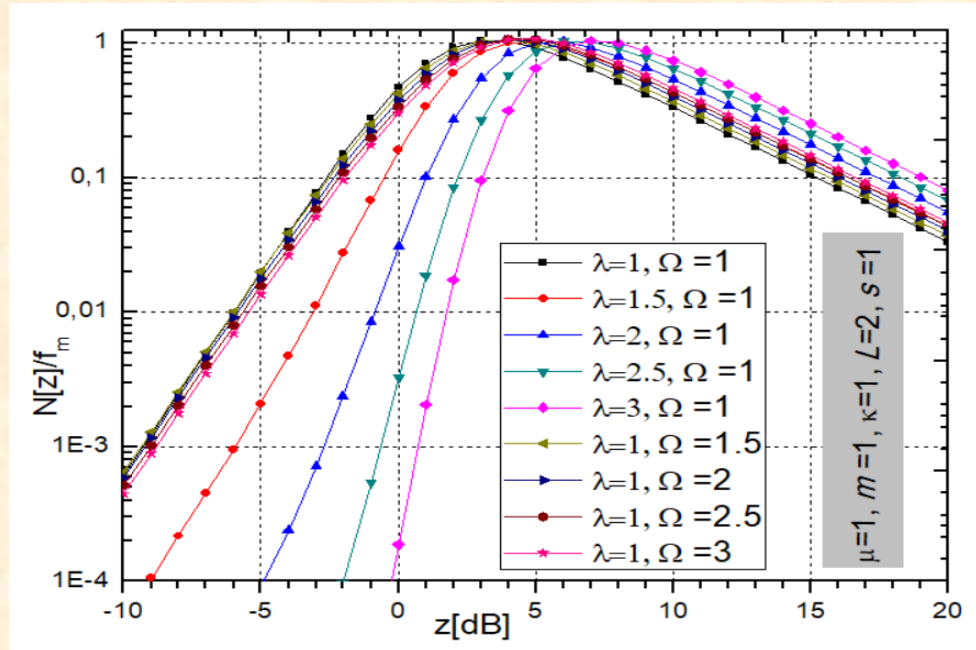
$$s_i = \bar{y}_i^2$$

DERIVATION OF SIR BASED LCR AT THE SC RECEIVER OUTPUT FOR BX FADING AND κ - μ CCI

- The final expression for LCR is:

$$\begin{aligned}
 N_{x_i|\Omega_i s_i}(z) &= \frac{L\sqrt{2\pi} f_m}{e^{(\mu\kappa+m\lambda^2/\Omega_i)L}} \times \\
 &\times \sum_{i_1=0}^{+\infty} \sum_{i_2=0}^{+\infty} \frac{\mu^{2i_2+\mu-1/2} m^{2i_1+m-1/2} \lambda^{2i_1} \kappa^{i_2} z_i^{2i_1+2m-1} \Omega_i^{i_2-i_1+\mu-1/2}}{i_1! i_2! \Gamma(i_1+m) \Gamma(i_2+\mu)} \times \\
 &\times \frac{s_i^{i_1+m-1/2} (1+\kappa)^{i_2+\mu-1/2} \Gamma(i_1+i_2+\mu+m-1/2)}{(\mu\Omega_i(1+\kappa) + ms_i z_i^2)^{i_1+i_2+\mu+m-1}} \times \\
 &\times \left(\sum_{i_3=0}^{+\infty} \sum_{i_4=0}^{+\infty} \frac{(\mu\kappa)^{i_4} \Gamma(i_3+i_4+m+\mu)}{i_3! i_4! \Gamma(i_3+m)} \left(\frac{m}{\Omega_i} \right)^{i_3} \times \right. \\
 &\left. \times \frac{\lambda^{2i_3}}{\Gamma(i_4+\mu)} B_{\frac{ms_i z_i^2}{\mu\Omega_i(1+\kappa) + ms_i z_i^2}}(i_3+m, i_4+\mu) \right)^{L-1}
 \end{aligned}$$

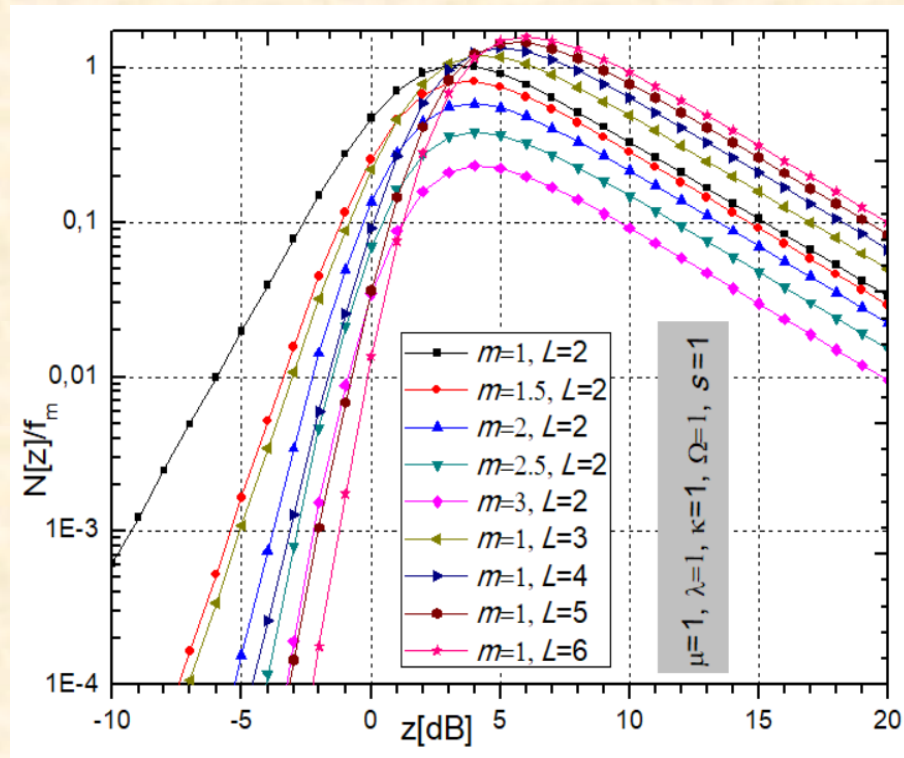
- In this figure, SIR based LCR, normalized by Doppler frequency f_m is presented for changeable BX fading parameter λ and power Ω . Other parameters remain constant, as well as CCI parameters.



Normalized LCR of SC receiver versus output SIR for different sets of fading parameters m_1 , λ_1 , and Ω .

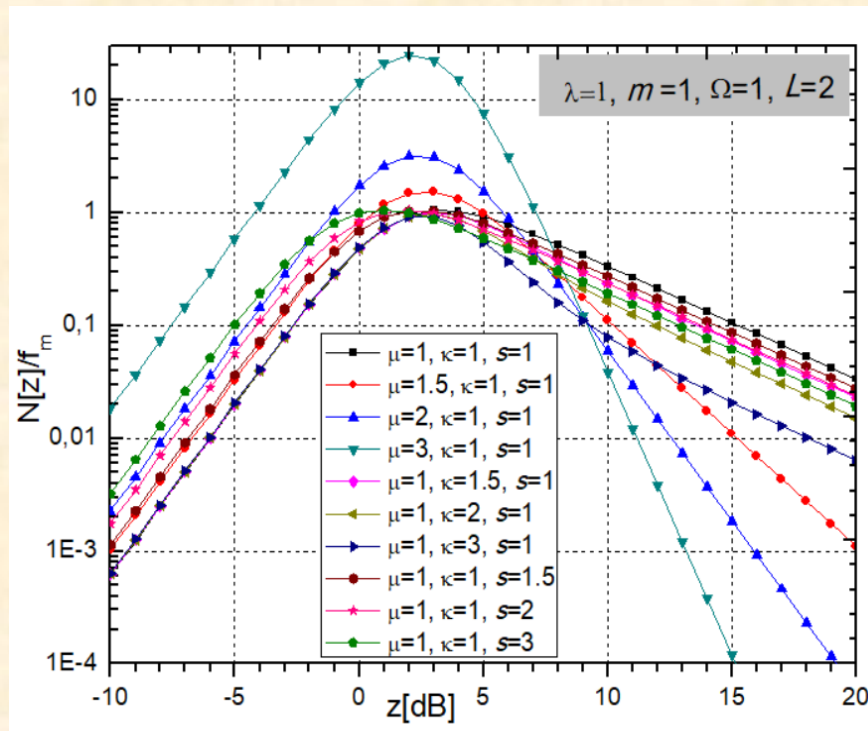
- An increase in λ causes a decrease of LCR for negative values of output SIR z (i.e. bigger CCI), while for positive values of SIR, LCR decreases.
- When LCR decreases, the system improves.
- Improvement also exists when the power Ω increases.

- In the next figure, the normalized LCR is shown for variable fading severity m and number of branches L at the input.
- We can see how increasing of L improves system characteristics. For higher values of z , the influence of increasing of L is not significant.
- When parameter m increases, the LCR is reducing and system has better performance.



Normalized LCR for various values of fading severity m and number of branches L .

- The impact of CCI parameters is shown in this figure:



Normalized LCR for various values of CCI parameters μ and κ and power s .

- When μ increases, the LCR for z less than zero.
- When CCI power s increases, performance deteriorates.
- When the parameter κ increases for negative z values, LCR is constant, whereas for positive z values it decreases and the system has better performance.

α - μ distribution

**Moment Generating Function Based
Calculation of Average Bit Error
Probability in an α - μ Fading
Environment with Selection Diversity
Receiver**

- We consider in this part the α - μ distribution
- The α - μ distribution is introduced as a small-scale fading model
- The α - μ distribution model includes the nonlinearity of the propagation medium since the assumption of a homogeneous diffuse scattering field is only an approximation
- Actually, this is generalized Gamma distribution that includes some other distributions

- Actually, α - μ distribution is generalized Gamma distribution that includes some other distributions
- Among them are:
 - Nakagami- m ($\alpha = 2$, $\mu = m$),
 - Weibull (when $\mu = 1$),
 - Rayleigh ($\mu = 1$ and $\alpha = 2$),
 - Negative exponential ($\mu = 1$ and $\alpha = 2$), and
 - One-sided Gaussian distribution ($\mu = 1$ and $\alpha = 2$).

- The influence of the CCI is studied along with the influence of fading
- We made in this paper MGF based calculation of the Average Bit Error Probability (ABEP) in an α - μ fading and CCI environment
- SC diversity receiver is used to mitigate the influences of these disturbances

- We choose to discuss MGFs since they are useful in analysis of sums of Random Variables (RVs).
- Namely, the MGF of RV gives all moments of that RV
- This fact gives the name to the Moment Generating Function
- Second, the MGF (if it exists) uniquely determines the distribution. So, if two RVs have the same MGF, then they must have the same distribution.
- Thus, if the MGF of a RV is found, its distribution is determined.

- The transmitted signal has the α - μ distribution:

$$p_{x_i}(x_i) = \frac{\alpha \mu_1^{\mu_1} x_i^{\alpha \mu_1 - 1}}{\Omega_i^{\mu_1} \Gamma(\mu_1)} e^{-\mu_1 \frac{x_i^\alpha}{\Omega_i}}$$

- Here, parameter α characterizes nonlinearity of the propagation environment; parameter
- μ is the number of clusters in that environment,
- $\Omega_i, i=1, 2, \dots, L$, represents the mean values of the input signals powers, and
- $\Gamma(\cdot)$ is the Gamma function.

- The CCI is also under α - μ distribution:

$$p_{y_i}(y_i) = \frac{\alpha \mu_2^{\mu_2} y_i^{\alpha \mu_2 - 1}}{s_i^{\mu_2} \Gamma(\mu_2)} e^{-\mu_2 \frac{y_i^\alpha}{s_i}}$$

- s_i are average powers of the CCI

- The PDFs of the SIRs z_i are mathematically given as:

$$p_{z_i}(z_i) = \int_0^\infty y_i p_{x_i}(z_i y_i) p_{y_i}(y_i) dy_i =$$

$$= \frac{\alpha \mu_1^{\mu_1} \mu_2^{\mu_2} z_i^{\alpha \mu_1 - 1} \Omega_i^{\mu_2} s_i^{\mu_1} \Gamma(\mu_1 + \mu_2)}{\Gamma(\mu_2) \Gamma(\mu_1) (\Omega_i \mu_2 + \mu_1 s_i z_i^\alpha)^{\mu_1 + \mu_2}}$$

- The expression for CDF of z_i is:

$$F_{z_i}(z_i) = \int_0^{z_i} p_{z_i}(t) dt =$$

$$= \frac{\Gamma(\mu_1 + \mu_2)}{\mu_1 \Gamma(\mu_2) \Gamma(\mu_1)} \sum_{j=0}^{+\infty} \frac{(\mu_1)_j (1 - \mu_2)_j}{j! (\mu_1 + 1)_j} \left(\frac{\mu_1 s_i z_i^\alpha}{\Omega_i \mu_2 + \mu_1 s_i z_i^\alpha} \right)^{j + \mu_1}$$

- In SC combining, the strongest signal from L received signals is processed to the user
- The PDF of the SIR z from SC receiver in our scenario is:

$$p_{z_i}(z) = \frac{L\alpha\mu_2^{\mu_2} z_i^{\alpha\mu_1-1} \Omega_i^{\mu_2} S_i^{\mu_1}}{\mu_1^{L-\mu_1-1} \left(\Omega_i\mu_2 + \mu_1 S_i z_i^\alpha \right)^{\mu_1+\mu_2}} \cdot \left(\frac{\Gamma(\mu_1 + \mu_2)}{\Gamma(\mu_2)\Gamma(\mu_1)} \right)^L \left(\sum_{j=0}^{+\infty} \frac{(\mu_1)_j (1-\mu_2)_j}{(\mu_1+1)_j j!} \left(\frac{\mu_1 S_i z_i^\alpha}{\Omega_i\mu_2 + \mu_1 S_i z_i^\alpha} \right)^{j+\mu_1} \right)^{L-1}$$

- The main formula for derivation the MGF is:

$$M_z(h) = \overline{e^{zh}} = \int_0^{\infty} e^{-zh} p_{z_i}(z) dz$$

- For our case MGF is:

$$M_z(h) = \frac{L\alpha}{2\sqrt{\pi}\mu_1^{L-1}} \left(\frac{\Gamma(\mu_1 + \mu_2)}{\Gamma(\mu_2)\Gamma(\mu_1)} \right)^L \left(\frac{\mu_2\Omega_i}{\mu_1 s_i} \right)^{\frac{\mu_1 L(\alpha-2)}{2}} \frac{1}{\Gamma(jL - j + \mu_1 L + \mu_2)} \cdot \left(\sum_{j=0}^{+\infty} \frac{(\mu_1)_j (1-\mu_2)_j}{(\mu_1+1)_j j!} \left(\frac{\mu_2\Omega_i}{\mu_1 s_i} \right)^{\frac{j(\alpha-2)}{2}} \right)^{L-1} G_{13}^{31} \left(\frac{h^2 \mu_2 \Omega_i}{4\mu_1 s_i} \left| \begin{matrix} 1 - \left(\frac{\alpha jL - \alpha j + \alpha \mu_1 L}{2} \right) \\ \left(\frac{(2-\alpha)(jL - j + \mu_1 L) + 2\mu_2}{2} \right) \end{matrix} \right. , 0, \frac{1}{2} \right)$$

where $G(\cdot)$ is the Meijer's G-function.

- The ABEP is among performance that describes the nature of the system behavior on the best manner and is used the most often to describe that behavior
- So, the determining the ABEP in the simplest possible way is of great importance.
- The difficulty in evaluating the ABEP is that the conditional BEP is a nonlinear function of the SNR or SIR
- The nonlinearity is a consequence of the modulation/detection scheme
- This is the reason for considering the MGF-based approach to determine ABEP.

- By utilizing the expression for MGF, the ABEP for non-coherent BFSK and BDPSK modulations will be:

$$P_{be}(\Omega_0)=0.5M_z(0.5)$$

$$P_{be}(\Omega_0)=0.5M_z(1)$$

- *A. Binary Frequency Shift Keying (BFSK) Modulation*

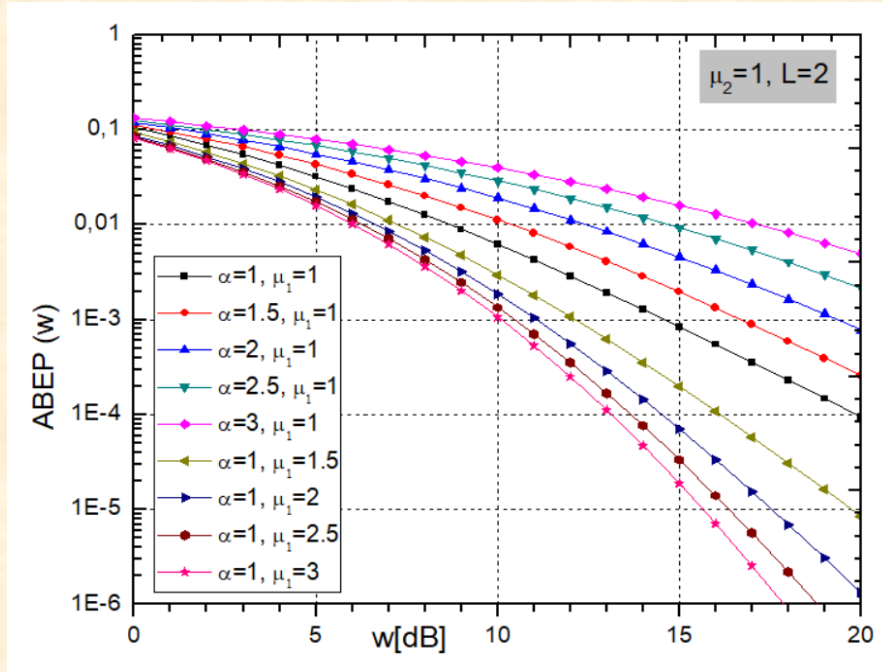


Fig 2. ABEP versus SIR for BFSK modulation when parameters α and μ_1 are changing.

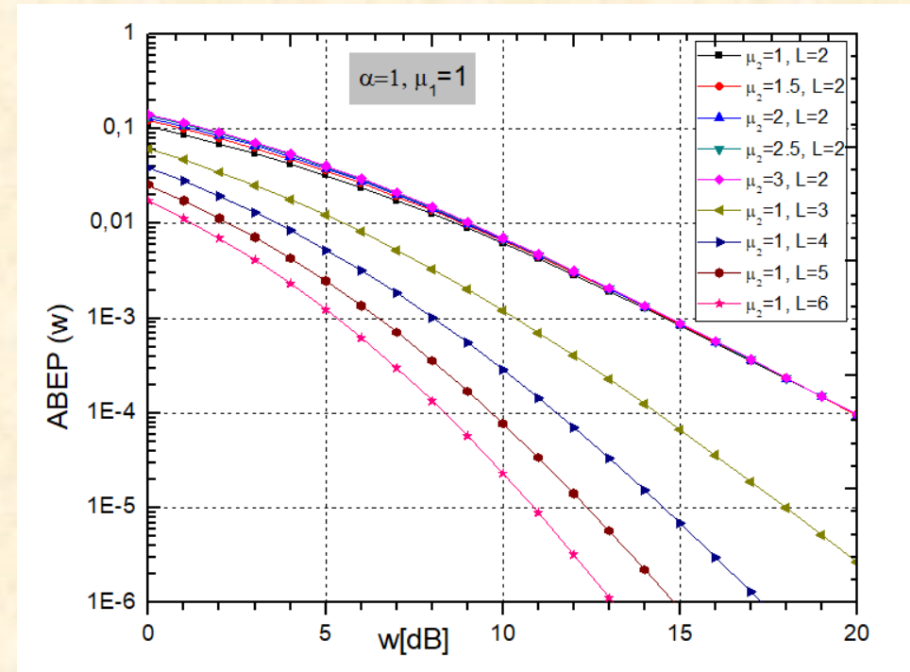
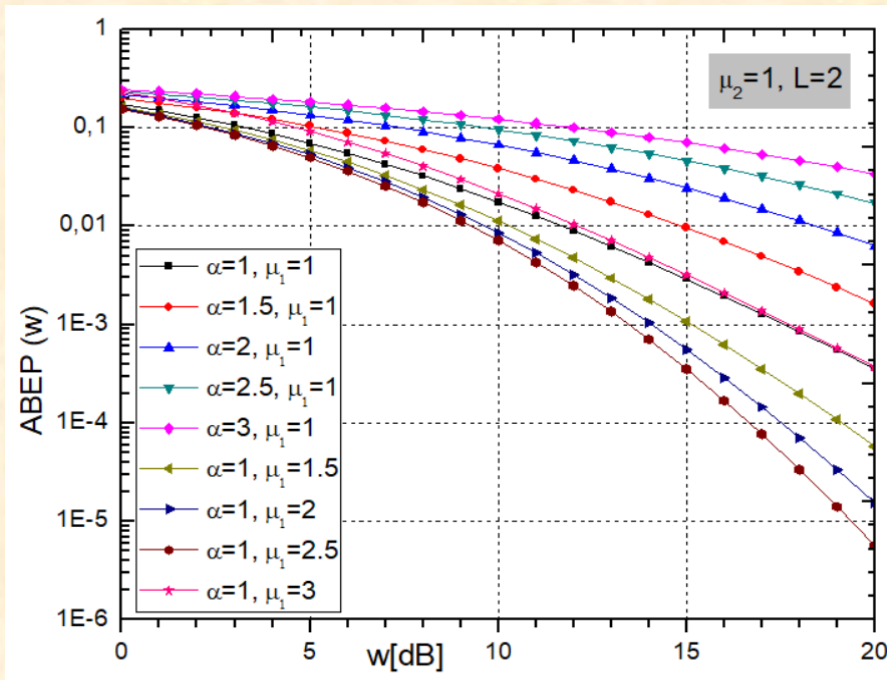


Fig 3. ABEP versus SIR for BFSK modulation with variable parameters μ_2 and L .

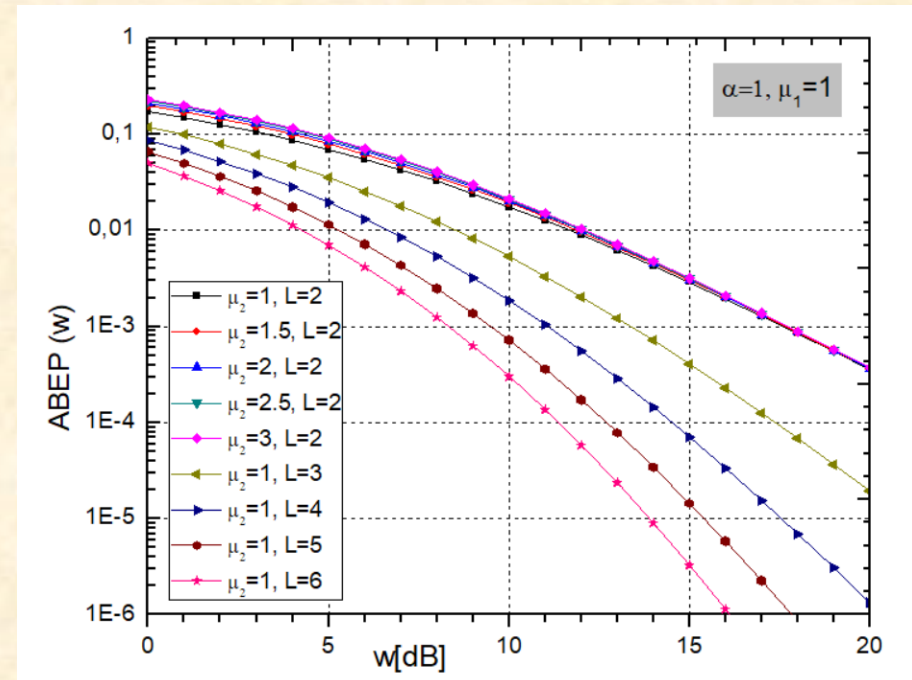
- In these figures, the curves for ABEP in the case with BFSK modulation, versus SIR, $w = \Omega/s$, at the output of the SC receiver with L branches, are presented.

- First, the ABEP is presented for BFSK modulation and dual branch SC receiver ($L=2$), with $\mu_2=1$, while parameters α and μ_1 are changing.
- The ABEP grows with an increasing in parameter α . This means that system performance gets worse. When the parameter μ_1 increases, the ABEP decreases and the system has better performance.
- Then, the ABEP is plotted versus SIR for BFSK modulation with variable parameters μ_2 and L , and $\alpha=1$, $\mu_2=1$.
- The increase in the parameter μ_2 has no greater influence on the ABEP, while with the increase in the number of branches L , the ABEP decreases significantly and the system has better performance.

- **B. Binary Differential Phase – Shift Keying (BDPSK) Modulation**
- Now, the curves for ABEP versus SIR at the output of SC receiver with L branches are presented for BDPSK modulation.



ABEP versus SIR for BDPSK modulation when parameters α and μ_1 are changing.



ABEP versus SIR for BDPSK modulation when parameters μ_2 and L are changing.

- First graph is for dual branch SC receiver ($L=2$).
- In this figure $\mu_2=1$, and parameters α and μ_1 took several values.
- It is visible from this figure that the ABEP grows when the parameter α increases. This happening makes the system performance to be worse.
- When the parameter μ_1 increases, the ABEP decreases, improving the system performance.

- In the last figure, the ABEP is presented versus SIR for BDPSK modulation with parameters μ_2 and L that are changing, and the parameters: $\alpha=1$, $\mu_1=1$.
- One can conclude that the increase in the parameter μ_2 is without significant impact on the ABEP, while with the increase in the number of branches L , the ABEP drops significantly.
- This leads to improvement of the system performance and is in accordance with theoretical assumptions.
- Comparing the results from these figures, we can conclude that the system has better performance with smaller ABEP in the case of using BDPSK than BFSK modulation.

- One of the main contributions of this result is that it can be used to determine the ABEP of wireless systems in the presence of other types of fading: Rayleigh, Nakagami- m , Weibull, and One-sided Gaussian, in the presence of CCI, by setting certain special values for parameters α and μ

- 7. **Dragana Krstic**, Suad Suljovic Devendra S. Gurjar, Suneel Yadav, “Moment Generating Function Based Calculation of Average Bit Error Probability in an α - μ Fading Environment with Selection Diversity Receiver”, IARIA Congress 2023: The 2023 IARIA Annual Congress on Frontiers in Science, Technology, Services, and Applications, 13-17 November, 2023, Valencia, Spain
- 8. **Dragana Krstic**, Suad Suljovic, Nenad Petrovic, Zoran Popovic, Sinisa Minic, „MGF Based Calculation and Simulation of ABEP for Multi-branch SC Receiver in an Environment under α - κ - μ Fading and Co-channel Interference“, Fifth International Balkan Conference on Communications and Networking, BalkanCom'22, August 22-24, 2022, Sarajevo, Bosnia and Herzegovina, pp. 26-30.
DOI: [10.1109/BalkanCom55633.2022.9900869](https://doi.org/10.1109/BalkanCom55633.2022.9900869)
- 9. **Dragana Krstic**, Suad Suljovic, Nenad Petrovic, Sinisa Minic, Zoran Popovic, „Determining the ABEP under the Influence of κ - μ Fading and CCI with SC combining at L-branch Receiver Using Moment Generating Function“, The 30th International Conference on Software, Telecommunications and Computer Networks (SoftCOM 2022), 22-24. September, Split, Croatia,
DOI: [10.23919/SoftCOM55329.2022.9911453](https://doi.org/10.23919/SoftCOM55329.2022.9911453),

- 10. **Dragana Krstic**, Nenad Petrovic, Suad Suljovic, and Issam Al-Azzoni, "AI-Enabled Framework for Mobile Network Experimentation Leveraging ChatGPT: Case Study of Channel Capacity Calculation for η - μ Fading and Co-Channel Interference", Electronics 2023, 12(19), 4088; <https://doi.org/10.3390/electronics12194088>
- 11. **Dragana Krstic**, Nenad Petrovic, Suad Suljovic, Gaurav K. Pandey, Devendra S. Gurjar, Suneel Yadav, "ChatGPT-Driven Approach for QoS Estimation Using LCR in 5G Network with α - η - μ Fading and CCI Environment", 2023 IEEE Silchar Subsection Conference, (SILCON), 3-5 November, 2023 (Hybrid Mode), National Institute of Technology Silchar, Silchar, India

The α - κ - μ distribution

- α - κ - μ distribution fits very well with experimental data
- It provides a general multipath model for a nonlinear line-of-sight (LOS) propagation scenario with two or more clusters
- Three parameters describe this distribution
 - parameter α (related to the non-linearity of propagation environment),
 - the Rician factor κ (the ratio of the powers of dominant and scattered components), and
 - μ (the number of clusters in the propagating channel)

- The α - κ - μ multipath fading is more severe for lower values of Rician κ factor and parameter μ
- When parameter μ tends to infinity, α - κ - μ fading channel becomes a channel without fading effects
The α - κ - μ multipath fading is more severe for lower values of Rician κ factor and parameter μ
- When parameter μ tends to infinity, α - κ - μ fading channel becomes a channel without fading effects

- Since α - κ - μ distribution is general distribution, some other fading distributions are included in α - κ - μ distribution as particular cases:
- α - μ distribution for $\kappa=0$,
- κ - μ , for $\alpha=0$,
- Weibull for $\kappa=0$ and $\mu=0$,
- Nakagami- m for $\alpha=2$ and $\kappa=0$,
- Ricean by placing $\mu=1$, and $\alpha=2$,
- Rayleigh by setting $\alpha=2$, $\kappa=0$ and $\mu=1$,
- and one-sided Gaussian.

Moment Generating Function Based
Calculation of Average Bit Error
Probability for Multi-branch SC Receiver
in an Environment under α - K - μ Fading
and Co-channel Interference

- Useful signal envelope is modelled by the α - κ - μ distribution:

$$p_{x_i}(x_i) = \frac{\alpha}{e^{\mu K}} \sum_{i_1=0}^{\infty} \frac{\mu^{2i_1+\mu} K^{i_1} x_i^{\alpha i_1 + \alpha\mu - 1}}{i_1! \Gamma(i_1 + \mu)} \left(\frac{1+K}{\Omega_i} \right)^{i_1+\mu} e^{-\frac{\mu(1+K)}{\Omega_i} x_i^\alpha}$$

- where parameter α represents nonlinearity of propagation environment, parameter K is Rician factor, parameter μ shows the number of clusters in propagation environment, Ω_i represents the mean values of the input signals powers, and $\Gamma(\cdot)$ is Gamma function.

- The CCI also follows α - κ - μ distribution:

$$p_{y_i}(y_i) = \frac{\alpha}{e^{\mu K}} \sum_{i_2=0}^{\infty} \frac{\mu^{2i_2+\mu} K^{i_2} y_i^{\alpha i_2 + \alpha \mu - 1}}{i_2! \Gamma(i_2 + \mu)} \left(\frac{1+K}{s_i} \right)^{i_2+\mu} e^{-\frac{\mu(1+K)}{s_i} y_i^\alpha}$$

- where s_i are average powers of the CCI y_i .
- The ratios of the desired signal and the co-channel interference at the i^{th} input branch of the SC combiner are:

$$z_i = x_i / y_i, x_i = z_i y_i$$

- The PDF of the SIRs z_i are derived as:

$$\begin{aligned}
 p_{z_i}(z_i) &= \int_0^{+\infty} y_i p_{x_i}(y_i z_i) p_{y_i}(y_i) dy_i = \\
 &= \frac{\alpha}{e^{2\mu K}} \sum_{i_1=0}^{\infty} \sum_{i_2=0}^{\infty} \frac{\mu^{i_1+i_2} K^{i_1+i_2} \Omega_i^{i_2+\mu} s_i^{i_1+\mu} z_i^{\alpha i_1+\alpha\mu-1} \Gamma(i_1+i_2+2\mu)}{i_1! i_2! (\Omega_i + s_i z_i^\alpha)^{i_1+i_2+2\mu} \Gamma(i_1+\mu) \Gamma(i_2+\mu)}
 \end{aligned}$$

- The expression for CDF of z_i is:

$$F_{z_i}(z_i) = \frac{1}{e^{2\mu K}} \sum_{i_1=0}^{\infty} \sum_{i_2=0}^{\infty} \sum_{i_3=0}^{\infty} \frac{(i_1+\mu)_{i_3} (1-i_2-\mu)_{i_3} (\mu K)^{i_1+i_2} \Gamma(i_1+i_2+2\mu)}{i_1! i_2! i_3! (i_1+\mu)(i_1+\mu+1)_{i_3} \Gamma(i_1+\mu) \Gamma(i_2+\mu)} \left(\frac{s_i z_i^\alpha}{\Omega_i + s_i z_i^\alpha} \right)^{i_1+i_3+\mu}$$

- The PDF of the output SIR z may be calculated using next formula:

$$p_{z_i}(z) = L p_{z_i}(z_i) \left(F_{z_i}(z_i) \right)^{L-1} =$$

$$= \frac{\alpha L}{e^{2\mu K}} \sum_{i_1=0}^{\infty} \sum_{i_2=0}^{\infty} \frac{(\mu K)^{i_1+i_2} \Omega_i^{i_2+\mu} s_i^{i_1+\mu} z_i^{\alpha i_1+\alpha\mu-1} \Gamma(i_1+i_2+2\mu)}{i_1! i_2! \Gamma(i_1+\mu) \Gamma(i_2+\mu) (\Omega_i + s_i z_i^\alpha)^{i_1+i_2+2\mu}}.$$

$$\cdot \left(\frac{1}{e^{2\mu K}} \sum_{i_3=0}^{\infty} \sum_{i_4=0}^{\infty} \sum_{i_5=0}^{\infty} \frac{(i_3+\mu)_{i_5} (1-i_4-\mu)_{i_5} (\mu K)^{i_3+i_4} \Gamma(i_3+i_4+2\mu)}{i_3! i_4! i_5! (i_3+\mu)(i_3+\mu+1)_{i_5} \Gamma(i_3+\mu) \Gamma(i_4+\mu)} \left(\frac{s_i z_i^\alpha}{\Omega_i + s_i z_i^\alpha} \right)^{i_3+i_5+\mu} \right)^{L-1}$$

- MGF is:

$$M_z(h) = M_z(h) = \int_0^\infty dz e^{-zh} p_{z_i}(z) = \frac{\alpha}{2\sqrt{\pi} e^{2\mu K}} \sum_{i_1=0}^\infty \sum_{i_2=0}^\infty \frac{(\mu K)^{i_1+i_2}}{i_1! i_2!} \left(\frac{\Omega_i}{s_i} \right)^{\frac{(i_1+L\mu)(\alpha-2)}{2}} \frac{\Gamma(i_1+i_2+2\mu)}{\Gamma(i_1+\mu)\Gamma(i_2+\mu)}$$

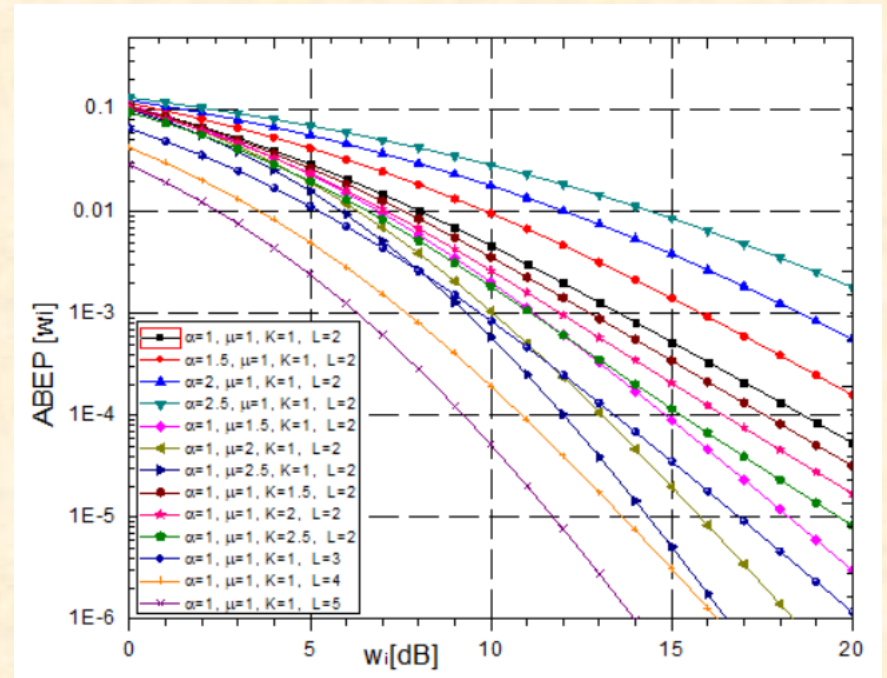
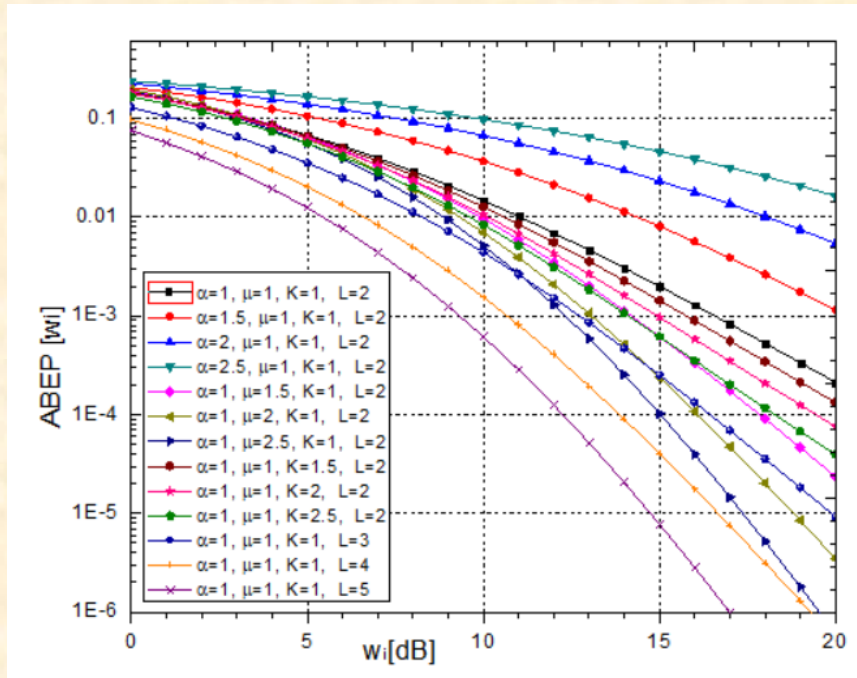
$$\cdot \left(\frac{1}{e^{2\mu K}} \sum_{i_3=0}^\infty \sum_{i_4=0}^\infty \sum_{i_5=0}^\infty \frac{(i_3+\mu)_{i_5}}{i_3! i_4! i_5! (i_3+\mu)} \frac{(1-i_4-\mu)_{i_5}}{(i_3+\mu+1)_{i_5}} \frac{(\mu K)^{i_3+i_4}}{\Gamma(i_3+\mu)\Gamma(i_4+\mu)} \left(\frac{\Omega_i}{s_i} \right)^{\frac{(i_3+i_5)(\alpha-2)}{2}} \right)^{L-1}$$

$$\cdot \frac{1}{\Gamma(i_1+i_2+Li_3-i_3+Li_5-i_5+L\mu+\mu)} G_{13}^{31} \left(\frac{h^2}{4} \left(\frac{\Omega_i}{s_i} \right) \left| \begin{array}{c} 1 - \left(\frac{\alpha(i_1+Li_3-i_3+Li_5-i_5+L\mu)}{2} \right) \\ \left(\frac{(2-\alpha)i_1+2i_2+(2L+\alpha-\alpha L-2)(i_3+i_5)+(2L-\alpha L+2)\mu}{2} \right) \end{array} \right. , 0, \frac{1}{2} \right)$$

- where $G[\cdot]$ is the Meijer's G-function.
- Using the MGF expression, the ABEP for non-coherent BFSK and BDPSK modulation can be calculated by:

$$P_{be}(\Omega_0) = 0.5 M_z(0.5); \text{ for BFSK,} \quad P_{be}(\Omega_0) = 0.5 M_z(1); \text{ for BDPSK}$$

- ABEP of multi-branch SC receiver output SIR in the presence of generalized α - κ - μ fading and CCI for two or more SC combiner inputs and two different modulations: BDPSK and BFSK modulations.



- For BFSK and BDPSK, it can be noticed that the ABEP grows and the system performance gets worse when parameter α increases.
- It is also visible that the ABEP decreases with increasing of the SIR w , Rician factor K , number of clusters μ and number of input branches in the SC combiner, L , and the system performance is significantly improved. Since parameter K represents the ratio between the total power of the dominant components and the total power of the scattered waves, it is clear that bigger values of K lead to smaller values of ABEP.
- Also, higher values of the SIR w give lower ABEP and better system performance. Further, an increasing of the number of input branches L means better possibilities for combiner to choose antenna with higher SIR.
- Finally, since within each cluster there is a dominant component, bigger number of clusters μ leads to lower values of ABEP. These conclusions follow theoretical assumptions. Comparing the results from these two figures, we can conclude that the system is more stable and the ABEP is smaller in the case of BDPSK than BFSK modulation.

***SC Diversity Receiver Outage Probability in
the Presence of Beaulieu-Xie Fading and
 κ - μ Co-Channel Interference***

- Each signal at the input of the SC diversity receiver has the probability density function (PDF) modelled by the BX distribution:

$$p_{X_i}(x_i) = 2e^{-\frac{m}{\Omega_i}(x_i^2 + \lambda^2)} \sum_{i_1=0}^{+\infty} \frac{\lambda^{2i_1} x_i^{2i_1+2m-1}}{i_1! \Gamma(i_1 + m)} \left(\frac{m}{\Omega_i} \right)^{2i_1+m}$$

- From Bealeu-Xie distribution, the Rician distribution may be obtained by putting parameter $m = 1$, and Nakagami- m distribution will be obtained if non-central parameter $\lambda = 0$.
- Also, if non-central parameter $\lambda = 0$, the Rician distribution converges to a Rayleigh model; the same result is achieved for $m=1$ applied to a Nakagami- m distribution.

- The CCI follows κ - μ distribution:

$$p_{y_i}(y_i) = \frac{2e^{-\frac{\mu(1+\kappa)}{s_i}y_i^2}}{e^{\mu\kappa}} \sum_{i_2=0}^{+\infty} \frac{\mu^{2i_2+\mu} \kappa^{i_2} y_i^{2i_2+2\mu-1}}{\Gamma(i_2 + \mu) i_2!} \left(\frac{1+\kappa}{s_i} \right)^{i_2+\mu}$$

- The κ - μ distribution, is characterized by two parameters, κ and μ .
- Here, parameter $\kappa=K$ is called Rician factor and equals to the ratio of the dominant and scattered components, parameter μ represents the number of clusters in the propagation environment, the mean square values of the CCI envelopes are denoted by s_i , $i=1,2, \dots, L$.
- The SIR z_i has the PDF given by:

$$p_{z_i}(z_i) = \int_0^\infty p_{x_i}(z_i / y_i) p_{y_i}(y_i) y_i dy_i = \frac{2}{e^{\mu\kappa + \frac{m}{\Omega}\lambda^2}} \sum_{i_1=0}^{+\infty} \sum_{i_2=0}^{+\infty} \frac{z_i^{2i_1+2m-1}}{i_1! i_2!} \cdot \left(\frac{1+\kappa}{s_i} \right)^{i_2+\mu} \frac{\lambda^{2i_1} \mu^{2i_2+\mu} \kappa^{i_2} \Omega^{i_2-i_1+\mu} s_i^{i_1+i_2+\mu+m} m^{2i_1+m} \Gamma(i_1 + m + i_2 + \mu)}{\Gamma(i_1 + m) \Gamma(i_2 + \mu) (\mu\Omega(1+\kappa) + m s_i z_i^2)^{i_1+i_2+\mu+m}}$$

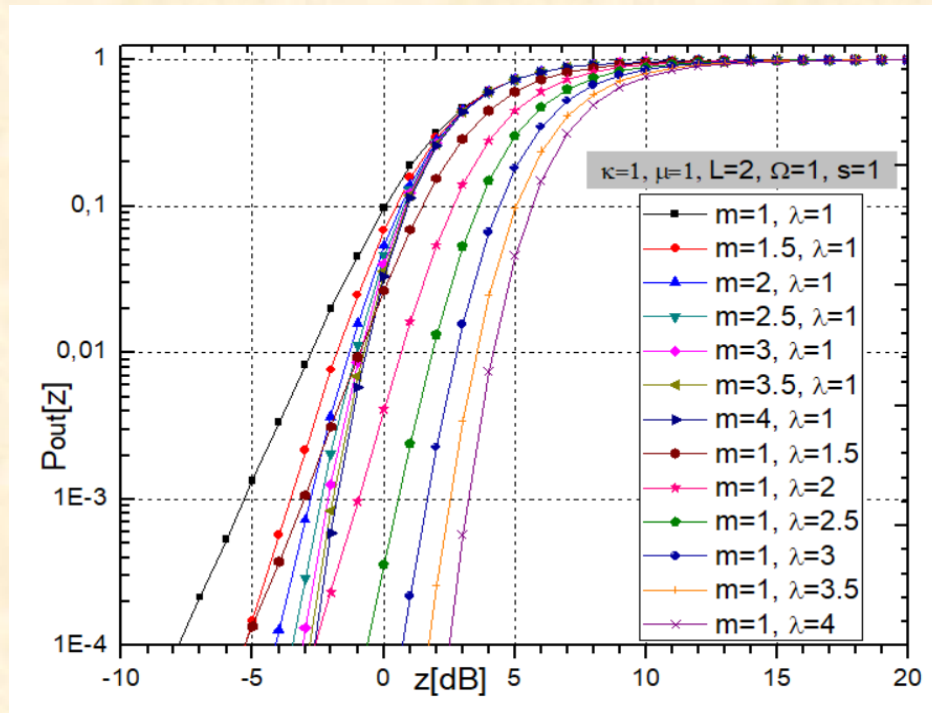
- The CDF is defined as:

$$\begin{aligned}
 F_{z_i}(z_i) &= \int_0^{z_i} p_{z_i}(t) dt = \\
 &= \frac{1}{e^{\mu\kappa + \frac{m}{\Omega}\lambda^2}} \sum_{i_1=0}^{+\infty} \sum_{i_2=0}^{+\infty} \frac{(\mu\kappa)^{i_2}}{i_1! i_2!} \left(\frac{m}{\Omega}\right)^{i_1} \cdot \\
 &\quad \cdot \frac{\lambda^{2i_1} \Gamma(i_1 + m + i_2 + \mu)}{\Gamma(i_1 + m) \Gamma(i_2 + \mu)} B_{\frac{ms_i z_i^2}{\mu\Omega(1+\kappa) + ms_i z_i^2}}(i_1 + m, i_2 + \mu)
 \end{aligned}$$

where $B_z(a, b)$ is the incomplete Beta function.

- The final exact expression for P_{out} on the SC receiver output is:

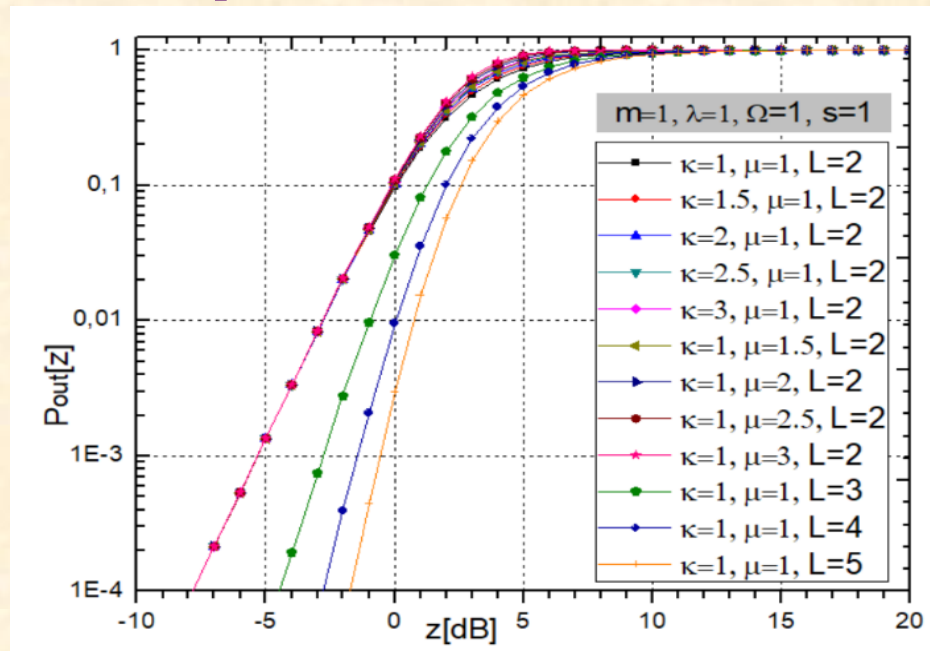
$$\begin{aligned}
 P_{out}(z) &= F_{z_i}(z) = \left(F_{z_i}(z_i) \right)^L = \\
 &= \left(\frac{1}{e^{\mu\kappa + \frac{m}{\Omega}\lambda^2}} \sum_{i_1=0}^{+\infty} \sum_{i_2=0}^{+\infty} \frac{\lambda^{2i_1} (\mu\kappa)^{i_2}}{i_1! i_2!} \cdot \right. \\
 &\quad \left. \frac{\Gamma(i_1 + i_2 + m + \mu)}{\Gamma(i_1 + m) \Gamma(i_2 + \mu)} \left(\frac{m}{\Omega} \right)^{i_1} B_{\frac{mS_i z_i^2}{\mu\Omega(1+\kappa) + mS_i z_i^2}}(i_1 + m, i_2 + \mu) \right)^L
 \end{aligned}$$



Normalized P_{out} of L -branch SC receiver depending on SIR considering different values of fading parameters m and λ .

- We can see that due to the increase in the parameters m and λ , the system P_{out} decreases and the system has better performance.

- Now we can see that larger value of L (number of input diversity branches) reduces the effect of the BX fading and conducts to decreasing of P_{out} and the system shows better performance.



Normalized P_{out} of L -branch SC receiver depending on SIR for variable parameters κ , μ and L

- From this figure it is visible that the P_{out} does not change significantly when the parameters κ and μ are changing.
- Therefore, we can conclude that these two parameters do not affect P_{out} considerably.

The η - μ Distribution

- The model for the fading with η - μ distribution is a signal composed of clusters of multipath waves which propagate in a non-homogeneous environment
- Inside any one cluster, the phases of the scattered waves are random with similar delay times
- The delay-time spreads of different clusters are relatively large

- Proposed η - μ distribution is a general probability distribution because it contains as particular cases the majority of the linear fading models presented in the available literature: Nakagami- m , exponential, one-sided Gaussian, Rayleigh, and Nakagami- q (Hoyt)
- The Rician and log-normal distributions can also be good approximated
- These distributions may be obtained from $\eta - \mu$ distribution putting appropriate values for parameters η and μ

- The one-sided Gaussian and the Rayleigh probability distributions may arise from the Nakagami- m distribution by assigning the Nakagami parameter to $m=0.5$ and $m=1$, respectively.
- Thus, in order to relate the η - μ distribution with the one-sided Gaussian and the Rayleigh distributions, it is sufficient to relate it with the Nakagami- m distribution.
- Nakagami- m distribution can be easily obtained from the η - μ distribution on many ways. An exact manner is by assigning parameter values $\mu = m$ and $\eta \rightarrow 0$ (or equivalently $\eta \rightarrow \infty$) or, in the same way, for $\mu = m/2$ and $\eta \rightarrow 1$. On the other side, the Hoyt distribution can be obtained by setting $\eta \rightarrow 1$ and $\mu = 1/2$

Determination the Level Crossing Rate of Mobile Systems Limited by α - η - μ Distributed Fading and Interference

- The signal's probability density function (PDF) is α - η - μ distributed:

■

$$p_{x_i}(x_i) = \frac{\alpha \sqrt{\pi} (\eta_1 + 1)^{2\mu_1}}{\Gamma(\mu_1) e^{\frac{(\eta_1 + 1)^2 \mu_1}{2\eta_1 \Omega_i} x_i^\alpha}} \sum_{i_1=0}^{+\infty} \frac{(1 - \eta_1^2)^{2i_1} x_i^{2\alpha i_1 + 2\alpha\mu_1 - 1} (\mu_1 / \Omega_i)^{2i_1 + 2\mu_1}}{2^{4i_1 + 2\mu_1 - 1} i_1! \eta_1^{2i_1 + \mu_1} \Gamma(i_1 + \mu_1 + 1/2)}$$

where $x_i \geq 0$; Ω_i are average powers $\Omega_i(x_i) = \bar{x}_i^2$; α is the non-linearity parameter; $\eta_1 \geq 0$ accounts for unequal powers of the components in-phase and quadrature, the number of the multipath clusters is μ ; and $\Gamma(\cdot)$ is the Euler gamma function.

- The CCI also follows α - η - μ distribution:

$$p_{y_i}(y_i) = \frac{\alpha \sqrt{\pi} (\eta_2 + 1)^{2\mu_2}}{\Gamma(\mu_2) e^{\frac{(\eta_2 + 1)^2 \mu_2}{2\eta_2 s_i} y_i^\alpha}} \sum_{i_2=0}^{+\infty} \frac{(1 - \eta_2^2)^{2i_2} y_i^{2\alpha i_2 + 2\alpha\mu_2 - 1} (\mu_2 / s_i)^{2i_2 + 2\mu_2}}{2^{4i_2 + 2\mu_2 - 1} i_2! \eta_2^{2i_2 + \mu_2} \Gamma(i_2 + \mu_2 + 1/2)}$$

s_i are CCI powers: .

$$s_i = \bar{y}_i^2$$

- The PDF of SIR z_i is calculated by formula:

$$\begin{aligned}
 p_{z_i}(z_i) &= \int_0^{\infty} y_i p_{x_i}(z_i y_i) p_{y_i}(y_i) dy_i = \\
 &= \frac{\alpha \pi (\eta_1 + 1)^{2\mu_1} (\eta_2 + 1)^{2\mu_2}}{\Gamma(\mu_1) \Gamma(\mu_2)} \sum_{i_1=0}^{+\infty} \sum_{i_2=0}^{+\infty} \frac{\mu_1^{2i_1+2\mu_1} \mu_2^{2i_2+2\mu_2}}{2^{2i_1+2i_2-2} i_1! i_2!} \cdot \\
 &\cdot \frac{(1-\eta_1^2)^{2i_1} (1-\eta_2^2)^{2i_2} \Gamma(2i_1+2i_2+2\mu_1+2\mu_2) \eta_1^{2i_2+\mu_1+2\mu_2} \eta_2^{2i_1+2\mu_1+\mu_2} \Omega_i^{2i_2+2\mu_2} S_i^{2i_1+2\mu_1} Z_i^{2\alpha i_1+2\alpha\mu_1-1}}{\Gamma(i_1+\mu_1+1/2) \Gamma(i_2+\mu_2+1/2) \left(\mu_2 \eta_1 \Omega_i (\eta_2+1)^2 + (\eta_1+1)^2 \mu_1 \eta_2 S_i Z_i^\alpha \right)^{2i_1+2i_2+2\mu_1+2\mu_2}}
 \end{aligned}$$

- The CDF of the SIR z_i by formula:

$$\begin{aligned}
 F_{z_i}(z_i) &= \int_0^{z_i} p_{z_i}(t) dt = \\
 &= \frac{\pi \eta_1^{\mu_1} \eta_2^{\mu_2}}{\Gamma(\mu_1) \Gamma(\mu_2)} \sum_{i_1=0}^{+\infty} \sum_{i_2=0}^{+\infty} \frac{(1-\eta_1^2)^{2i_1} (1-\eta_2^2)^{2i_2}}{2^{2i_1+2i_2-2} (\eta_1+1)^{4i_1+2\mu_1} (\eta_2+1)^{4i_2+2\mu_2}} \\
 &\times \frac{\Gamma(2i_1+2i_2+2\mu_1+2\mu_2)}{i_1! i_2! \Gamma(i_1+\mu_1+1/2) \Gamma(i_2+\mu_2+1/2)} \\
 &\times B \frac{\mu_1 \eta_2 s_i (\eta_1+1)^2 z_i^\alpha}{\mu_2 \eta_1 \Omega_i (\eta_2+1)^2 + \mu_1 \eta_2 s_i (\eta_1+1)^2 z_i^\alpha} (2i_1+2\mu_1, 2i_2+2\mu_2)
 \end{aligned}$$

- $B_z(a, b)$ is incomplete Beta function

- The first derivative of SIR z_i :

$$\dot{z}_i = \frac{1}{y_i} \dot{x}_i - \frac{x_i}{y_i^2} \dot{y}_i$$

- The variance is given by:

$$\sigma_{\dot{z}_i}^2 = \frac{1}{y_i^2} \sigma_{\dot{x}_i}^2 + \frac{x_i^2}{y_i^4} \sigma_{\dot{y}_i}^2 = \frac{4\pi^2 f_m^2}{\alpha^2 z_i^{\alpha-2} y_i^\alpha} \left(\frac{\Omega_i \eta_1}{\mu_1 (1 + \eta_1)} + z_i^\alpha \frac{s_i \eta_2}{\mu_2 (1 + \eta_2)} \right)$$

- The conditional PDFs for $\dot{z}_i$ and z_i are:

$$p_{\dot{z}_i}(\dot{z}_i | z_i y_i) = \frac{1}{\sqrt{2\pi} \sigma_{\dot{z}_i}} e^{-\frac{\dot{z}_i^2}{2\sigma_{\dot{z}_i}^2}}, \quad p_{z_i}(z_i | y_i) = \left| \frac{dx_i}{dz_i} \right| p_{x_i}(z_i y_i) = y_i p_{x_i}(z_i y_i)$$

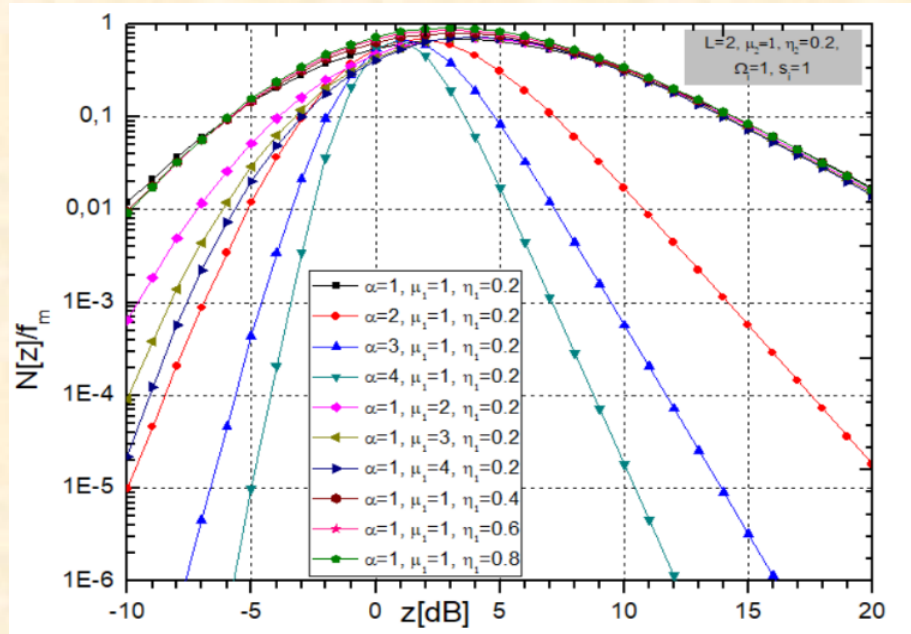
- The joint PDF of z_i and $\dot{z}_i$ is:

$$p_{\dot{z}_i z_i}(\dot{z}_i z_i) = \int_0^\infty p_{\dot{z}_i z_i y_i}(\dot{z}_i z_i y_i) dy_i = \int_0^\infty p_{\dot{z}_i}(\dot{z}_i | z_i y_i) p_{y_i}(y_i) y_i p_{x_i}(z_i y_i) dy_i$$

- The LCR of the output signal is obtained as the mean value of the first derivative of that output signal:

$$\begin{aligned}
 N_{z_i}(z) = & \int_0^\infty \dot{z}_i p_{\dot{z}_i z_i}(\dot{z}_i z_i) d\dot{z}_i = \frac{2 f_m \pi \sqrt{2\pi} (\eta_1 + 1)^{2\mu_1 - \frac{1}{2}}}{\Gamma(\mu_1) \Gamma(\mu_2) (\eta_2 + 1)^{\frac{1}{2} - 2\mu_2}} \sum_{i_1=0}^{+\infty} \sum_{i_2=0}^{+\infty} \frac{z_i^{\frac{4\alpha i_1 + 4\alpha \mu_1 - \alpha}{2}}}{2^{\frac{4i_1 + 4i_2 - 1}{2}} i_1! i_2!} \\
 & \frac{(1 - \eta_1^2)^{2i_1} (1 - \eta_2^2)^{2i_2} \mu_1^{2i_1 + 2\mu_1 - \frac{1}{2}} \mu_2^{2i_2 + 2\mu_2 - \frac{1}{2}} \left(\mu_2 \eta_1 \Omega_i (1 + \eta_2) + \mu_1 \eta_2 s_i (1 + \eta_1) z^\alpha \right)^{\frac{1}{2}}}{\Gamma(i_2 + \mu_2 + 1/2) \Gamma(i_1 + \mu_1 + 1/2)} \\
 & \frac{\eta_1^{\frac{4i_2 + 2\mu_1 + 4\mu_2 - 1}{2}} \eta_2^{\frac{4i_1 + 4\mu_1 + 2\mu_2 - 1}{2}} \Omega_i^{\frac{4i_2 + 4\mu_2 - 1}{2}} s_i^{\frac{4i_1 + 4\mu_1 - 1}{2}} \Gamma\left(\frac{(4i_1 + 4i_2 + 4\mu_1 + 4\mu_2 - 1)}{2}\right)}{\left(\mu_1 \eta_2 s_i (\eta_1 + 1)^2 z^\alpha + \mu_2 \eta_1 \Omega_i (\eta_2 + 1)^2 \right)^{\frac{4i_1 + 4i_2 + 4\mu_1 + 4\mu_2 - 1}{2}}}
 \end{aligned}$$

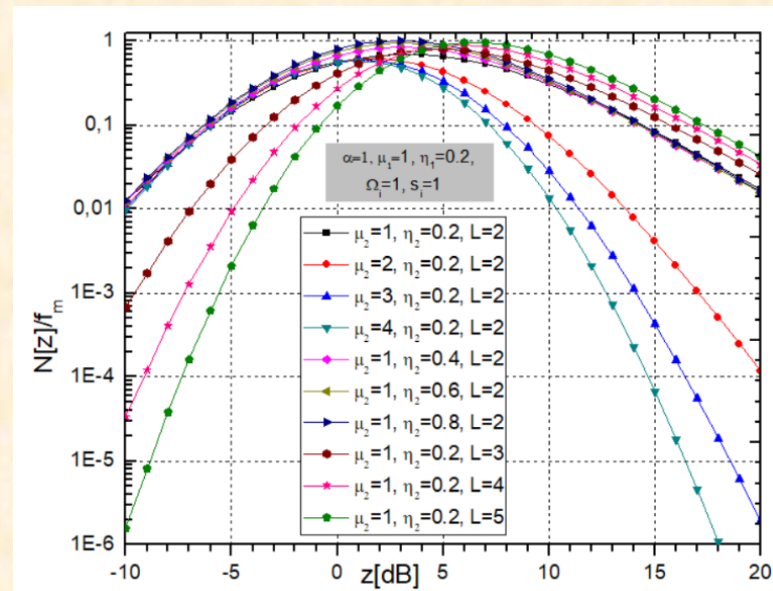
- The normalized LCR versus SIR at the SC receiver output.



LCR versus SIR for several values of parameters μ_1 , η_1 and α .

- For increasing values of the number of clusters μ_1 , LCR decreases improving the system performance. When parameter η_1 increases, LCR also increases and system performance deteriorates. Better performance is obtained for increased values of the SIR and larger parameter α . The changing of LCR is small when parameters η_1 is changing.

- From this figure one can conclude that system is improved for greater number of branches at the input, since the receiver chooses the branch with the best SIR and forward to the user.



LCR versus SIR for variable parameters μ_2 and η_2 , and the number of input branches L .

- When μ_2 increases for negative values of SIR z [dB] no major changes occur in LCR, while for positive values of z [dB] the LCR decreases and the situation is more favorable in terms of performance. When η_2 changes, no considerable change in the LCR.

**THANK YOU FOR YOUR
ATTENTION!**

