

# **A HYBRID MODEL APPLIED TO THE VEHICLES ROUTING PROBLEM WITH SIMULTANEOUS PICKUPS AND DELIVERIES - VRPSPD.**

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## Brief summary of the resume of Pedro Pablo Ballesteros Silva



**P.P. Ballesteros Silva.** Industrial Engineer, Production Engineering Specialist of the Francisco José de Caldas District University of Bogotá; Master in Operations Research and Statistics, PhD. in Engineering from the Technological University of Pereira.

Pedro Pablo has worked in many private and official companies: Colcarga, Temecal S.A., Industria Militar, Colombian Institute for the Promotion of Higher Education - ICFES. He has worked as a university professor at the Francisco José de Caldas District University, Cundinamarca University, Los Libertadores University Foundation and Technological University of Pereira. In this university he has been titular professor and has guided the subjects Times and Methods, Linear Programming, Operations Management I and II, Logistics. He has authored more than fifty scientific articles in the areas of Logistics, administration of Operations, Simulation and others. He was the creator and coordinator of the Postgraduate Program Specialization in Business Logistics. He has been a visiting professor at several universities abroad such as Universidad Nacional del Sur in Bahía Blanca (Argentina) where he directed the course The logistics from a point of business view at the Doctorate of Engineering; at the Technological University of San Juan del Río in the state of QUERETARO (Mexico) where he directed the course "The importance of reverse logistics in the rescue of the environment". He has been an international speaker at events such as EPIO-ENDIO, CLAIO.

Ph. D. Ballesteros Silva obtained from the Ministry of Interior of Colombia in 2018 the Certificate of Logic Support Registry - Software awarded for the work entitled "Application of Matheuristic Techniques for the Solution of the Vehicle Routing Problem with Simultaneous Deliveries and Pickups - VRPSPD".

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## Projects of the research group "Logistics: strategy of the supply chain"

Logistics research group projects: supply chain strategy	Start year
Integrated management model of info-knowledge and innovation for the Center for Innovation and Technological Development of Pereira, Colombia.	2016
Intangible Assets in Technological Development Centers in Colombia	2014
Design, development and validation of the transportation and messaging system of Audifarma S.A. (S.I.T.A)	2012
Design of a model for the management of reverse logistics of packaging, packaging and hazardous waste in the companies of Pereira and Dosquebradas as a tool for the protection of the environment.	2013
Diploma in logistics management	2009
Creation and implementation of the regional center for logistics research	2009
Characterization of the Sources that Finance innovation in Risaralda	2007
Application of Metaheuristic Techniques for the solution of the vehicle routing problem with simultaneous deliveries and pickups	2020

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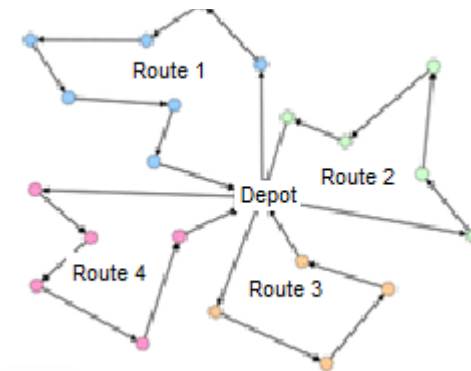


# 1

## INTRODUCTION

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INTRODUCTION



The description of the processes that can be carried out with the software and the implemented applications that have been programmed in the Java, Python and C ++ languages for the Windows platform is presented. It can be run on any system that has the Java virtual machine installed, and the respective compilers available on the websites:

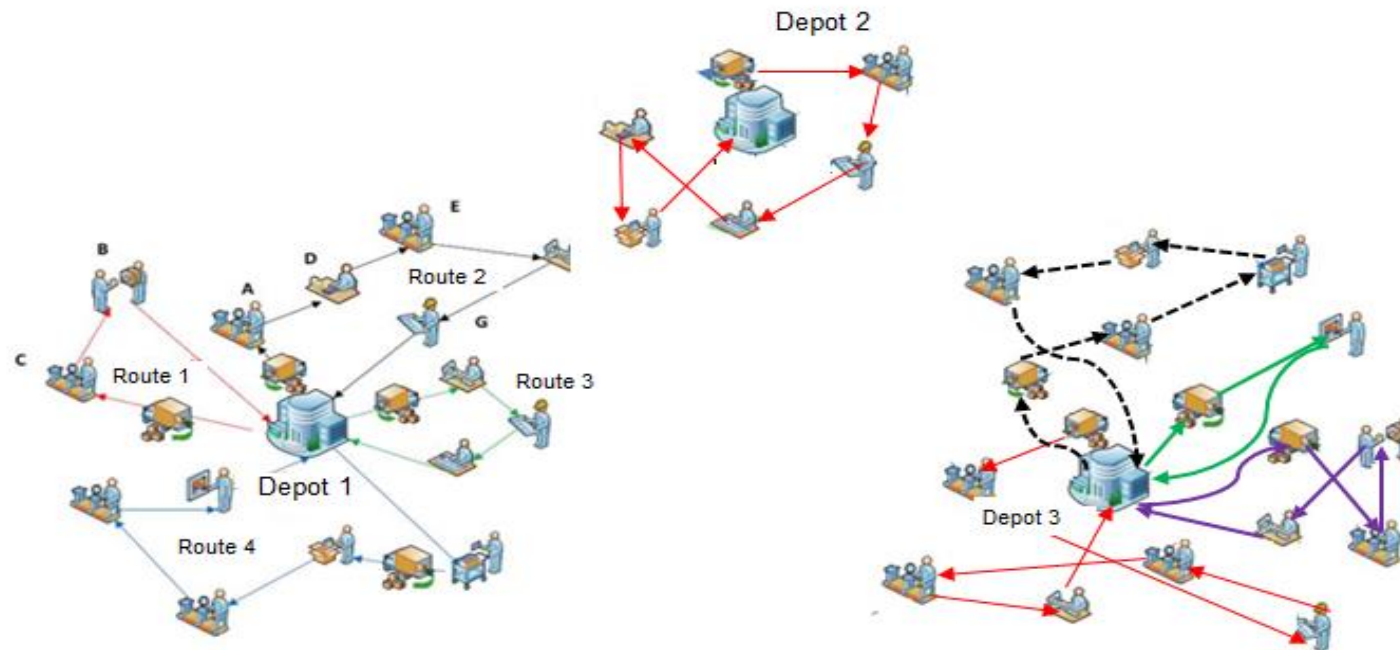
<http://www.java.com> o <http://www.java.com/es>,  
<https://www.python.org/> y <http://www.mingw.org/>.

1

## INTRODUCTION

Meaning of work:

Solving the **M-VRPSPD-MD** means looking for routes or delivery and pickup routes that are feasible and that satisfy a certain objective function, respecting restrictions on the capacity of both tanks and vehicles.





1

INTRODUCTION

The objective of the problem:

To find a series of routes with minimal cost

To provide a depot service or several depots to some clients in the most appropriate way possible

That complies with the transport capacity restriction

For products (or people) that must be pickups and / or delivered to each client (node).



1

## INTRODUCTION

**Scope:** mathematical techniques → solve the problem of routing vehicles with deliveries and pickups considering **environmental impacts**.

**Limitations:** complexity of the VRPSPD problem, it is not convenient to apply exact methods due to the number of variables and restrictions that are part of the mathematical formulation of the model.

The **M-VRPSPD-MD** is a NP-hard type combinatorial optimization problem.

The real cases of VRPSPD application are always large problems (P. Toth and D. Vigo).

For this reason, heuristic and metaheuristic techniques have been applied.  
→ good solutions to the optimization problem in reasonable computational times.

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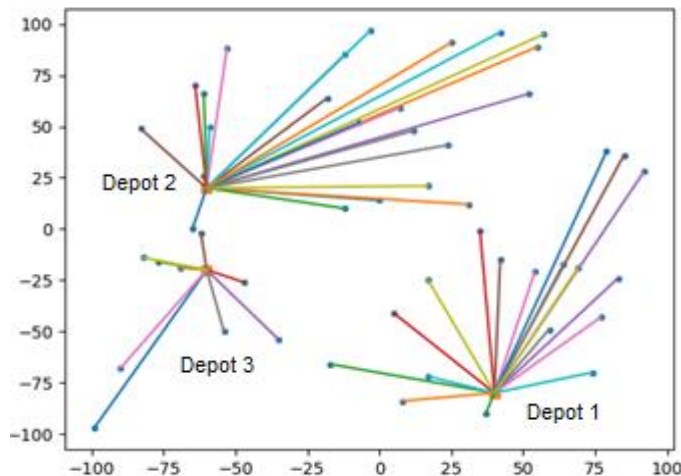
## DESIGN AND APPLICATION OF THE CLUSTERING ALGORITHM

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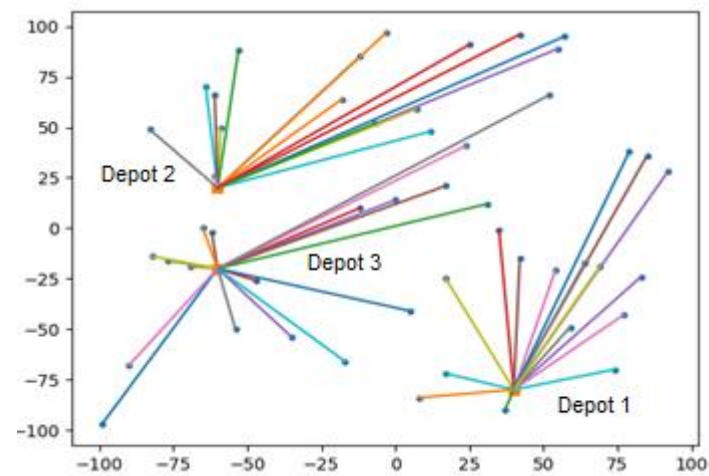
## DESIGN AND APPLICATION OF THE CLUSTERING ALGORITHM

This algorithm makes it easy to assign customers to depots.  
It has the following phases:

Phase 1



Phase 2

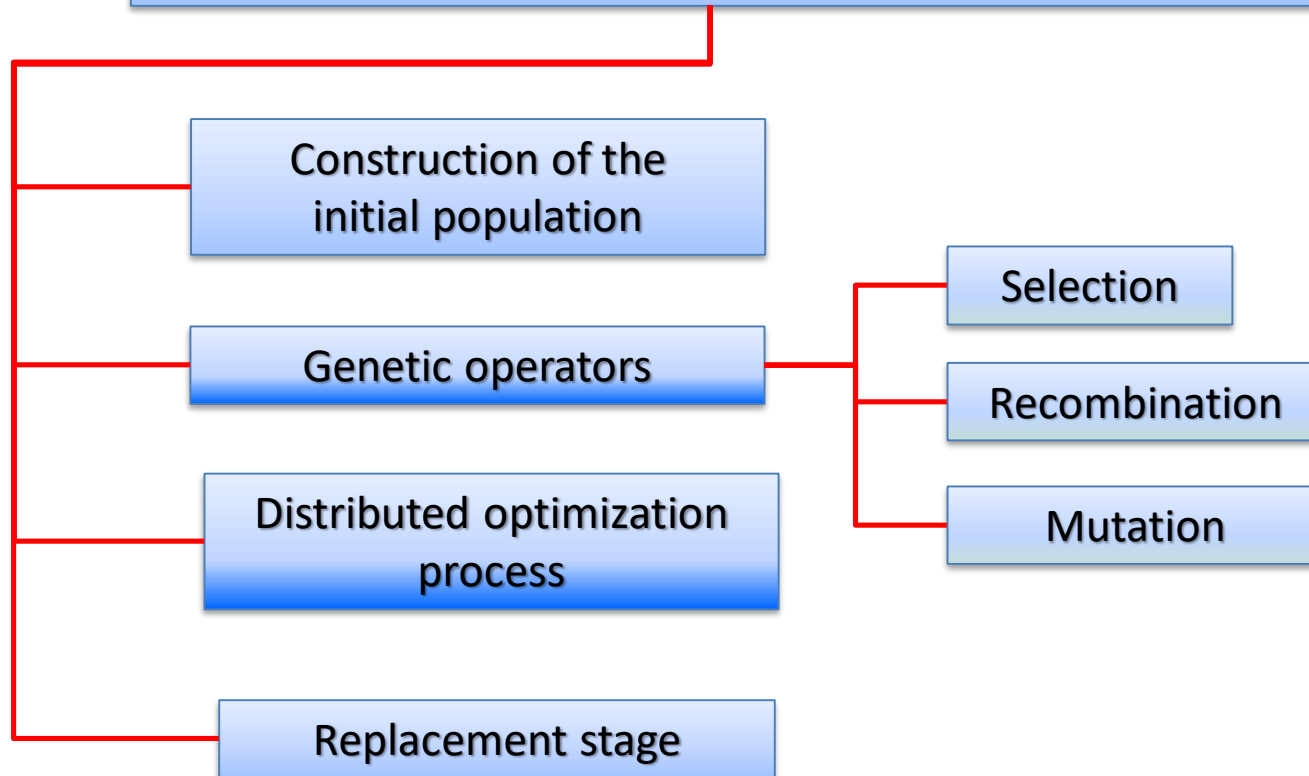


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## **SOFTWARE DESIGN FOR THE IMPLEMENTATION OF THE CHU - BEASLEY GENETIC ALGORITHM TO SOLVE THE M-VRPSPD-MD**

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## SOFTWARE DESIGN FOR THE IMPLEMENTATION OF THE CHU - BEASLEY GENETIC ALGORITHM TO SOLVE THE M-VRPSPD-MD



3

## SOFTWARE DESIGN FOR THE IMPLEMENTATION OF THE CHU - BEASLEY GENETIC ALGORITHM TO SOLVE THE M- VRSPD-MD (Continuation)

### 3.1 Construction of the initial population

Constructive  
heuristics

Random  
settings

### 3.2 Genetic operators

Selection

Tournament

Recombination

PMX - OBX

Mutation

Shift - Swap

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## SOFTWARE DESIGN FOR THE IMPLEMENTATION OF THE CHU - BEASLEY GENETIC ALGORITHM TO SOLVE THE M- VRPSPD-MD (Continuation)

Shift (1,1)

	Route 1				Route 2					Route 3			O.f.
$P_i$	6	7	5	4	9	10	11	12	8	1	2	3	549



	Route 1				Route 2					Route 3			O.f.
$P_j$	6	7	5	9	10	11	12	8	1	2	3	4	505

Swap (1,1)

	Route 1				Route 2					Route 3			O.f.
$P_i$	6	7	5	4	9	10	11	12	8	1	2	3	549

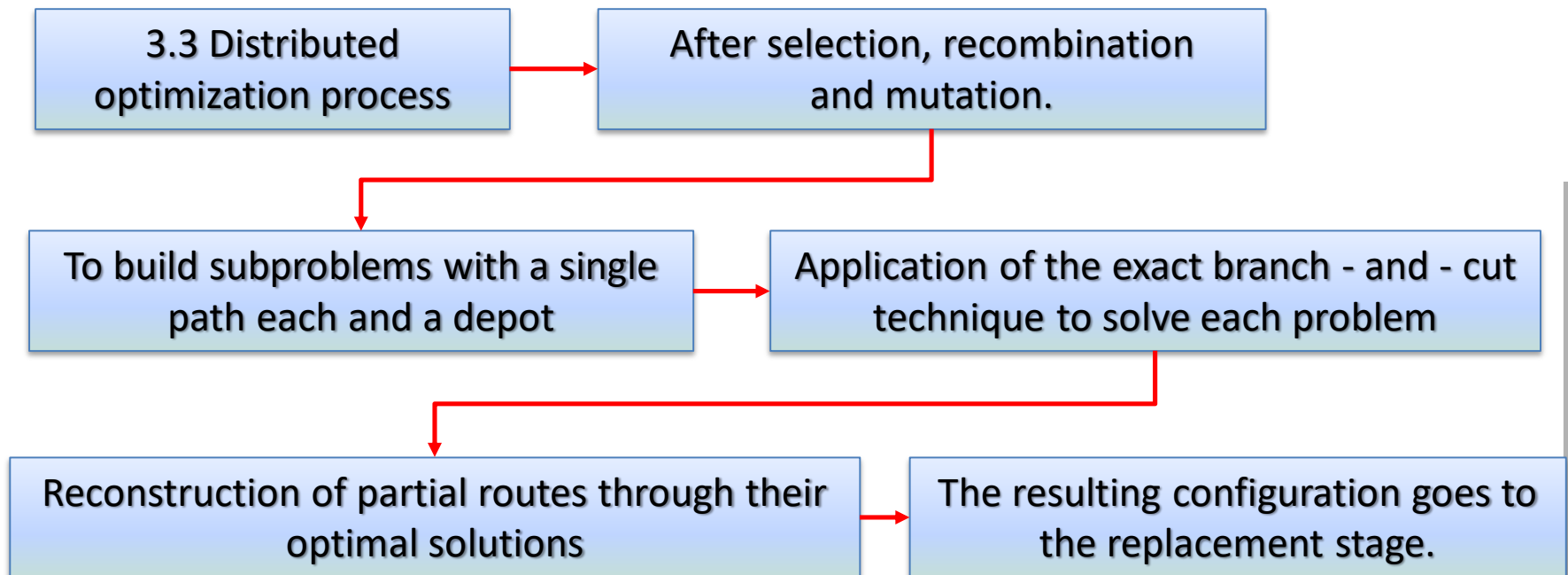


	Route 1				Route 2					Route 3			O.f.
$P_j$	6	7	5	3	9	10	11	12	8	1	2	4	499



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## SOFTWARE DESIGN FOR THE IMPLEMENTATION OF THE CHU - BEASLEY GENETIC ALGORITHM TO SOLVE THE M- VRPSPD-MD (Continuation)



3

## SOFTWARE DESIGN FOR THE IMPLEMENTATION OF THE CHU - BEASLEY GENETIC ALGORITHM TO SOLVE THE M- VRPSPD-MD (Continuation)

### 3.4 Replacement stage

Comparison of the resulting configuration with the individual with the lowest objective function in the initial population, prioritizing feasibility over infeasibility.

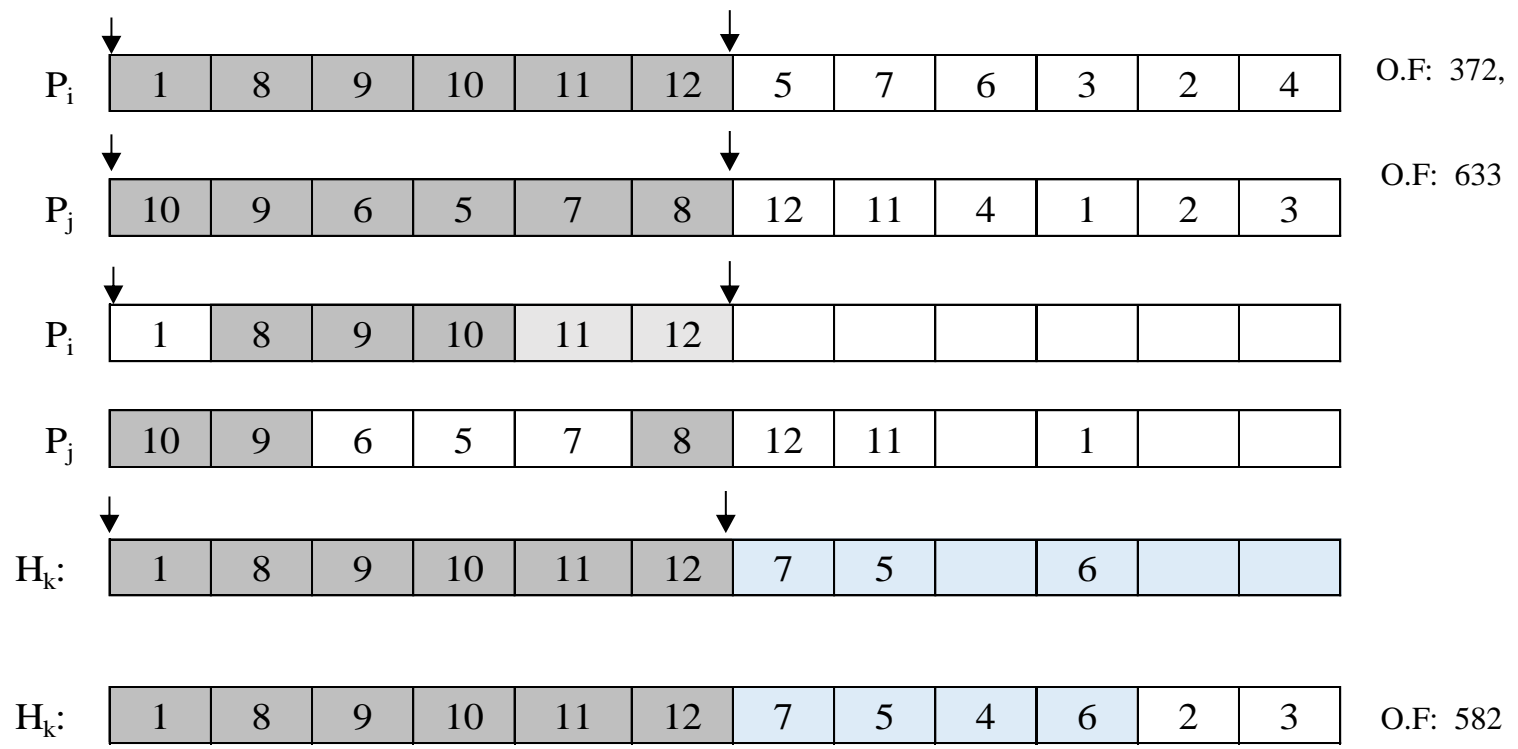
Reapplying the Chu Beasley Genetic Algorithm



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SOFTWARE DESIGN FOR THE IMPLEMENTATION OF THE CHU - BEASLEY  
GENETIC ALGORITHM TO SOLVE THE M-VRSPD-MD (Continuation)

Application of PMX recombination in  
a configuration of twelve clients.



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## DESIGN OF THE MATRIX GENERATOR ALGORITHM

4

## DESIGN OF THE MATRIX GENERATOR ALGORITHM

With this algorithm, distance matrices are generated with the quantities to be delivered and pick upped for each vehicle, for each of the configurations obtained with the AGCB

[0-17-27-14-40-10-25-8-22-35-34-32-24-15-16-1-36-48-29-33-44-18-6-26-13-28-11-47-5-37-38-50-31-23-3-7-46-42-49-20-41-45-2-21-12-39-30-43-19-4-9-0]

Each matrix becomes a small problem that is solved with the exact technique through the GAMS software or the mathematical model encoded in C ++.

Distance matrix for vehicle 1

	i0	i1	i2	i3	i4
i0	0.00	31.00	29.50	23.30	44.10
i1	37.90	0.00	6.00	41.50	58.70
i2	29.50	8.90	0.00	34.10	49.80
i3	23.30	41.50	34.10	0.00	16.30
i4	29.40	39.20	33.20	10.90	0.00

Quantities to be delivered  $d_i$

	$d_i$
i0	0
i1	41.9
i2	59.2
i3	92.1
i4	79.1

Quantities to pickup  $p_i$

	$p_i$
i0	0
i1	47.4
i2	32.5
i3	49.6

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## DESIGN OF THE MATRIX GENERATOR ALGORITHM (Continuation)

Each matrix becomes a small problem that is solved with the exact technique through the GAMS software or the mathematical model encoded in C++.

Distance matrix for vehicle 2

	i0	i1	i2	i3	i4	i5	i6
i0	0.00	58.70	42.40	43.00	59.30	59.80	59.00
i1	48.00	0.00	37.00	33.00	52.60	70.90	85.60
i2	42.40	24.60	0.00	6.00	35.10	57.70	36.50
i3	28.70	33.00	7.40	0.00	33.10	47.70	39.80
i4	39.50	78.80	35.10	33.10	0.00	13.70	8.60
i5	73.10	86.70	38.50	47.70	20.60	0.00	8.00
i6	59.00	57.10	44.60	39.80	10.50	8.00	0.00

Quantities to be  
delivered  $d_i$

	$d_i$
i0	0
i1	14.94
i2	35.53
i3	39.17
i4	56.67
i5	66.21
i6	56.48

Quantities to  
pickup  $p_i$

	$p_i$
i0	0
i1	8.37
i2	20.25
i3	57.71
i4	36.01
i5	52.17
i6	39.64

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## APPLICATION OF MATHEURISTICS TO SOLVE THE M-VRPSPD-MD WITH ENVIRONMENTAL IMPACT



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APPLICATION OF MATHEURISTICS TO SOLVE  
THE M-VRSPD-MD WITH ENVIRONMENTAL IMPACT

This mathematical model was proposed by Dell'Amico, Righini, & Salani, 2006 and applied by Subramanian, Satoru & Uchoa, 2010 with the following notation:

$A$  = set of arcs which consist of the pairs  $(i, j) \in (j, i)$  for each edge  $\{i, j\} \in E_k$ .

$G = (V_k, E_k)$  = complete graph with vertexes  $V = \{0, 1, 2, \dots, n\}$ , where the vertex 0 represents the depot and the rest corresponds to the customers. Each edge  $\{i, j\} \in E_k$  has a non-negative cost and each customer  $i \in V' = V - \{0\} = \{1, 2, 3, \dots, n\}$ .

$d_i$  = amount of merchandise or product that has to be delivered to the customer  $i$ .

$p_i$  = amount of merchandise or product that has to be picked up from customer  $i$ .

$c_{ij}$  = matrix of travelling costs or distances,  $i, j \in V$ .

$C = \{1, 2, \dots, m\}$  = set of  $m$  homogeneous vehicles with capacity  $Q$ .

$E_k$  = subset of  $V_k * V_k$  that comprises all the possible arcs.

Decision variables:

$$x_{ij} = \begin{cases} 1, & \text{if the } k \text{ vehicle travels the arc } (i, j) \in \\ 0, & \text{in any other case.} \end{cases}$$

$D_{ij}$  = amount of products or merchandise pending to be delivered, which is transported in the arc  $(i, j)$ .

$P_{ij}$  = amount of products or merchandise pending to be picked up, which is transported in the arc  $(i, j)$ .

$Q$  = capacity of the homogeneous vehicles.

APPLICATION OF MATHEURISTICS TO SOLVE  
THE M-VRPSPD-MD WITH ENVIRONMENTAL IMPACT

The objective function and the constraints are:

$$Z_{\min} = \min \sum_{i \in V} \sum_{j \in V} c_{ij} x_{ij}$$

(1) Subject to:

$$\sum_{j \in V} x_{ij} = 1 \quad \forall i \in V' \quad (2)$$

$$\sum_{j \in V} x_{ji} = 1 \quad \forall i \in V'$$

$$(3) \quad \sum_{j \in V} x_{0j} \leq m \quad (4)$$

$$\sum_{j \in V} D_{ji} - \sum_{j \in V} D_{ij} = d_i \quad \forall i \in V'$$

$$(5) \quad \sum_{j \in V} P_{ij} - \sum_{j \in V} P_{ji} = p_i \quad \forall i \in V' \quad (6)$$

$$D_{ij} + P_{ij} \leq Q x_{ij} \quad \forall (i, j) \in A$$

$$(7) \quad D_{ij} \geq 0 \quad \forall (i, j) \in A \quad (8)$$

$$P_{ij} \geq 0 \quad \forall (i, j) \in A$$

$$(9) \quad x_{ij} \in \{0, 1\} \quad \forall (i, j) \in A \quad (10)$$

$$d_j x_{ij} \leq D_{ij} \leq (Q - d_i) x_{ij} \quad \forall (i, j) \in A$$

$$(11) \quad p_i x_{ij} \leq P_{ij} \leq (Q - p_j) x_{ij} \quad \forall (i, j) \in A \quad (12)$$

$$D_{ij} + P_{ij} \leq (Q - \max \{0, p_j - d_j, d_i - p_i\}) x_{ij} \quad \forall (i, j) \in A$$

$$(13) \quad x_{ij} + x_{ji} \leq 1 \quad \forall i, j, i < j, \in V' \quad (14)$$

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APPLICATION OF MATHEURISTICS TO SOLVE  
THE M-VRSPD-MD WITH ENVIRONMENTAL IMPACT

Solutions obtained with the mathematical model proposed by Dell'Amico, Righini, & Salani, (2006) for the CON 3-8 instance for a depot.

Routes	Routes sequence	Matheuristic solution	Chu Beasley Solution	Best possible solution
Vehicle route 1	dep 25 27 42 36 37 24 4 9 2 43 50 31 29 39 dep	194.71	196.63	190.26
Vehicle route 2	dep 14 21 22 12 6 41 18 15 26 19 48 49 3 dep	128.74	126.73	116.52
Vehicle route 3	dep 35 45 16 5 10 30 1 13 17 32 44 28 33 7 46 20 47 8 11 dep	183.73	184.62	183.73
Vehicle route 4	dep 40 23 38 34 dep	31.30	31.30	31.30
Total matheuristic distance traveled		<b>538.48</b>	<b>539.28</b>	
Total distance from Chu Beasley:		<b>539.28</b>		
Optimal solution according to Subramanian (2012)		<b>523.05</b>		

## COMPUTATIONAL RESULTS IN THE IMPLEMENTATION OF THE CHU-BEASLEY GENETIC ALGORITHM USING THE CON 3-8 INSTANCE OF DETHLOFF

**Technical characteristics for the application of the AGCB.**

Description	Depot 1	Depot 2	Depot 3
Number of clients	16	16	18
Number of vehicles	3	3	3
Number of configurations	2.000	6.000	4.000
Vehicle capacity	320 units	350 units	350 units
Recombination rate	1,0	1,0	1,0
Mutation rate	0,05	0,05	0,05
Interval for PMX	2-4; 6-8; 12-14	2-4; 6-8; 12-14	2-5; 8-11; 12-15
Best result	715,05 length units	1.183,42 length units	907,01 length units
Generation cycles	437.618	1.588.507	160.253
Processing time	649.924 milliseconds	5.086.714 milliseconds	792.074 milliseconds

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## COMPUTATIONAL RESULTS IN THE IMPLEMENTATION OF THE CHU-BEASLEY GENETIC ALGORITHM USING THE CON 3-8 INSTANCE OF DETHLOFF

Number of experiments	Population size	Last generation in which the best objective function was maintained	A	B	C(s)
1	200	9,546,568	539.28	539.22	826.58
2	200	169,195,900	631.47	634.56	6022.13
3	200	14,797,376	604.41	606.86	621.71
4	200	18,281,904	601.59	604.66	865.83
5	200	25,751,135	566.4	569.15	1296.69
6	200	3,875,938	618.01	620.88	272.66
7	200	21,615,848	606.92	608.98	750.13
8	200	13,634,095	538.78	543.38	612.28
9	200	6,847,952	543.45	545.59	413.04
10	200	2,136,193	690.19	692.52	196.78
11	200	1,557,888	601.54	604.33	101.57
12	200	1,795,437	610.41	614.69	107.84

## COMPUTATIONAL RESULTS IN THE IMPLEMENTATION OF THE CHU-BEASLEY GENETIC ALGORITHM USING THE CON 3-8 INSTANCE OF DETHLOFF (Cont.)

Number of experiments	Population size	Last generation in which the best objective function was maintained	A	B	C(s)
13	200	139,029	676.67	701.18	51.40
14	200	7,860,846	612.69	613.77	216.72
15	200	6,991,854	546.82	550.75	411.33
16	200	594,774	605.87	609.91	134.55
17	200	16,078,477	644.01	646.70	523.78
18	200	41,765,529	593.76	597.38	1131.61
19	200	44,159,790	551.33	553.53	1378.16
20	200	12,396,611	584.14	586.64	803.85
21	200	17,381,286	537.36	541.40	866.87
22	200	19,271,996	602.20	604.85	851.25
23	200	105,346,307	567.76	579.51	4786.89
24	200	50,972,211	571.41	573.79	1836.32

A: Best value of the objective function in the respective generation

B: Worst value of the objective function in the respective generation.

C: Computation time for the related number of generations (seconds)

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## EXPERIMENTAL RESULTS OF MATHEURISTICS

### Solution obtained with AGCB.

Routes	Route sequences by clients									AGCB
Vehicle route 1	Dep1	35	34	44	23	22	Dep1	-	-	210,47
Vehicle route 2	Dep1	45	46	48	50	Dep1	-	-	-	189,60
Vehicle route 3	Dep1	28	25	37	41	20	29	14	Dep1	314,98
Depot 1 total distance										<b>715,05</b>
Vehicle route 1	Dep2	24	2	47	38	36	Dep2	-	-	512,34
Vehicle route 2	Dep2	11	10	17	39	43	27	Dep2	-	279,77
Vehicle route 3	Dep2	13	26	42	40	9	Dep2	-	-	391,31
Depot 2 total distance										<b>1.183,42</b>
Vehicle route 1	Dep3	21	4	1	31	33	5	Dep3	-	265,60
Vehicle route 2	Dep3	15	32	7	8	19	Dep3	-	-	376,51
Vehicle route 3	Dep3	49	12	3	6	16	18	30	Dep3	264,90
Depot 3 total distance										<b>907,01</b>
Total distance with the AGCB for the three depot										<b>2.805,48</b>



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## EXPERIMENTAL RESULTS OF MATHEURISTICS

### Solution obtained with matheuristics

Routes	Route sequences by clients									Matheuristic
Vehicle route 1	Dep1	35	34	44	22	23	Dep1	-	-	208,58
Vehicle route 2	Dep1	46	48	45	50		Dep1	-	-	155,89
Vehicle route 3	Dep1	28	25	37	29	20	41	14	Dep1	300,47
Depot 1 total distance										<b>664,94</b>
Vehicle route 1	Dep2	38	24	47	2	36	Dep2	-	-	318,69
Vehicle route 2	Dep2	43	39	17	10	11	27	Dep2	-	278,34
Vehicle route 3	Dep2	40	42	13	9	26	Dep2	-	-	321,91
Depot 2 total distance										<b>918,94</b>
Vehicle route 1	Dep3	5	31	1	4	21	33	Dep3	-	252,03
Vehicle route 2	Dep3	7	32	15	8	19	Dep3	-	-	334,76
Vehicle route 3	Dep3	12	49	18	30	3	16	6	Dep3	243,99
Depot 3 total distance										<b>830,78</b>
Total distance with the matheuristics for the three depots										<b>2.414,68</b>

**Difference between AGCB and Matheuristic = 390,82 length units**

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CONCLUSIONS

## CONCLUSIONS

- ❑ The incorporation of the algorithms designed by the author as a clustering algorithm and the matrix generator that are integrated with the Chu Beasley genetic algorithm and the exact technique, facilitates the solution of the M-VRPSPD-MD in very reasonable computation times with results efficient.
- ❑ On the other hand, since the first version of the VRPSPD dating from 1989 was proposed (Min, 1989) to date, its growing evolution has been known with the design of a large number of variants of the problem, which has allowed to expand this important area research.

## CONCLUSIONS

- ❑ The proposed mathematics considers the costs of the routes (measured in distances) and their environmental impact (generation of CO<sub>2</sub>).
- ❑ The field of action of the hybrid algorithm proposed to solve this class of problems is very broad and depends fundamentally on the knowledge and ability of researchers to achieve a very good implementation.
- ❑ The parameterization of the Chu - Beasley genetic algorithm requires a lot of time and dedication, to achieve its fine-tuning and its corresponding synchronization.

## CONCLUSIONS

- ❑ The fact of incorporating environmental aspects is another of the contributions of the work and from there other lines of research emerge, which can enrich the exciting field of action of the vehicle routing problem with simultaneous deliveries and pickups.
- ❑ The results achieved with the applied mathematics can become a starting point for new research, since at present there are no known investigations that include the proposed mathematics.
- ❑ It should be borne in mind that Matheuristics do not guarantee obtaining an optimal global solution to problems, but they do generate good solutions in very reasonable computational times.

**Thank you very much**

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