Lossless Data Compression
Standard Applications and the MapReduce Web Computing Framework

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Internet as a Distributed System

Modern distributed systems may nowadays consist of hundreds of thousands of computers.

Decentralization is the fundamental approach to reach unprecedented scales, towards millions or even billions of elements using Internet as an extreme distributed system.
Web Computing

Web Computing is a special kind of distributed computing and involves internet-based collaboration of an arbitrary number of remotely located applications.

MapReduce is the framework proposed by Google to realize distributed applications on the Internet, making distributed computing more accessible.
The MapReduce Framework

MapReduce (MR) is a framework for processing parallelizable problems across huge datasets using a large number of computers (nodes).

Nodes could be either on the same local network or shared across geographically and administratively more heterogenous distributed systems.
The MR Programming Paradigm

\[ P = \mu_1 \rho_1 \ldots \mu_R \rho_R \] is a sequence where \( \mu_i \) is a mapper and \( \rho_i \) is a reducer for \( 1 \leq i \leq R \).

Mapper \( \mu_i \) receives the input from reducer \( \rho_{i-1} \) for \( 1 < i \leq R \).

For \( 1 \leq i \leq R \), such input is a multiset \( U_{i-1} \) of \((\text{key}, \text{value})\) pairs (\( U_0 \) is given) and each pair \((k, v) \in U_{i-1}\) is mapped by \( \mu_i \) to a set of new \((\text{key}, \text{value})\) pairs.
The MR Programming Paradigm (2)

The input to reducer $\rho_i$ is $U_i'$, where $U_i'$ is the union of the sets output by mapper $\mu_i$.

For each key $k$, $\rho_i$ reduces the subset of pairs with $k$ as key component to a new set of pairs with key still equal to $k$.

$U_i$ is the union of these new sets and is output by $\rho_i$ as input to $\mu_{i+1}$. 
Distributed Implementation

A key is associated with a processor.

Each pair with a given key is processed by the same node.

More keys can be associated with the same node in order to lower the scale of the distributed memory system of nodes involved in the Web computer network.
Input/Output Phases

The sequence $P = \mu_1 \rho_1 \ldots \mu_R \rho_R$ does not include the I/O phases ($\mu_1$ can be seen as a second step of the input phase).

Extend $P$ to $\mu_0 \mu_1 \rho_1 \ldots \mu_R \rho_R \mu_{R+1} \rho_{R+1}$.

$\mu_0 \mu_1$ is the input phase.

$\mu_{R+1} \rho_{R+1}$ is the output phase.
Input Phase

There is a unique \((key, value)\) pair input to \(\mu_0\), where the value component is the input data.

\(\mu_0\) distributes the data generating \(U_0\).

\(\mu_1\) distributes the data furtherly or is the identity function.
Output Phase

\( \mu_{R+1} \) maps the multiset \( U_R \) to a multiset where all the pairs have the same key \( k \).

\( \rho_{R+1} \) reduces such multiset to the unique pair \( (k, y) \).

\( y \) is the output data.
Complexity Requirements

- R is polylogarithmic in the input size
- sublinear number of nodes
- sub-linear work-space for each node
- mappers running time is polynomial
- reducers running time is polynomial

Karloff, Suri and Vassilvistkii, A Model of Computation for MapReduce, SODA 2010
Stronger Requirements

R is constant (< 10).

Total running time of mappers and reducers times the number of processors must be equal to sequential time.

The MR implementations of Standard Lossless Data Compression satisfy these stronger requirements with R < 3 and guarantee a linear speed-up.
Lossless Data Compression

Standard lossless data compression applications are based on Lempel-Ziv methods (Zip compressors or Unix Compress and Dos Stuffit).

The Zip family is based on sliding window (SW) compression.

Unix Compress and Dos Stuffit implement the Lempel, Ziv and Welch method (LZW compression).
SW Factorization and Compression

**SW factorization:** string $S = f_1 f_2 \ldots f_i \ldots f_k$ where $f_i$ is either the longest substring occurring previously in $f_1 f_2 \ldots f_i$ or an alphabet character.

**SW compression** is obtained by encoding $f_i$ with a pointer to a previous copy.

**Bounded memory factorization:** $f_i$ is determined by the last $w$ characters ($w=32K$ in the Zip compressor).
LZW Factorization and Compression

Finite alphabet $A$, dictionary $D$ initially equal to $A$, $S$ in $A^*$. 

LZW factorization: $S = f_1 f_2 \ldots f_i \ldots f_k$ where $f_i$ is the longest match with the concatenation of a previous factor and the next character.

LZW compression is obtained by encoding $f_i$ with a pointer to a copy of such concatenation previously added to $D$. 
Bounded Memory Factorizations

**LZW-FREEZE:** $f_i$ is the longest match with one of the first $d$ factors ($d=64\text{K}$).

**LZW-RESTART:** $f_i$ is determined by the last $(i \mod d)$ factors.

To improve LZW-RESTART the dictionary of $d$ factors is frozen and the restart operation happens when the compression ratio starts deteriorating.
Distributed Memory SW Factorization

The SW factorization process is applied independently to different blocks of the input string (no communication cost).

A block of 300kB guarantees robustness (about ten times the window length of a Zip compressor).

1000 processors $\rightarrow$ 300 megabytes
The MapReduce Implementation

Input $X$; MR sequence $\mu_0 \mu_1 \rho_1 \mu_2 \rho_2$; $R=1$.

- $\mu_0$ maps $(0, X)$ to $U_0 = \{(1, X_1), \ldots (m, X_m)\}$
- $\mu_1$ is the identity function ($U_0' = U_0$)

- $\rho_1$ reduces $U_0'$ to $U_1 = \{(1, Y_1), \ldots (m, Y_m)\}$ where $Y_i$ is the SW coding of $X_i$, $1 \leq i \leq m$

- $\mu_2$ maps $U_1$ to $U_1' = \{(0, Y_1), \ldots (0, Y_m)\}$
- $\rho_2$ reduces $U_1'$ to $(0, Y)$ where $Y = Y_1 \ldots Y_m$ ($Y$ is the output).
Distributed Memory LZW Factorization

If LZW factorization is applied in parallel, blocks of 600 kB are needed to guarantee robustness (300 kB to learn the dictionary and 300 kB to compress statically).

Static compression is extremely scalable.

According to the scale of the network, different approaches can be described.

The MapReduce implementations require two iterations (R=2).
LZW Static Optimal Factorization

The *dictionary* of $d$ factors built by the LZW algorithm is *prefix*, that is, every prefix of a factor is in the dictionary.

Since the dictionary is not *suffix*, the greedy factorization of the input string after the dictionary has been *frozen* is not optimal.
Greedy vs Optimal: Example

**string:** \(aba^n\)

**dictionary**
\[a, b, ab, ba^k \text{ for } 0 < k < n+1\]

**greedy factorization**
\[ab, a, a, \ldots, a\]

**optimal factorization**
\[a, ba^n\]
Semi-Greedy (Optimal) Factorization

$x_{t+1}, x_{t+2}, \ldots$ is the suffix of the input string after the dictionary has been frozen

\[ j = t; i := t \]

repeat forever

\[ \text{for } k = j + 1 \text{ to } i + 1 \text{ compute} \]

\[ h(k) : x_k \ldots x_{h(k)} \text{ is the longest match} \]

\[ \text{let } k' : h(k') \text{ is maximum} \]

\[ x_j \ldots x_{k'-1} \text{ is a factor} \]

\[ j = i+1; i = h(k') \]
Distributed Approximation Scheme

Processors store blocks of length $m(k+2)$ overlapping on $2m$ characters where $m$ is the maximum factor length.

Each processor computes the boundary matches ending furthest to the right.

Proc. 1 2 3 4 5 6 7 etc.
Distributed Approximation Scheme (2)

Each processor applies the semi-greedy procedure from the first position of the match computed on the left boundary of its block to the position of the match on the right boundary.

Stopping the parsing at the beginning of the match on the right boundary might create for each block a surplus phrase with respect to the optimal parsing of the string (this might happen when the last factor could be extended to cover the right boundary). Therefore, the approximation factor is \((k+1)/k\).
Distributed Approximation Scheme (3)

If $k \geq 10$, there is no relevant loss of compression effectiveness with respect to the optimal factorization.

The approximation scheme is robust on large scale and extreme distributed systems since $m \approx 10$.

Greedy compares to optimal in practice.

If $k \geq 100$, there is no need of computing boundary matches.
The MapReduce Implementation

MR sequence $\mu_0\mu_1\rho_1\mu_2\rho_2\mu_3\rho_3$; $R=2$.

**Input Phase** $\mu_0\mu_1$ (input string $X$)

$\mu_0$ maps $(0, X)$ to $U_0 = \{(1, X_1), \ldots, (m, X_m)\}$

- $X_i = Y_i Z_i$ with $|Y_i| = |Z_i| \geq 300K$ for $1 \leq i \leq m$
- $Z_i = B_{i,1} \ldots B_{i,r}$ with $|B_{i,j}| = c \geq 1000$ for $1 \leq j \leq r$

$\mu_1$ maps $U_0$ to $U_0' = \{(i, Y_i), ((i,j), B_{i,j}) \}$ for $1 \leq i \leq m$, $1 \leq j \leq r$
The MapReduce Implementation (2)

MR sequence $\mu_0\mu_1\rho_1\mu_2\rho_2\mu_3\rho_3$

**Computational phase** $\rho_1\mu_2\rho_2$

**first step**

$\rho_1$ reduces $U_0'$ to $U_1 = \{ (i, C_i), (i, D_i), ((i,j), B_{i,j}) \text{ for } 1 \leq i \leq m, 1 \leq j \leq r \}$.

$C_i$ is the LZW coding of $Y_i$ and $D_i$ is the dictionary learned during the coding process for $1 \leq i \leq m$. 
The MapReduce Implementation (3)

MR sequence $\mu_0 \mu_1 \rho_1 \mu_2 \rho_2 \mu_3 \rho_3$

Computational phase $\rho_1 \mu_2 \rho_2$

second step

$\mu_2$ maps $U_1$ to $U_1' = \{ (i, C_i), (i, j), D_i), (i, j), B_{i,j}) for \ 1 \leq i \leq m, 1 \leq j \leq r \}.$

$\rho_2$ reduces $((i,j), B_{i,j})$ to $((i,j), C_{i,j})$ with $C_{i,j}$ compressed form of $B_{i,j}$ using $D_i$ as a static dictionary to produce $U_2.$
The MapReduce Implementation (4)

MR sequence $\mu_0\mu_1\rho_1\mu_2\rho_2\mu_3\rho_3$

**Output phase** $\mu_3\rho_3$

$\mu_3$ maps $U_2$ to $U_2' = \{(0, C_i), (0, C_{i,j}) \text{ for } 1 \leq i \leq m, 1 \leq j \leq r\}$.

$\rho_3$ reduces $U_2'$ to $(0, C = C_1C_{1,1}...C_{1,r}...C_mC_{m,1}...C_{m,r})$ where $C$ is the output.
The central node distributes blocks of 600kB (the first half of 300kB to the adjacent processors and a sub-block of the second half to a leaf).
Robustness and Communication

Low communication cost is required between the two phases of the computation.

If the input phase broadcasts sub-blocks of size between 100 bytes and one kylobyte the computation of boundary matches enhance robustness (extreme case beyond large scale).
Beyond Large Scale

**Input Phase** \( \mu_0 \mu_1 \) (input string \( X \))

\( \mu_0 \) maps \((0, X)\) to \( U_0 = \{(1, X_1), \ldots, (m, X_m)\} \)

\( X_i = Y_i Z_i, \quad Z_i = B_{i,1} \ldots B_{i,r} \) with \( 100 \leq |B_{i,j}| \ll 1000 \)

\( \mu_1 \) maps \( U_0 \) to \( U_0' = \{(i, Y_i), (i,j), B_{i,j}'\} \) for \( 1 \leq i \leq m, 1 \leq j \leq r \)

\( B_{i,j}' \) extends \( B_{i,j} \) by overlapping \( B_{i-1,j} \) and \( B_{i+1,j} \) on \( m \) characters.
Compressing Large Size Files

Scalability, robustness and communication are not an issue with SW and LZW compression of very large size files (gigabytes or beyond).

Bounding the dictionary of the LZW compressor with a relaxed version of the least recently used strategy has been proposed as the most efficient approach.

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Future Work

The main goal is the design of a robust and scalable approach for lossless compression of arbitrary size files.

Such goal can be achieved experimentally for a specific type of files by decreasing the dictionary size.

A more ambitious project is the design of a new general procedure for distributed memory compression.
Thank you