Power grids: Small Signal Stability vs. Dynamical Parameters

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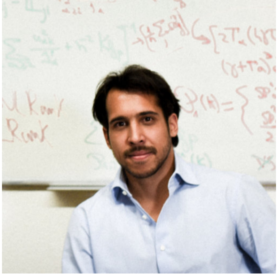
References

MT, Jacquod *IEEE Control Systems Letters* **5** (3), 929-934 (2020)
MT, Pagnier, Jacquod *Science Advances* **5** (11):eaaw8359 (2019)
MT, Jacquod *Physical review E* **101** (3), 032303 (2019)

Dr. Melvyn Tyloo

Melvyn obtained his master degree and PhD in Theoretical Physics at the Swiss Federal Institute of Technology in Lausanne (EPFL) respectively in 2016 and 2020. He is currently working as a postdoc researcher at the University of Geneva (UNIGE). His research focuses on complex network-coupled dynamical systems and the identification of their local/global vulnerabilities against external perturbations. He also recently developed methods for inferring coupling network from time-series and for locating line and node disturbances in diffusively coupled agents.



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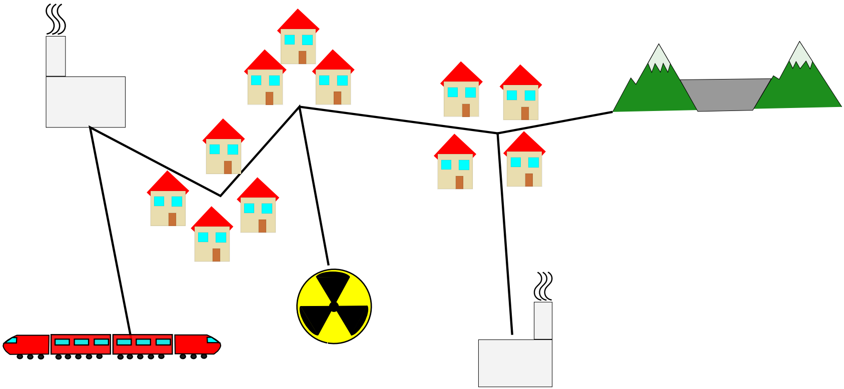
Motivation: Energy Transition

Traditional transmission power grid



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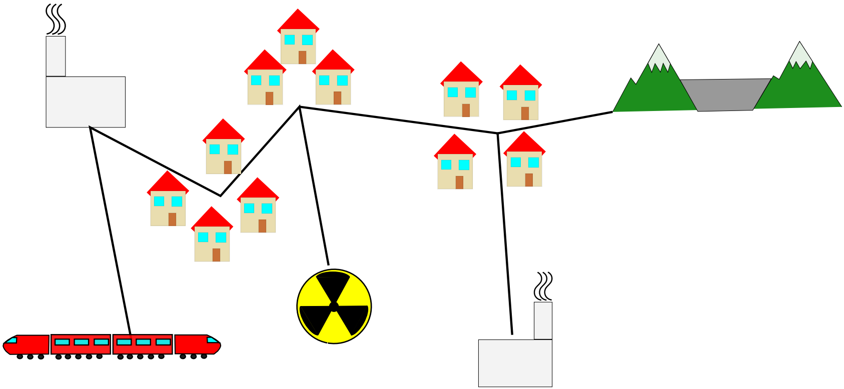
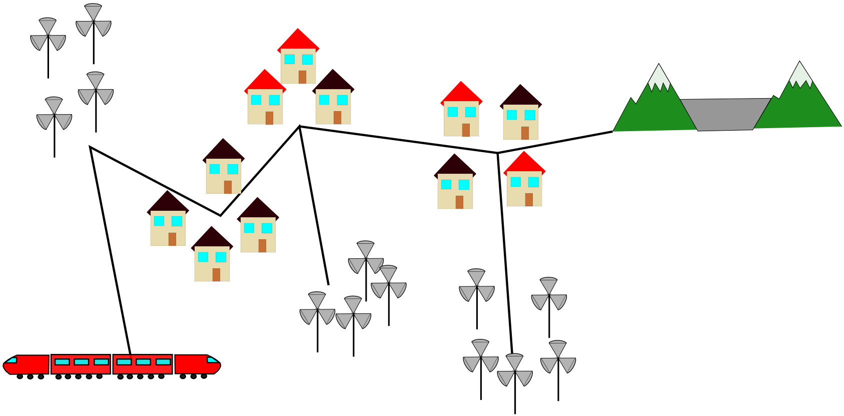


Future transmission power grid



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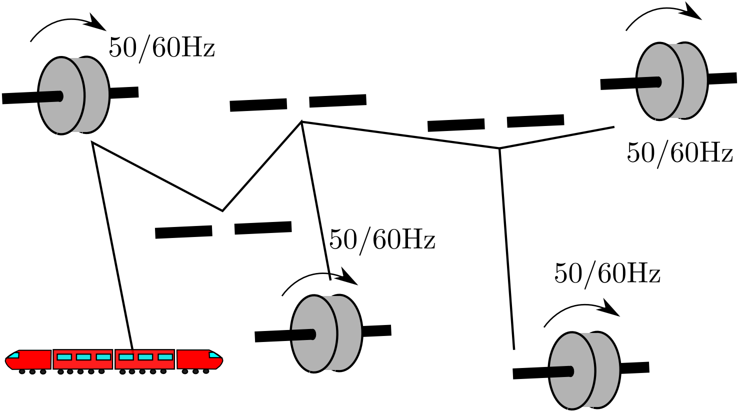

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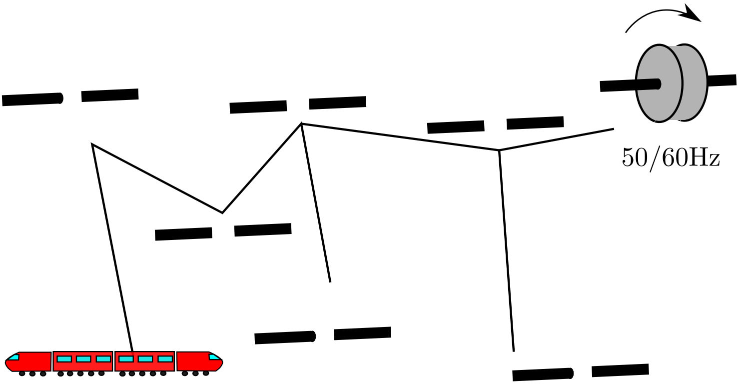
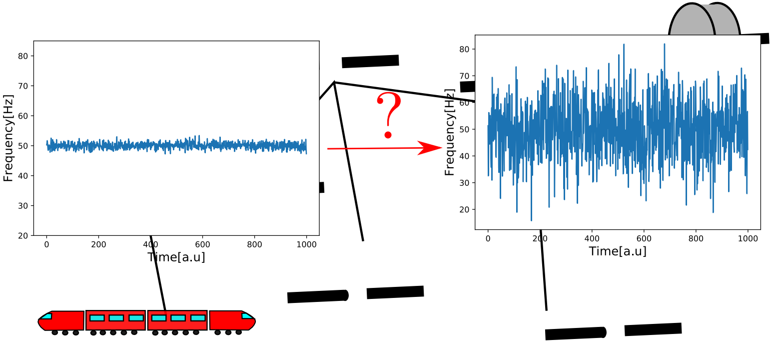


How do inertia and damping affect performances?



Swing equations in the lossless line approximation

Voltage phase dynamics is given by

$$m_i \dot{\omega}_i + d_i \omega_i = P_i - \sum_j b_{ij} \sin(\theta_i - \theta_j) , \quad i \in \text{Generators} , \tag{1}$$

$$d_i \omega_i = P_i - \sum_j b_{ij} \sin(\theta_i - \theta_j) , \quad i \in \text{Loads} . \tag{2}$$

b_{ij} : line capacity.
m_i : inertia.
d_i : damping.
$\omega_i = \dot{\theta}_i$.

Bergen, Hill, "A structure preserving model for power system stability analysis," IEEE Trans. Power App. Syst., vol. PAS-100, no. 1, pp. 25–35, Jan. 1981.

High-Voltage Power Networks Phase Dynamics

Swing equations in the lossless line approximation:
The common assumption on dynamical parameters

$$\gamma^{-1}\, d_i\, \dot{\omega}_i + d_i\, \omega_i \;=\; P_i - \sum_j b_{ij} \sin(\theta_i - \theta_j)\ .$$

b_{ij} : line capacity.
m_i : inertia.
d_i : damping.
$\omega_i = \dot{\theta}_i$.
Usual assumptions that allow analytical treatment: inertia-to-damping constant ratio $\gamma^{-1} = m_i/d_i\ ,\ \forall i$.

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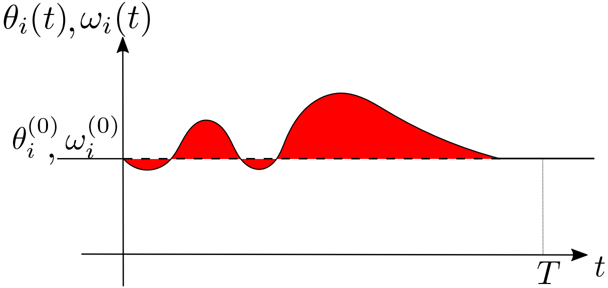
Usual assumptions that allow analytical treatment: inertia-to-damping constant ratio $\gamma^{-1} = m_i/d_i\,,\,\forall i$.
We also take this assumption... but eventually say something about realistic power networks!

Robustness Assessment

- Quadratic performance metrics: $\mathcal{H}_2$ norms $\rightarrow$ Quantify the amplitude of the transient response following a disturbance.



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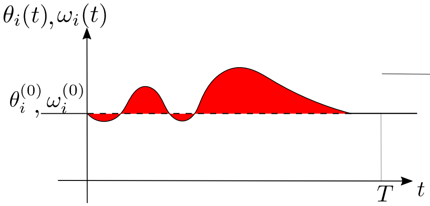

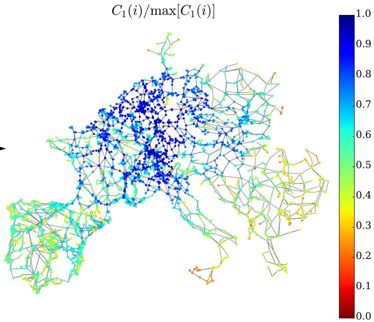

Performance vs. Topology $\rightarrow$ Generalized Kirchhoff indices Kf_n and resistance Centralities $C_n(k)$.

MT, Coletta, Jacquod *Physical review letters* **120** (8), 084101 (2018)
MT, Pagnier, Jacquod *Science Advances* **5** (11):eaaw8359 (2019)
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Response to Perturbations: Linearization

Swing equations in the lossless line approximation:

$$\gamma^{-1}\, d_i\, \ddot{\theta}_i + d_i\, \dot{\theta}_i \;\; = \;\; P_i - \sum_j b_{ij} \sin(\theta_i - \theta_j)\ .$$

Linear response: Perturbation of the injected/consumed powers.

- $P_i(t) = P_i^{(0)} + \delta P_i(t) \to \theta_i(t) = \theta_i^{(0)} + \delta\theta_i(t)$:

$$\gamma^{-1}\, \delta\ddot{\boldsymbol{\varphi}}(t) + \delta\dot{\boldsymbol{\varphi}}(t) = D^{-1/2}\delta P(t) - D^{-1/2}\mathbb{L}(\{\theta_i^{(0)}\})D^{-1/2}\delta\boldsymbol{\varphi}(t)\ ,$$

where $\delta\boldsymbol{\varphi}(t) = D^{1/2}\delta\boldsymbol{\theta}(t)$.

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$\mathbb{L}(\{\theta_i^{(0)}\})$: the weighted Laplacian matrix,

$$[D^{-1/2}\,\mathbb{L}\,D^{-1/2}]_{ij} = \begin{cases} -\frac{b_{ij}}{\sqrt{d_i\, d_j}} \cos(\theta_i^{(0)} - \theta_j^{(0)})\,, & i \neq j\,, \\ \frac{1}{d_i} \sum_k b_{ik} \cos(\theta_i^{(0)} - \theta_k^{(0)})\,, & i = j\,. \end{cases}$$

Response to Perturbations: Linearization

Linear response: Perturbation of the injected/consumed powers.

- $P_i(t) = P_i^{(0)} + \delta P_i(t) \to \theta_i(t) = \theta_i^{(0)} + \delta\theta_i(t)$:

$$\gamma^{-1}\,\delta\ddot{\boldsymbol{\varphi}}(t) + \delta\dot{\boldsymbol{\varphi}}(t) = D^{-1/2}\delta \boldsymbol{P}(t) - D^{-1/2}\mathbb{L}(\{\theta_i^{(0)}\})D^{-1/2}\delta\boldsymbol{\varphi}(t)\ ,$$

Solution:

$$\begin{aligned}\delta\varphi_i(t) = \sum_\alpha \gamma e^{\frac{-\gamma-\Gamma_\alpha}{2}t}\int_0^t e^{\Gamma_\alpha t_1} \\ \times \int_0^{t_1} [D^{-1/2}\delta \boldsymbol{P}(t_2)]^\top \mathrm{u}_\alpha^D e^{\frac{\gamma-\Gamma_\alpha}{2}t_2}\mathrm{d}t_2\mathrm{d}t_1\ u_{\alpha,i}^D\ ,\end{aligned} \tag{3}$$

Time-correlated power fluctuations:

$$\langle \delta P_i \rangle = 0\,,\ \langle \delta P_i(t)\delta P_j(t')\rangle = \delta_{ij}\delta P_0^2 \exp[-|t-t'|/\tau_0]\,.$$

Primary control effort:

$$\begin{aligned}\mathcal{P}(T) &= \lim_{T\to\infty} T^{-1}\int_0^T (\boldsymbol{\omega}^\top - \overline{\boldsymbol{\omega}}^\top) D(\boldsymbol{\omega} - \overline{\boldsymbol{\omega}})\,\mathrm{d}t\,,\\ &= \lim_{T\to\infty} T^{-1}\int_0^T (\delta\boldsymbol{\varphi}^\top - \overline{\delta\boldsymbol{\varphi}}^\top)(\delta\boldsymbol{\varphi} - \overline{\delta\boldsymbol{\varphi}})\,\mathrm{d}t\,.\end{aligned}$$

Linear system $\rightarrow$ analytical solution!

Primary Control Effort

Primary control effort:

$$\overline{\mathcal{P}^{\infty}} = \sum_{\alpha \geq 2} \frac{\sum_{i \in N_n} \delta P_{0i}^2 {u_{\alpha,i}^D}^2 d_i^{-1}}{\lambda_\alpha^D \tau_0 + 1 + \gamma^{-1} \tau_0^{-1}} \,,$$

with λ_α^D the eigenvalue associated with the eigenvector u_α^D of $\mathbb{L}^D$.

Primary Control Effort

Short noise correlation time: $\tau_0 \ll \gamma^{-1}, {\lambda_\alpha^D}^{-1}$

$$\overline{\mathcal{P}^\infty} = \tau_0 \sum_{i\in N_n} \delta P_{0i}^2 \left(1/m_i - 1/\sum_j m_j\right) .$$

No dependence on damping nor network connectivity!

Long correlation time: $\tau_0 \gg \gamma^{-1}, {\lambda_\alpha^D}^{-1}$

$$\overline{\mathcal{P}^\infty} = \tau_0^{-1} \sum_{\alpha\geq 2} \frac{\sum_{i\in N_n} \delta P_{0i}^2 {u_{\alpha,i}^D}^2 d_i^{-1}}{\lambda_\alpha^D} .$$

No dependence on inertia!

Primary Control Effort

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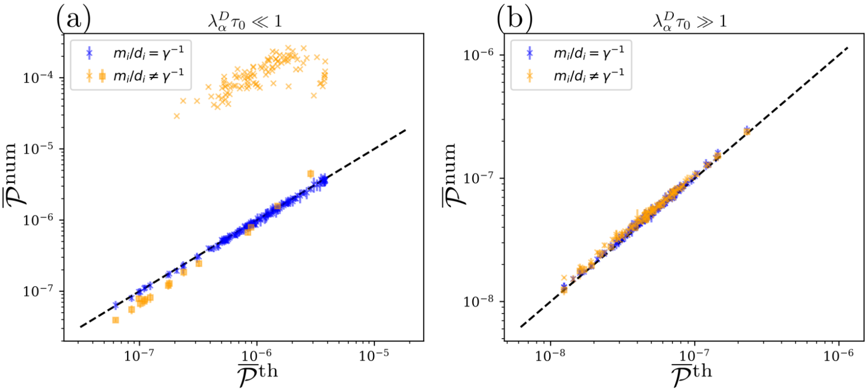
No dependence on inertia!

Realistic high-voltage power networks: ${\lambda_\alpha^D}^{-1} < 0.5s$ and $\gamma^{-1} \cong 2.5s$.

Renewable power sources fluctuate on time scales of few seconds.

IEEE 118-Bus Test Case:



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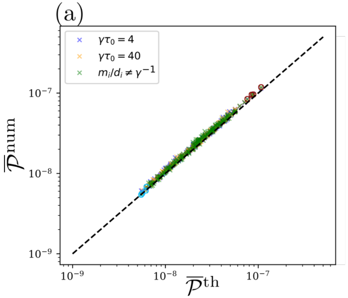

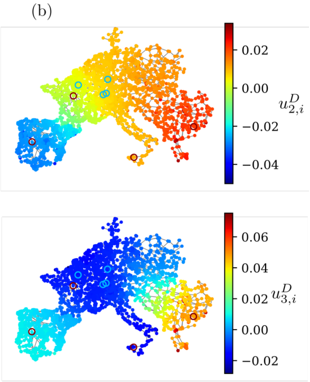

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Conclusion

Description of realistic power networks

- Consider $D^{-1/2}\mathbb{L}D^{-1/2}$ instead of $M^{-1/2}\mathbb{L}M^{-1/2}$,
- Time-correlated noise instead of white-noise,

$\rightarrow$ Primary control effort for power networks with inhomogeneous dynamical parameters.

- Inertia does not impact much primary control effort.

$\rightarrow$ Focus on damping/control.

MT, Jacquod *IEEE Control Systems Letters* **5** (3), 929-934 (2020)
MT, Pagnier, Jacquod *Science Advances* **5** (11):eaaw8359 (2019)
MT, Jacquod *Physical review E* **101** (3), 032303 (2019)