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Solving Stationary Gas Transport Problems with Compressors of Piston and Generic Type

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Transport network problems

- network is an oriented graph
- linear Kirchhoff equations $\Sigma_i Q_i = 0$ (conservation of flows, per node)
- non-linear element equations $f(P_{in}, P_{out}, Q) = 0$, per edge
- where P_{in} / P_{out} – nodal variables (e.g., pressures)
- Q – flow variable per edge (different normalizations, mass flow, molar flow, volumetric flow, etc.)
- generalized resistivity:

  $\partial f / \partial P_{in} > 0, \partial f / \partial P_{out} < 0, \partial f / \partial Q < 0$, (+––) signature
- required for existence of a single solution (reachable by stabilized Newton method with an arbitrary starting point)

Ref: T. Clees et al., Making Network Solvers Globally Convergent, Advances in Intelligent Systems and Computing, 676:140-153, Springer 2018.

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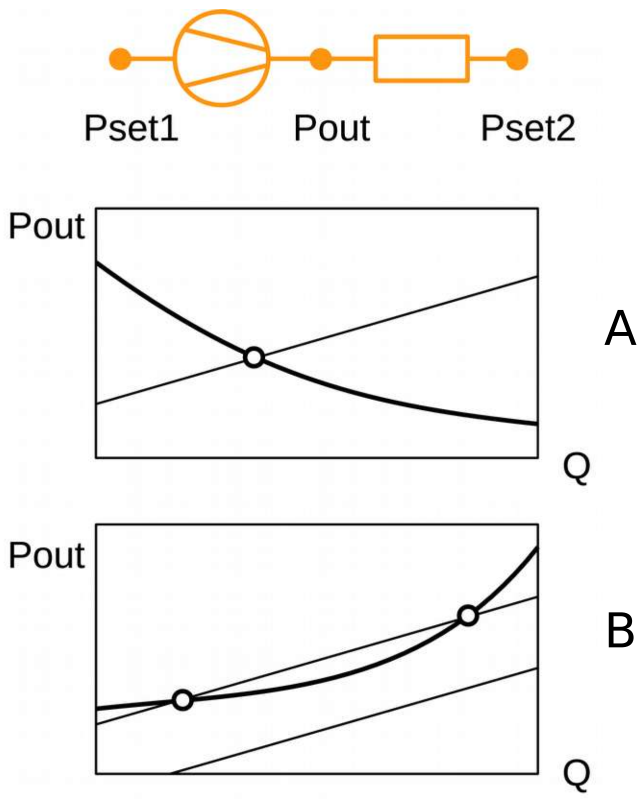
Generalized resistivity, illustration

- simple network with equation
- $P_{out}(P_{set1}, Q) = P_{set2} + RQ$
- $R>0$
- case A: resistivity condition satisfied, P_{out} decreases with Q, single solution exists
- case B: resistivity condition not satisfied, P_{out} increases with Q, multiple solutions or no solution possible



Modeling of gas compressors

- elements increasing pressure
- free model:

$$\max(\min(P_{in} - P_L, -P_{out} + P_H, -Q + Q_H), P_{in} - P_{out}, -Q) + \epsilon(P_{in} - P_{out} - Q) = 0,$$

where parameters P_L , P_H , Q_H define target values,

for example, P_H = *SPO* for specified output pressure,

or upper and lower limits for other controlled values,

ε – regularization parameter

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Modeling of gas compressors

- advanced model:

$$\max(\min(\ P_{in} - P_L, -P_{out} + P_H, -Q + Q_H, \\ \underline{f_1, \ldots, f_n}\ ), \\ P_{in} - P_{out}, -Q) + \epsilon(P_{in} - P_{out} - Q) = 0,$$

- where f_i – additional conditions

Ref: T. Clees et al. Modeling of Gas Compressors and Hierarchical Reduction for Globally Convergent Stationary Network Solvers, Int. J. on Advances in Systems and Measurements, vol. 11, 2018, pp. 61-71.

A. Baldin et al., Topological Reduction of Stationary Network Problems: Example of Gas Transport, Int. J. on Advances in Systems and Measurements, vol. 13, 2020, pp. 83-93.

Modeling of gas compressors

- advanced model, additional internal variables:

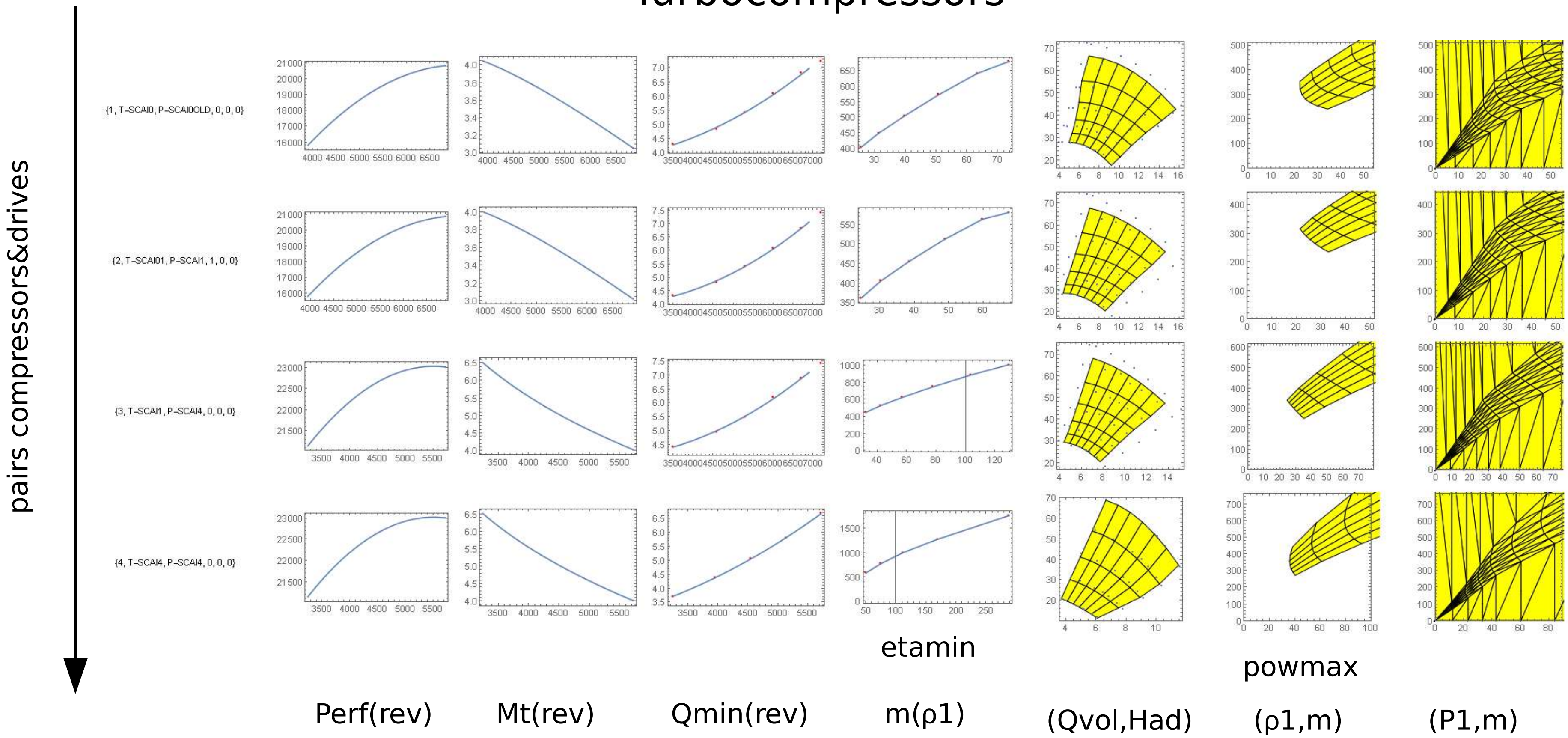
$$P = \rho R T z/\mu, \; Q_m = Q_{vol}\rho_{in},$$
$$H_{ad} = P_{in}/(\rho_{in}\alpha) \cdot ((P_{out}/P_{in})^{\alpha} - 1),$$
$$Perf = Q_m H_{ad}/\eta, \; M_t = Perf/(2\pi \cdot rev),$$
$$\alpha = (\kappa - 1)/\kappa, \; 0 < \alpha < 1, \; 0 < \eta < 1,$$

- revolution number *rev*, adiabatic enthalpy increase H_{ad} , performance *Perf* , efficiency η, torque M_t ,density ρ, universal gas constant R, absolute temperature T , compressibility factor z, molar mass μ, adiabatic exponent $\kappa > 1$

Ref: A. Baldin et al., AdvWarp: A Transformation Algorithm for Advanced Modeling of Gas Compressors and Drives, in Proc. of SIMULTECH 2021, pp. 231-238, SciTePress, 2021.

Modeling of gas compressors

Turbocompressors



additional relationships between internal variables based on the calibration procedure

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Modeling of gas compressors

- piston compressors:

$$Q_{vol} = V \cdot rev$$

$$f_1 = rev_{max} - rev \geq 0,$$

$$f_2 = M_{t,max} - M_t \geq 0,$$

$$f_3 = Perf_{max} - Perf \geq 0,$$

$$f_4 = rel_{max} - P_{out}/P_{in} \geq 0,$$

$$f_5 = \Delta P_{max} - (P_{out} - P_{in}) \geq 0,$$

where $Perf_{max}(rev)$ determined by the characteristics of the compressor drive, other limits are constant

Modeling of gas compressors

- stability analysis:

PATCH SIGNATURES OF PISTON COMPRESSOR

patch	sgn	condition
f_1	$(+\ 0\ -)$	
f_2	$(+\ -\ 0)$	$P_{out}/P_{in} < \beta$
f_3	$(+\ -\ -)$	$P_{out}/P_{in} < \beta,\ \partial M_{t,drv}/\partial\, rev < 0$
f_4	$(+\ -\ 0)$	
f_5	$(+\ -\ 0)$	

the required signature (+––) is marginally satisfied, enforced by ε-regularization

Modeling of gas compressors

- generic compressors (with constant limits):

$$f_1 = Q_{vol,max} - Q_{vol} \geq 0,$$
$$f_2 = H_{ad,max} - H_{ad} \geq 0,$$
$$f_3 = Perf_{max} - Perf \geq 0$$

- stability analysis:

PATCH SIGNATURES OF GENERIC COMPRESSOR

patch	sgn	condition
f_1	$(+\ 0\ -)$	
f_2	$(+\ -\ 0)$	$\partial z_{in}/\partial P_{in} < 0$ or small
f_3	$(+\ -\ -)$	$\partial z_{in}/\partial P_{in} < 0$ or small

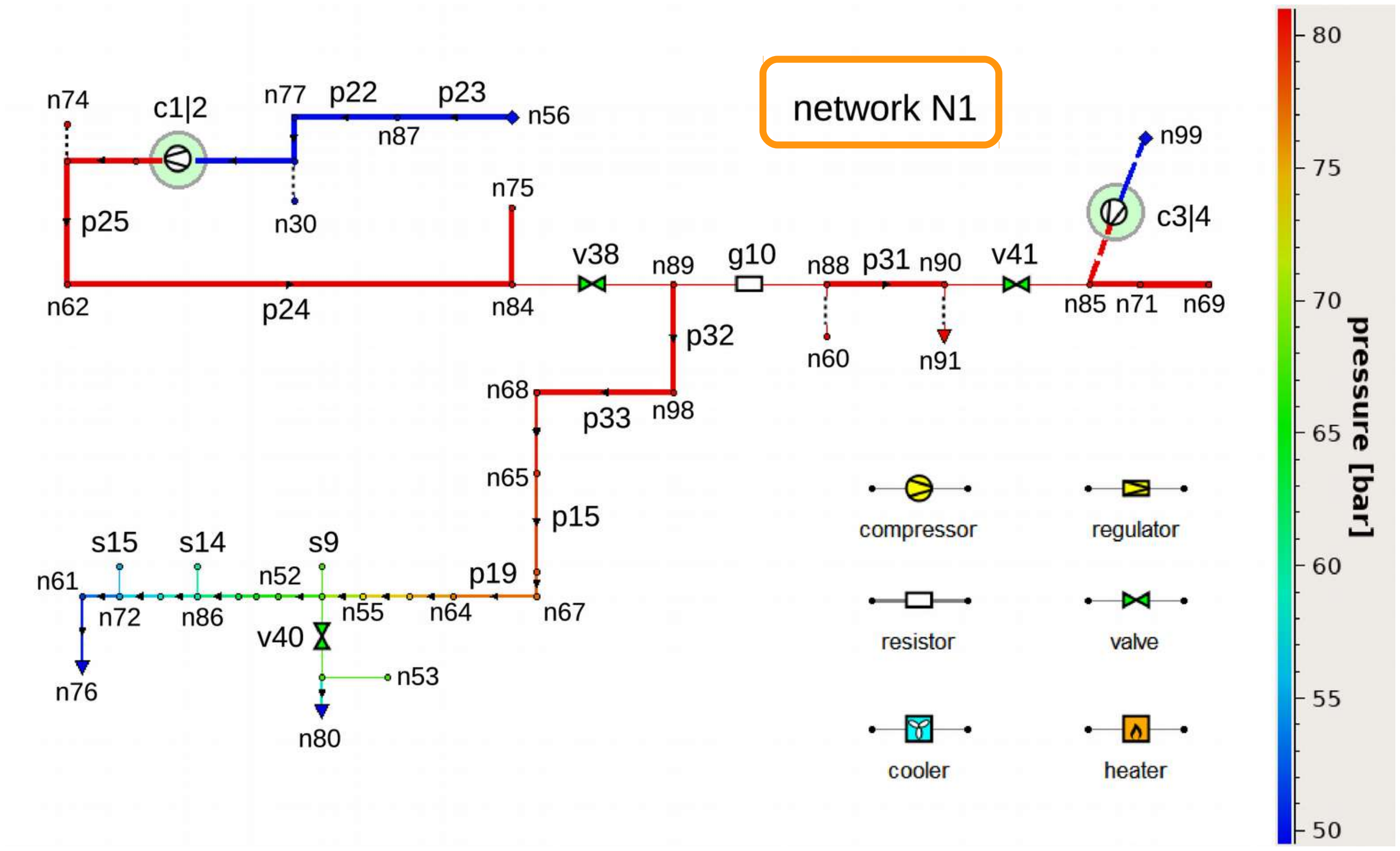
the required signature (+––) is marginally satisfied, enforced by ε-regularization

Numerical tests

- Test networks

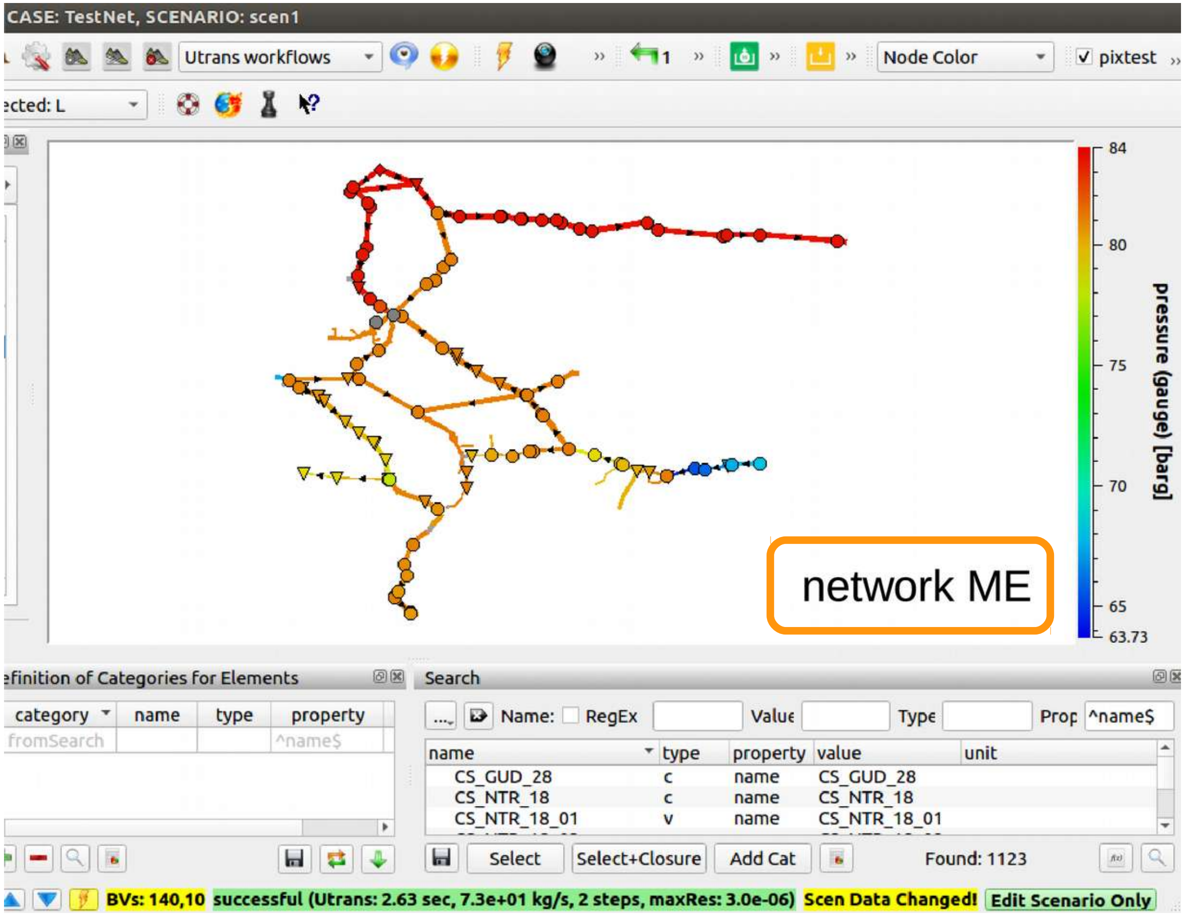
network	nodes	edges	pipes	compressors	regulators	Psets	Qsets
N1	100	111	34	4	4	2	3
ME	437	482	370	20	24	3	164
N85/L	3232-3886	3305-3974	2406-2835	1-7	59-77	6-7	625-843
N85/H	2914-3818	2989-3952	1498-1937	16-42	59-107	5-9	328-505

Numerical tests



Examples of test networks

Numerical tests



Examples of test networks

Numerical tests

- N1: piston compressors in station c1|2, generic – in station c3|4

TIMING FOR DIFFERENT PHASES OF THE SOLUTION PROCEDURE*

phase	translate	solve
init	15	8
free	15	7
adv	17	20
total	47	35

* in milliseconds, for 2.6 GHz Intel i7 CPU 16 GB RAM computer.

- ME&N85: realistic networks, each containing up to 3 piston and generic compressors
- convergence not influenced by their presence

Conclusion

- modeling of piston and generic gas compressors was carried out
- stability analysis is performed, working regions are identified
- numerical tests on a number of realistic gas networks have been done, stability is confirmed
- the modeling enhances our network solver MYNTS

Thank you for your attention!

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