Super-resolving a Single Blurry Image Through Blind Deblurring Using ADMM

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Single image super-resolution (SISR)

aims to recover a high-resolution (HR) image $x \in \mathbb{R}^{N_h}$ from a low-resolution (LR) input image $y \in \mathbb{R}^{N_l}$

$$y = DBx + n$$

$D : \mathbb{R}^{N_h} \to \mathbb{R}^{N_l} (N_l < N_h)$ is the downsampling matrix

$B : \mathbb{R}^{N_h} \to \mathbb{R}^{N_h}$ is the blurring matrix

$n \in \mathbb{R}^{N_l}$ is the additive noise

The SISR problem is typically severely ill-posed!
Single image super-resolution (SISR)

\[ y = DBx + n \]

If \( B \) is the identity, then SISR reduces to the Image interpolation.

Most SISR cases assume \( B \) is known or predefined:

- Gaussian blur [Begin and Ferrie, 2004]
- Bicubic interpolation (BI) [Glasner, et al., 2009; Yang, et al., 2010]
- Gaussian blur followed by BI [Freeman and Liu, 2011]
- Simple pixel averaging [Fattal, 2007]
- .......
Two important works

- An accurate blur model is critical to the success of SISR algorithms [Efrat et al., 2013]
- The PSF of camera is the wrong blur kernel from the LR image [Michaeli and Irani, 2013]

Both seek accurate blur kernels based on existing SISR algorithms, thus their complexities are even more than those of the SISR ones.
Single blind image super-resolution (SBISR)

\[ y = DBx + n \]

If \( B \) is unknown, then SISR becomes the single blind image super-resolution (SBISR).

Only a few works dedicated to the SBISR problem, have restrictive assumptions on the blur kernel:

- A parametric Gaussian model with unknown width [Begin and Ferrie, 2004; Qiao, et al., 2006; Wang, et al., 2005]
- Multiple parametric models [He, et al., 2009]
- A nonparametric model assuming the kernel has a single peak [He, et al., 2009]
In this paper

We address the SBISR problem via a blind image deblurring (BID) method, bridge the gap between SBISR and BID, benefit from that some BID methods are arguably faster and easier to understand, than state-of-the-art SISR/SBISR methods, and reach competitive speed and restoration quality.
SBISR and BID

SBISR:
recover a HR image $x \in \mathbb{R}^{Nh}$ from a LR image $y \in \mathbb{R}^{N_l}$

\[ y = DBx + n \]

$D: \mathbb{R}^{Nh} \rightarrow \mathbb{R}^{N_l} (N_l < N_h)$ is the downsampling matrix

$B: \mathbb{R}^{Nh} \rightarrow \mathbb{R}^{Nh}$ is the blurring matrix

$n \in \mathbb{R}^{N_l}$ is the additive noise

BID:
recover a sharp image $x \in \mathbb{R}^{Nh}$ from a blurry image $z \in \mathbb{R}^{Nh}$

\[ z = Bx + s \]

$B: \mathbb{R}^{Nh} \rightarrow \mathbb{R}^{Nh}$ is the blurring matrix

$s \in \mathbb{R}^{Nh}$ is the additive noise
SBISR and BID

With the same $B$ and $x$

BID: $z = Bx + s \iff Bx = z - s$

$\downarrow$

SBISR: $y = DBx + n$

$\downarrow$

$y = D(z - s) + n$

$= Dz + (n - Ds)$

Due to the introduce of $D$, the length of $y$ is less than that of $z$, namely, $y$ has fewer known samples than $z$.

we can solve the SBISR problem in an easier way via reformulating it into a BID problem.
Reformulating SBISR into BID

SBISR: \( y = DBx + n \)

The idea is to first interpolate the LR image \( y \in \mathbb{R}^{N_l} \) as \( u \in \mathbb{R}^{N_h} \)

\[ u = Uy = UDBx + Un \]

\( U: \mathbb{R}^{N_l} \rightarrow \mathbb{R}^{N_h} \) is the interpolation operator
(e.g. bicubic or bilinear)

The resulting BID: \( u = Kx + e \)

\( K = UDB: \mathbb{R}^{N_h} \rightarrow \mathbb{R}^{N_h} \) is the new blurring matrix

\( e \in \mathbb{R}^{N_h} \) is the interpolation of \( n \)

Instead of super-resolving \( x \) from \( y \), the HR image can be obtained via blind deblurring \( x \) from \( u \).
The resulting BID problem

The regularization problem

\[
(\hat{x}, \hat{k}) = \arg \min_{x, k} \frac{\lambda}{2} \|Kx - u\|_2^2 + \phi_{\text{GTV}}(x) + \iota_{\mathcal{S}}(k)
\]

\[
\phi_{\text{GTV}}(x) = \sum_i \left| [D_h x]_i \right|^p + \left| [D_v x]_i \right|^p, 0 \leq p \leq 1
\]

\(D_h\) and \(D_v\) denote the horizontal and vertical derivative operator, respectively.

\(\iota_{\mathcal{S}}\) is the indicator of the set \(\mathcal{S}\) which is defined as

\[
\mathcal{S} = \{k: k \geq 0, \|k\|_1 = 1\}
\]
The algorithmic framework

**Algorithm Proposed algorithmic framework**

1. **Input:** Observed LR image $y$, $\lambda$ and $\alpha > 1$.
2. **Step I:** Interpolate $y$ via $u = Uy$.
3. **Step II:** Blind estimation of blur filter $k$ from $u$, by alternative loop over coarse-to-fine levels:
   4. ▶ Update the image estimate
      \[
      \hat{x} \leftarrow \arg \min_x \frac{\lambda}{2} \left\| \hat{K}x - u \right\|_2^2 + \phi_{GTV}(x) \tag{8}
      \]
      where $\hat{K}$ is the convolution matrix constructed by $\hat{k}$ obtained from the blur filter estimation below.
   5. ▶ Update the blur filter estimate
      \[
      \hat{k} \leftarrow \arg \min_k \frac{\lambda}{2} \left\| \hat{X}k - u \right\|_2^2 + \iota_S(k) \tag{9}
      \]
      where $\hat{X}$ is the convolution matrix constructed by $\hat{x}$ obtained from the image estimation above.
   6. ▶ Increase the parameter $\lambda$
      \[
      \lambda \leftarrow \alpha \lambda. \tag{10}
      \]
4. **Step III:** Non-blind estimation of HR image $x^*$ from $u$ through solving (8) with final $\hat{k}$ (obtained by Step II).
5. **Output:** the HR image $x^*$ and the blur estimate $\hat{k}$.

Can be efficiently solved by alternating direction method of multipliers (ADMM)
The alternating direction method of multipliers (ADMM)  
[Gabay and Mercier, 1976; Boyd et al., 2011; Almeida and Figueiredo, 2013]  

ADMM has been as a popular tool to solving imaging inverse problems

\[ \min_x \sum_{j} g_j(B^{(j)}x) \]  \hspace{1cm} (11)

**Algorithm ADMM for solving (11)**

1. Set \( k = 0, \beta > 0, v^{(1)}_0, \ldots, v^{(j)}_0, d^{(1)}_0, \ldots, d^{(j)}_0 \).
2. repeat
3. \( r_k = \sum_{j=1}^{J} (B^{(j)})^T (v^{(j)}_k + d^{(j)}_k) \)
4. \( x_{k+1} = \left[ \sum_{j=1}^{J} (B^{(j)})^T B^{(j)} \right]^{-1} r_k \)
5. for \( j = 1, \ldots, J \)
6. \( v^{(j)}_{k+1} = \text{Prox}_{g_j/\tau} \left( B^{(j)} x_{k+1} - d^{(j)}_k \right) \)
7. \( d^{(j)}_{k+1} = d^{(j)}_k - (B^{(j)} x_{k+1} - v^{(j)}_{k+1}) \)
8. end for
9. \( k \leftarrow k + 1 \)
10. until some stopping criterion is satisfied.

In line 6 of above algorithm, the proximity operator of \( g_j/\tau \): \( \text{Prox}_{g_j/\tau} \) is defined as

\[ \text{Prox}_{g_j/\tau} (v) = \arg \min_x \left( g_j (x) + \frac{\tau}{2} \| x - v \|^2 \right) . \]  \hspace{1cm} (12)
x update using the ADMM

\[
\hat{x} \leftarrow \arg \min_x \frac{\lambda}{2} \| \hat{K} x - u \|_2^2 + \phi_{GTV}(x) \quad (8)
\]

\[
g_1(\cdot) = \frac{\lambda}{2} \| \cdot - u \|_2^2, \quad g_2(\cdot) = g_3(\cdot) = \| \cdot \|_p^p,
\]

\[
B^{(1)} = \hat{K}, \quad B^{(2)} = D_h, \quad B^{(3)} = D_v
\]

Algorithm ADMM for solving (8)
1. Initialize \( k = 0, \tau_1 > 0, v_0^{(1)}, v_0^{(2)}, v_0^{(3)}, d_0^{(1)}, d_0^{(2)}, d_0^{(3)}. \)
2. repeat
3. \( z_k^{(1)} = v_k^{(1)} + d_k^{(1)} \)
4. \( z_k^{(2)} = v_k^{(2)} + d_k^{(2)} \)
5. \( z_k^{(3)} = v_k^{(3)} + d_k^{(3)} \)
6. \( r_k = \hat{K}^T z_k^{(1)} + D_h^T z_k^{(2)} + D_v^T z_k^{(3)} \)
7. \( x_{k+1} = \left[ \hat{K}^T \hat{K} + D_h^T D_h + D_v^T D_v \right]^{-1} r_k \)
8. \( v_{k+1}^{(1)} = \operatorname{Prox}_{g_1/\tau_1} \left( \hat{K} x_{k+1} - d_k^{(1)} \right) \)
9. \( d_{k+1}^{(1)} = d_k^{(1)} - (\hat{K} x_{k+1} - v_{k+1}^{(1)}) \)
10. \( v_{k+1}^{(2)} = \operatorname{Prox}_{g_2/\tau_1} \left( D_h x_{k+1} - d_k^{(2)} \right) \)
11. \( d_{k+1}^{(2)} = d_k^{(2)} - (D_h x_{k+1} - v_{k+1}^{(2)}) \)
12. \( v_{k+1}^{(3)} = \operatorname{Prox}_{g_3/\tau_1} \left( D_v x_{k+1} - d_k^{(3)} \right) \)
13. \( d_{k+1}^{(3)} = d_k^{(3)} - (D_v x_{k+1} - v_{k+1}^{(3)}) \)
14. \( k \leftarrow k + 1 \)
15. until some stopping criterion is satisfied.
**k update using the ADMM**

\[
\hat{k} \leftarrow \arg \min_k \frac{\lambda}{2} \| \hat{X}k - u \|^2_2 + \nu_S(k) \tag{9}
\]

\[
g_1(\cdot) = \frac{\lambda}{2} \| \cdot - u \|^2_2, \quad g_2(\cdot) = \nu_S(\cdot),
\]

\[
B^{(1)} = \hat{X}, \quad B^{(2)} = I,
\]

**Algorithm ADMM for solving (9)**

1. Initialize \( k = 0, \tau_2 > 0, v_0^{(1)}, v_0^{(2)}, d_0^{(1)}, d_0^{(2)}. \)
2. repeat
3. \( z_k^{(1)} = v_k^{(1)} + d_k^{(1)} \)
4. \( z_k^{(2)} = v_k^{(2)} + d_k^{(2)} \)
5. \( r_k = \hat{X}^T z_k^{(1)} + z_k^{(2)} \)
6. \( k_{k+1} = \left[ \hat{X}^T \hat{X} + I \right]^{-1} r_k \)
7. \( v_{k+1}^{(1)} = \text{Prox}_{g_1/\tau_2} \left( \hat{X}k_{k+1} - d_k^{(1)} \right) \)
8. \( d_{k+1}^{(1)} = d_k^{(1)} - (\hat{X}k_{k+1} - v_{k+1}^{(1)}) \)
9. \( v_{k+1}^{(2)} = \text{Prox}_{g_2/\tau_2} \left( k_{k+1} - d_k^{(2)} \right) \)
10. \( d_{k+1}^{(2)} = d_k^{(2)} - (k_{k+1} - v_{k+1}^{(2)}) \)
11. \( k_{k+1} \leftarrow k + 1 \)
12. until some stopping criterion is satisfied.
On synthetic blurry images

Test the baby image (size: 512×512) blurred by eight PSFs provided by [Levin et al., 2009]. In the algorithm, the operator $U$ here has two options:

(a) PSNR (dB): 18.008  
(b) PSNR (dB): 20.861

Figure 1. Estimated HR images, PSFs and PSNRs. (a) are input LR blurry image (size: 256 × 256, obtained by (1)) and one of the eight PSFs (corresponding to $B$ in (1)); (b) and (c) are estimated HR images, PSFs (corresponding to $K$ in (5)) and PSNRs by the proposed method with the bicubic and bilinear interpolation operators, respectively.
On synthetic blurry images

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<th>Other seven PSFs</th>
<th>Estimated PSFs</th>
<th>PSNR (dB)</th>
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Figure 2. Other seven PSFs and their corresponding estimated PSFs and PSNRs by the proposed method with the *bicubic* and *bilinear* operators, respectively.
On real images

![Image of real images](image)

- LR blurry
- ScSR, 5424 sec.
- ScSR+BID, 5501 sec.
- SRCNN, 230 sec.
- SRCNN+BID, 306 sec.
- Proposed, 78 sec.

Figure 3. Results on a real LR blurry image (size: 900 × 540).
On real images


Figure 4. Results on a real LR image (size: $324 \times 464$).
Conclusions

• Have proposed a new approach for single blind image super-resolution (SBISR) via a blind image deblurring (BID) method, bridging the gap between SBISR and BID, benefitting from that some BID methods are arguably faster and easier to understand, than state-of-the-art SISR/SBISR methods, and reaching competitive speed and restoration quality.

• Experiments on synthetic and real images show that the effectiveness and competitiveness of the proposed method.

Thanks for your attention!