

# Inverse Convolution Method for Periodic Media under Deterministic and Stochastic Condition

Authors: Xuefeng Li, Mohamed Ichchou, Abdelmalek Zine, Nouredine Bouhaddi, Christophe Droz

Presenter: Xuefeng Li

Email: [xuefeng.li@ec-lyon.fr](mailto:xuefeng.li@ec-lyon.fr)

LTDS - Ecole Centrale de Lyon, France

Laboratoire de  
Tribologie et  
Dynamique des  
Systèmes

LTDS UMR 5513

<http://www.ltds.fr>

## Profile

Name Xuefeng Li  
Date Nov. 04, 1993  
University Ecole Centrale de Lyon  
Lab Vibroacoustics & Complex Media  
Research Group in LTDS



## Research

Direction: Periodic media, Wave propagating, Vibration control  
Supervisors: N.BOUHADDI, M.N.ICHCHOU, A.-M. ZINE

- I. The background of INCOME
- II. Inverse methods for wavenumber extraction
- III. The theory of 1D deterministic INCOME
- IV. Application cases
- V. The prospection of 1D stochastic INCOME

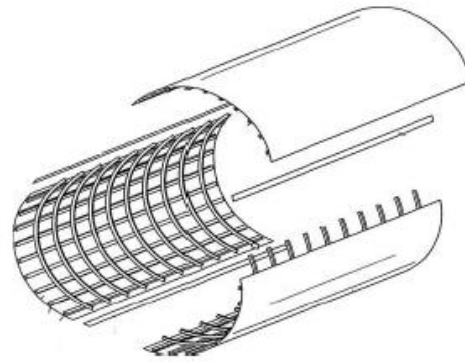
# The background of INCOME

• Background

• Periodic structure



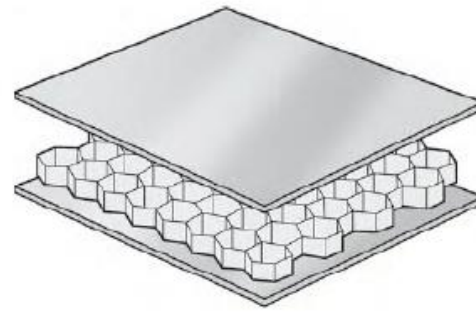
(a) Train rail



(b) Fuselage of an aircraft



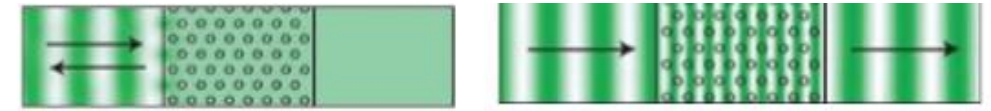
(c) Perforated plate



(d) Honeycomb sandwich

Fig.1 Periodic structure

• Characteristic: Attenuation band



(a) Band gap

(b) Pass band

Fig.2 Schematic diagram of the band structure

• **K-space**-----Dynamical behavior

- ✓ Structural optimization in aerospace and civil engineering: vibration isolation, unbalance filters.
- ✓ The arrival estimation for sonar and radar, protection of electrical power lines and so on.

• **Uncertainties**-----Practical meaning

Achieve more realistic k-space characteristics' identification

• Block diagram of Inverse methods

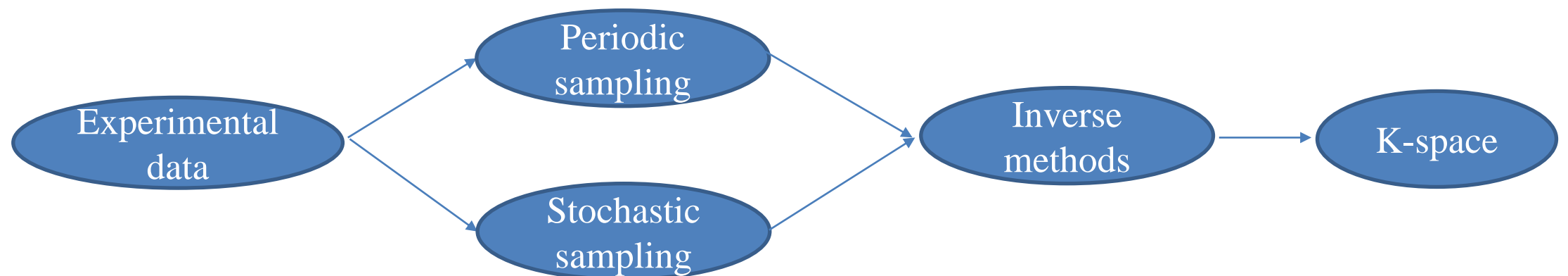
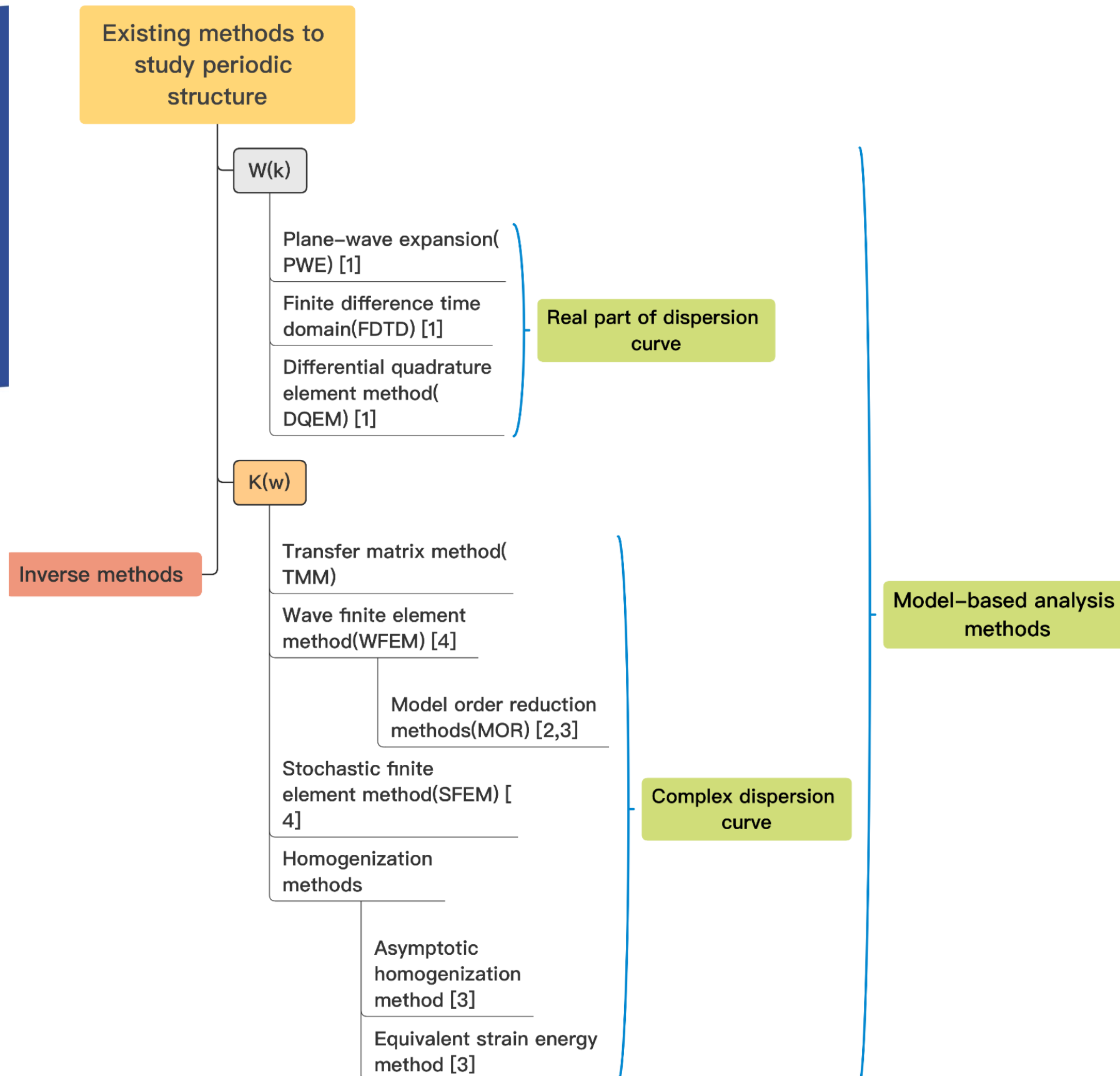


Fig.3 Block diagram of inverse methods



# Inverse methods for wavenumber extraction

• Existing methods to study periodic structure



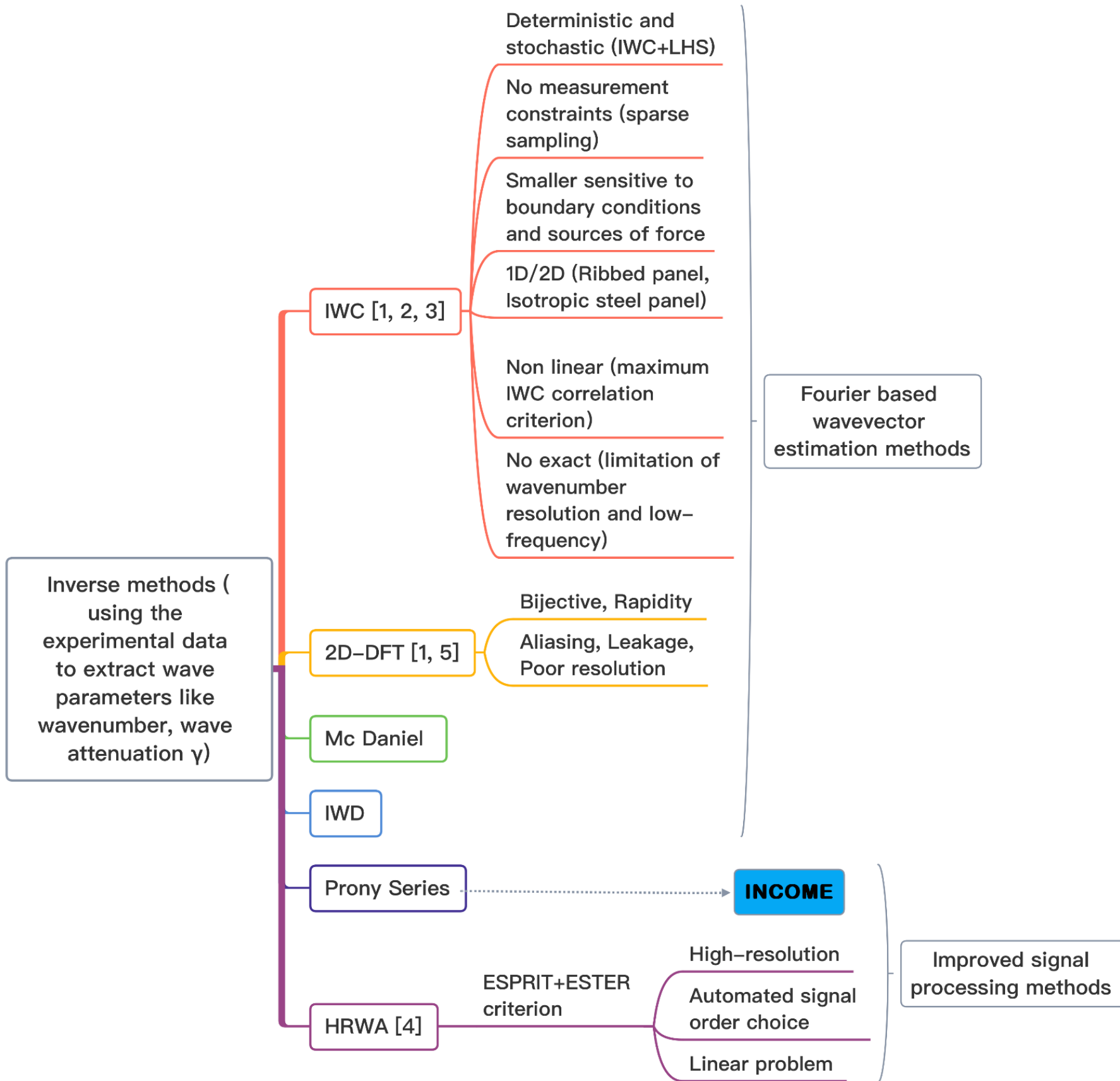
[1] Shi Zhaifei, "Periodic structure theory and its application in vibration isolation and vibration reduction," C. Science Press, 2017-06-01.

[2] Droz C, Zhou C, Ichchou M, et al, "A hybrid wave-mode formulation for the vibro-acoustic analysis of 2D periodic structures," J. Journal of Sound & Vibration, 363:285-302, 2016.

[3] Zhou C, "Wave and modal approach for multi-scale analysis of periodic structures," D. Ecole centrale de lyon, 2014.

[4] Wim Desmet, Mohamed Ichchou et al, "Mid-frequency CAE methodologies for mid-frequency analysis in vibration and acoustics," C. Katholieke University Leuven, 2012.

• K-space analysis methods



[1] Mckay M D , Conover R J B J , “A Comparison of Three Methods for Selecting Values of Input Variables in the Analysis of Output from a Computer Code,” *J. Technometrics*, 21(2):239-245, 1979.

[2] Ichchou M N, Berthaut J, Collet M, “Multi-mode wave propagation in ribbed plates: Part I, wavenumber-space characteristics,” *J. International Journal of Solids & Structures*, 45(5):1179-1195, 2008.

[3] Bouazizi M L , et al, “Inhomogeneous Wave Correlation for Propagation Parameters Identification in Presence of Uncertainties,” *C. Design and Modeling of Mechanical Systems—III*, pp 823-833, 2018.

[4] Margerit P , Arthur Lebé, Jean-François Caron, et al, “The High-Resolution Wavevector Analysis for the characterization of the dynamic response of composite plates,” *J. Journal of Sound and Vibration*, 458:177-196, 2019.

[5] Ramzi L , Chikhaoui K , Bouazizi M L , et al, “Robust 2D-Spatial Fourier Transform Identification of Wavenumber-Space Characteristics of a Composite Plate,” *M. Design and Modeling of Mechanical Systems - IV*. 2020.



# The theory of 1D deterministic INCOME

• **Deterministic**

INCOME: Modeling + Wavenumber identification

• Assumed modeling:  $U_n = \sum_{m=1}^{n_w} b_m \exp(jk_m x_n) + e = \sum_{m=1}^{n_w} b_m \lambda_m^n + e$   $e = 0$  (1)

**Bloch principle:**  $\lambda_m = \exp(jk_m \Delta x)$   $x_n = n\Delta x$  (2)

**Corresponding characteristic polynomial:**

$$\psi(\lambda) = \prod_{m=1}^{n_w} (\lambda - \lambda_m) = \sum_{m=0}^{n_w} a_m \lambda^{n_w-m}, a_0 = 1$$
 (3)

• Recursive difference equation :  $U_n - \sum_{m=1}^{n_w} a_m U_{n-m} = 0$   $n_w \leq n \leq N$  (4)

• Convolutional matrix formula:  $(U_j) * (a_i) = 0$   $(U_j)_{j \in [0, N]}$ ,  $(a_i)_{i \in [0, n_w]}$ ,  $a_0 = 1$  (5)

$$UA = 0 \quad U = \begin{bmatrix} U_{n_w} & U_{n_w-1} & \dots & U_0 \\ U_{n_w+1} & U_{n_w} & \dots & U_1 \\ \vdots & \vdots & \dots & \vdots \\ U_N & U_{N-1} & \dots & U_{n_w} \end{bmatrix} \quad A = \begin{bmatrix} a_0 \\ a_1 \\ \vdots \\ a_{n_w} \end{bmatrix}$$
 (6)

• Least square method:  $U^* U x = 0$  (7)

• Bloch principle:  $k_m = \frac{j \ln \lambda_m}{\Delta x}$  (8)

# Application cases

- A longitudinal propagating wave case

*propagating positive wave*

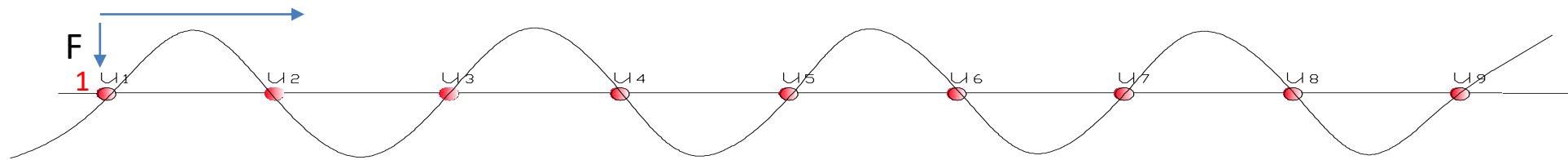


Fig.4 Longitudinal motion generated by a harmonic point force acting on a finite bar.

- Non-dimensional dispersion relation

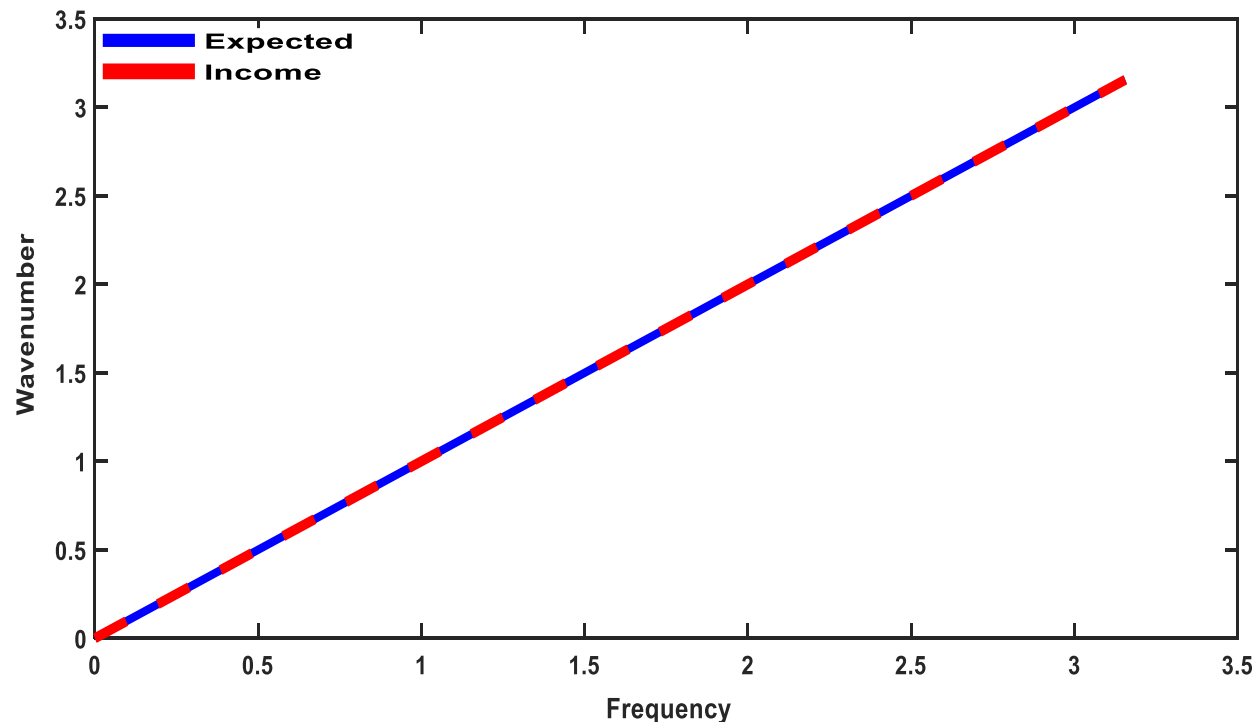


Fig.5 The real part of dispersion curve

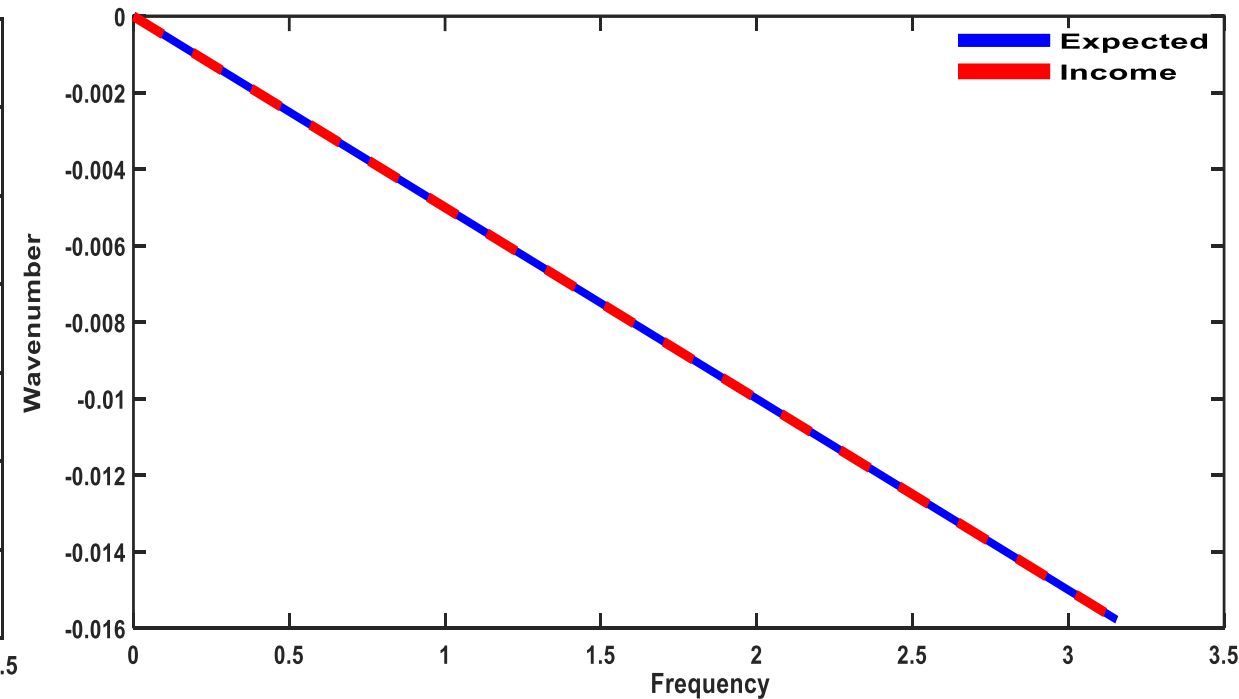


Fig.6 The imaginary part of dispersion curve

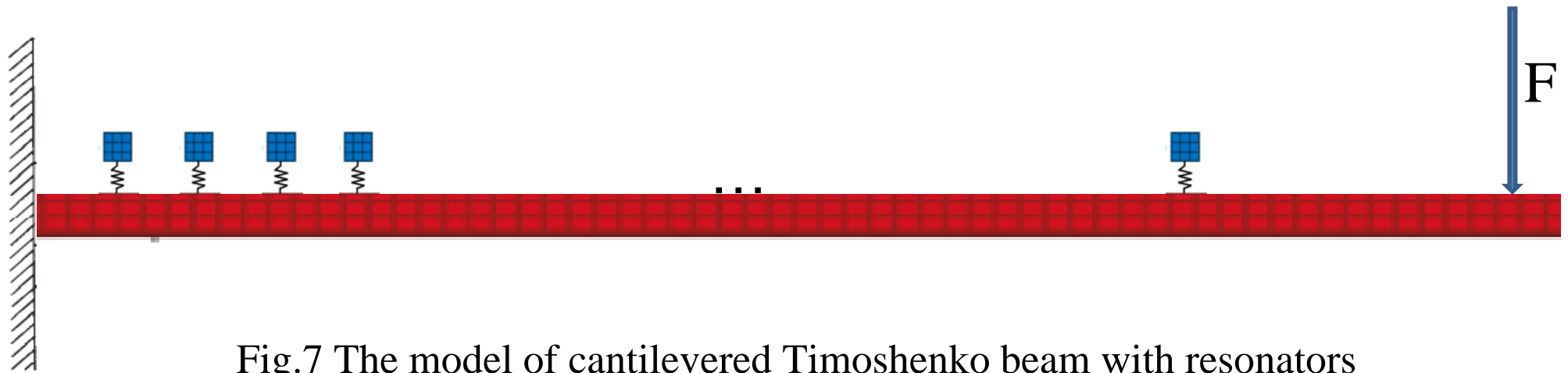


Fig.7 The model of cantilevered Timoshenko beam with resonators

- **Resonators**

Natural frequency of resonators: 500 Hz

Damping of resonators: 0.05

- **Harmonic excitation**

$$F = 10 \sin(\omega t)$$

WFEM: Expected dispersion curve

comparison

FEM: Displacement curve

INCOME: INCOME dispersion curve



- Frequency dispersion curve

Frequency=**500Hz** (the natural frequency of local resonator)

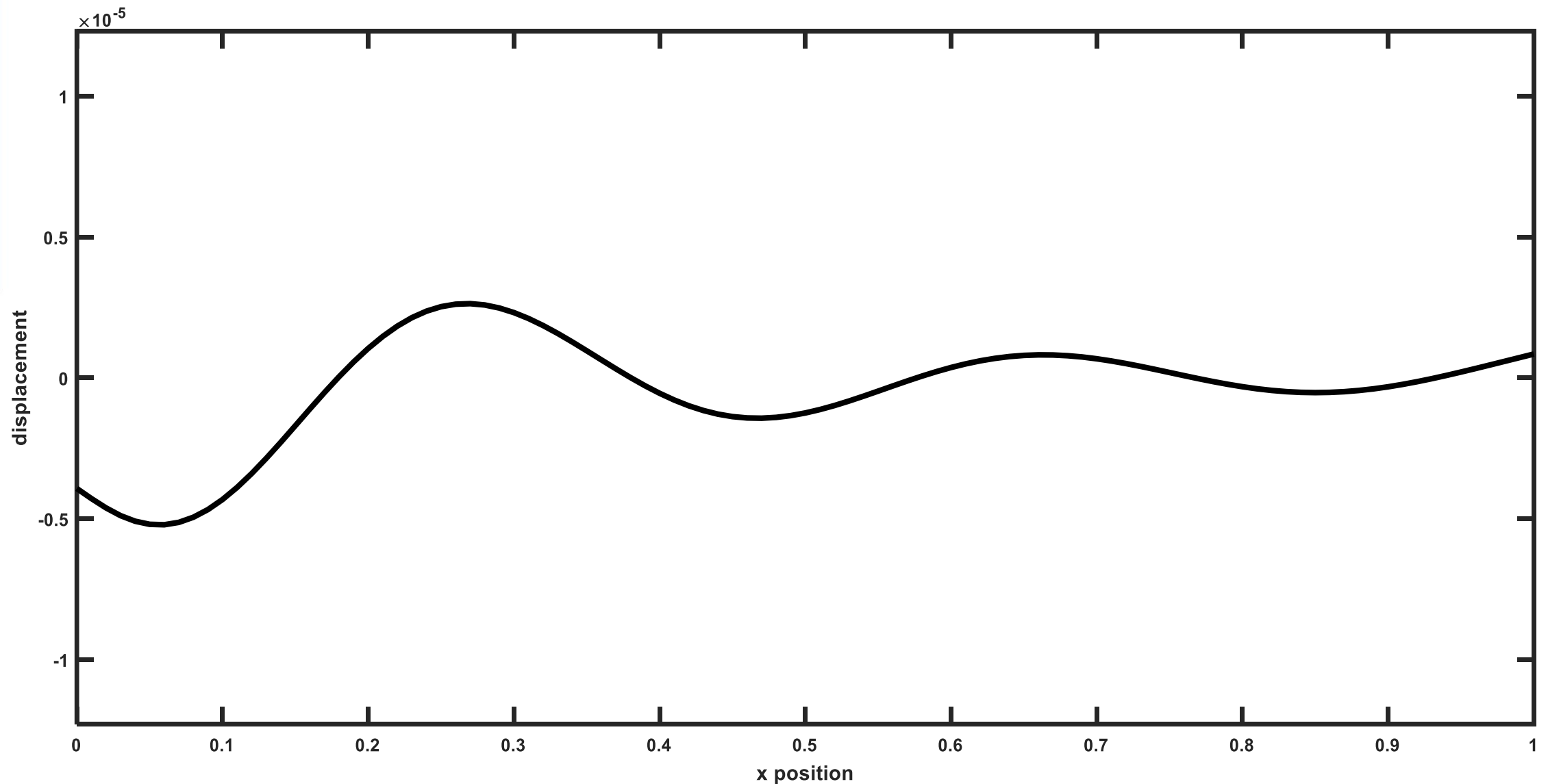


Fig.8 The frequency response curve

- The local resonator resonates with the main beam, causing the energy to decay exponentially, so that the wave cannot propagate, a band gap is generated and the displacement tends to zero with distance

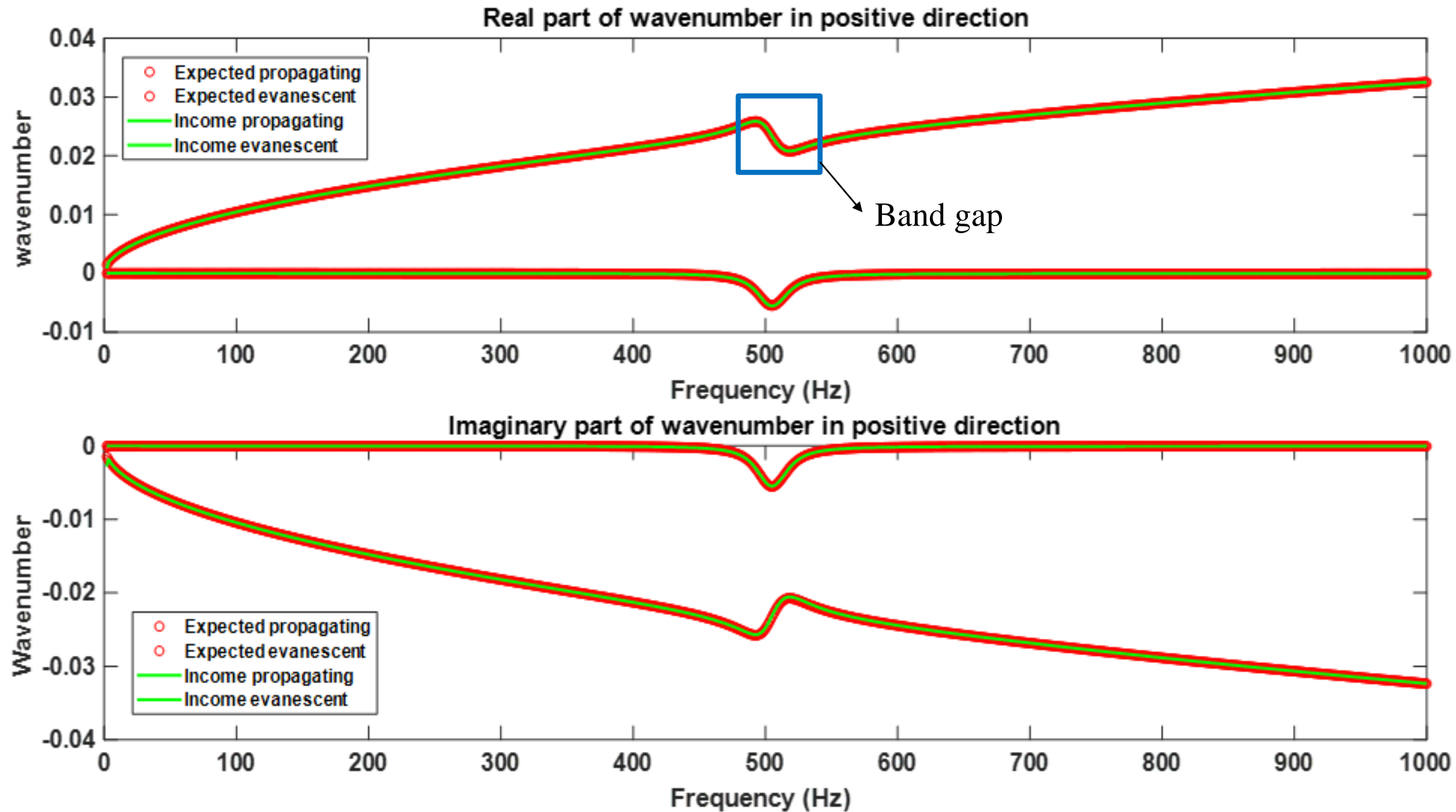


Fig.9 The complex dispersion curve of positive propagating wave

- The first band gap: 480Hz---520Hz

# The prospection of 1D stochastic INCOME

- Assumed modeling

$$U_n = \sum_{m=1}^{n_w} b_m \exp(jk_m x_n) + e \quad e \neq 0$$

Uncertainty factors

Noise: white Gaussian noise

Non-periodicity measurements

- Wavenumber identification
- ✓ A sample-based uncertainty propagating method
- ✓ An automated estimation of signal order  $n_w$

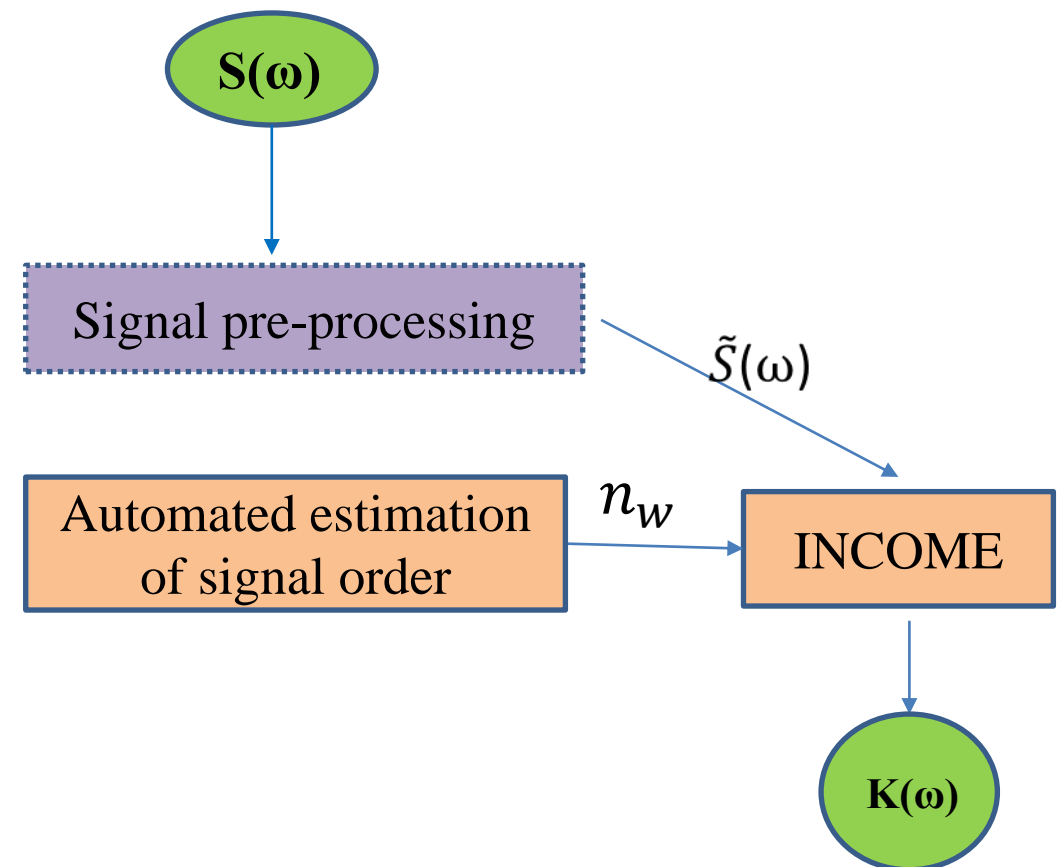


Fig.10 Block diagram of INCOME

*Thank you all for  
your time !*