Handling Uncertainty as a Human Factor in Transportation Problems

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Mauro Dell'Orco – Polytechnic University of Bari (Italy)

''Only certainty is that nothing is certain."

Chinese fortune cookie

"An important source of bad decisions is illusion of certainty. "

Kenneth Boulding

NATURE OF UNCERTAINTY

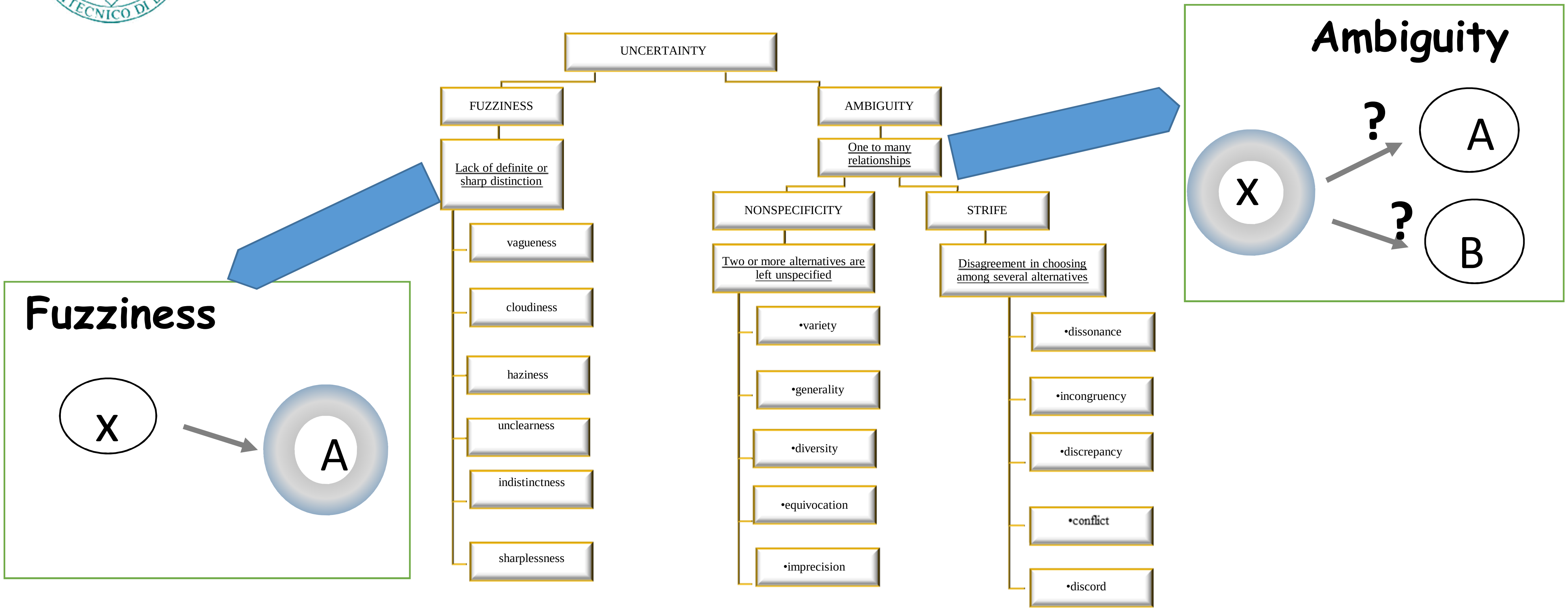

Fig. 1 -Different facets of Uncertainty

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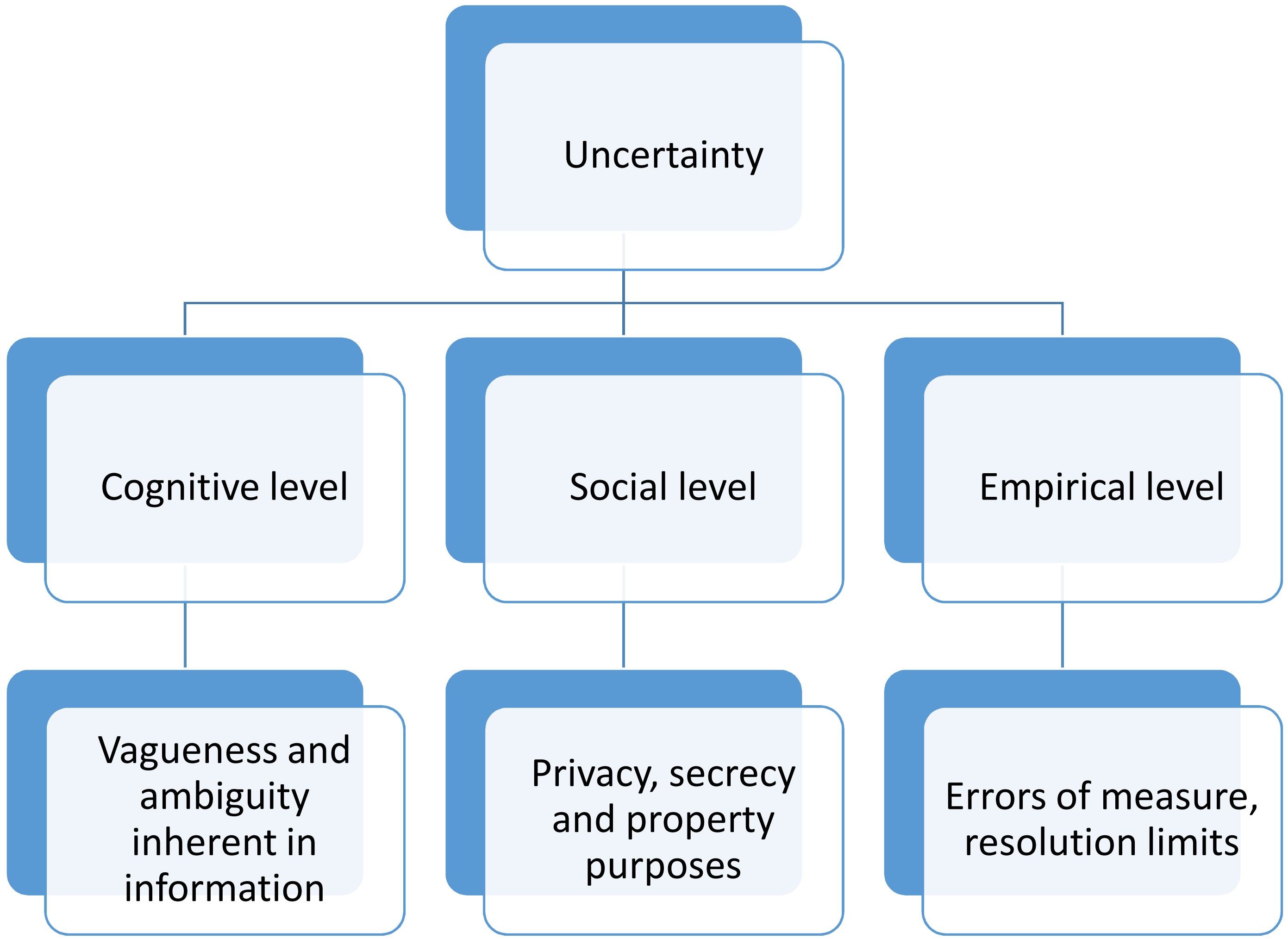

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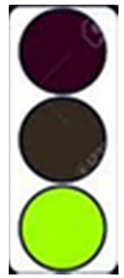
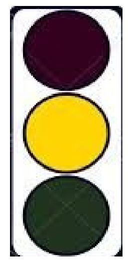
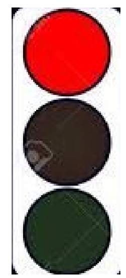
Green:  go

Yellow:  clear the intersection

Red:  stop

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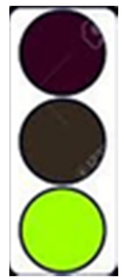
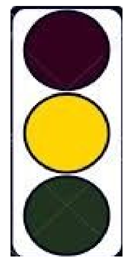
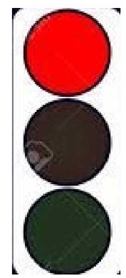
Green:  go

Yellow:  Speed up

Red:  Speed up more

Handling Uncertainty as a Human Factor in Transportation Problems

- Gödel's theorems of incompleteness.

The theorems demonstrate the inherent limitations of every formal axiomatic system containing basic arithmetic.

- Shackle [1961] :

"In a predestinate world, decision would be *illusory*; in a world of a perfect foreknowledge, *empty*; in a world without natural order, *powerless*. Our intuitive attitude to life implies non-illusory, non-empty, non-powerless decision.... Since decision in this sense excludes both perfect foresight and anarchy in nature, it must be defined as choice in face of bounded uncertainty."

- Smithson [1989]:

"Western intellectual culture has been preoccupied with the pursuit of absolutely certain knowledge or, barring that, the nearest possible approximation of it."

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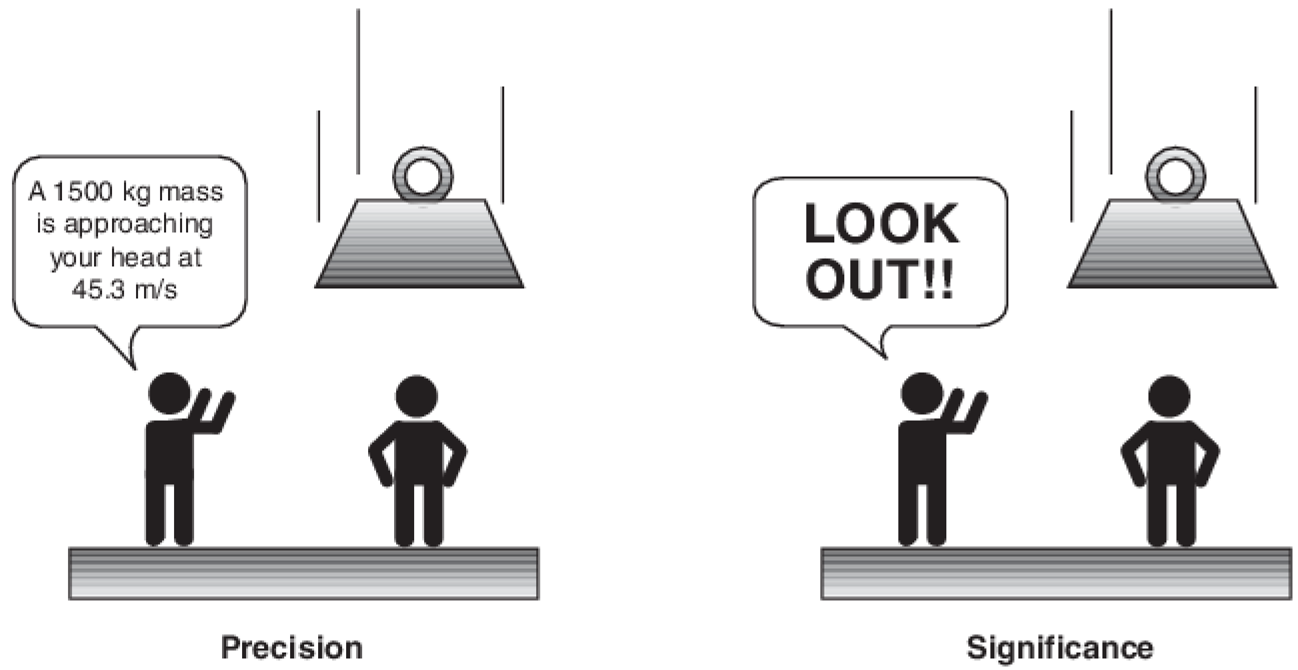


Handling Uncertainty as a Human Factor in Transportation Problems

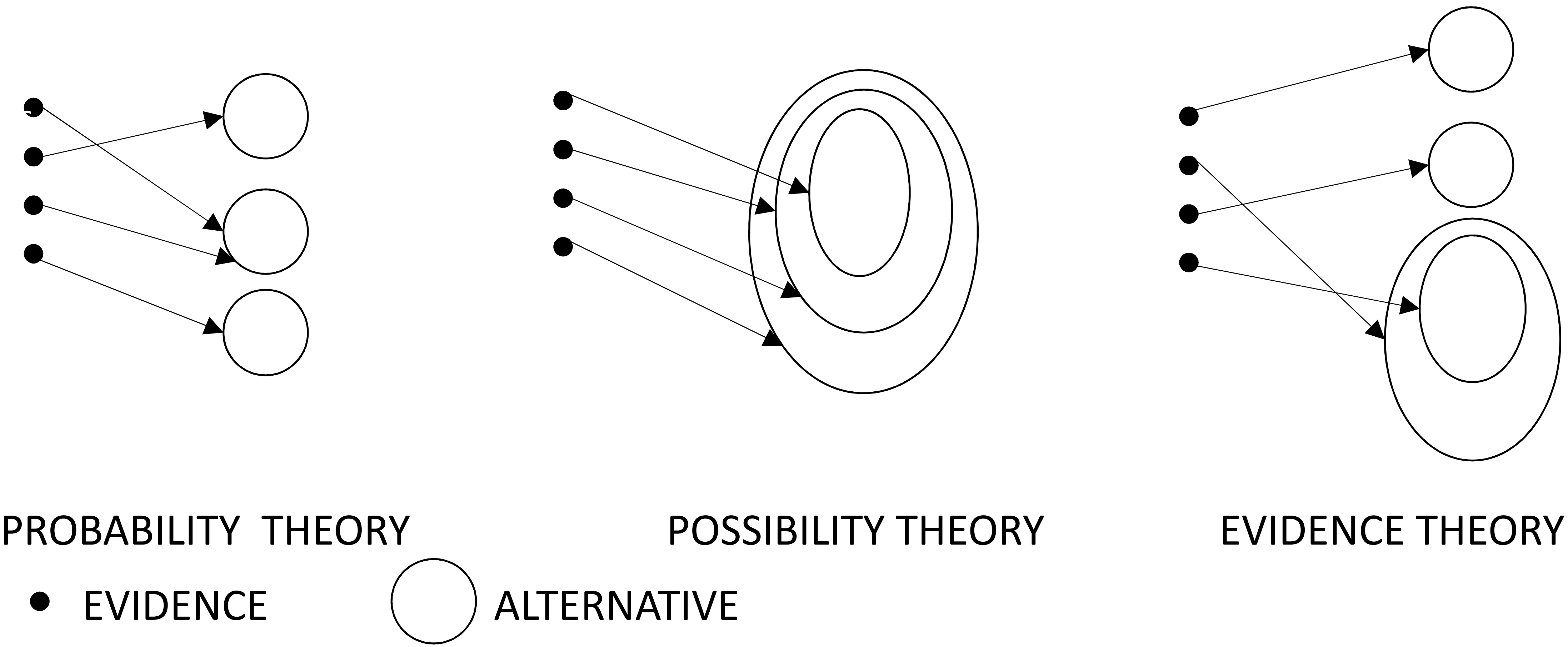
Uncertainty in Transportation Problems resides in:

- Data (numerical, descriptive, perceptive)
- Measurement
- Human perception
- Understanding of objectives and goals
- Reasoning logic based on similarity and association
- Accuracy level required for planning and design

How to measure Uncertainty?

How do you want it - the crystal ball or probability?

UNCERTAINTY MEASURES PATTERN



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$$U(S) = a \cdot \log_b |S| \qquad \text{(Hartley, 1928)}$$

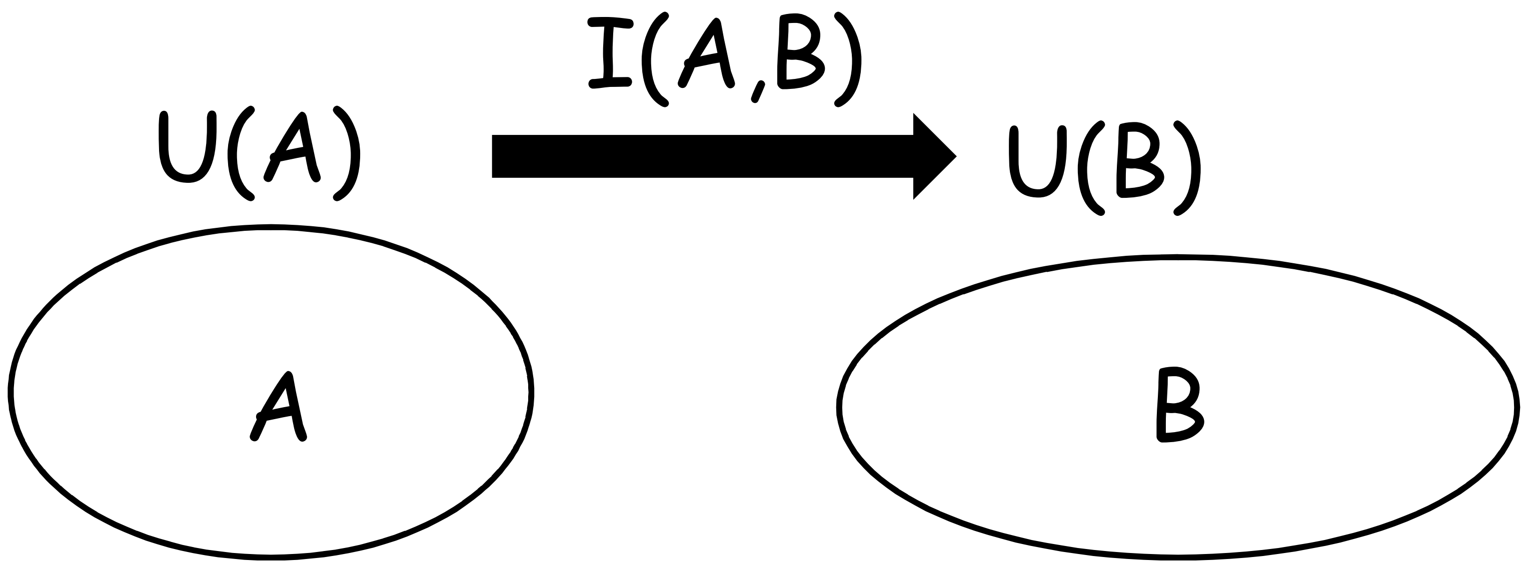
where:

S is a generic finite set

$|S|$ is the cardinality of S;

a and b are positive constants ($a>0$, $b>1$) that determine the unit of measure of uncertainty.

Information Theory-based Uncertainty measures



$$I(A,B) = U(A)-U(B) = \log_2(|A|/|B|)$$

$$|B| = 1 \longrightarrow I(A,B) = \log_2(|A|) = U(A)$$

Uncertainty as Information associated with a message x_k:

$$I_k = -\log_2 P\{x_k\}, \qquad \text{(Shannon, 1948)}$$

where $P\{x_k\}$ is the probability associated with the selection of the message x_k. The average Information (Uncertainty) is:

$$H = -\sum_{k=1}^{n} P\{x_k\} \log_2 P\{x_k\} \qquad \textbf{Shannon entropy}$$

Possibility Theory

Given a set X and its Power set $\mathcal{P}(X)$, a Possibility distribution is a function

$$r: X \longrightarrow [0,1]$$

the Possibility measure is

$$\mathrm{Poss}(A) = \max_{x \in A} r(x) \quad \forall A \in \mathcal{P}(X)$$

and the Necessity measure is

$$\mathrm{Nec}(A) = 1 - \mathrm{Poss}(\mathrm{not}A)$$

With the following axioms:

Axiom 1: $\mathrm{Poss}(\emptyset) = 0$

Axiom 2: $\mathrm{Poss}(X) = 1$

Axiom 3: $\mathrm{Poss}(U \cup V) = \max(\mathrm{Poss}(U), \mathrm{Poss}(V)$ for any disjoint subsets U and V.

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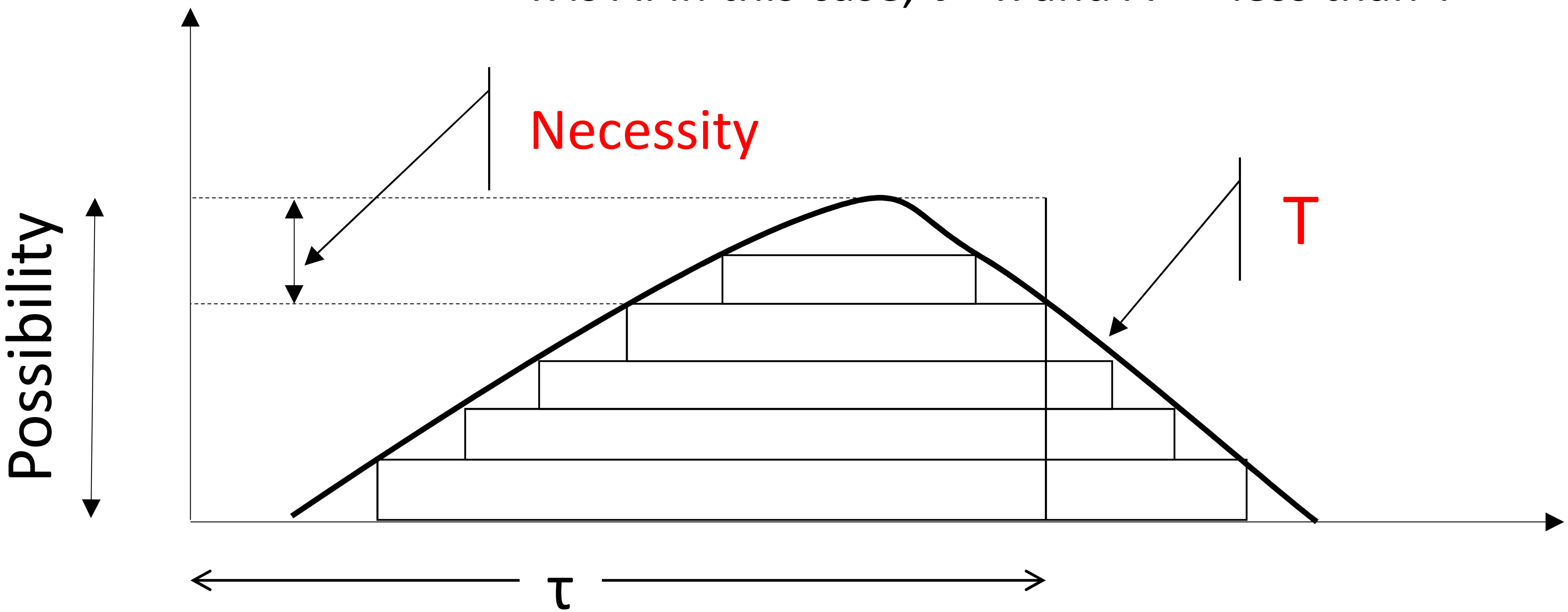
x is A: in this case, τ = x and A = «less than T»



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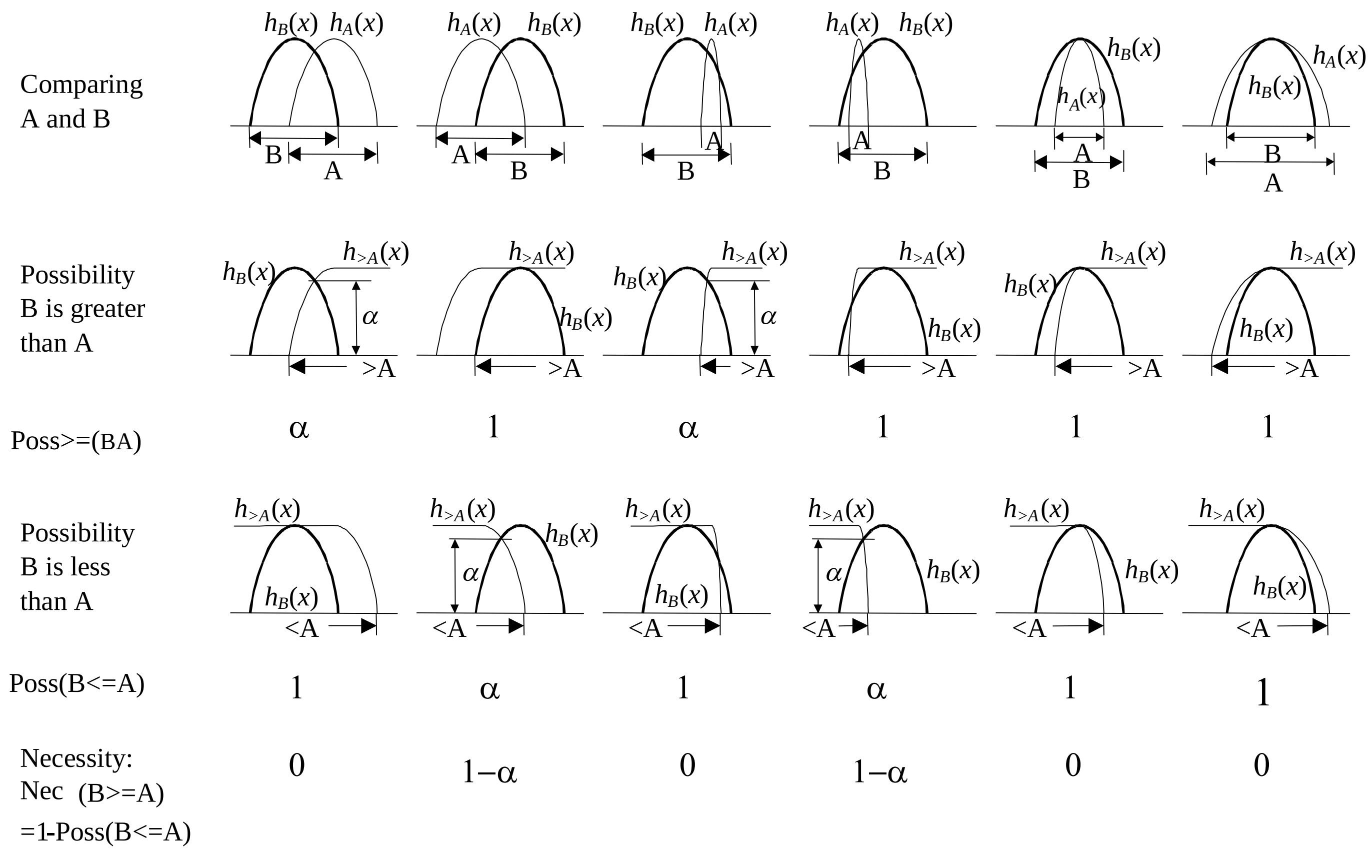
$$Poss(B \geq A) = \text{Max Min } (Poss_B(x), Poss_{\geq A}(x)) \text{ for } x \in X$$



Note: h = Poss

$$Poss(A \cup B) = \max\{Poss(A), Poss(B)\}$$

$$Nec(A \cap B) = \min\{Nec(A), Nec(B)\}$$

U-Uncertainty

$$U(A) = \sum_{i=2}^{n} log_2(i) \cdot [Poss(x_i) - Poss(x_{i+1})]$$

with $Poss(x_1) = 1$, $Poss(x_{n+1}) = 0$ by convention

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Principle of Uncertainty Invariance:

$$H = U$$

$$-\sum_{i=1}^{n} P(x_i) \cdot \log_2 P(x_i)$$
$$= \sum_{i=2}^{n} log_2(i) \cdot [Poss(x_i) - Poss(x_{i+1})]$$

- Probabilistic normalization:

$$\sum_{k=1}^{n} P(x_k) = 1$$

- Possibilistic normalization

$$Max(Poss(x_i)) = 1$$

- The log-interval scale transformation has the form:

$$Poss(x_i) = \beta \cdot P(x_i)^{\alpha} \qquad i = 1, 2, \ldots..n$$

where α and β are positive components.

From probabilistic normalization, we obtain $\beta^{1/\alpha} = \Sigma_i\, Poss(x_i)^{1/\alpha}$ and then, with $\gamma = 1/\alpha$:

$$P(x_i) = \frac{Poss(x_i)^{\gamma}}{\sum_{i=1}^{n} Poss(xi\ )^{\gamma}}$$

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$$-\sum_{k=1}^{n}\frac{Poss(x_k)^{\gamma}}{\sum_{i=1}^{n}Poss(x_i)^{\gamma}}\log_2\frac{Poss(x_k)^{\gamma}}{\sum_{i=1}^{n}Poss(x_i)^{\gamma}}$$

$$=$$

$$\sum_{i=2}^{n} log_2(i)\cdot[Poss(x_i)-Poss(x_{i+1})]$$

AN EXAMPLE

MODELLING PARKING CHOICE BEHAVIOUR USING POSSIBILITY THEORY

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The generalised cost for the parking facility j is defined as:

$$PFC_j = c_j + m_j \cdot \frac{1}{\alpha}$$

Where:

$$c_j = TV_{o,j} + TR_j + CT_{j,d}$$

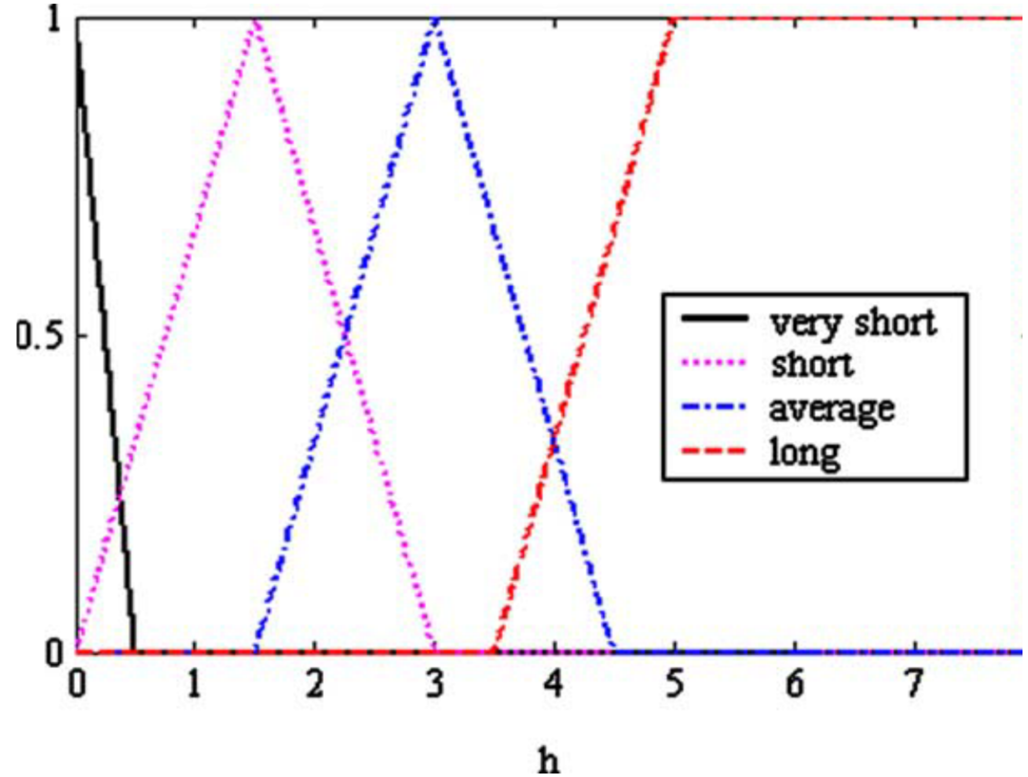
$$m_j = \begin{cases} 0 & for\ free\ parking \\ MU \cdot \left[min\left(1, FC_j \cdot DS_j\right)\right] & for\ illegal\ parking \\ TAR_j \cdot minimum\ integer \geq DS_j & for\ charged\ parking \end{cases}$$

DS_j = dwell time (hours)

MODELLING PARKING CHOICE BEHAVIOUR USING POSSIBILITY THEORY

Scenario	TAR (€/h)	MU (€)	DS	FC
1	1	50	short	weak
2	1	50	average	average
3	1	50	average	strong

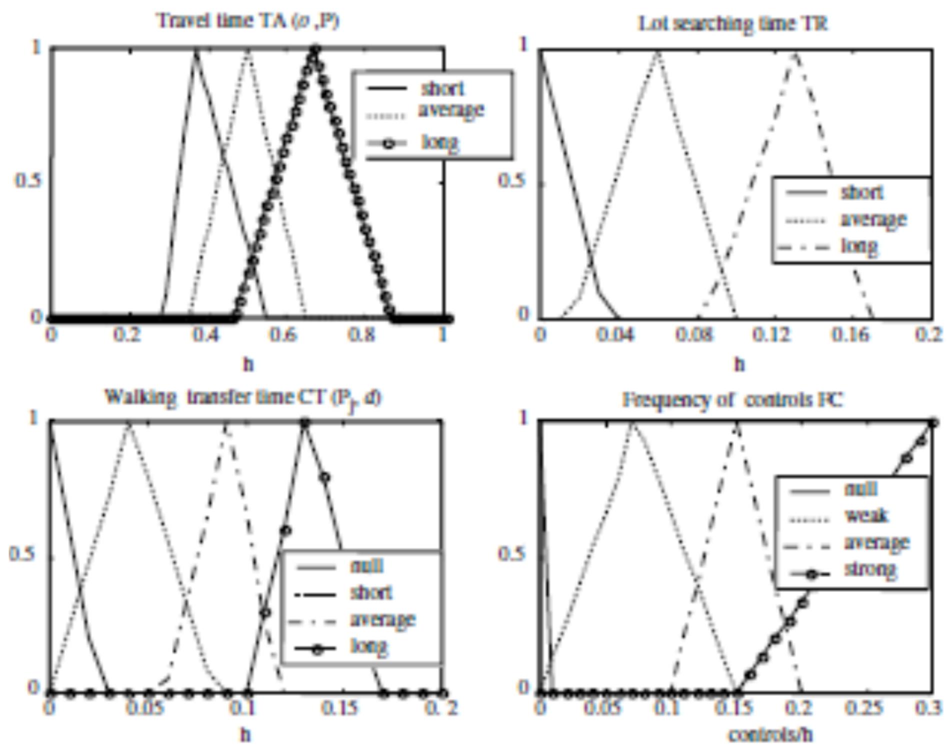
MODELLING PARKING CHOICE BEHAVIOUR USING POSSIBILITY THEORY



Possibility values for ‘Dwell Time’ (DS)

MODELLING PARKING CHOICE BEHAVIOUR USING POSSIBILITY THEORY

- Values of the perceived costs



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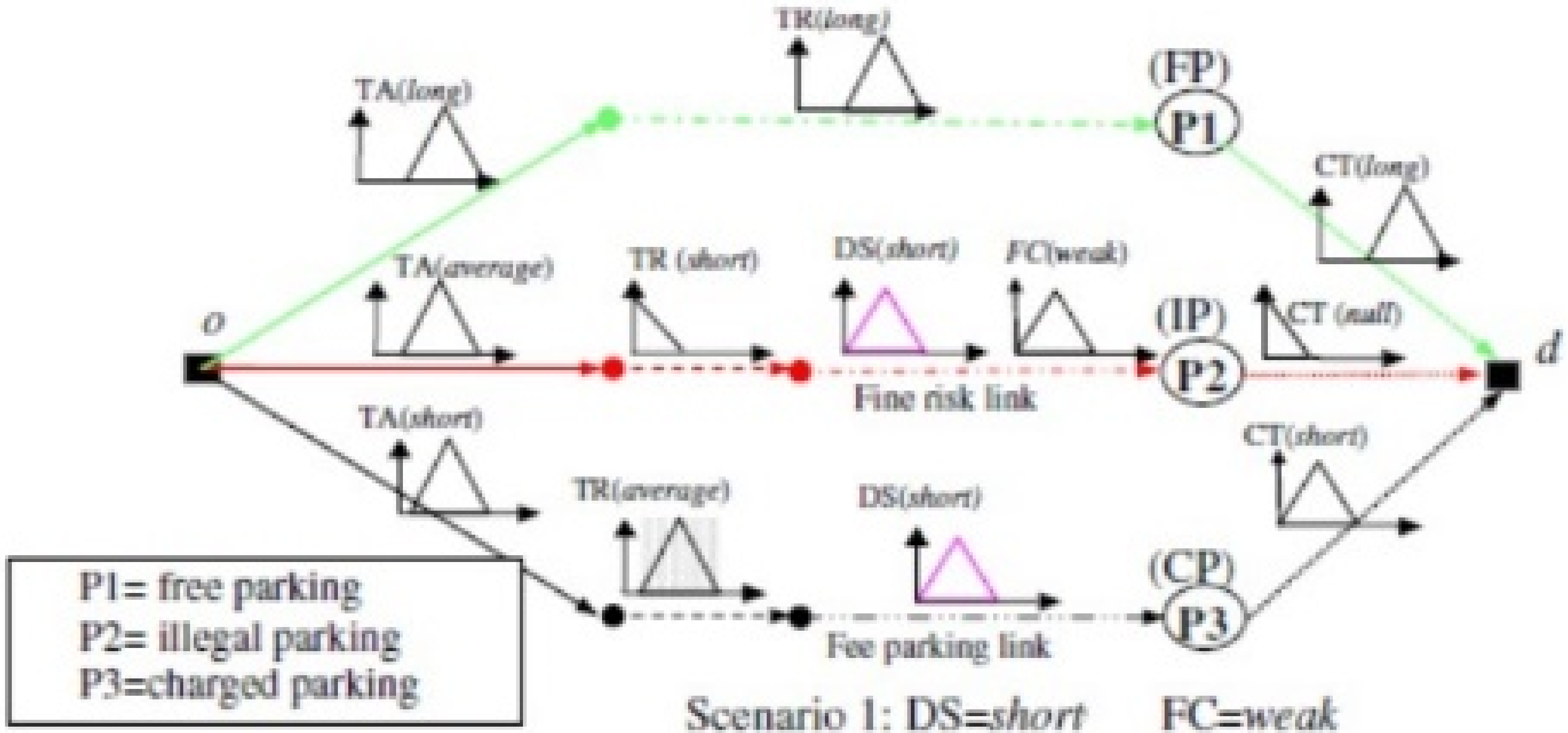

MODELLING PARKING CHOICE BEHAVIOUR USING POSSIBILITY THEORY

	Scenario		
Parking facility	1	2	3
1 free parking	0.06 (0.56)	0.19 (0.89)	0.32 (0.89)
2 illegal parking	0.55 (1.00)	0.28 (0.93)	0.10 (0.74)
3 charged parking	0.39 (0.92)	0.53 (1.00)	0.58 (1.00)

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- Conclusions

In my opinion, the Possibility Theory and the Fuzzy Set Theory are not a «cure-all» for whichever problem. There are two level of analysis: the level of the analyst , who knows statistics and probability calculations; and the level of the decision-makers, who often ignore average, standard deviation, probabilities etc.. They make decision on the basis of approximate reasonings, of their information and uncertainty about the problem. Thus, when dealing with models of decision-makers' behavior, I believe that the Possibility Theory and the Fuzzy Set Theory show all their potential

Knowing ignorance is strength.
Ignoring knowledge is sickness.

Tao Te Ching

by Lao Tzu

Thank you!