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Exploring Analog Emulation of Quantum Computation Using Quadrature Modulation

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Faster Logic, LLC

Fourteenth International Conference on Quantum, Nano/Bio, and Micro Technologies ICQNM November 21-25, 2020

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Presenter biography

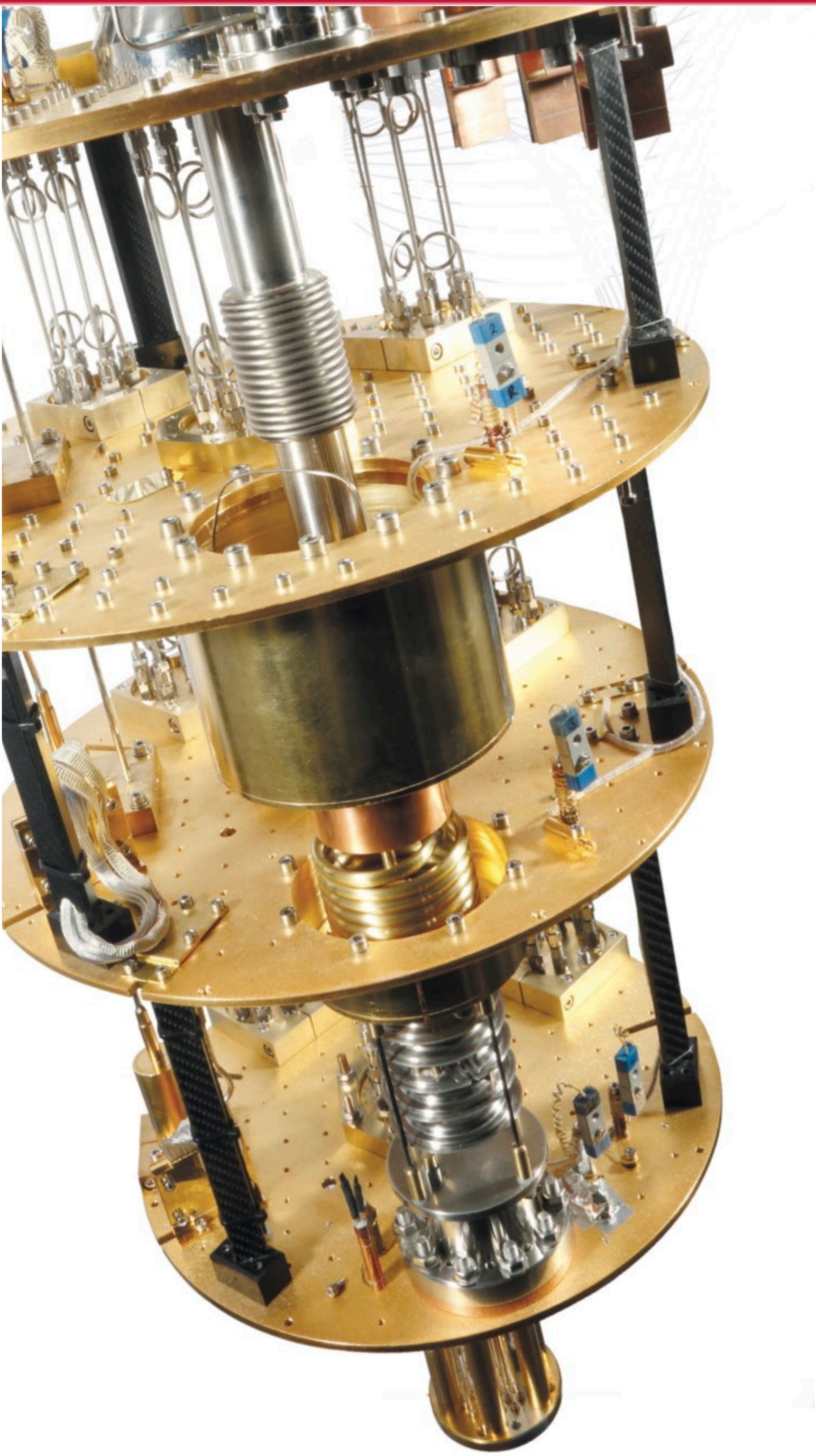
SDSU Investigator: **Dr. Kyle Sundqvist**, Assistant Professor of Physics, SDSU. 43 published peer-reviewed journal articles, conference proceedings, and book chapters. Internationally recognized expert on low-temperature condensed matter physics. Experienced undergraduate and graduate educator in physics and engineering. Expert in superconducting thin-film devices, as well as electron and hole transport phenomena in germanium originating from work on an experimental dark matter collaboration (Cryogenic Dark Matter Search -- CDMS).

Research Interests: Superconducting circuits and Josephson junctions; Engineered microwave quantum-optical systems Design and characterization of nano-electronic devices; Cryogenics at millikelvin temperatures; Low-noise instrumentation, at both low and microwave frequencies. Nonequilibrium semiconductor physics of carrier transport and recombination

Ph.D., Physics, with a Designated Emphasis in Nano-scale Science and Engineering,
University of California, Berkeley, Berkeley, CA, (2012).
Thesis: Carrier Transport and Related Effects in Detectors of the Cryogenic Dark Matter Search.

San Diego State University
Assistant Professor (Aug. 2016 -- present)
Research: Presently establishing a program of research in microwave-photonic devices using superconducting circuits for quantum information.
Teaching: Electronics for Scientists, Condensed Matter Physics.

Topics of research interest



Dr. Sundqvist's SDSU lab is funded and en-route to developing a cryrogenic laboratory capable of staging superconducting circuits for traditional quantum information efforts. Our lab will be able to test a broad range of microelectronics at low temperatures for various campus and industry collaborations. Sundqvist's collaboration with Faster Logic, LLC and the U.S. Navy's Office of Naval Research centers on emulating qubits. Exploring emulated qubit operations will also feed back to guide future physics experiments aiding research into innovative device architectures.

Faster Logic, LLC is collaborating to innovate with digital logic as an "active component" for interaction with the qubit-emulating oscillators. We can use this for automating a sequence of testing for each lab-produced coupled-oscillator, so that it can be "rung out".

Dr. Moberly is an expert in signal processing and cyber security for embedded systems as applicable to software-defined radio (SDR) architectures.

Outline:

A. Emulating a two-level atom using analog electronics

B. Exploring quadrature modulation for multiple, coupled qubits

C. Exploring digital possibilities for simulation and emulation

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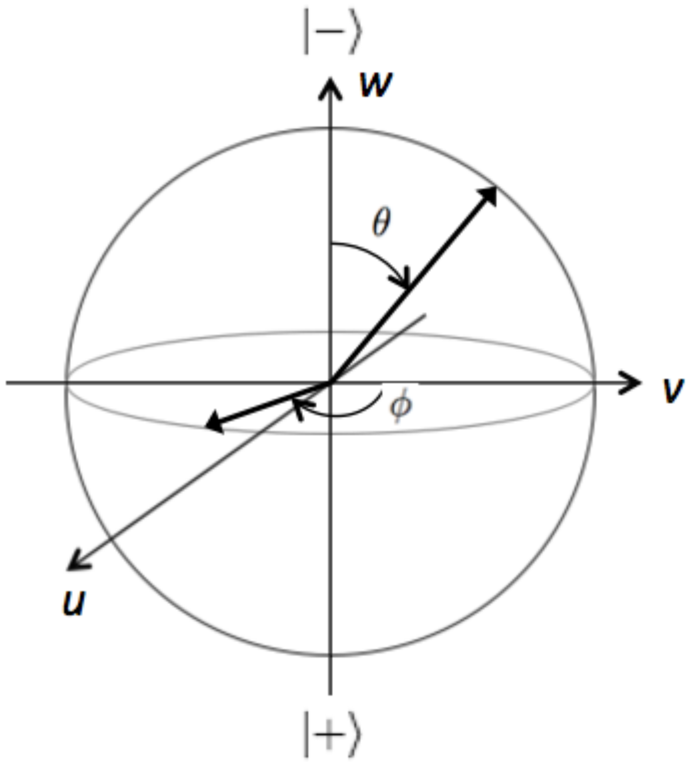
A. Emulating a two-level atom using analog electronics

Bloch equations describe the time evolution behavior of the Bloch vector.

Bloch Equations:

$$\frac{d}{dt}\begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} -1/T_2 & -\delta & 0 \\ \delta & -1/T_2 & A \\ 0 & -A & -1/T_1 \end{bmatrix}\begin{bmatrix} u \\ v \\ w \end{bmatrix}$$



- Created to describe nuclear magnetic resonance by Bloch
- Gives rise to Rabi oscillations
- Multiple relaxation times were added phenomenologically:
  - T_1 longitudinal or spin-lattice
  - T_2 transverse or spin-spin
- Rabi Frequency *A* controls rotation around *u*-axis
- Detuning δ, difference between frequency splitting and applied frequency, controls rotation around *w*-axis

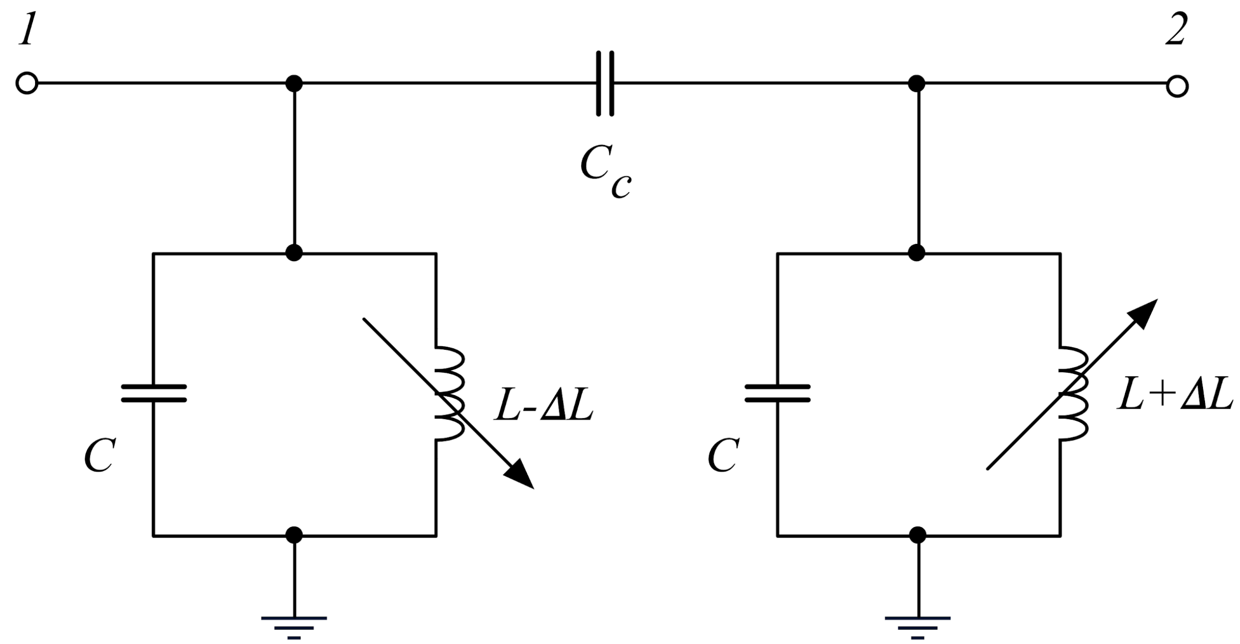
$$u = \bar{a}\bar{b}^* + \bar{a}^*\bar{b}$$

$$v = i\left(\bar{a}\bar{b}^* - \bar{a}^*\bar{b}\right)$$

$$w = \bar{a}\bar{a}^* - \bar{b}\bar{b}^*$$

Circuit dynamics of coupled LC oscillators can emulate Bloch equations.

- This analogy uses flux as displacement variable.
- Describe coupling due to controlled inductance contribution, ΔL
- Coupling occurs in the derivative terms
- It is natural to convert to representation in normal modes



$$I_1(t) = C\ddot{\phi}_1 + C_c\left(\ddot{\phi}_1 - \ddot{\phi}_2\right) + \frac{1}{R}\dot{\phi}_1 + \frac{1}{R_c}\left(\dot{\phi}_1 - \dot{\phi}_2\right) + \frac{1}{L+\Delta L}\phi_1$$

$$I_2(t) = C\ddot{\phi}_2 - C_c\left(\ddot{\phi}_1 - \ddot{\phi}_2\right) + \frac{1}{R}\dot{\phi}_2 - \frac{1}{R_c}\left(\dot{\phi}_1 - \dot{\phi}_2\right) + \frac{1}{L-\Delta L}\phi_2$$

Normal modes are obtained under weak coupling approximation.

$$I_2(t) + I_1(t) = I_+(t)$$
$$I_2(t) - I_1(t) = I_-(t)$$

gives the normal mode equations of motion

Split apart the normal mode frequencies into carrier frequency and coupling frequency:

$$\omega_+^2 = \omega_0^2 + \omega_c^2$$
$$\omega_-^2 = \omega_0^2 - \omega_c^2$$
$$\Delta\omega = \omega_+ - \omega_- \approx \frac{\omega_c}{\omega_0^2}$$

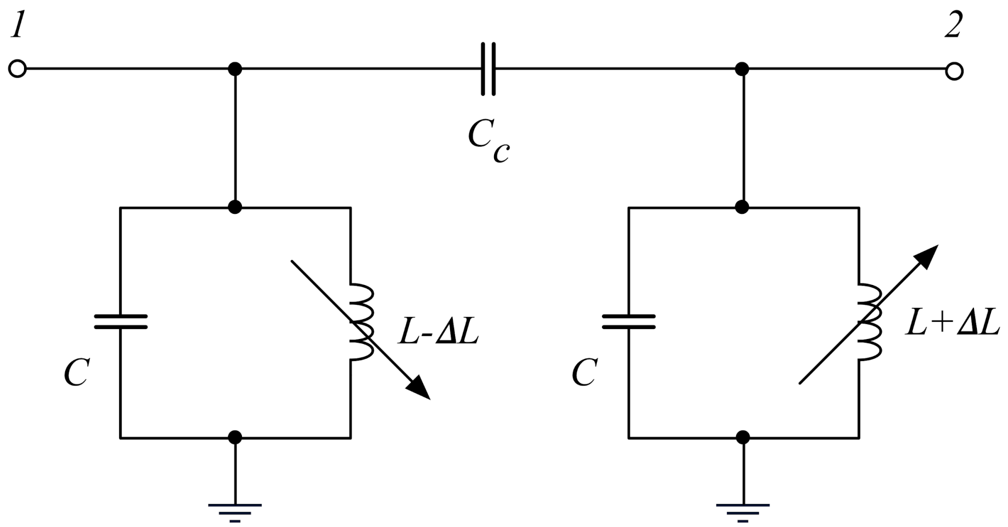
Weak coupling is required to have equal detuning frequencies:

$$\omega_d^2 = \frac{\Delta L}{L^2 C} \approx \frac{\Delta L}{L^2\left(C + 2C_c\right)}$$

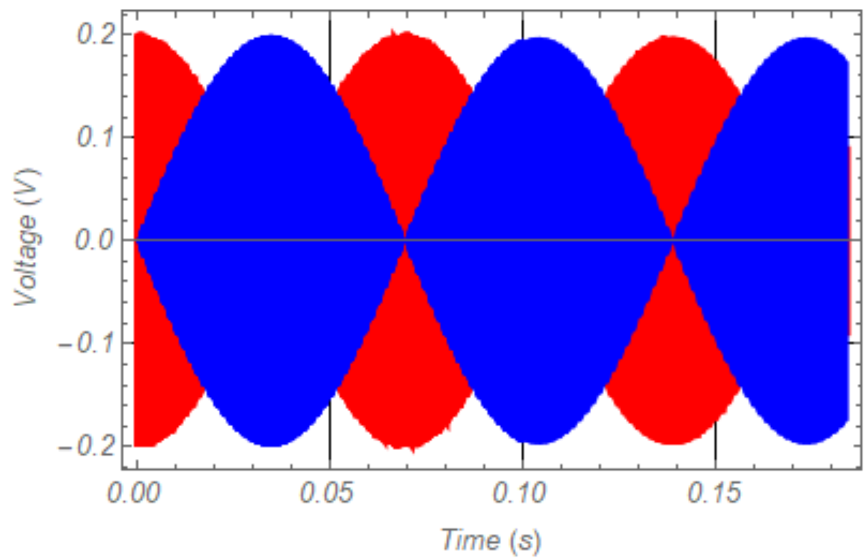
$$\left(\frac{d^2}{dt^2} + \omega_0^2\right)\begin{bmatrix} \phi_- \\ \phi_+ \end{bmatrix} + \frac{d}{dt}\begin{bmatrix} \gamma_2 & 0 \\ 0 & \gamma_1 \end{bmatrix}\begin{bmatrix} \phi_- \\ \phi_+ \end{bmatrix} + \begin{bmatrix} -\omega_c^2 & \omega_d^2 \\ \omega_d^2 & \omega_c^2 \end{bmatrix}\begin{bmatrix} \phi_- \\ \phi_+ \end{bmatrix} = \begin{bmatrix} f_- \\ f_+ \end{bmatrix}$$

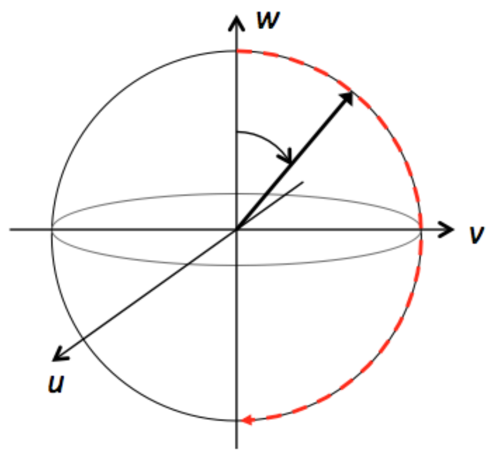
Normal mode dynamics can be manipulated by driving voltage-controlled inductors.

Our initial circuit simulation work:

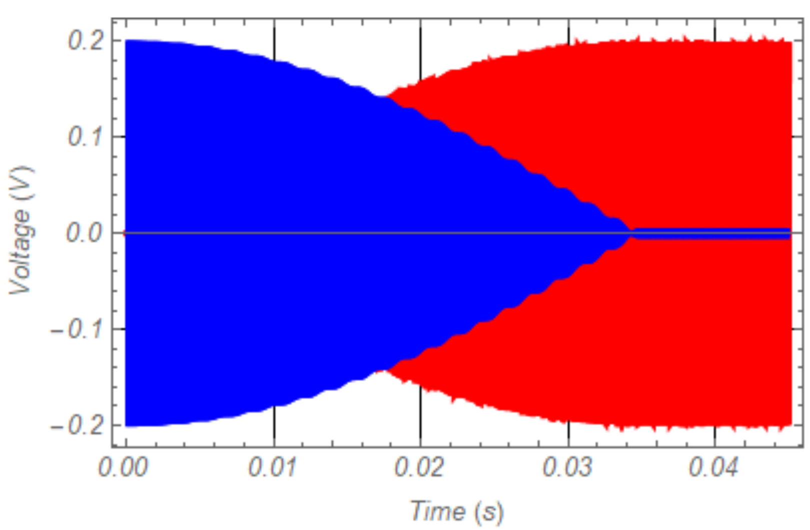


Rabi Oscillations

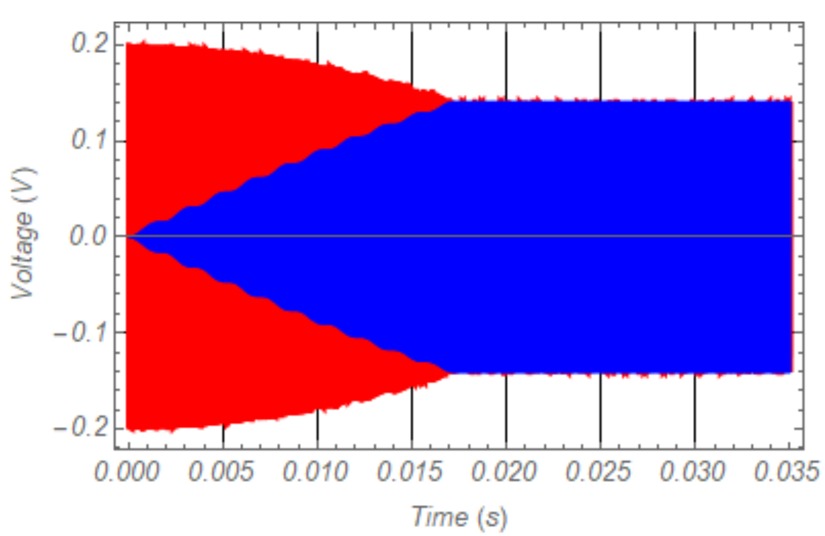




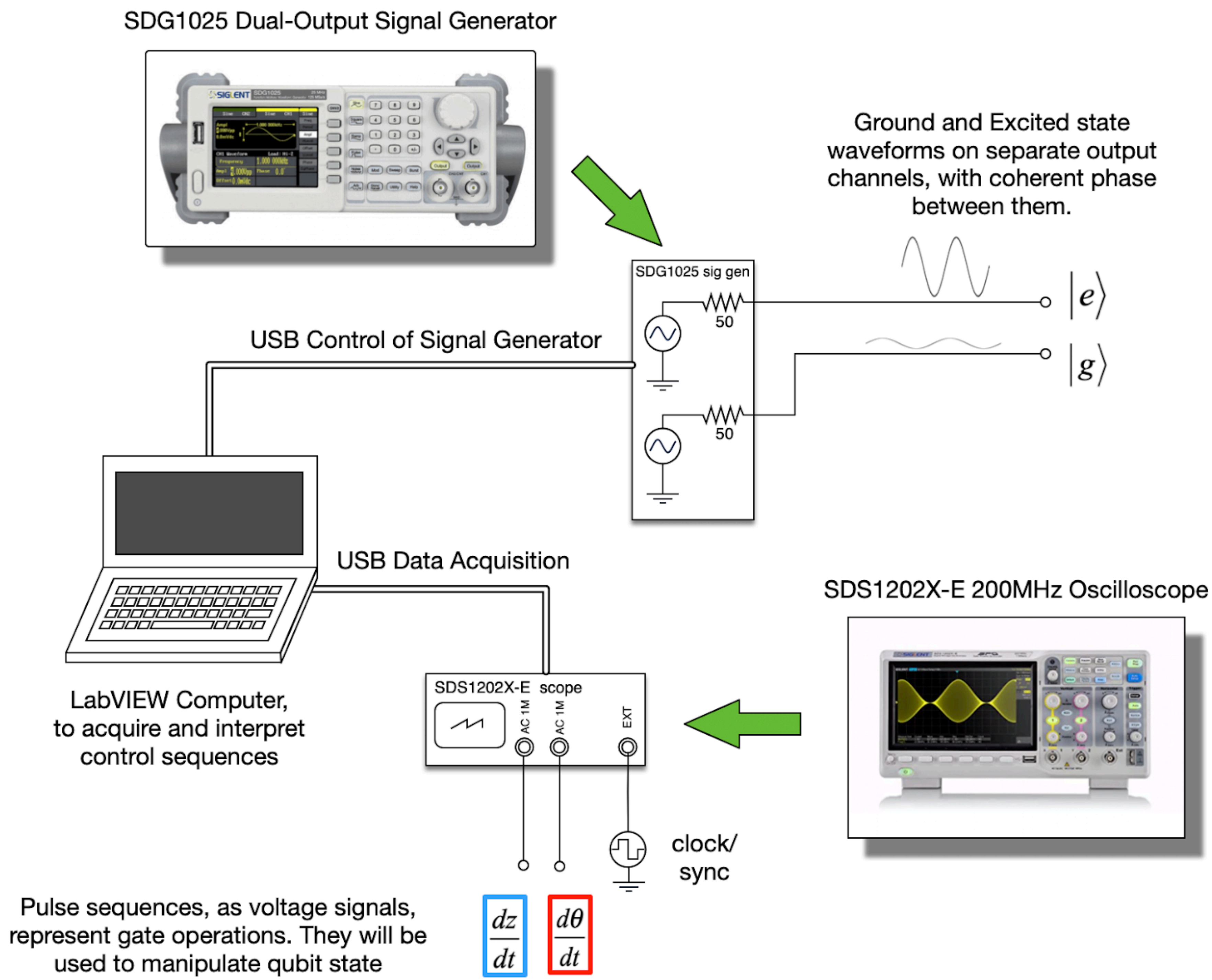
π-pulse switches states



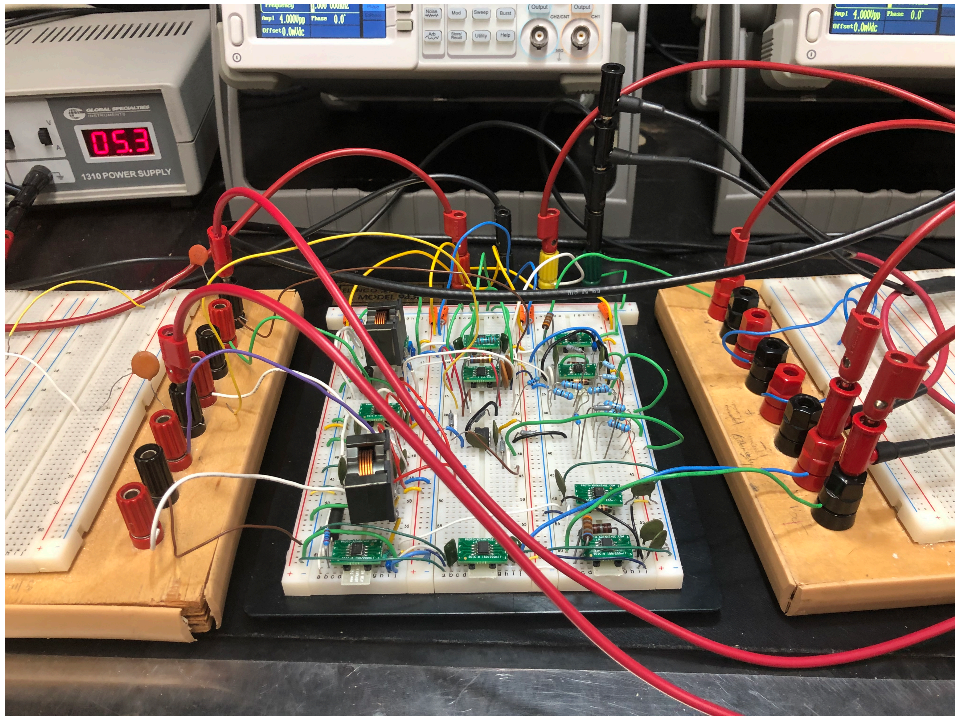
π/2-pulse end on equator

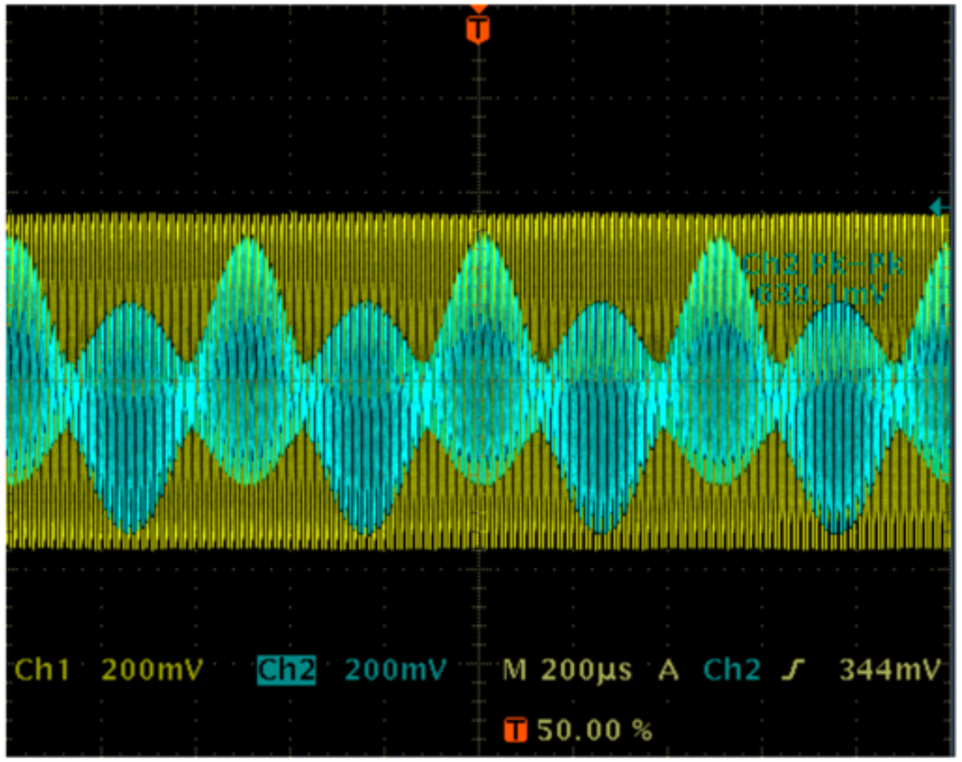


Computer control for initial conditions and field interaction is enabled by LabVIEW software.



Physical implementation using analog components has been carried out in the laboratory.



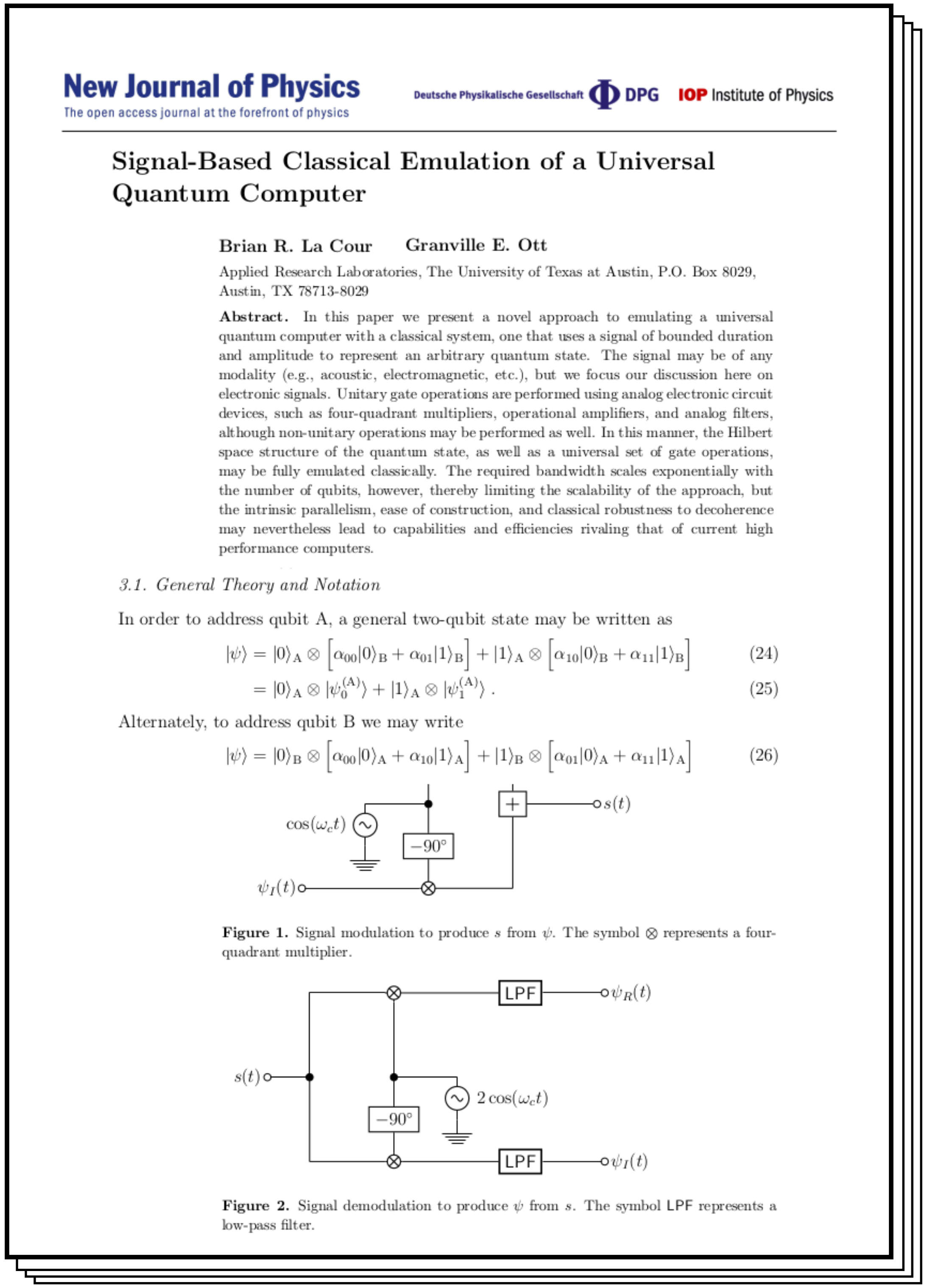


- Voltage-controlled, synthetic inductors were developed using analog integrated circuits.
- Pulse sequences were controlled using LabVIEW-enabled arbitrary waveform generators
- Resistive dissipation was countered by Q-enhancement techniques using operational-amplifiers.
- The resulting system was able to evidence emulated quantum behaviors such as Rabi oscillations and Ramsey fringes.

B. Exploring quadrature modulation for multiple, coupled qubits.

For many modulated qubits, harmonics are being simulated using LabVIEW.

Isaac Grubb, SDSU



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Signal-Based Classical Emulation of a Universal Quantum Computer

Brian R. La Cour Granville E. Ott
Applied Research Laboratories, The University of Texas at Austin, P.O. Box 8029, Austin, TX 78713-8029

Abstract. In this paper we present a novel approach to emulating a universal quantum computer with a classical system, one that uses a signal of bounded duration and amplitude to represent an arbitrary quantum state. The signal may be of any modality (e.g., acoustic, electromagnetic, etc.), but we focus our discussion here on electronic signals. Unitary gate operations are performed using analog electronic circuit devices, such as four-quadrant multipliers, operational amplifiers, and analog filters, although non-unitary operations may be performed as well. In this manner, the Hilbert space structure of the quantum state, as well as a universal set of gate operations, may be fully emulated classically. The required bandwidth scales exponentially with the number of qubits, however, thereby limiting the scalability of the approach, but the intrinsic parallelism, ease of construction, and classical robustness to decoherence may nevertheless lead to capabilities and efficiencies rivaling that of current high performance computers.

3.1. General Theory and Notation

In order to address qubit A, a general two-qubit state may be written as

$$|\psi\rangle = |0\rangle_A \otimes \left[\alpha_{00}|0\rangle_B + \alpha_{01}|1\rangle_B\right] + |1\rangle_A \otimes \left[\alpha_{10}|0\rangle_B + \alpha_{11}|1\rangle_B\right] \quad (24)$$

$$= |0\rangle_A \otimes |\psi_0^{(A)}\rangle + |1\rangle_A \otimes |\psi_1^{(A)}\rangle . \quad (25)$$

Alternately, to address qubit B we may write

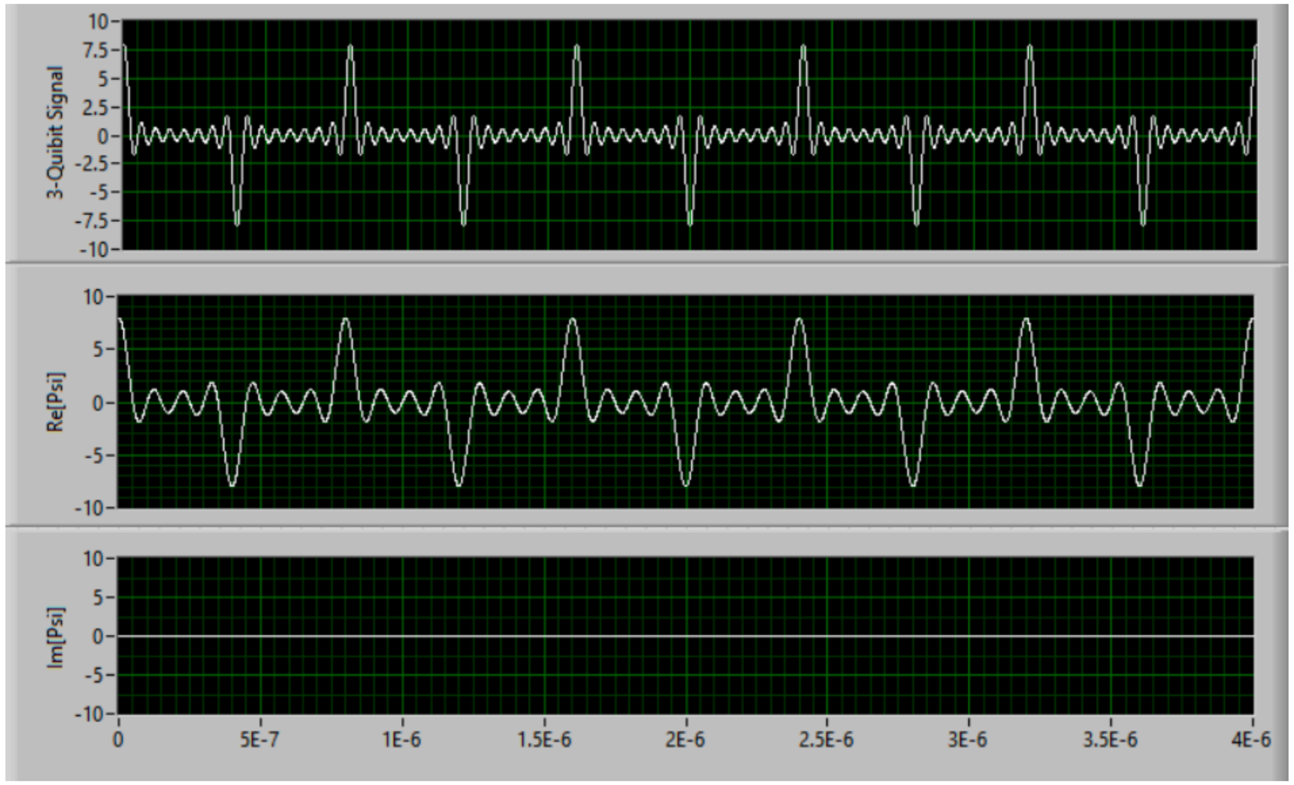
$$|\psi\rangle = |0\rangle_B \otimes \left[\alpha_{00}|0\rangle_A + \alpha_{10}|1\rangle_A\right] + |1\rangle_B \otimes \left[\alpha_{01}|0\rangle_A + \alpha_{11}|1\rangle_A\right] \quad (26)$$

$\cos(\omega_c t)$ $-90°$ $s(t)$ $\psi_I(t)$

Figure 1. Signal modulation to produce s from ψ. The symbol $\otimes$ represents a four-quadrant multiplier.

LPF $\psi_R(t)$ $s(t)$ $2\cos(\omega_c t)$ $-90°$ LPF $\psi_I(t)$

Figure 2. Signal demodulation to produce ψ from s. The symbol LPF represents a low-pass filter.



- We are studying a quadrature modulation scheme based on La Cour 2015.
- We stage this using numerous phase-coherent sinusoidal tones within a LabVIEW routine as a simulation tool

Brian R La Cour and Granville E Ott 2015 *New J. Phys.* 17 053017

Signal generators produce a stable and reproducible state for Quantum Emulation

The 2-level coherent state can be represented by way of an I and Q signal [Ferry] and extending this with the modulation techniques in [La Cour]. Using LabVIEW virtual instruments along with the Siglent signal generator, models 1025 and 1032X, to set the frequencies. The carrier and *n*-frequencies represent *n* qubits, the Hilbert-space compound states can be represented as signals with staggered frequencies. The first signal generator produces a carrier to which the emulated Qubit states are synchronized.

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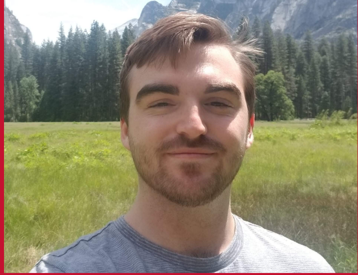
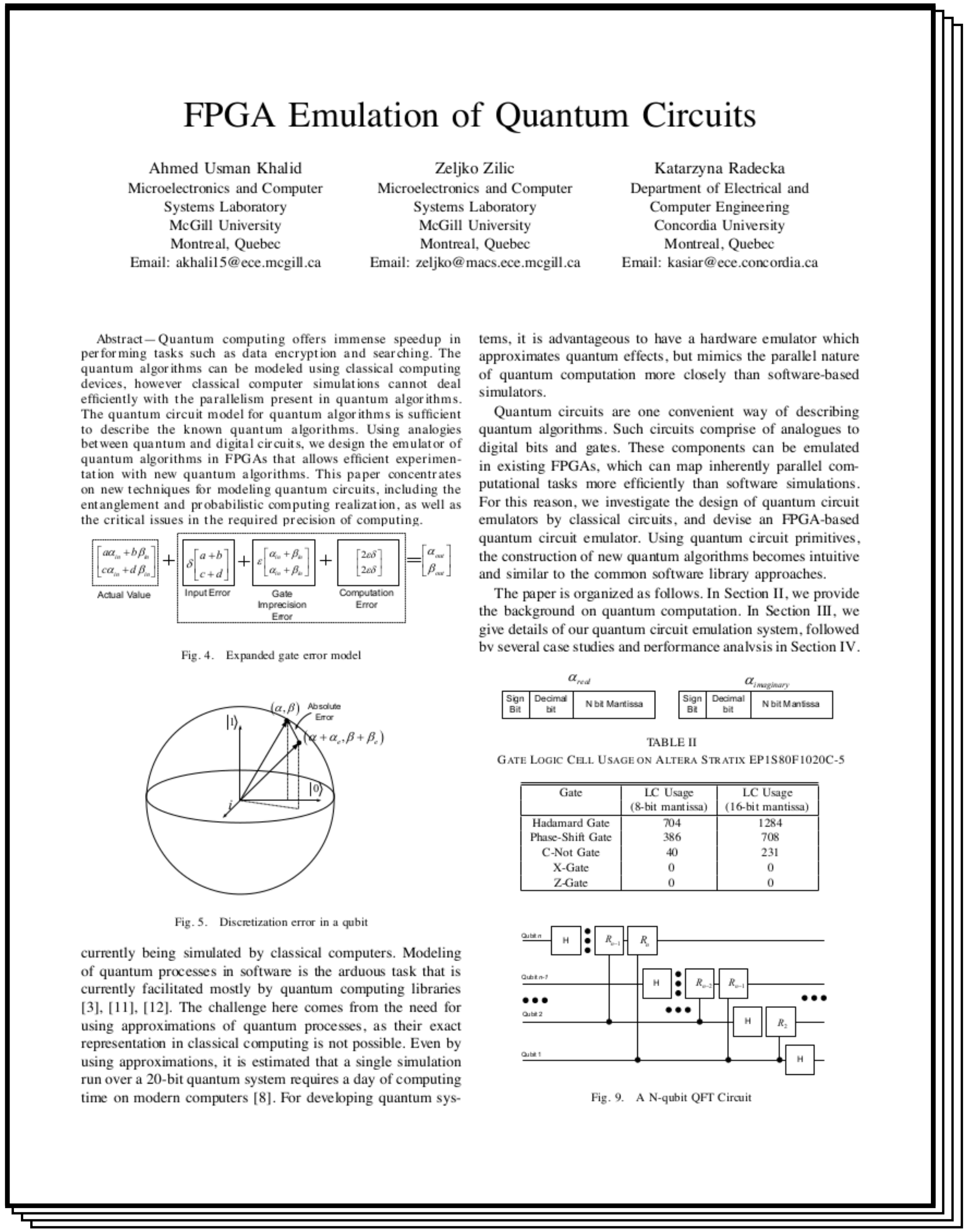
C. Exploring digital possibilities for simulation and emulation

Digital Logic has been used to Simulate Quantum Circuits at the Hardware Level



Kristopher McBrian, SDSU



FPGA Emulation of Quantum Circuits

Ahmed Usman Khalid, Microelectronics and Computer Systems Laboratory, McGill University, Montreal, Quebec, Email: akhali15@ece.mcgill.ca

Zeljko Zilic, Microelectronics and Computer Systems Laboratory, McGill University, Montreal, Quebec, Email: zeljko@macs.ece.mcgill.ca

Katarzyna Radecka, Department of Electrical and Computer Engineering, Concordia University, Montreal, Quebec, Email: kasiar@ece.concordia.ca

Abstract—Quantum computing offers immense speedup in performing tasks such as data encryption and searching. The quantum algorithms can be modeled using classical computing devices, however classical computer simulations cannot deal efficiently with the parallelism present in quantum algorithms. The quantum circuit model for quantum algorithms is sufficient to describe the known quantum algorithms. Using analogies between quantum and digital circuits, we design the emulator of quantum algorithms in FPGAs that allows efficient experimentation with new quantum algorithms. This paper concentrates on new techniques for modeling quantum circuits, including the entanglement and probabilistic computing realization, as well as the critical issues in the required precision of computing.

Fig. 4. Expanded gate error model

Fig. 5. Discretization error in a qubit

currently being simulated by classical computers. Modeling of quantum processes in software is the arduous task that is currently facilitated mostly by quantum computing libraries [3], [11], [12]. The challenge here comes from the need for using approximations of quantum processes, as their exact representation in classical computing is not possible. Even by using approximations, it is estimated that a single simulation run over a 20-bit quantum system requires a day of computing time on modern computers [8]. For developing quantum systems, it is advantageous to have a hardware emulator which approximates quantum effects, but mimics the parallel nature of quantum computation more closely than software-based simulators.

Quantum circuits are one convenient way of describing quantum algorithms. Such circuits comprise of analogues to digital bits and gates. These components can be emulated in existing FPGAs, which can map inherently parallel computational tasks more efficiently than software simulations. For this reason, we investigate the design of quantum circuit emulators by classical circuits, and devise an FPGA-based quantum circuit emulator. Using quantum circuit primitives, the construction of new quantum algorithms becomes intuitive and similar to the common software library approaches.

The paper is organized as follows. In Section II, we provide the background on quantum computation. In Section III, we give details of our quantum circuit emulation system, followed by several case studies and performance analysis in Section IV.

TABLE II

GATE LOGIC CELL USAGE ON ALTERA STRATIX EP1S80F1020C-5

Gate	LC Usage (8-bit mantissa)	LC Usage (16-bit mantissa)
Hadamard Gate	704	1284
Phase-Shift Gate	386	708
C-Not Gate	40	231
X-Gate	0	0
Z-Gate	0	0

Fig. 9. A N-qubit QFT Circuit

- Some Simulation work is labeled as Emulation
- The parallel nature of an FPGA (Field-Programmable Gate Arrays) lends to simulating quantum entangled systems.
- Stemming from prior simulation efforts using FPGAs, which allow for rapid highly-parallelized calculations, we reproduced the matrix operation set using Verilog targeted for an Intel(formerly Altera) DE-10 Lite development board.
- Matrix-matrix multiplication, a computationally expensive part of simulations, permits exploring algorithmic trade-offs which may give insight into speeding up quantum computing and simulations in general.
- Suitable for our future low-temperature circuitry
  - as a Test Architecture
  - for controller logic

A. U. Khalid, Z. Zilic and K. Radecka, "FPGA emulation of quantum circuits," IEEE International Conference on Computer Design: VLSI in Computers and Processors, 2004. ICCD 2004. Proceedings., San Jose, CA, USA, 2004, pp. 310-315

Today's Simulation technologies incorporate the latest in Python and GPU acceleration

Rohit De, DNHS

Royal Society **Publishing**

Informing the science of the future

Rapid solution of problems by quantum computation

By David Deutsch[1] and Richard Jozsa[2]†

[1] *Wolfson College, Oxford OX2 6UD, U.K.*

[2] *St Edmund Hall, Oxford OX1 4AR, U.K.*

A class of problems is described which can be solved more efficiently by quantum computation than by any classical or stochastic method. The quantum computation solves the problem with certainty in exponentially less time than any classical deterministic computation.

The operation of any computing machine is necessarily a physical process. Nevertheless, the standard mathematical theory which is used to study the possibilities and limitations of computing (e.g. based on Turing machines) disallows quantum mechanical effects, in particular the presence of coherent superpositions during the computational evolution. A suitable notion of a *quantum computer*, which, like the Turing machine, is idealized as functioning faultlessly and having an unlimited memory capacity, but which is able to exploit quantum effects in a programmable way, has been formulated by one of us (Deutsch 1985). Quantum computers cannot compute any function which is not turing-computable, but they do provide new modes of computation for many classes of problem. In this paper we demonstrate the importance of quantum processes for issues in computational complexity. We describe a problem which can be solved more efficiently by a quantum computer than by any classical computer. The quantum computer solves the problem with certainty in exponentially less time than any classical deterministic computer, and in somewhat less time than the expected time of any classical stochastic computer.

$$
\begin{aligned}
|\phi\rangle &\xrightarrow{U_f} \frac{1}{\sqrt{(2N)}} \sum_{i=0}^{2N-1} |i, f(i)\rangle \\
&\xrightarrow{S} \frac{1}{\sqrt{(2N)}} \sum_{i=0}^{2N-1} (-1)^{f(i)} |i, f(i)\rangle \\
&\xrightarrow{U_f} \frac{1}{\sqrt{(2N)}} \sum_{i=0}^{2N-1} (-1)^{f(i)} |i, 0\rangle \equiv |\psi\rangle.
\end{aligned} \tag{9}
$$

The magnitude of the inner product

$$|\langle\phi|\psi\rangle| = \frac{1}{2N}\left|\sum_{i=0}^{2N-1}(-1)^{f(i)}\right| \tag{10}$$

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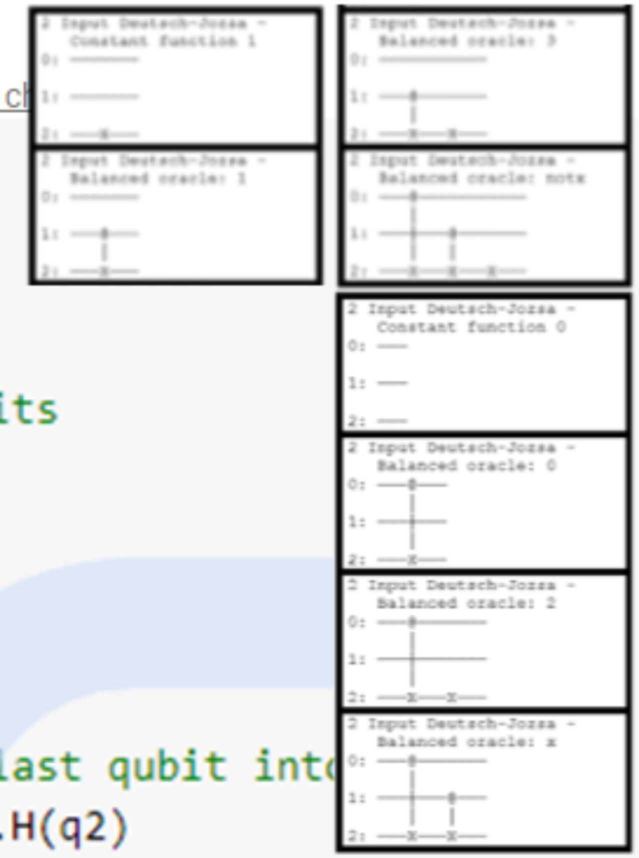
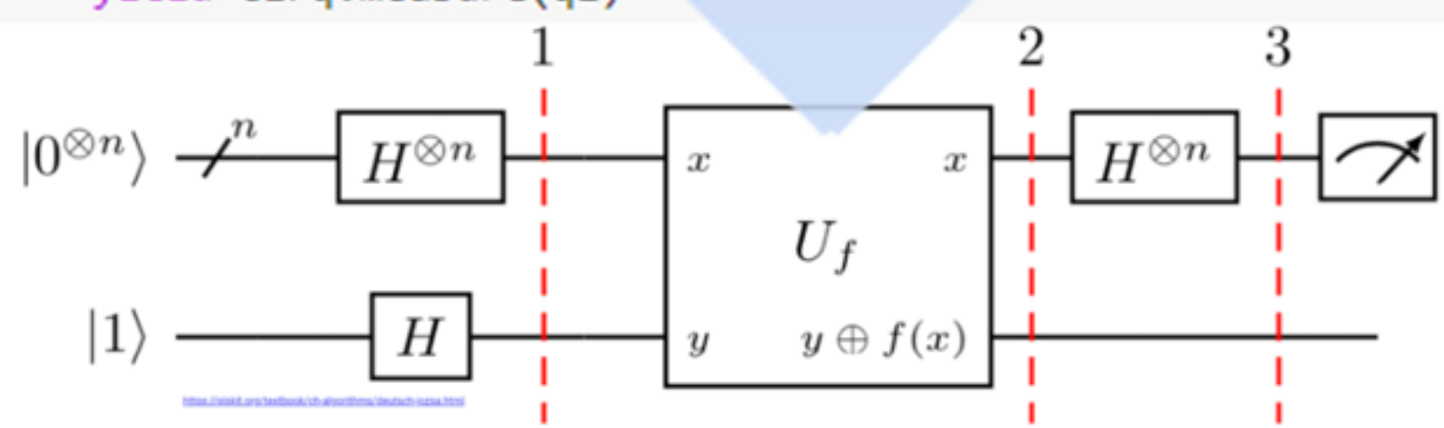
D. Deutsch and R. Jozsa, Proc. R. Soc. London, A 439, 553 (1992).

Deutsch-Jozsa Algorithm.ipynb

File Edit View Insert Runtime Tools Help All c

```python
# 2-input-qubit oracles
    # phase kickback trick
    yield cirq.X(q2), cirq.H(q2)

    # equal superposition over input bits
    yield cirq.H(q0), cirq.H(q1)

    # query the function
    yield oracle

    # interference to get result, put last qubit int
    yield cirq.H(q0), cirq.H(q1), cirq.H(q2)

    # a final OR gate to put result in final qubit
    yield cirq.X(q0), cirq.X(q1), cirq.CCX(q0, q1, q2)
    yield cirq.measure(q2)
```





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