Curves Similarity Based on Higher Order Derivatives

Classification in vector bundles

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The context

Runway adherence

- A critical information for landing aircraft;
- Low adherence will result in higher braking distance or deviation from runway axis;
- Direct measurement requires closing the runway for 15 to 30 minutes.

Indirect estimation

- Use of radar tracks to detect slipping.

Radar tracks example

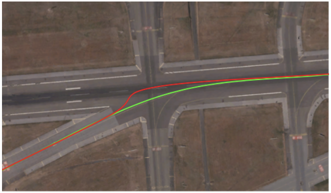

Figure: Runway clearing trajectories

Adherence inference

- Deviation from mean line indicates slipping.
- Is it possible to derive a runway condition indicator ?

Indirect adherence measurement

Naive procedure

- Collect landing trajectories data.
- Use a clustering algorithm to split the set of trajectories into a nominal and abnormal ones.
- When a new track is acquired and falls within the abnormal classes, trigger an alert.

What is wrong with it

Deviation from mean line does not always indicates slipping, but may result from pilot actions.

A problem with two facets

The shape of trajectories

- Part of the information lies within the shape of the landing and taxiing tracks.
- When comparing shapes, the aircraft velocity is not taken into account.
- Some clustering algorithms where especially tailored for this problem [1, 2].

The deceleration law

- Apart from the geometry of the curves, the longitudinal deceleration is an extremely important information.
- A large deviation from nominal trajectory but with high deceleration does not indicates a bad adherence condition.

The scope of the present work

Decoupling geometry and longitudinal acceleration

- The position and the velocity must be simultaneously taken into account.
- The state space of the tracks must be defined accordingly.
- Principle: consider a trajectory as a manifold, with an added velocity information.
- This is exactly how a vector bundle is defined !

Ultimate goal

- Release an automated tool triggering an alert when bad adherence is suspected.
- Thanks to the bundle approach, probability of false alarms is expected to be low enough.

The model

The geometric part

- Curves are defined as one dimensional manifolds with boundaries in $\mathbb{R}^2$ (general case in the article).
- There are parametrized using the arclength divided by the length of the curve.
- The resulting model is a mapping $\gamma\colon [0,1] \to \mathbb{R}^2$ such that $\forall \eta \in]0,1[, \|\gamma'(\eta)\| = l$ with l the length of the curve.

The velocity part

- An immersed vector bundle $\mathcal{E}$ with base curve γ has elements the couples (η, v) with $\eta \in]0,1[$ and $v \in \mathbb{R}^2$.
- The model of a trajectory is such a bundle:
  - The base curve is related to the shape only;
  - The attached vector at each point describes the velocity.

Similarity between sections I

Let $\mathcal{E}_0, \mathcal{E}_1$ be immersed vector bundles on respective immersions γ_0, γ_1 and let s_0, s_1 be respective sections , that will represent the vector samples along the curves γ_0, γ_1. An immersed path between s_0 and s_1 is a smooth mapping $\phi \colon [0,1] \times [0,1] \to \mathbb{R}^2 \times \mathbb{R}^2$ such that:

- For all $s \in [0,1]$, the mapping $t \in [0,1] \mapsto \phi(s,t)$ is a smooth section the trivial bundle $\mathbb{R}^2 \times R^2 \mapsto^{\pi} \mathbb{R}^2$;
- For all $s \in [0,1]$, $\pi \circ \phi(s, \bullet)$ is a smooth immersion in $\mathbb{R}^2$;
- For all $t \in [0,1]$, $\phi(0,t) = s_0(t), \phi(1,t) = s_1(t)$.

Similarity between sections II

- The mapping $t \in [0,1] \to \pi \circ \phi(s,t)$ defines an immersion from $[0,1] \to \mathbb{R}^2$ that interpolates between γ_1, γ_2.
- This immersion defines an immersed bundle, denoted by $\mathcal{E}_s$.
- It is also assumed that a metric g_s is available on $\mathcal{E}_s$ for each s.
- The energy of ϕ is defined as:

$$E(\phi) = \int_0^1 \int_0^1 g_s \left(\frac{\partial \phi}{\partial s}, \frac{\partial \phi}{\partial s} \right) dsdt$$

Similarity between sections III

- A path that minimizes the energy between two fixed sections s_0, s_1 is called a geodesic.
- If the family of metrics g_s is well chosen, such a path will always exist.
- Its energy is a measure of similarity: the lower the energy, the more similar are the sections.
- It may thus be used in clustering algorithms like the k-medoids.

Back to the original problem

Contact mechanics

- The braking force exerted by the tires of the landing gear is governed by Coulomb's law of friction.
- It is bounded by $\mu_s g M$ with M the aircraft mass and μ_s the static friction coefficient.
- When slipping occurs, it has a constant norm $\mu_d g M$ with μ_d the dynamic friction coefficient.

Control law in case of low adherence

- Since the braking force is a constant when slipping occurs, it is primary used for guiding.
- The ratio between the longitudinal and normal acceleration is as high as possible.

The associated bundle model

- Let $\gamma\colon [0,1] \to \mathbb{R}^2$ be the trajectory (with the η parametrization) and $v(\eta), v'(\eta)$ the respective velocity and acceleration at position $\gamma(\eta)$.
- A bundle metric can be derived as:

$$g((u(\eta), u'(\eta)), (v(\eta), v'(\eta)) = \langle u(\eta)_{\mathcal{N}}, v(\eta)_{\mathcal{N}} \rangle \left(1 + \kappa^2(\eta)\right) + \det\left(D_\eta \gamma(t), D_{\eta\eta}\gamma(s)\right) \quad (1)$$

  where $\kappa(\eta)$ is the curvature of the base curve at η.
- It is the sum of a shape variation and a proper velocity variation, which was the original goal.

Implementation I

Preprocessing

- Trajectories are available as sequences of couples $(t_i, x_i), i = 1 \dots N$ with t_i the sampling time and $x_i \in \mathbb{R}^2$ the position.
- In a first step, the arclength is computed using a numerical quadrature formula at each t_i.
- The trajectories are then expanded on a cubic spline basis using the η parametrization.

Implementation II

Geodesics computation

- A path ϕ between two sections is discretized on an evenly spaced grid in $[0,1] \times [0,1]$ and represented using piecewise polynomials.
- All the derivatives become linear combinations of the samples.
- The minimal energy problem turns into a standard numerical optimization program with variables the values of the path on the discretization grid.
- A limited memory BFGS is applied to solve it.

Implementation III

Clustering

- For all couples of trajectories, the energy of geodesics joining them is computed.
- A k-mediod algorithm is used to perform the clustering.
- It does not requires computing linear combinations of samples, and thus is more adapted to the case addressed.

Conclusion and future work

- The theoretical part of the work is completed.
- A simple implementation was done, not optimized for operational applications.
- Testing is in progress, but will done only on simulated data as the adherence condition is not known on real datasets.
- A realistic landing and taxiing simulator was developed and released recently to address the previous problem.

P. W. Michor and D. Mumford, *Riemannian geometries on spaces of plane curves*, J. Eur. Math. Soc. (JEMS), 8 (2006), pp. 1–48.

S. Puechmorel, *Geometry of curves with application to aircraft trajectory analysis.*, Annales de la faculté des sciences de Toulouse, (2015).