



# Modeling Damage Paths and Repairing Objects in Critical Infrastructure Systems

by

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# Agenda

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## Introduction

- Critical infrastructure systems are those that are considered extremely critical to the function of a government or a country.
- Many of these systems are now connected to the internet, which makes them vulnerable to attacks.
- Prevention methods are not enough to fully secure a system and it is necessary to prepare for post attack activities, including damage assessment and recovery.

## Related Work

- **Evaluating Synergistic Effects of Failures in CIS** [*Rehak et al.*, 2016]:  
Establishes a model with elements and linkages of varying criticality.
- **Post-Failure Network Recovery** [*Bartolini et al.*, 2016]:  
Efficiently restores damaged element paths by recursively breaking demand flows into smaller problems.
- **Recovery with Progressive Damage Assessment** [*Ciavarella et al.*, 2017]:  
Uses centrality rankings to determine which damaged elements should be repaired first.

## Objectives

- Establish a model that allows for fast, accurate, and efficient damage assessment.
- Define methods to give repair priority to more important sections of a system based on criticality and centrality metrics.
- Develop an algorithm to determine repair order using the aforementioned methods.

## Model Definitions: Graphs

- Given two objects  $O_i$  and  $O_j$  in a system, if the value of  $O_j$  is calculated using the value of  $O_i$ , we say that there is information flow from  $O_i$  to  $O_j$ .
  - This defines the concept of information flow in a system.
- The **Possible Paths Graph (PPG)**
  - For a node  $N$  in each object  $O$ , there exists an edge  $E_{ij}$  between  $N_i$  and  $N_j$  if there is a possibility that  $N_j$  can be modified by  $N_i$

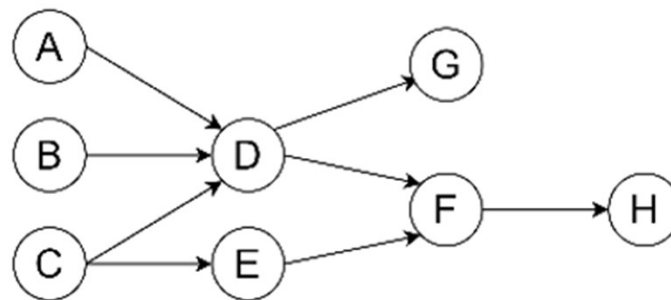


Figure 1a. The Possible Paths Graph (PPG)

## Model Definitions: Graphs

### ➤ The **Active Paths Graph (APG)**

- Each edge  $E_{ij}$  in the APG represents actual information flow between  $N_i$  and  $N_j$ .  
The APG must fully exist within the PPG

### ➤ The **Damage Spread Graph (DSG)**

- Contains edges and nodes that have been damaged through information flow.  
The DSG must fully exist within the APG

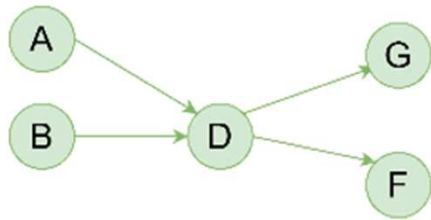


Figure 1b. The Active Paths Graph (APG)

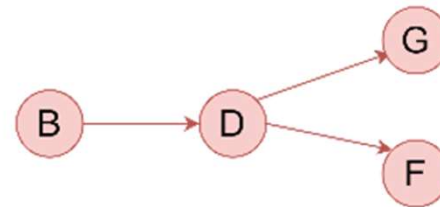


Figure 1c: The Damage Spread Graph (DSG)

## Model Definitions: Metrics

- The **criticality** of a node  $N$  is its predetermined level of importance to the system's functions.
- The **repair time** of a node  $N$  is how many inward-flowing edges  $E^i$  it is receiving damage from.
  - For a node to be repaired, all of its parent nodes must be repaired first.
- The **centrality** of a node  $N$  is the number of outward-flowing edges  $E^o$  it has.
  - Nodes with higher centrality will reduce the repair time of more nodes when repaired than nodes with lower centrality.



# Model Description

- We use the three defined graphs to model damage assessment and prepare for recovery
- The PPG is preprocessed with all nodes and dependency paths in a system.
  - The APG is built by including the all paths that were used in the period between the initial attack and the current time.
  - Finally, the DSG is built by following the dependency paths that use the initially damaged nodes to make updates. Damage spread must follow two criteria:
    - There is a damaged node  $N_i$  that has an edge  $E_{ij}$  flowing from it to node  $N_j$
    - $E_{ij}$  is used for a transaction while  $N_i$  is damaged

# Model Description

- The goal of the model is to find the optimal sequence of repairs to restore the most important operations of a system as quickly as possible.
- We must first plan to repair nodes in order of their criticality.
- Between two or more nodes with equal criticality, the one with the lowest repair time is selected.
- If two or more nodes also have an equal repair time, the one with the highest criticality is selected.
  - When the first node is picked, its parent nodes must be fully repaired first before it can be repaired.
  - This selection process repeats for all the parent nodes of the first node until a repair can be made.

# Notations Table

Notations	Descriptions
$P = (V, E)$	Possible Path Graph
$A = (V_A, E_A)$	Active Path Graph ( $V_A \subseteq V, E_A \subseteq E$ )
$D = (V_D, E_D)$	Damage Spread Graph ( $V_D \subseteq V_A, E_D \subseteq E_A$ )
$D = (V_C, E_C)$	Critical Node Graph ( $V_C \subseteq V_D, E_C \subseteq E_D$ )
$\delta_{ij}$	Decision to fix edge $i$ to $j$
$\delta_i$	Decision to fix node $i$
$t_i$	Time to fix node $i$
$c_i$	Centrality of node $i$
$P_{ij}$	Dependency indicator of node $i$ and $j$

Our objective is to find  $\min \sum_{i \in V_D} t_i \delta_i$  subject to:

- $\delta_i \sum_{j \in V_C} P_{ij} \leq \sum_{j \in V_C} P_{ij} \delta_j \quad \forall i, j \in V_C \quad (1)$
- $\delta_i c_i \geq \sum_{(i,j) \in E_C} \delta_{ij} \quad \forall i \in V_C \quad (2)$
- $P_{ij} \in \{0,1\} \quad \forall i, j \in V_C \quad (3)$
- $\delta_i, \delta_{ij} \in \{0,1\} \quad \forall i \in V_C, (i,j) \in E_C \quad (4)$

# Initialization Algorithm

## **Algorithm 1: Initialization for object set repair**

**Result:** Queue of objects ordered by repair priority

- 1 Initialize set of damaged objects  $O$
- 2 Preprocess object priority using criticality, repair time, and centrality
- 3 Initialize repair queue  $Q$
- 4 while  $O$  has damaged nodes remaining
  - 4.1 Select the highest critical node(s)  $N$  within  $O$
  - 4.2 if Two or more nodes are tied for highest criticality
    - 4.2.1 Select the node(s)  $N$  with the lowest repair time  $R$  within  $O$
  - 4.3 if Two or more nodes are tied for lowest repair time
    - 4.3.1 Select the node(s)  $N$  with the highest centrality within  $O$
  - 4.4 if Two or more nodes are tied for highest centrality
    - 4.4.1 Select a single node at random from those still tied
  - 4.5 Update repair queue( $N_0, O, Q$ )  $\rightarrow Q$
- 5 Print  $Q$

# Recursive Repair Algorithm

## Algorithm 1.1: Recursive repair function

**Result:** Schedules a node  $N$  for repairs and returns the updated repair queue  $Q$

1 Update repair queue(*Selected node  $N$ , object set  $O$ , repair queue  $Q$* ):

2 while *Current object has unrepaired dependencies*:

2.1 Create subset of damaged nodes  $O'$  of all nodes  $N'$  and edges  $E'$  that  $N$  is dependent on

2.2 Select the highest critical node(s)  $N'$  within  $O'$

2.3 if *Two or more nodes are tied for highest criticality*

2.3.1 Select the node(s)  $N'$  with the lowest repair time  $R$  within  $O$

2.4 if *Two or more nodes are tied for lowest repair time*

2.4.1 Select the node(s)  $N'$  with the highest centrality within  $O$

2.5 if *Two or more nodes are tied for highest centrality*

2.5.1 Select a single node at random from those still tied

2.6 Update repair queue( $N', O', Q$ )  $\rightarrow Q$

2.7 Remove the most recent object in repair queue from  $O$

3 Repair  $N$

4 Add  $N$  to  $Q$

5 Return  $Q$

## Conclusion

- We have presented a method to repair data objects that prioritizes quick recovery for the most important components of a system.
- This allows for the partial restoration of functions during the recovery process with an emphasis on restoring service to the most necessary functions.
- Our work is most applicable to protecting critical infrastructure systems where services need to be restored as quickly as possible to avoid economic or societal disruptions.