STP-Net: Semi-Tensor Product Neural Network for Image Compressive Sensing

Youhao Yu, Richard M. Dansereau

Department of Systems and Computer Engineering,

Carleton University, Ottawa, Canada

e-mail: youhaoyu@sce.carleton.ca rdanse@sce.carleton.ca

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1. Compressive sensing (CS)

- Reconstructs a signal from far fewer samples than the **Shannon-Nyquit sampling theorem**.
- Finds solutions through **optimization**.
- Relies on **sparsity** and **incoherence**.

2. Semi-tensor product (STP)

- If $A \in \mathbb{R}^{m\times n}$ and $B \in \mathbb{R}^{p\times 1}$, $nt = p$, then

$$\boldsymbol{A \ltimes B = (A \otimes I_t) B} \qquad \in \mathbb{R}^{\boldsymbol{mt\times 1}}.$$

- Reshape: $\begin{cases} B \in \mathbb{R}^{nt\times 1} \quad \Rightarrow \quad B_r \in \mathbb{R}^{t\times n}. \\ A \ltimes B \in \mathbb{R}^{mt\times 1} \quad \Rightarrow \quad (A \ltimes B)_r \in \mathbb{R}^{t\times m}. \end{cases}$

- Then, $\boldsymbol{(A \ltimes B)_r = B_r A^T} \qquad \in \mathbb{R}^{\boldsymbol{t\times m}}.$

- **Application:** view B_r as an image and A as the measurement matrix for CS.

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3. Example

- **Image size:** 256 × 256 (65536 pixels);   **Sampling rate:** 1% (655 measurements).
- Measurement matrix $\boldsymbol{\Phi} \in \mathbb{R}^{655 \times 65536}$ (needs more than **300 MB** of memory to store).

- **According to the STP method:**
  - If t is 256, then Φ with size ***ceiling*(256 × 1%) × 256** can be used.
  - only needs **6 kB** of memory.

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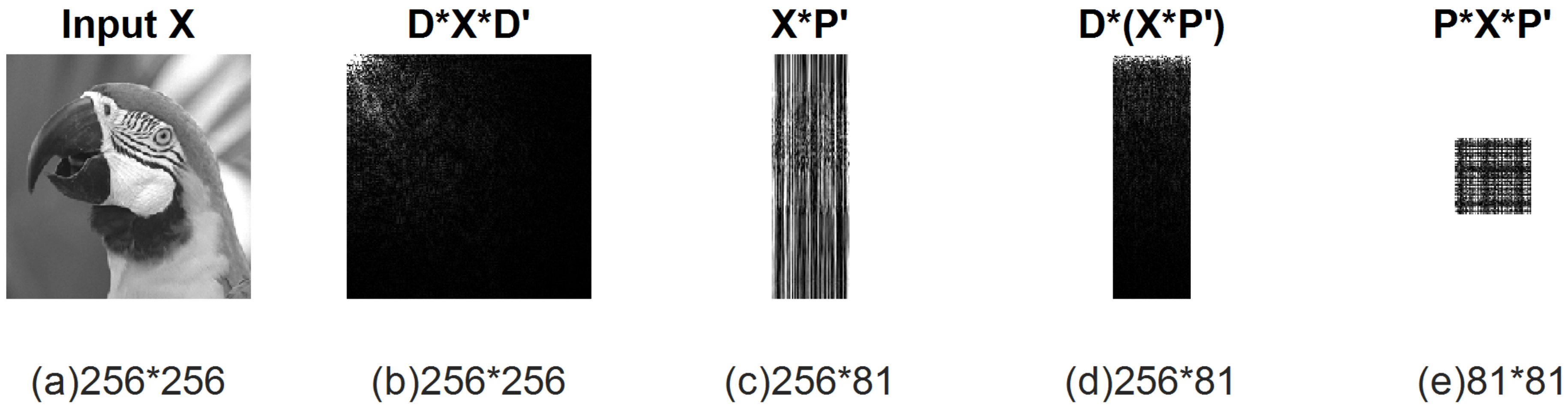
4. Minimum energy reconstruction (MER)

---often used as initial reconstruction

- If $\Phi \in \mathbb{R}^{m\times n}$, $y \in \mathbb{R}^{m\times 1}$, MER of $\boldsymbol{\Phi x = y}$ is the solution of $\boldsymbol{\Phi^T \Phi x = \Phi^T y}$.
- If Φ has orthonormal rows, then $\mathbf{MER} = \boldsymbol{\Phi^T y}$

- **STP maintains the property:**

- If $y = \Phi(t) \ltimes x$, $\mathbf{MER} = \boldsymbol{\tilde{x}} = [\boldsymbol{\Phi}(\boldsymbol{t})]^{\boldsymbol{T}} \ltimes \boldsymbol{y}$
- Written in matrix form: $\boldsymbol{\tilde{X}} = \boldsymbol{Y} \cdot \boldsymbol{\Phi}(\boldsymbol{t})$



- Figure 1. Image measurements and sparsity
- **(a)** original image, **(b)** 1D sparsifying basis *D* applied along rows and columns,
- **(c)** measurement of (a) using random matrix *P*, **(d)** result showing measurement is still sparse, and **(e)** image sampled along rows and columns.
- Sampling rate = $(81/256)^2 \approx 10\%$.

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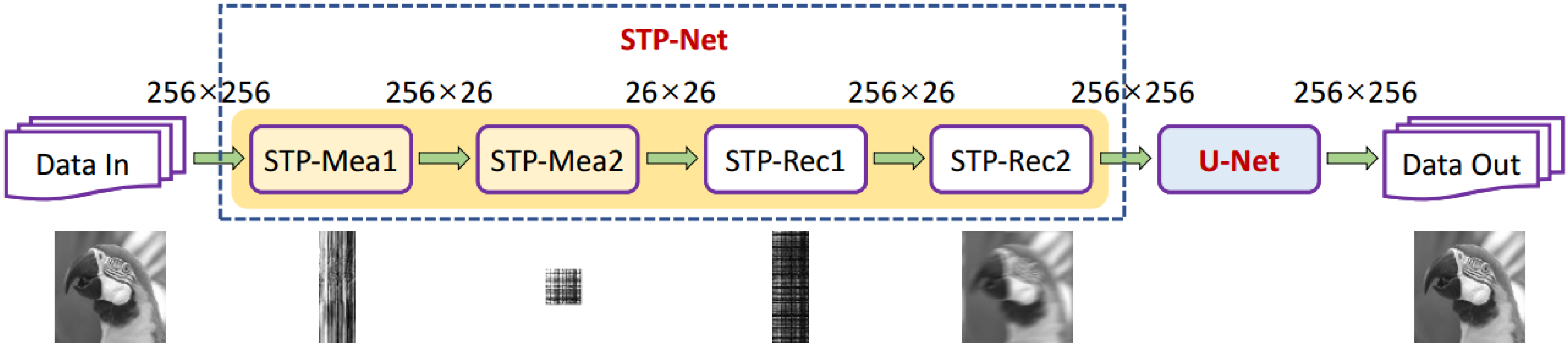
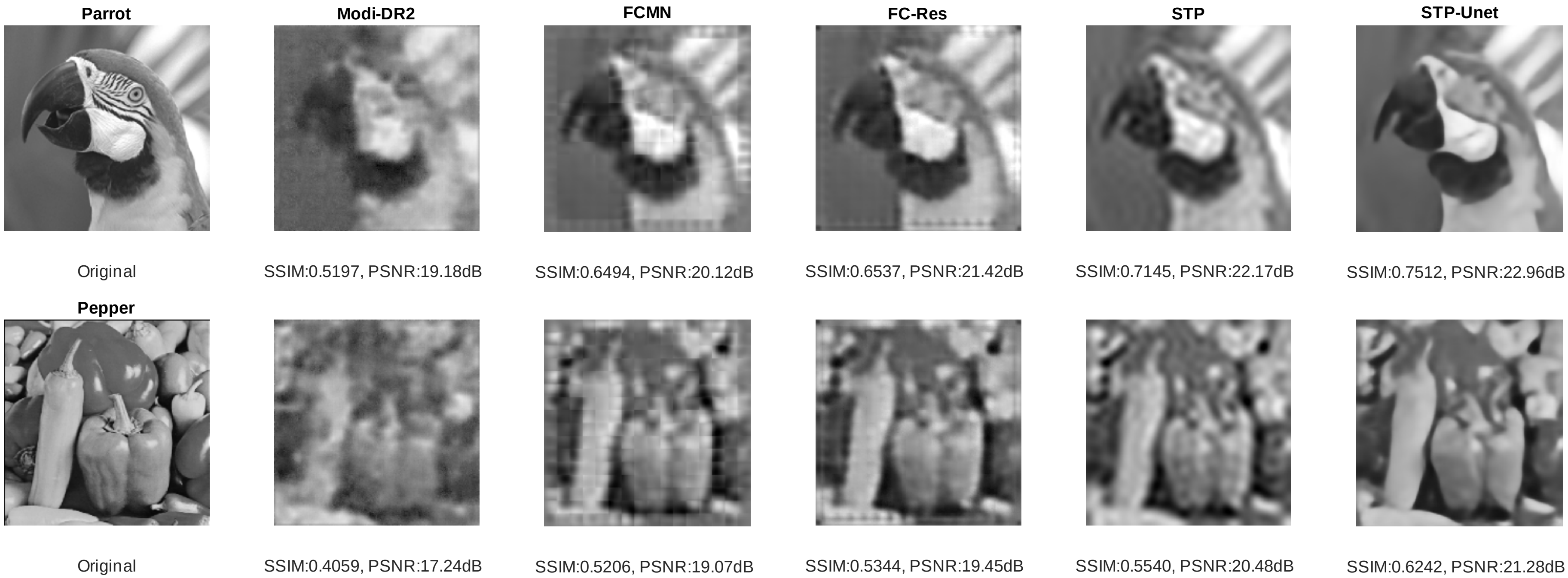


- Figure 2. STP-Net connected with U-Net for deblurring (Sampling rate:1%)

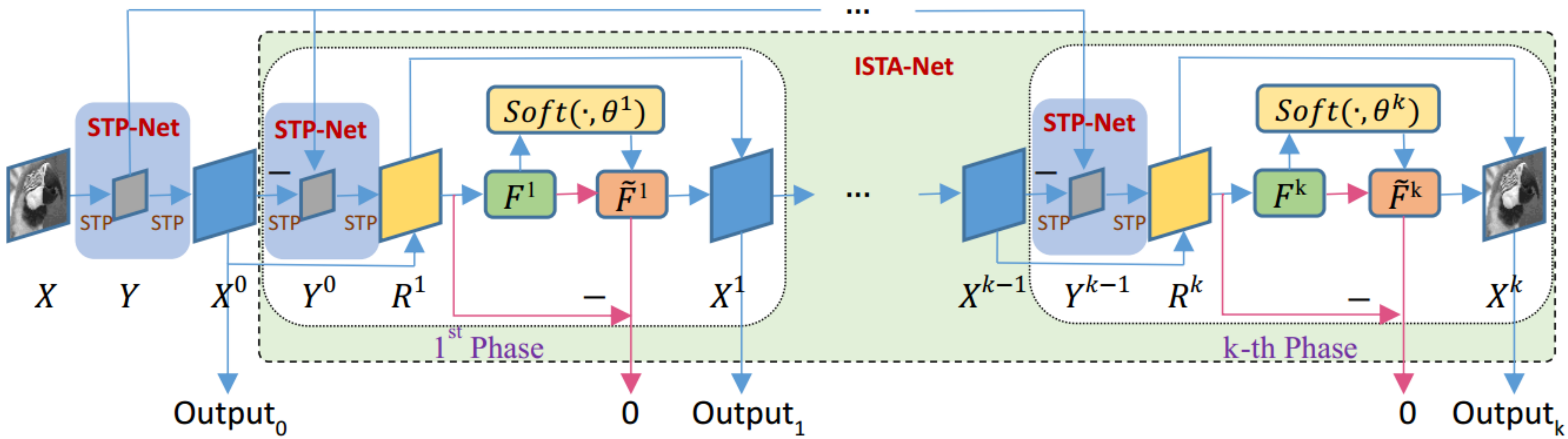


- Figure 3. Reconstruction results for parrot and pepper from noiseless CS measurements at measurement rate of 1%.

5. Iterative shrinkage thresholding algorithm (ISTA)

$$\begin{cases} r^{(k)} = x^{(k-1)} - \rho \Phi^T \left(\Phi x^{(k-1)} - y \right) \\ x^{(k)} = \underset{x}{arg\,min} \left\{ \frac{1}{2} \left\| x - r^{(k)} \right\|_2^2 + \lambda \| \Psi x \|_1 \right\} \end{cases}$$

- where k is the ISTA iteration index, $\rho \geq 0$ is the step size. The first equation is essentially the gradient descent method that contracts the immediate reconstruction along the gradient of the data fidelity term.



- Figure 4. Connect STP-Net with ISTA-Net and use STP layers for measurement and reconstruction in every phase

Table I. PSNR Values in dB on Set11 with Different Algorithms at 1% Measurement Rate

Image Name	ReconNet +BM3D [13]	DR^2-Net [14]	DR^2+BM3D [14]	Modified DR^2-Net [14]	FCMN [16]	FC-Res [16]	STP-Net	STP-UNet
Barbara	19.08	18.65	19.10	19.02	20.38	20.97	21.83	**22.10**
Boat	18.83	18.67	18.95	18.82	19.96	20.57	21.46	**22.23**
Cameraman	17.49	17.08	17.34	17.72	19.16	19.68	20.14	**21.25**
Fingerprint	14.88	14.73	14.95	14.92	15.56	15.83	**16.16**	**16.16**
Flintstones	14.08	14.01	14.18	13.29	14.46	14.77	15.28	**15.37**
Foreman	20.33	20.59	21.08	22.54	21.08	23.72	**27.15**	27.00
House	19.52	19.61	19.99	20.61	20.93	22.38	23.16	**24.47**
Lena	18.05	17.97	18.40	18.51	20.49	21.15	21.95	**22.72**
Monarch	15.49	15.33	15.50	15.52	17.20	17.58	18.28	**18.79**
Parrot	18.30	18.01	18.41	19.18	20.12	21.42	22.17	**22.96**
Pepper	16.96	16.90	17.11	17.24	19.07	19.45	20.48	**21.28**

- (For ReconNet, we use the results reported in [13]. For DR^2-Net and DR^2+BM3D, we use the results reported in [14]. For the other algorithms, the experiments use MATLAB with networks trained from the same dataset with the same images.)

TABLE II. SSIM VALUE FOR 11 EXTRA IMAGES

Image Name	ReconNet [13]	Modified DR^2-Net [14]	FCMN [16]	FC-Res [16]	STP-Net	STP-UNet
Barbara	0.3730	0.3578	0.4555	0.4575	0.5024	**0.5271**
Boat	0.4140	0.3838	0.4729	0.4771	0.4950	**0.5587**
Cameraman	0.4517	0.4391	0.4998	0.5389	0.5503	**0.6565**
Fingerprint	0.1641	0.0708	0.0853	0.0858	0.0884	**0.0886**
Flintstones	0.2733	0.1789	0.2386	0.2429	0.2580	**0.2871**
Foreman	0.5647	0.6078	0.6680	0.6849	0.7536	**0.7869**
House	0.5278	0.5282	0.5809	0.5948	0.6291	**0.7056**
Lena	0.4418	0.4344	0.5364	0.5489	0.5765	**0.6324**
Monarch	0.3802	0.3427	0.4683	0.4816	0.5003	**0.5578**
Parrot	0.5329	0.5197	0.6494	0.6537	0.7145	**0.7512**
Pepper	0.4002	0.4059	0.5206	0.5344	0.5540	**0.6242**
Mean SSIM	0.4112	0.3881	0.4705	0.4819	0.5111	**0.5615**

- (For ReconNet, we calculate the values of SSIM from the images the authors provide.)

TABLE III. RESULTS OF 3000 TEST IMAGES

Evaluation index	Modified DR^2-Net [14]	FCMN [16]	FC-Res [16]	STP-Net	STP-UNet
Mean SSIM	0.3696	0.4347	0.4563	0.4911	0.5301
Mean PSNR	18.72	19.26	20.45	21.50	22.06
Elapsed Time(s)	56.62	10.70	22.02	9.36	41.33

- (The number in the table are the mean of 10 experiments.)

TABLE IV. AVERAGE PSNR (DB) PERFORMANCE COMPARISONS ON SET11 WITH DIFFERENT CS RATIOS

Sampling rate	ReconNet +BM3D [13]	DR^2-Net [14]	DR^2+BM3D [14]	FCMN [16]	FC-Res [16]	ISTA-Net [18]	ISTA-Net+ [18]	STP-Net	STP-ISTA-Net
1%	17.55	17.44	17.73	18.95	19.77	17.30	17.34	20.65	**21.30**
4%	20.44	20.80	21.29	23.14	24.22	21.23	21.31	23.39	**24.92**
10%	23.23	24.32	24.71	25.36	27.30	25.80	26.64	26.02	**28.65**
25%	25.92	28.66	29.06	28.69	31.15	31.53	32.57	30.06	**33.54**

- (The best performance is labeled in bold.)

6. Conclusion

- Present an STP-based neural network to CS image reconstruction.
- Efficient (without dividing the image into blocks and vectorizing).
- Superior quality reconstructions at different measurement rates.
- Does not have block artifacts
- Provides good initial reconstruction for subsequent network (U-Net or ISTA-Net).

End.

Thank you.

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