Call-level Performance Analysis of Wired and Wireless Networks

TUTORIAL

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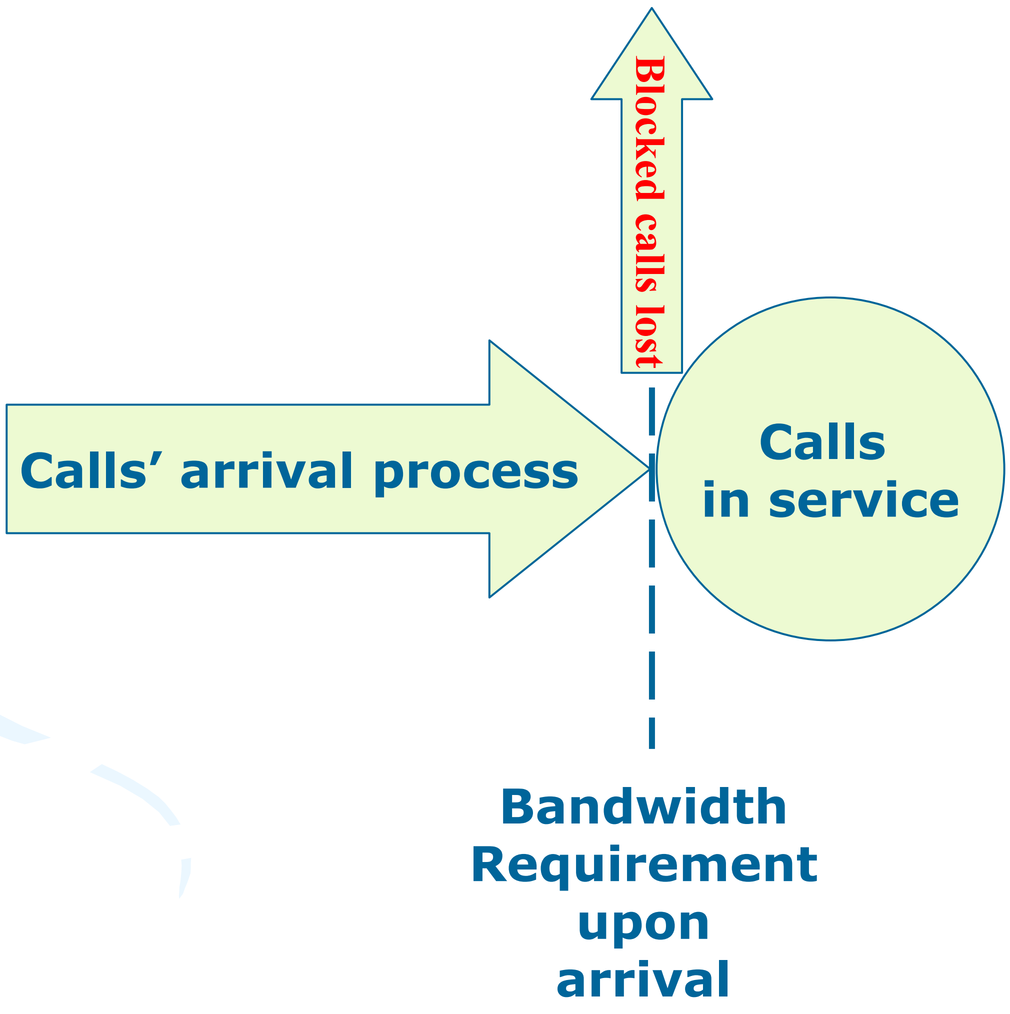
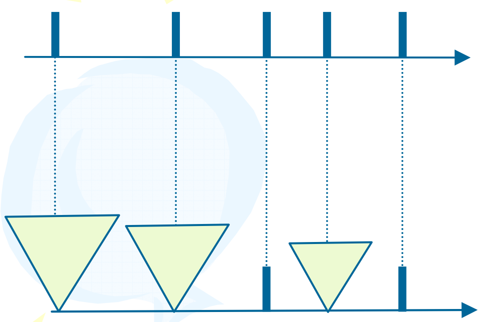
Preamble

A Loss Service System



Preamble (cont.1)

Call Arrival Process

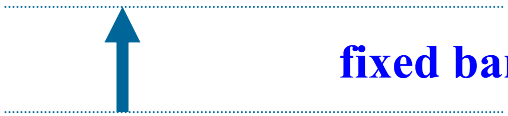


- **Random arrivals – traffic *(infinite number of traffic sources).***
- **Quasi-random arrivals – traffic *(finite number of traffic sources).***

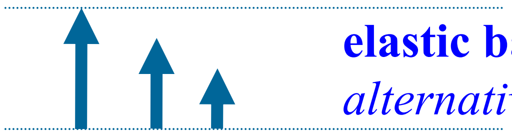
- **Batch Poisson arrivals (*infinite number of traffic sources*).**
  Calls from different service-classes arriving in batches, while batches arriving randomly.

Preamble (cont.2)

Bandwidth requirement upon call arrival



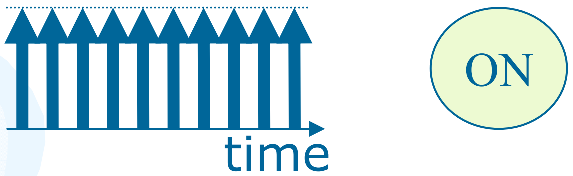
fixed bandwidth



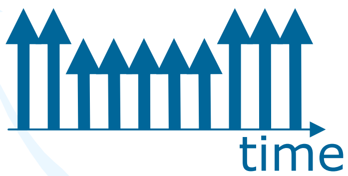
elastic bandwidth: *calls have several, alternative, contingency bandwidth requirements*

Preamble (cont.3)

Call's behavior while in service



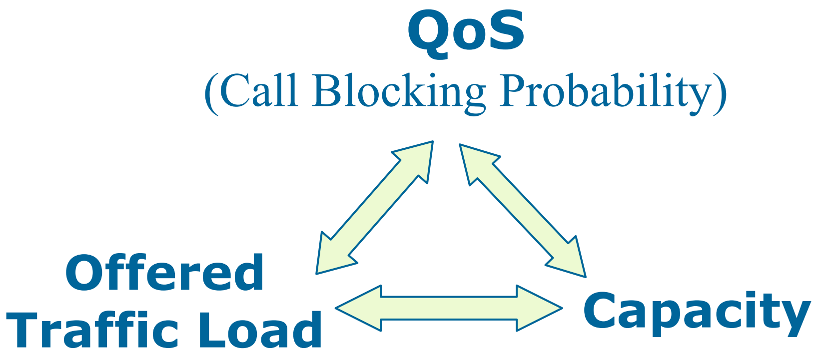
constant-bit-rate/stream traffic

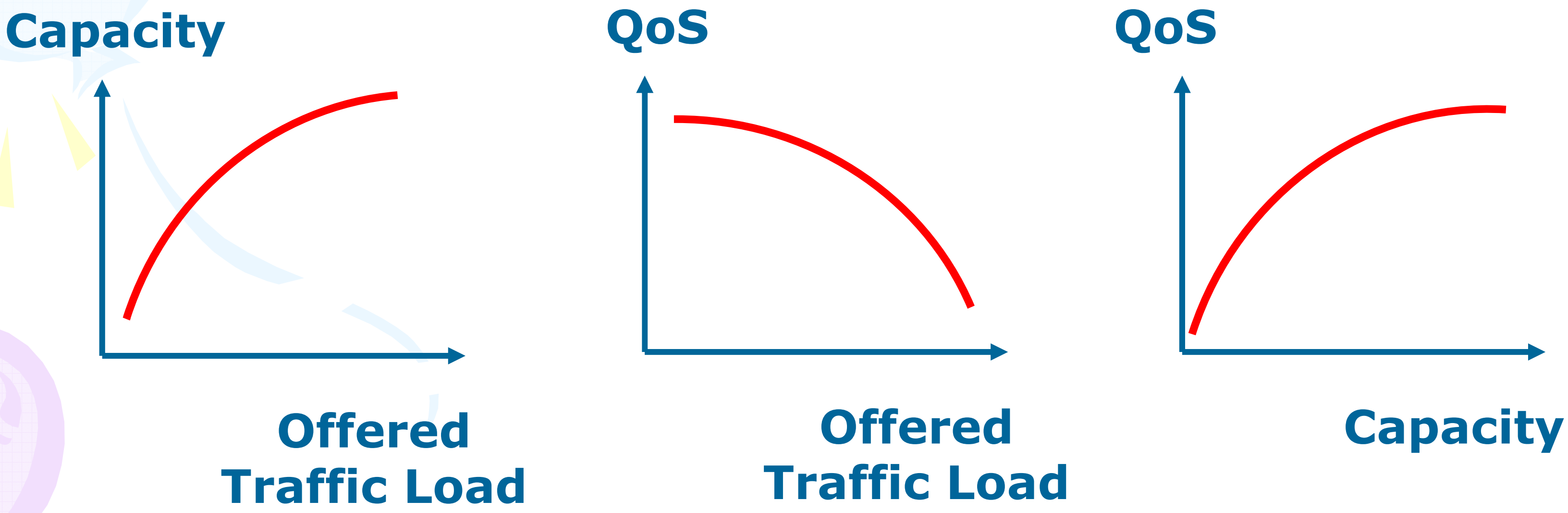


bandwidth compression/expansion

Preamble (cont.4)

Teletraffic (Loss) Models





Preamble (cont.5)

Teletraffic (Loss) Models

- **Importance of QoS assessment through teletraffic models:**
  - Bandwidth allocation among service-classes ➡ QoS Guarantee.
  - Avoidance of too costly over-dimensioning of the network.
  - Prevention of excessive network throughput degradation, through traffic engineering mechanisms.

- **A sine qua non of teletraffic loss models:**
  The efficient calculation of Call Blocking Probability ➡ Recursive formula

- **Applicability:**
  - Connection Oriented Communication Networks, in general.
  - IP based networks with resource reservation capabilities.
  - Cellular networks (e.g. UMTS).
  - All-optical core networks (MPλS/GMPLS).

STRUCTURE

Teletraffic Models for:

- **(A) Random Traffic**
- **(B) Quasi-random Traffic**
- **(C) Batched Poisson Traffic**

STRUCTURE (cont.1)

- **(A) Random Traffic**
  - **(A1) Random arriving calls with either fixed (certain) or elastic bandwidth requirements upon arrival, and constant use of the assigned bandwidth (constant-bit-rate/stream traffic) while in service.**
  - **(A2) Random arriving calls with either fixed or elastic bandwidth requirements upon arrival, and elastic bandwidth (compression/expansion) while in service.**

STRUCTURE (cont.2)

- **(B) Quasi-random Traffic**
  - **(B1) Quasi-random arriving calls with either fixed or elastic bandwidth requirements upon arrival, and constant use of the assigned bandwidth (constant-bit-rate/stream traffic) while in service.**
  - **(B2) Quasi-random arriving calls with either fixed or elastic bandwidth requirements upon arrival, and elastic bandwidth (compression/expansion) while in service.**

STRUCTURE (cont.3)

- **(C) Batched Poisson Traffic**
  - **(C1) Batched Poisson arriving calls with fixed bandwidth requirements and continuous use of the assigned bandwidth (constant-bit-rate/stream traffic) while in service.**
  - **(C2) Batched Poisson arriving calls with fixed bandwidth requirements upon arrival, and elastic bandwidth (compression/expansion) while in-service.**

STRUCTURE – Where We Are

We are here!

- **(A) Random Traffic**
  - **(A1) Constant-bit-rate/stream traffic**
  - **(A2) Elastic/adaptive traffic while in service**
- **(B) Quasi-random Traffic**
  - **(B1) Constant-bit-rate/stream traffic**
  - **(B2) Elastic/adaptive traffic while in service**
- **(C) Batched Poisson Traffic**
  - **(C1) Constant-bit-rate/stream traffic**
  - **(C2) Elastic/adaptive traffic while in service**

(A) Random Traffic

(A1) *Random arriving calls with either fixed (certain) or elastic bandwidth requirements upon arrival, and constant use of the assigned bandwidth (constant-bit-rate/stream traffic) while in service.*

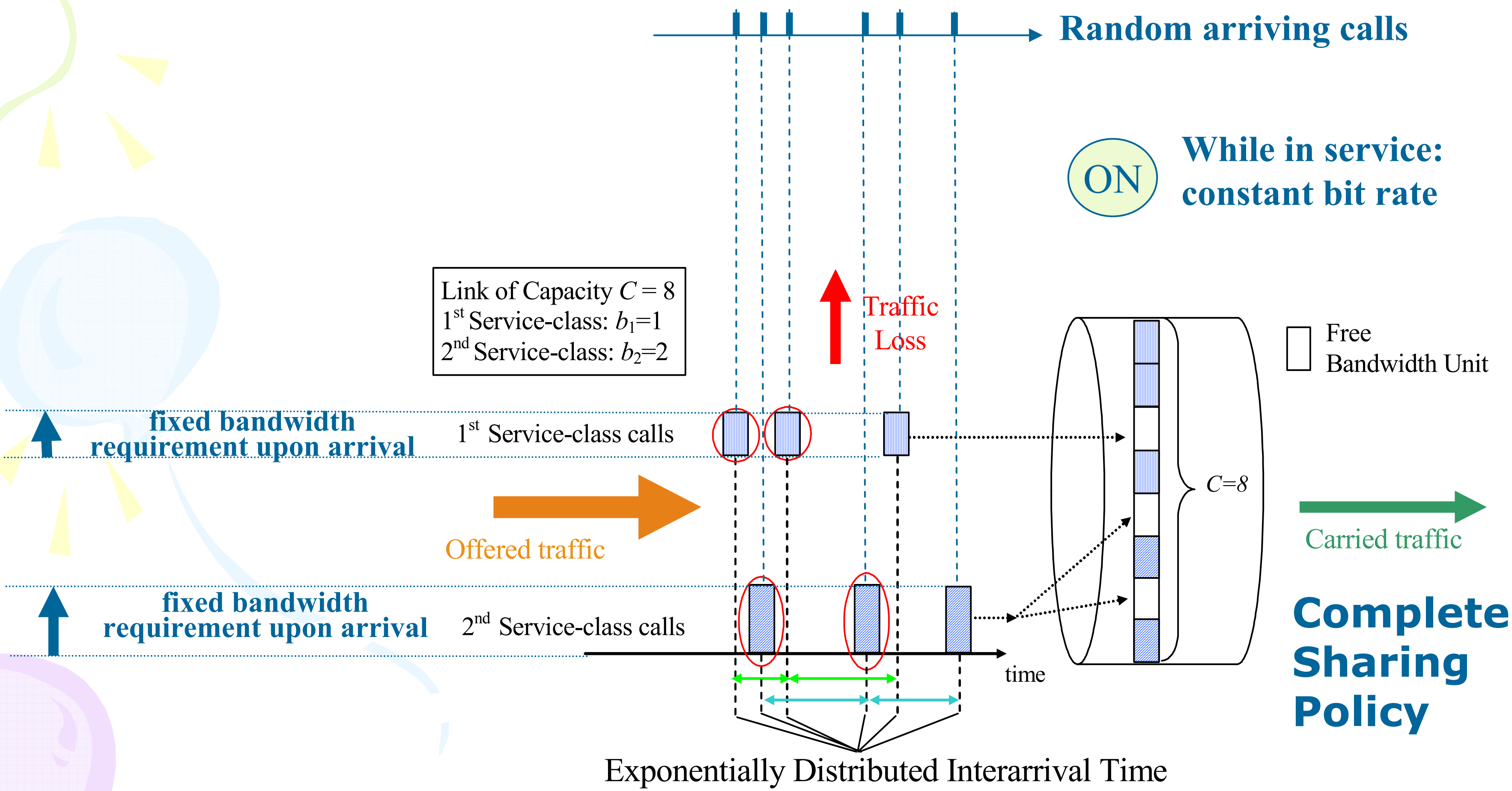
State of the art

- **The Erlang Multi-rate Loss Model (EMLM)** *1981*
- **The Retry Models** *1992*

Furthermore

- **The Connection Dependent Threshold Model (CDTM)** *2002*
- **The CDTM under the Bandwidth Reservation Policy** *2002*

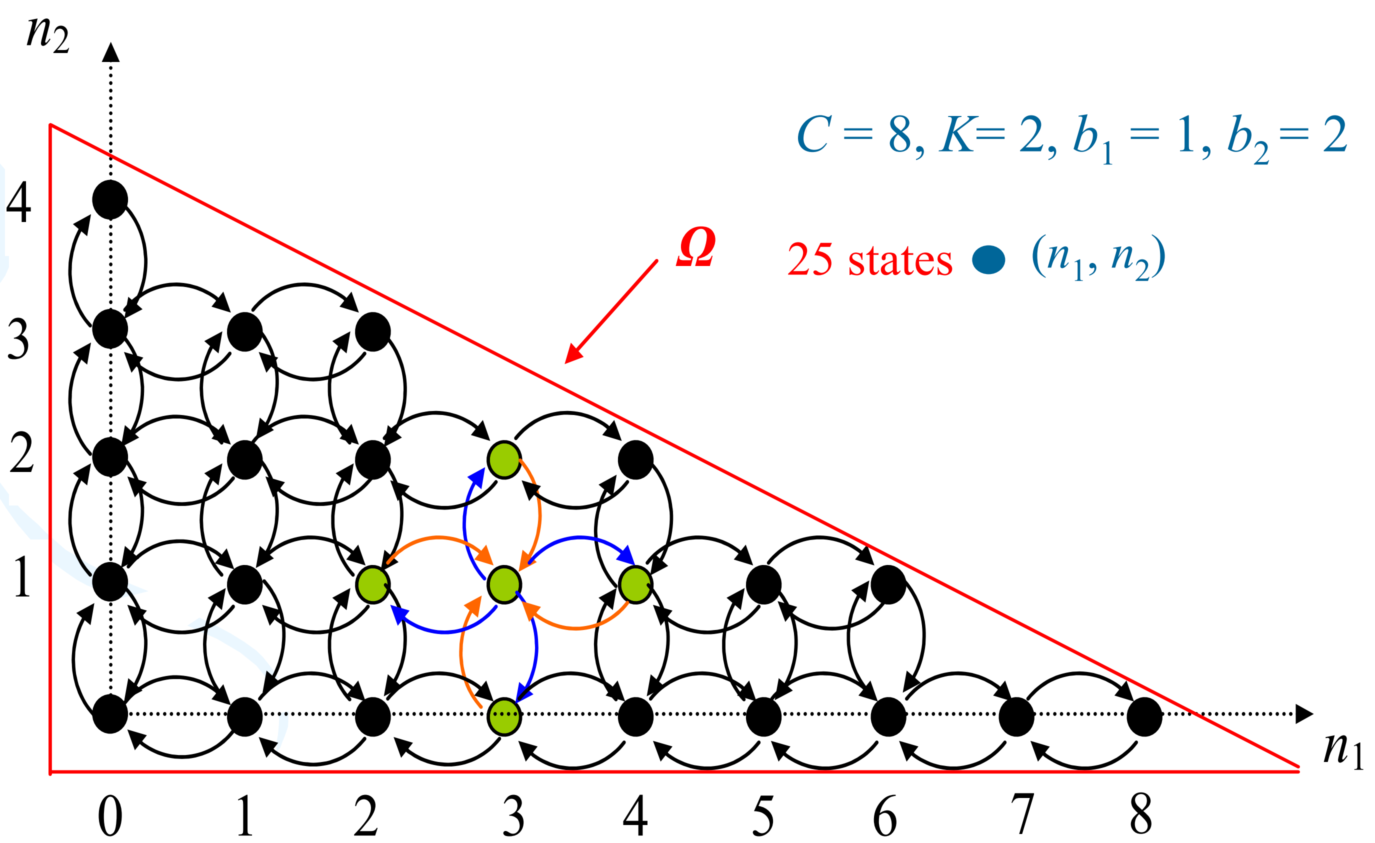
The Erlang Multi-rate Loss Model (EMLM)



Complete Sharing Policy

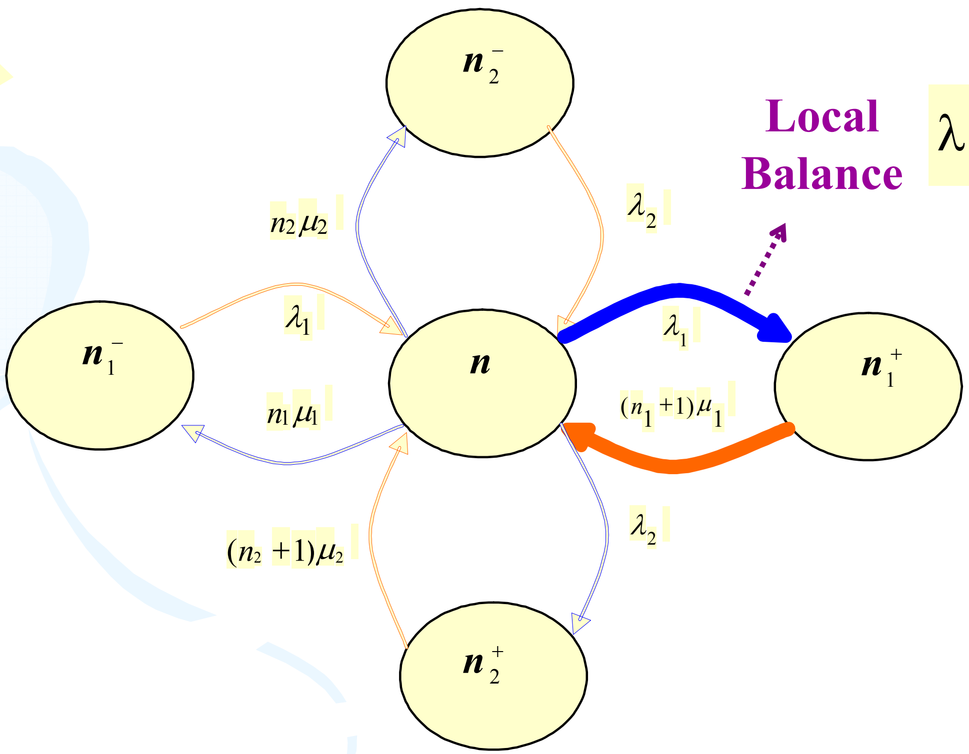
EMLM Analysis – Classical Method

State Space *Ω*
Complete Sharing Policy – A coordinate convex policy
Global Balance (rate_in=rate_out) - Statistical equilibrium



EMLM Analysis – Classical Method (cont.1)

Local Balance (Rate_up = rate_down)



$$\lambda_1 P(\boldsymbol{n}) = (n_1+1)\mu_1 P(\boldsymbol{n}_1^+)$$

λ: arrival rate (Poisson)
μ: service rate

EMLM Analysis – Classical Method (cont.2)

Product Form Solution

Product Form Solution of the State Probabilities

$$P(\boldsymbol{n}) = G^{-1}\left(\prod_{k=1}^{K}\frac{a_k^{n_k}}{n_k!}\right)$$

where $\boldsymbol{n} = (n_1, n_2, \ldots n_k, \ldots, n_K)$,

$\alpha_k = \lambda_k / \mu_k$ (offered traffic load, in erl)

$$G \equiv G(\Omega) = \sum_{\boldsymbol{n}\in\Omega}\left(\prod_{k=1}^{K}\frac{a_k^{n_k}}{n_k!}\right)$$

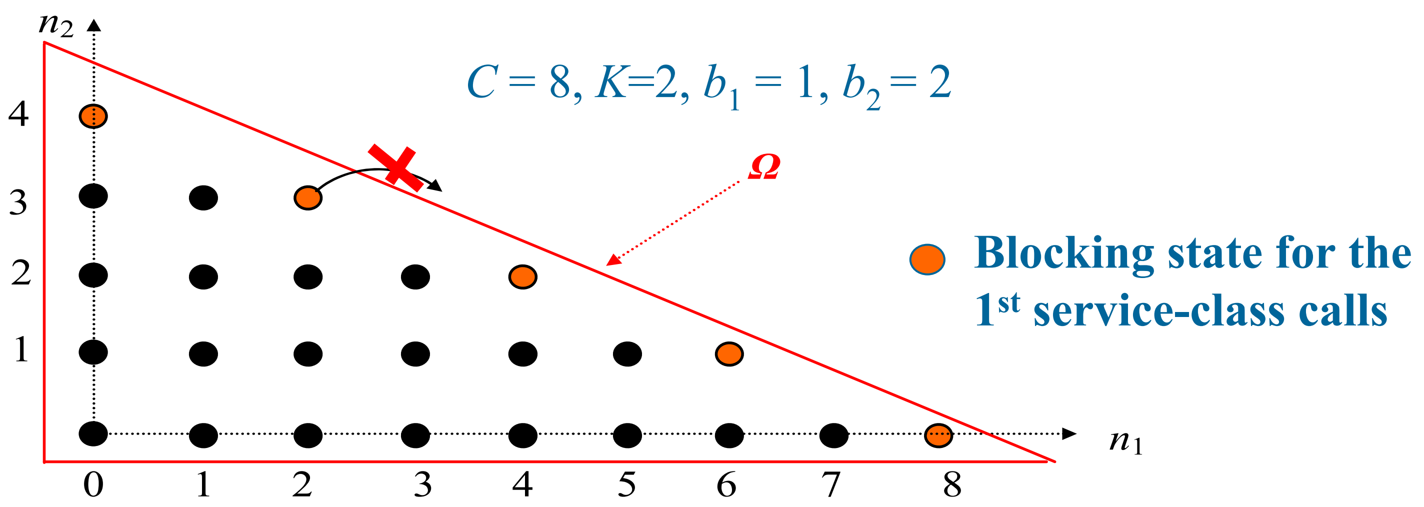
normalization constant

Product Form ⟺ Local Balance ⟺ Reversible Markov Chain

⇒ High accuracy in Call Blocking Probability calculation

EMLM Analysis – Classical Method (cont.3)

Call Blocking Probability Determination – Classical Method



Call Blocking Probability:

$$P_{b_k} = \sum_{n \in B_k^+} P(\boldsymbol{n})$$

$$B_k^+ = \{\boldsymbol{n} \in \Omega : \boldsymbol{n}_k^+ \notin \Omega\}$$

Remind: $P(\boldsymbol{n}) = G^{-1}\left(\prod_{k=1}^{K} \frac{a_k^{n_k}}{n_k!}\right)$ $\quad G \equiv G(\boldsymbol{\Omega}) = \sum_{\boldsymbol{n} \in \boldsymbol{\Omega}} \left(\prod_{k=1}^{K} \frac{a_k^{n_k}}{n_k!}\right)$

EMLM Analysis – Classical Method (cont.4)

Call Blocking Probability Determination – Classical Method

$K=2,\ b_1 = 1,\ b_2 = m$

Example of formulas for Call Blocking Probability Calculation

$$P_{b_1} = P_{00} \sum_{j=0}^{s} \frac{\alpha_1^{C-mj}}{(C-mj)!} \frac{\alpha_2^{j}}{j!}$$

$$P_{b_2} = P_{00} \left(\frac{\alpha_2^{2}}{s!} \sum_{i=0}^{k} \frac{\alpha_1^{i}}{i!} + \sum_{j=0}^{s-1} \sum_{i=C-mj-m+1}^{C-mj} \frac{\alpha_1^{i}}{i!} \frac{\alpha_2^{j}}{j!} \right)$$

where $k = C \pmod{m}$

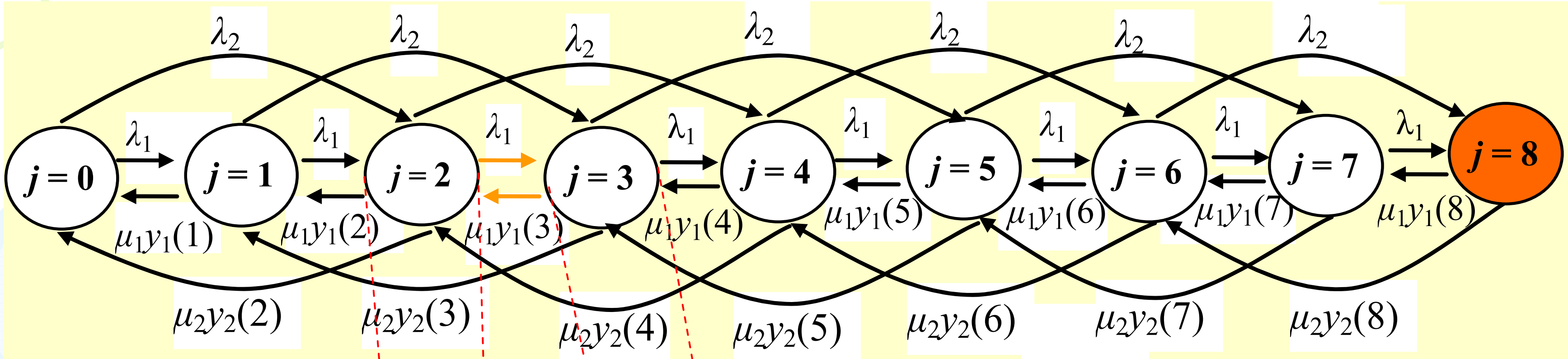
Necessity for recursive formulas

EMLM Analysis – Recursive formula

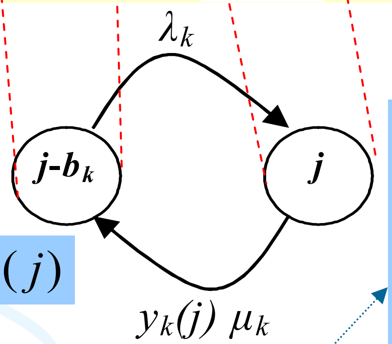
Kaufman, IEEE Trans. on Commun. 1981

Macro-states – One-dimensional Markov chain

$C = 8$, $K=2$, $b_1 = 1$, $b_2 = 2$ Macro-state $j=n_1 b_1+n_2 b_2$ denotes the occupied link bandwidth



local balance



$$\lambda_k q(j-b_k) = y_k(j)\mu_k q(j)$$

Link occupancy distribution

$$q(j) = \begin{cases} 1 & \text{for } j = 0 \\ \frac{1}{j}\sum_{k=1}^{K} \alpha_k b_k q(j-b_k) & \text{for } j = 1,\ldots,C \\ 0 & \text{otherwise} \end{cases}$$

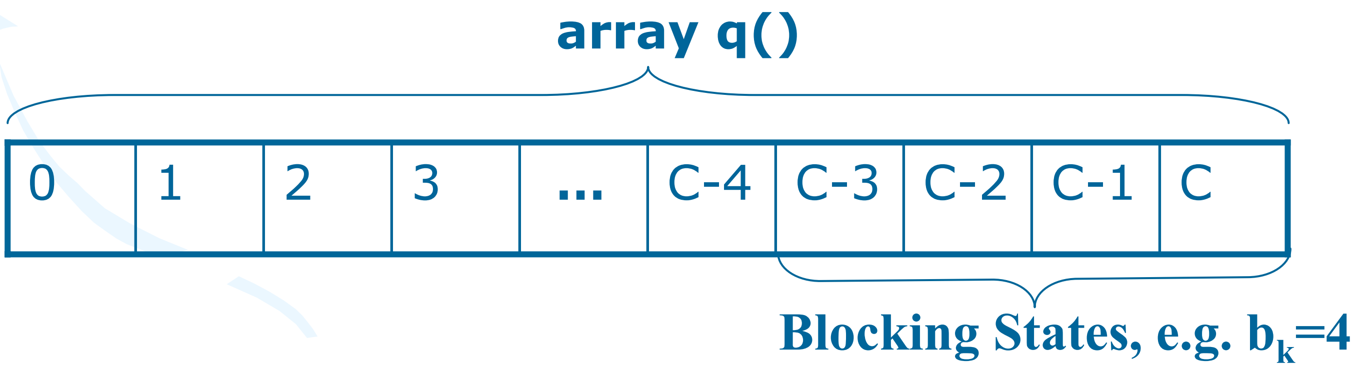
"Kaufman / Roberts Recursion"

EMLM Analysis – Recursive formula (cont.)

Call Blocking Probability – Recursive Calculation

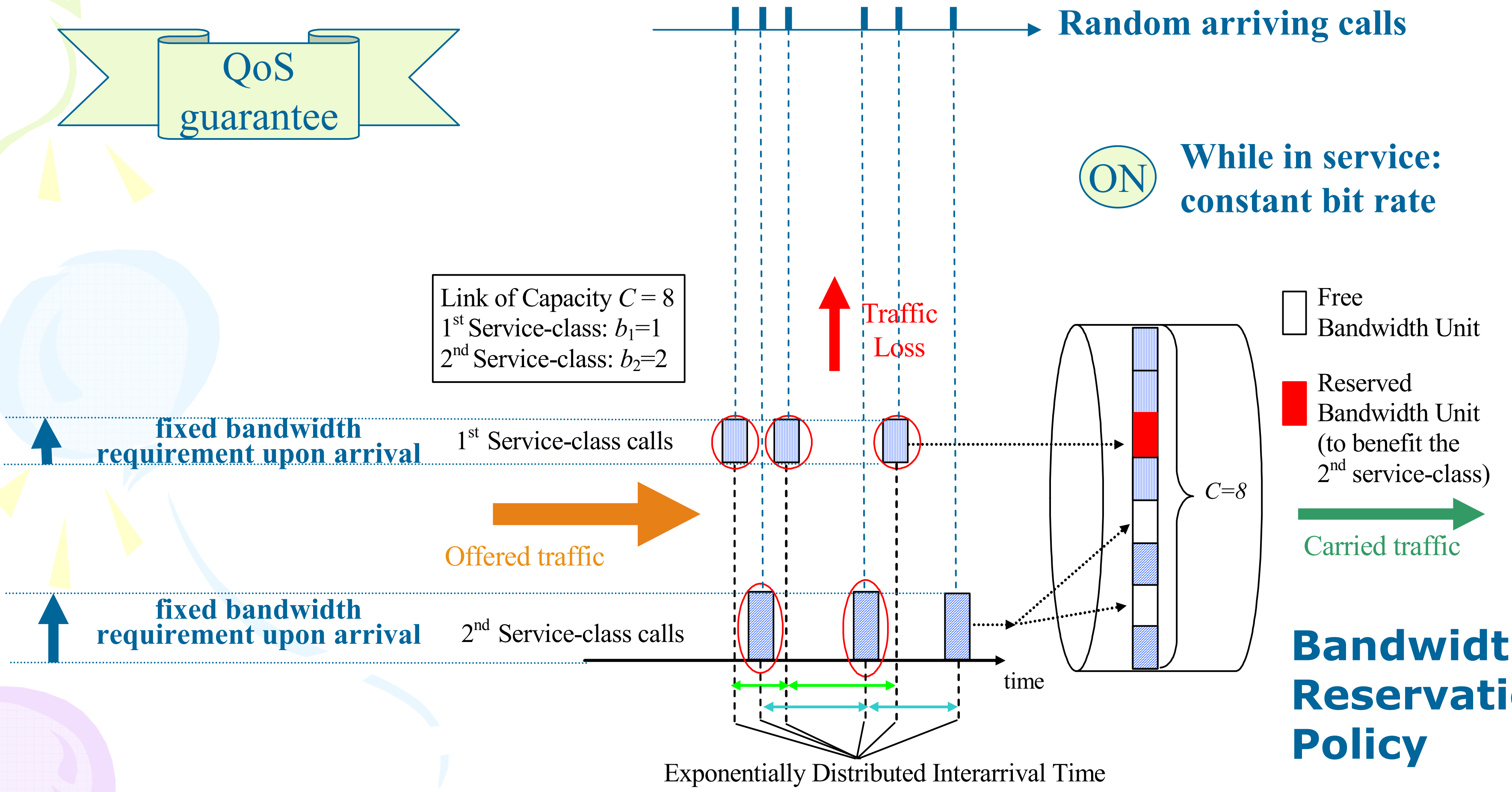
Call Blocking Probability: $$P_{b_k} = \sum_{j=C-b_k+1}^{C} G^{-1} q(j) \quad \text{where} \quad G = \sum_{j=0}^{C} q(j)$$

q(j)/G – Macro-state Probabilities



Link Utilization: $$U = \sum_{j=1}^{C} j\, q(j)$$

The EMLM under Bandwidth Reservation Policy (EMLM/BR)

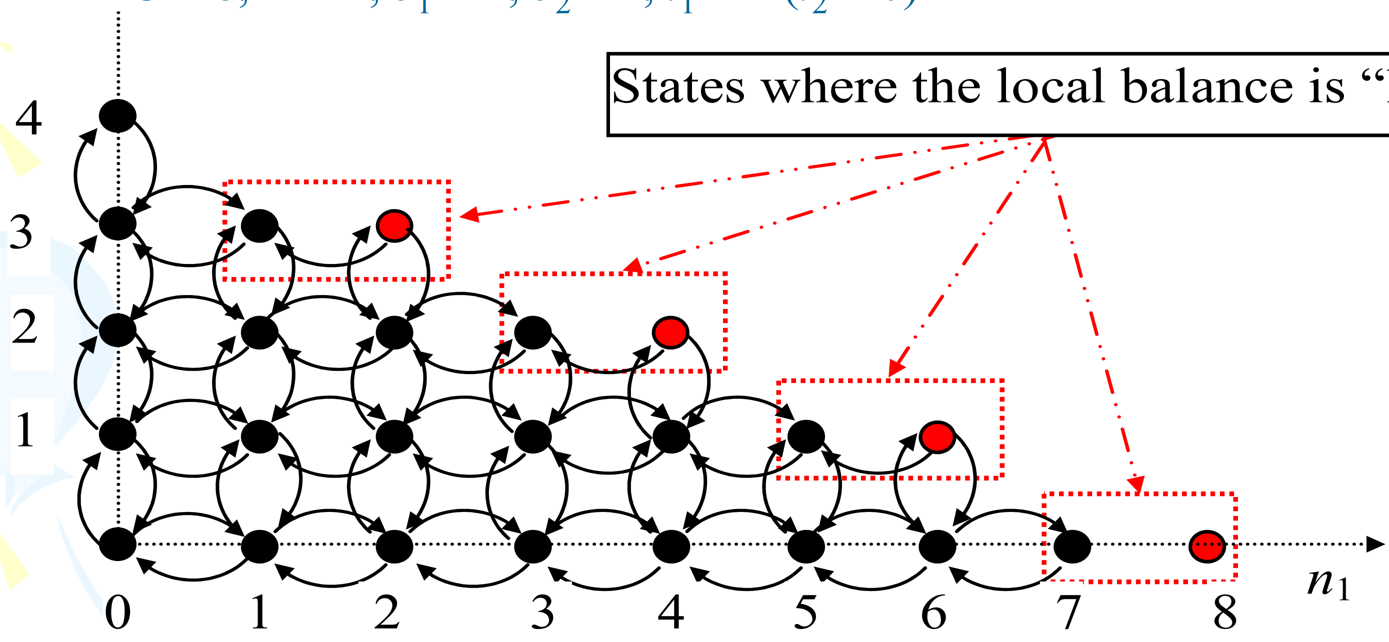


Bandwidth Reservation Policy

EMLM/BR Analysis

State Space Ω, Local-Global Balance? Product Form Solution?

$C = 8, K = 2, b_1 = 1, b_2 = 2, t_1 = 1\ (t_2 = 0)$



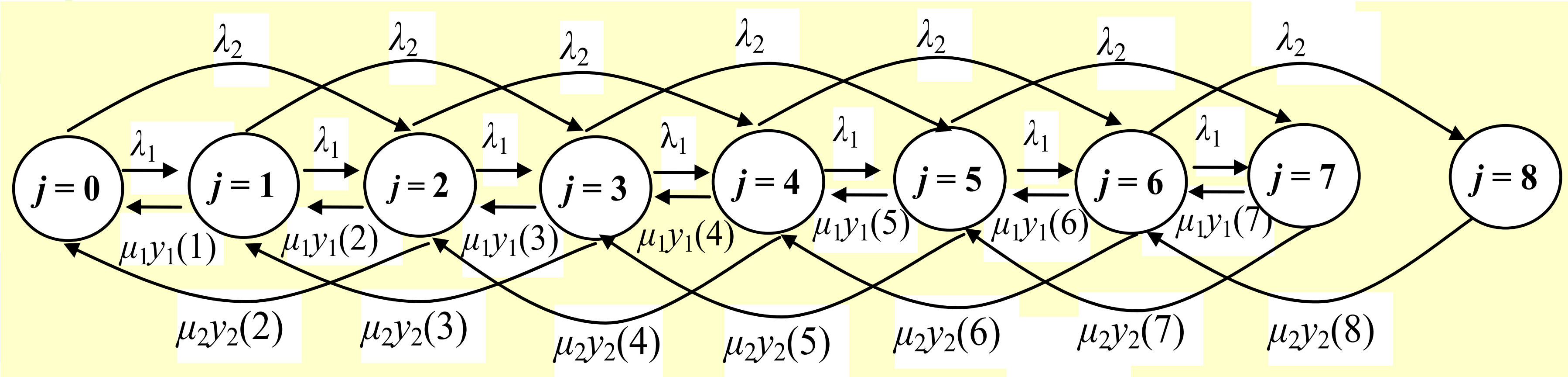
Local Balance ⟹ Product Form Solution ⟹ $\approx P_{bk}$

EMLM/BR – Roberts' Method

Roberts, International Teletraffic Congress 1983

Macro-states – One-dimensional Markov chain

$C = 8$, $K=2$, $b_1 = 1$, $b_2 = 2$, $t_1 = 1$ ($t_2 = 0$)



$$q(j) = \begin{cases} 1 & \text{for} \quad j = 0 \\ \dfrac{1}{j} \sum_{k=1}^{K} a_k D_k(j - b_k) q(j - b_k) & \text{for} \quad j = 1, \ldots, C \\ 0 & \text{otherwise} \end{cases}$$

where $D_k(j - b_k) = \begin{cases} b_k & \text{when } j \le C - t_k \\ 0 & \text{when } j > C - t_k \end{cases}$

approximation

$$y_k(j) = \begin{cases} \dfrac{a_k q(j - b_k)}{q(j)} & \text{for } j \le C - t_k \\ 0 & \text{for } j > C - t_k \end{cases}$$

EMLM/BR – Roberts’ Method (cont.)

Call Blocking Probability – Recursive Calculation

$$P_{b_k} = \sum_{j=C-b_k-t_k+1}^{C} G^{-1} q(j) \quad \text{where} \quad G = \sum_{j=0}^{C} q(j)$$

$K=3,\ b_1 = 1,\ b_2 = 2\ ,\ b_3 = 4$

$t_1 = 3,\ \ t_2 = 2,\ \ t_3 = 0$

Call Blocking equalization

array q()

0	1	2	3	...	C-4	C-3	C-2	C-1	C

1^{st} service-class: blocking states $b_1 + t_1 = 4$

2^{nd} service-class: blocking states $b_2 + t_2 = 4$

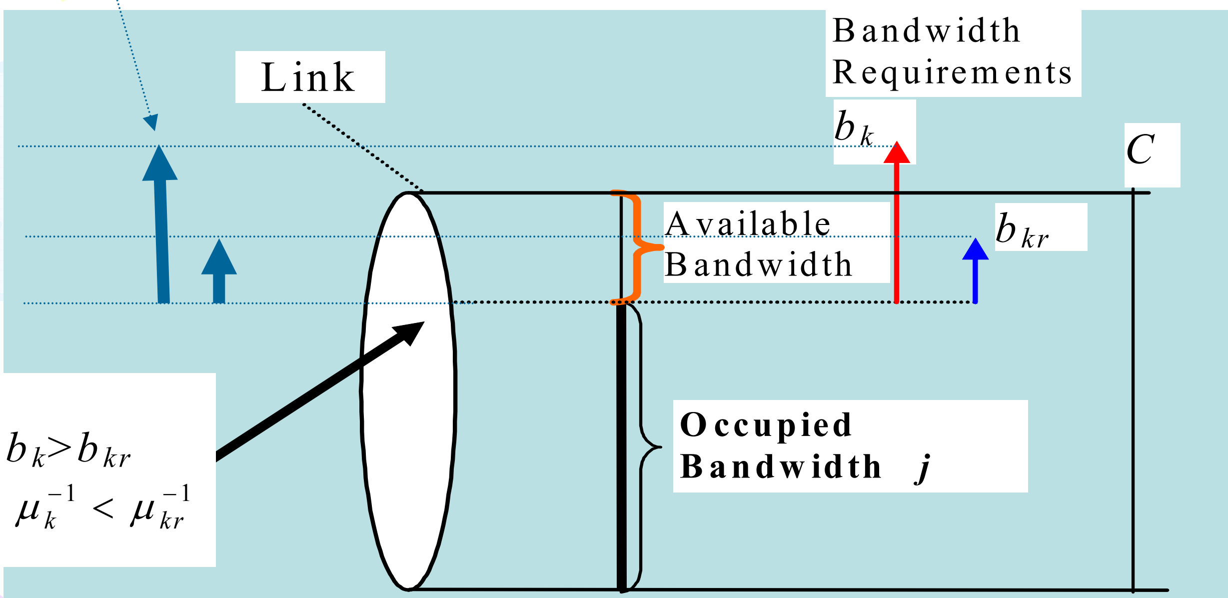
3^{rd} service-class: blocking states $b_3 + t_3 = 4$

The Retry Models

Random arrivals

Elastic bandwidth requirements upon arrival

Single Retry - Multiple Retries



ON **While in service: constant bit rate (stream traffic)**

Call with b_{kr} is admitted when $C\text{-}b_k < j \leq C\text{-}b_{kr}$

~~Local Balance~~ ⇒ ~~Product Form Solution~~ ⇒ $\approx P_{bk}$

The Retry Models (cont.)

Kaufman, IEEE INFOCOM 1992, Performance Evaluation 1992

Assumptions – Approximations

- **Local Balance**
- **When $j \le C - b_{kr_{s-1}} + b_{kr_s}$ (migration space) then $y_{kr_s}(j) = 0$ (Migration Approximation, M.A.)**

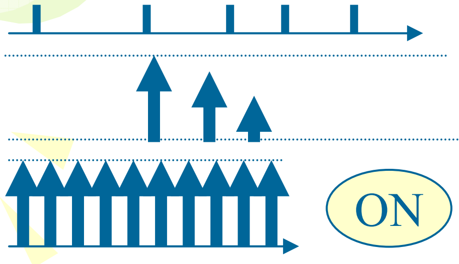
$$q(j) = \begin{cases} 1 \text{ for } j = 0 \\ \dfrac{1}{j}\left(\sum_{k=1}^{K} a_k b_k q(j-b_k) + \sum_{k=1}^{K} \sum_{s=1}^{S(k)} a_{kr_s} b_{kr_s} \delta_{k_s}(j) q(j - b_{kr_s}) \right) \text{ for } j = 1,\dots,C \\ 0 \text{ otherwise} \end{cases}$$

EMLM **$S(k)$ retries**

$a_{kr_s} = \lambda_k \mu_{kr_s}^{-1}$, $\delta_{k_s}(j) = 1$ for $j > C - (b_{kr_{s-1}} - b_{kr_s})$ otherwise $\delta_{k_s}(j) = 0$

Call Blocking Probability: $P_{b_k} = \sum_{j=C-b_{kr_{S(k)}}+1}^{C} G^{-1} q(j)$ where $G = \sum_{j=0}^{C} q(j)$

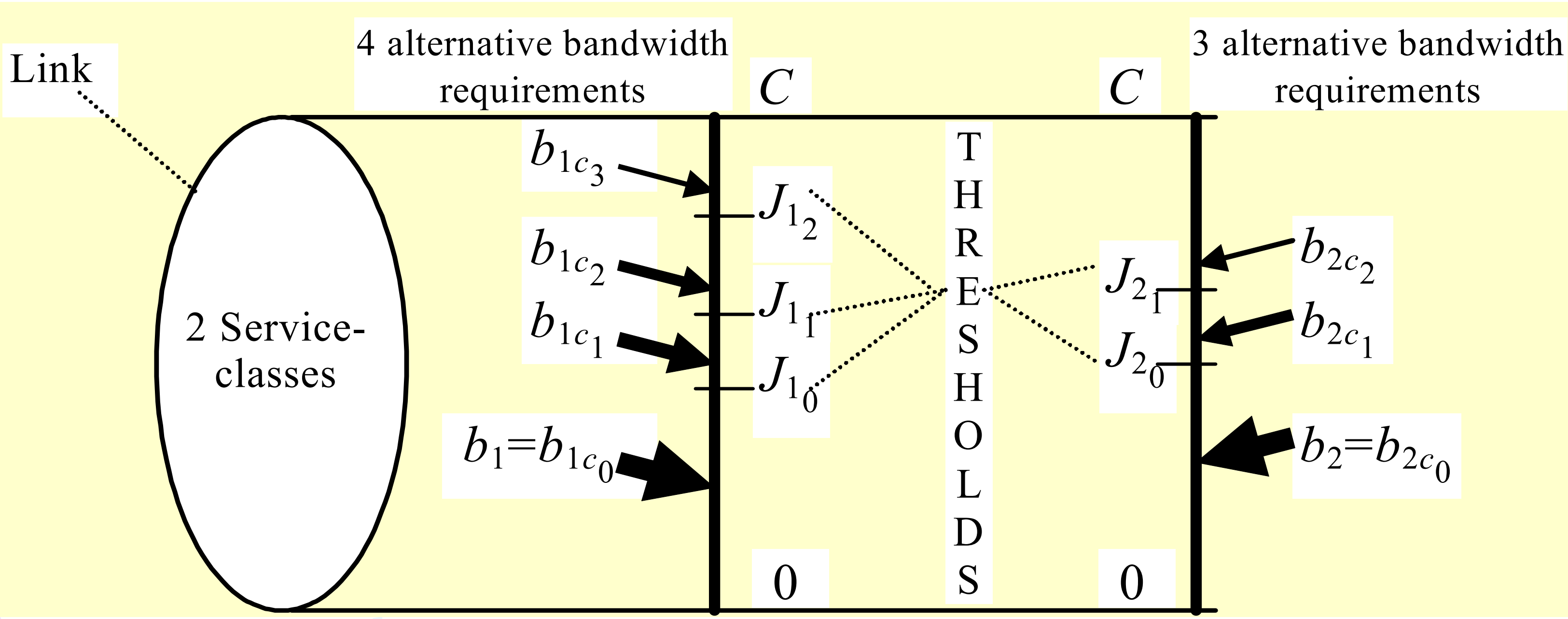
The Connection Dependent Threshold Model (CDTM)



Random arrivals

Elastic bandwidth requirements

Constant bit rate (stream traffic)



~~Local Balance~~ ⟹ ~~Product Form Solution~~ ⟹ $\approx P_{bk}$

CDTM - The analytical model

Moscholios et al. Performance Evaluation 2002

Assumptions – Approximations

1) **Local Balance**
2) **Migration Approximation, M.A** $(\delta_{kc_s}(j))$
3) **Upward migration Approximation, U.A** $(\delta_k(j))$

$$q(j) = \begin{cases} 1 & for \;\; j = 0 \\ \frac{1}{j}\left(\sum_{k=1}^{K} a_k b_k \delta_k(j) q(j-b_k) + \sum_{k=1}^{K} \sum_{s=1}^{S(k)} a_{kc_s} b_{kc_s} \delta_{kc_s}(j) q(j-b_{kc_s}) \;\; for \;\; j = 1,\dots,C \right) \\ 0 & otherwise \end{cases}$$

$$a_{kc_s} = \lambda_k \mu_{kc_s}^{-1} \qquad \delta_k(j) = \begin{cases} 1 & (\text{if } 1 \le j \le J_{k0} + b_k \text{ and } b_{kc_s} > 0) \text{ or } (\text{if } 1 \le j \le C \text{ and } b_{kc_s} = 0) \\ 0 & \text{otherwise} \end{cases}$$

U.A

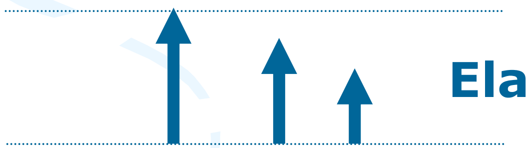
$$\delta_{kc_s}(j) = \begin{cases} 1 & \text{if } J_{ks} + b_{kc_s} \ge j > J_{ks-1} + b_{kc_s} \text{ and } b_{kc_s} > 0 \\ 0 & \text{otherwise} \end{cases}$$

M.A

Call Blocking Probability: $P_{b_k} = \sum_{j=C-b_{kc_{S(k)}}+1}^{C} G^{-1} q(j)$ where $G = \sum_{j=0}^{C} q(j)$

Importance of the CDTM

- **Generalizes the models of Thresholds, Retries and the EMLM**
  - **Incorporates the Thresholds models, by setting the same set of thresholds for all service-classes.**
  - **Incorporates the Retries models, when each service-class *k* has threshold: $J_{k_s-1} = C-b_{k_{cs}-1}$**
  - **Incorporates the EMLM by setting for each service-class *k* the threshold $J_{k_s-1} = C$**
- **The CDTM models elastic traffic at the call setup phase**



Elastic bandwidth requirements

STRUCTURE – Where We Are

- **(A) Random Traffic**
  - (A1) Constant-bit-rate/stream traffic
  - **(A2) Elastic/adaptive traffic while in service**
- **(B) Quasi-random Traffic**
  - **(B1) Constant-bit-rate/stream traffic**
  - **(B2) Elastic/adaptive traffic while in service**
- **(C) Batched Poisson Traffic**
  - **(C1) Constant-bit-rate/stream traffic**
  - **(C2) Elastic/adaptive traffic while in service**

We are here!

(A) Random Traffic

(A2) *Random arriving calls with either fixed or elastic bandwidth requirements upon arrival, and elastic bandwidth (compression/expansion) while in service.*

State of the art

- **The Extended Erlang Multi-rate Loss Model (E-EMLM)** *1997*

Furthermore

- **The E-EMLM for elastic and adaptive traffic** *2002*
- **The Extended Connection Dependent Threshold Model (E-CDTM)** *2007*

The Extended Erlang Multiple Rate Loss Model (E-EMLM)

Parameters

- C : link bandwidth capacity
- K : service-classes
- λ_k : arrival rate (Poisson)
- b_k : peak bandwidth requirement
- μ_k : service rate, μ_k^{-1} : service time (exponential)

  If compression: "Bandwidth * Service-time" ⇒ constant ⇒ elastic traffic

- j : total bandwidth demand ($0 \le j \le T$)
- T : maximum total bandwidth demand ($T \ge C$)
- s : real bandwidth allocation ($0 \le s \le C$)

Number of occupied b.u. if all in-service calls were receiving the requested bandwidth (without bandwidth compression)

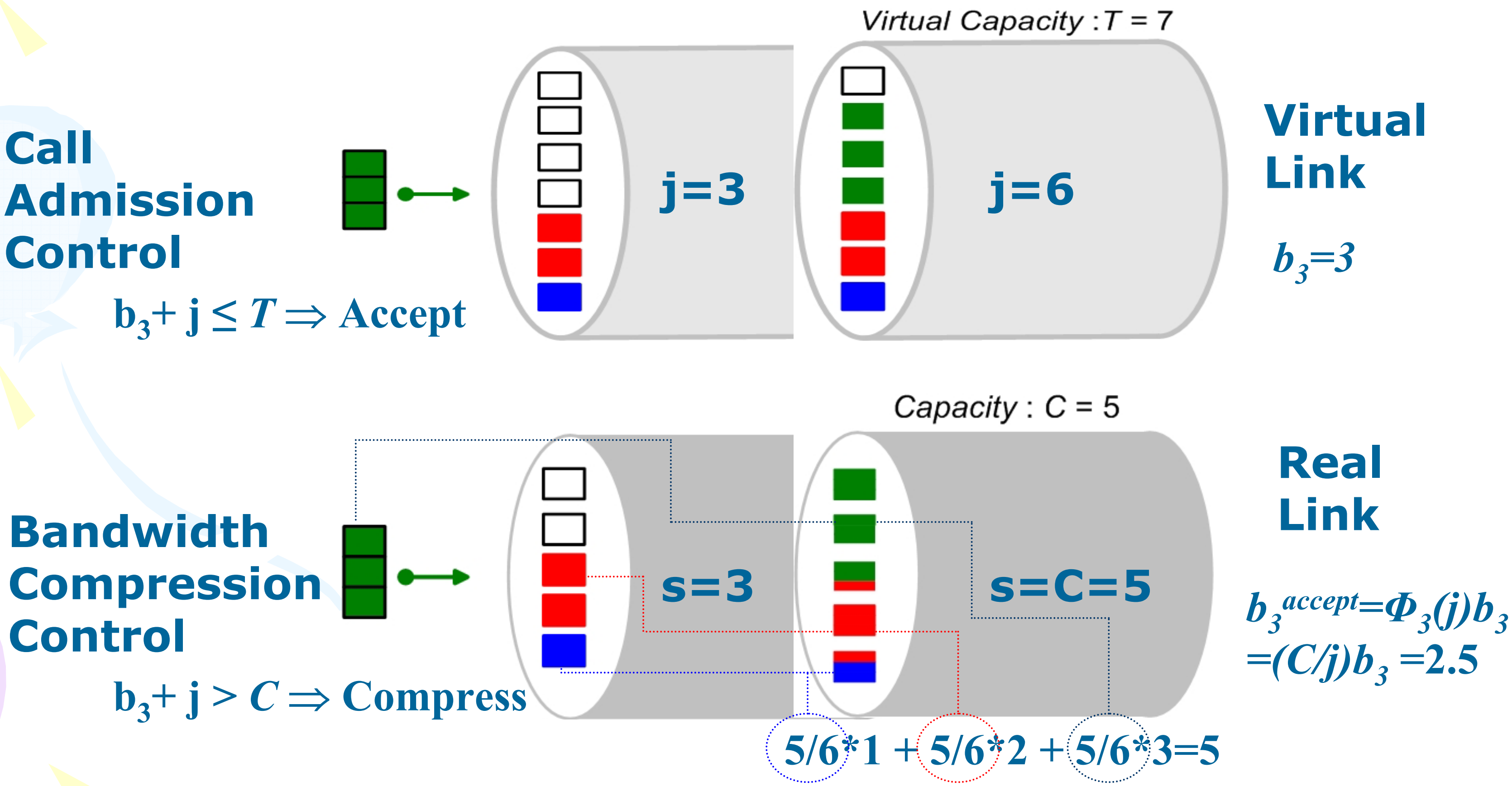
The Extended Erlang Multiple Rate Loss Model (E-EMLM) (cont).

Transmission link: $C = 5$, $T = 7$
In-service calls: $b_1 = 1$, $b_2 = 2$
Arriving call: $b_3 = 3$

j : system macro state, $0 \leq j \leq T$

s : real bandwidth allocation, $0 \leq s \leq C$

example



E-EMLM – The analytical model for elastic traffic

Stamatelos & Koukoulidis, IEEE/ACM Trans. Networking 1997

Total bandwidth demand: $$j = \sum_{k=1}^{K} n_k b_k$$

Real bandwidth allocation: $$s = \sum_{k=1}^{K} n_k b_k \Phi_k(\boldsymbol{n})$$

Where $b_k \Phi_k(\boldsymbol{n})$ is the actual allocated bandwidth to service-class k calls

$\Phi_k(\boldsymbol{n})$: service-class k and state $\boldsymbol{n}$ dependent factor

$$\Phi_k(\boldsymbol{n}) = \begin{cases} 1 & \text{for } 0 \le j \le C \\ \dfrac{x(\boldsymbol{n}_k^-)}{x(\boldsymbol{n})} & \text{for } C < j \le T \\ 0 & \text{otherwise} \end{cases}$$

$x(\boldsymbol{n})$: state multiplier or weight associated with the state $\boldsymbol{n}$

$$x(\boldsymbol{n}) = \begin{cases} 1 & \text{for } 0 \le j \le C \\ \dfrac{1}{C} \sum_{k=1}^{K} n_k b_k x(\boldsymbol{n}_k^-) & \text{for } C < j \le T \\ 0 & \text{otherwise} \end{cases}$$

E-EMLM – The analytical model for elastic traffic (cont.)

Link Occupancy Distribution

$$q(j) = \frac{1}{\min(C,j)} \sum_{k=1}^{K} \alpha_k b_k q(j - b_k), \quad j = 0, \ldots, T$$

$q(x)=0$ for $x < 0$ and $\sum_{j=0}^{C} q(j) = 1$

No product form solution

Call Blocking Probabilities (CBP)

CBP of service-class k: $P_{b_k} = \sum_{j=0}^{b_k - 1} q(T - j)$

E-EMLM – The analytical model for elastic and adaptive traffic

Racz, Gero and Fodor, Performance Evaluation 2002

$$q(j) = \frac{1}{\min(C,j)} \sum_{k \in K_e} \alpha_k b_k q(j - b_k) + r(j) \sum_{k \in K_a} a_k b_k q(j - b_k), \quad j = 0, \ldots, T$$

$q(x)=0$ for $x < 0$, $\sum_{j=0}^{C} q(j) = 1$ and $r(j) = \min(1, \frac{C}{j})$

where K_e is the set of elastic service-classes

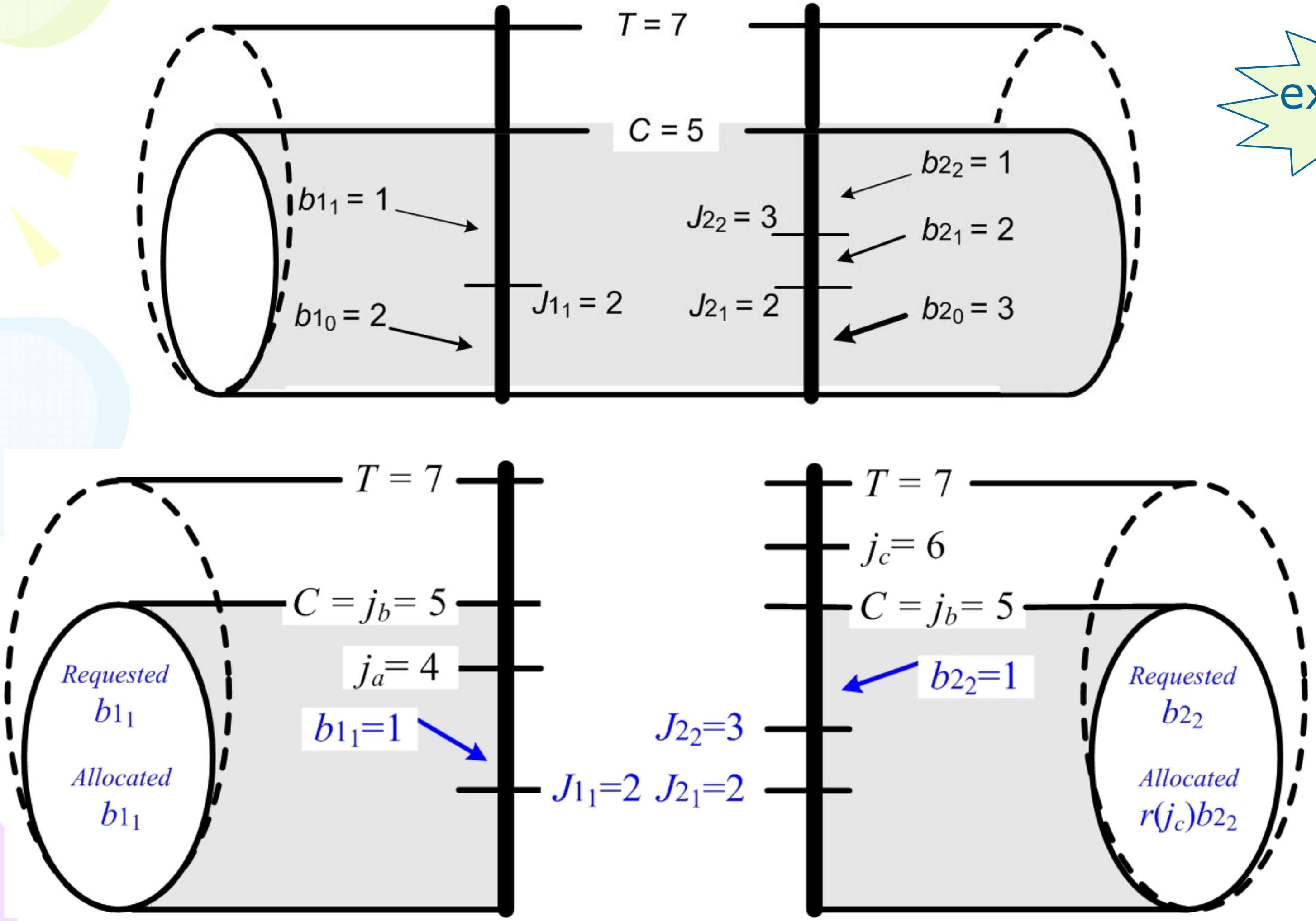
and K_a is the set of adaptive service-classes

No product form solution

CBP of service-class k : $B_k = \sum_{j=0}^{b_k - 1} q(T - j)$

The Extended Connection Dependent Threshold Model (E-CDTM)

example



Compression rate=C/j= 5/6

E-CDTM – The analytical model

Vassilakis et al., Int. Journal of Commun. Systems 2012

Link occupancy distribution

$$q(j) = \begin{cases} 1 & \text{for } j = 0 \\ \frac{1}{\min(C,j)} \sum_{k \in K_e} \sum_{l=0}^{S_k} \alpha_{k_l} b_{k_l} \delta_{k_l}(j)\, q(j - b_{k_l}) + \\ \quad + \frac{1}{j} \sum_{k \in K_a} \sum_{l=0}^{S_k} \alpha_{k_l} b_{k_l} \delta_{k_l}(j)\, q(j - b_{k_l}) & \text{for } j = 1,\ldots,T \\ 0 & \text{otherwise} \end{cases}$$

un-normalized

$$G = \sum_{j=0}^{T} q(j)$$

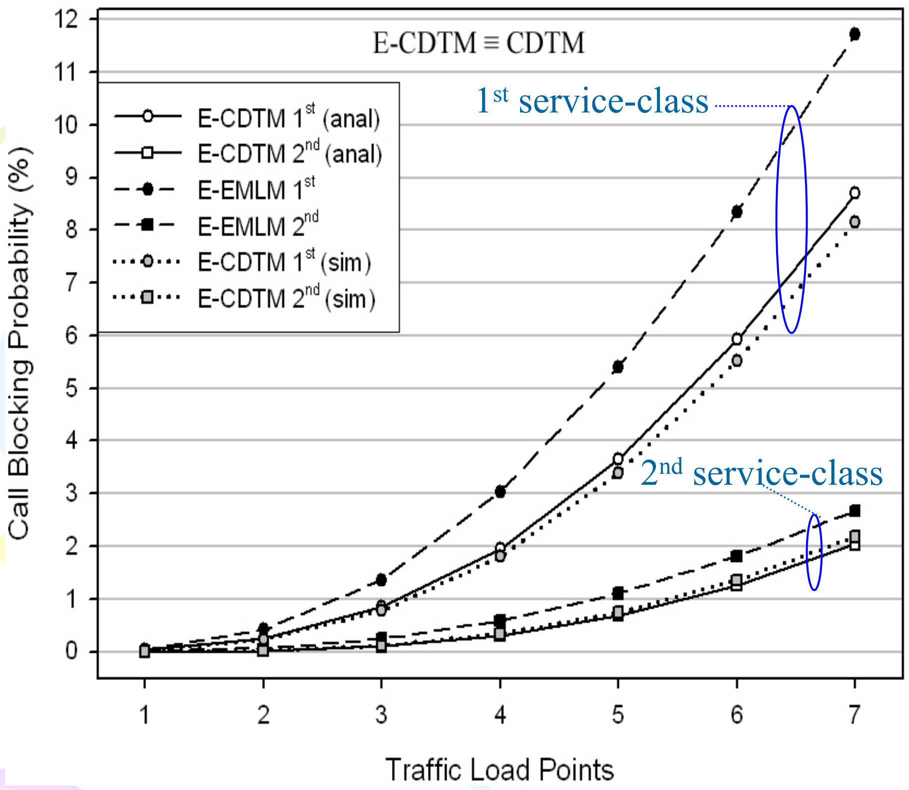
Call Blocking Probability

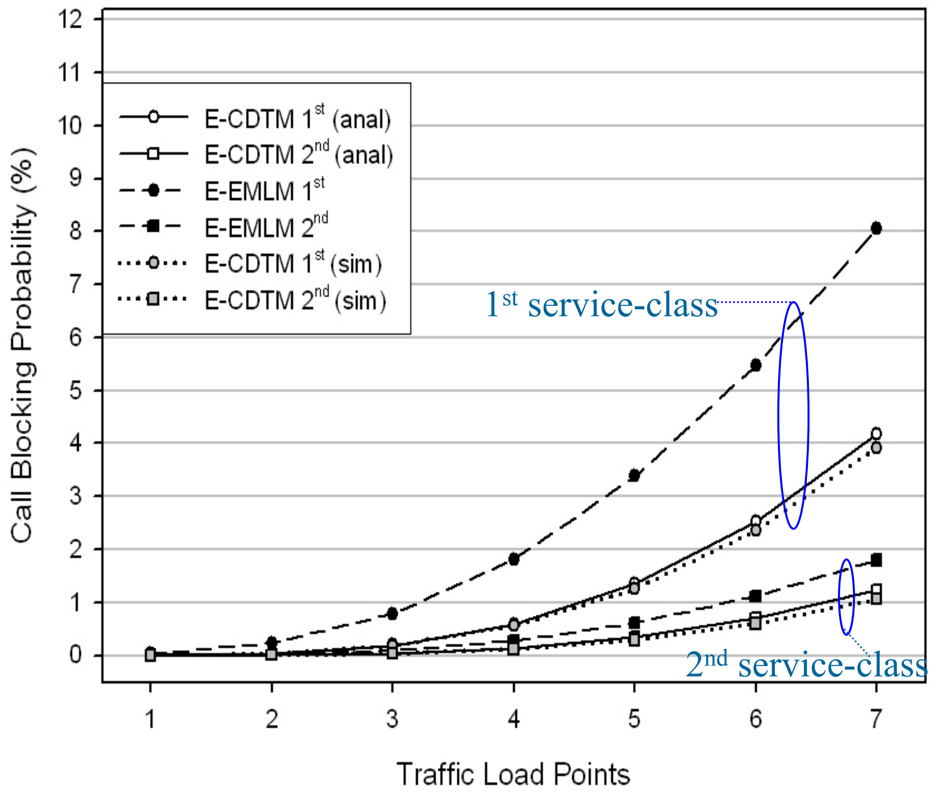
$$P_{b_k} = \sum_{j=T-b_{k_{S_k}}+1}^{T} G^{-1} q(j)$$

Link Utilization

$$U = \sum_{j=1}^{C} j G^{-1} q(j) + \sum_{j=C+1}^{T} G^{-1} C\, q(j)$$

E-CDTM versus E-EMLM





C=T = 80 **T = C + 10**

STRUCTURE – Where We Are

- (A) Random Traffic
  - (A1) Constant-bit-rate/stream traffic
  - (A2) Elastic/adaptive Traffic while in service
- **(B) Quasi-random Traffic**
  - **(B1) Constant-bit-rate/stream traffic**
  - **(B2) Elastic/adaptive Traffic while in service**
- **(C) Batched Poisson Traffic**
  - **(C1) Constant-bit-rate/stream traffic**
  - **(C2) Elastic/adaptive Traffic while in service**

We are here!

(B) Quasi-random Traffic

(B1) *Quasi-random arriving calls with either fixed or elastic bandwidth requirements upon arrival, and constant use of the assigned bandwidth (constant-bit-rate/stream traffic) while in service.*

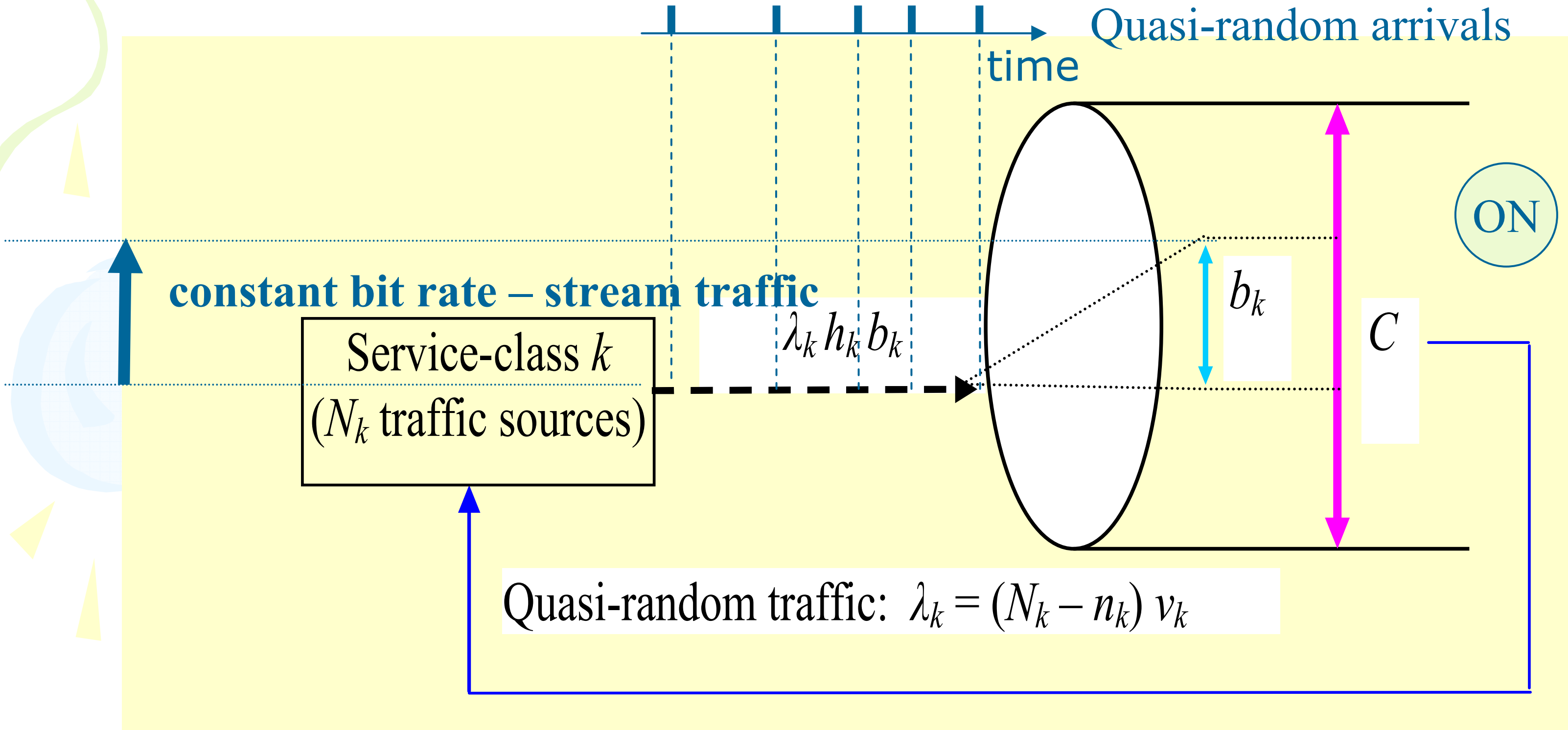
State of the art

- **The Engset Multi-rate Loss Model (EnMLM)** *1994*
- **The Single Retry Model for finite population (f-SRM)** *1997*

Furthermore

- The EnMLM for elastic and adaptive traffic
- The EnMLM under the Bandwidth Reservation Policy
- The f-SRM under the Bandwidth Reservation Policy
- The Multi Retry Model for finite population(f-MRM)
- The f-MRM under the Bandwidth Reservation Policy
- **The CDTM for finite population (f-CDTM)**
- The f-CDTM under the Bandwidth Reservation Policy
- **The Generalized f-CDTM when random and quasi-random traffic coexist**

The Engset Multi-rate Loss Model (EnMLM)



While in service: constant bit rate (stream traffic)

n_k : number of service-class k calls (sources) which are in service
v_k : fixed arrival rate per «free» source (not in service yet)
λ_k : mean arrival rate of service-class k calls
h_k : holding (service) time of service-class k calls

EnMLM – The Analytical Model

A Product Form Solution model

$$P(\boldsymbol{n}) = G^{-1}\left(\prod_{k=1}^{K}\binom{N_k}{n_k}a_k^{n_k}\right)$$

Where G=G(Ω)= $\sum_{\boldsymbol{n}\in\boldsymbol{\Omega}}\left(\prod_{k=1}^{K}\binom{N_k}{n_k}a_k^{n_k}\right)$

Macro-states – One-dimensional Markov chain

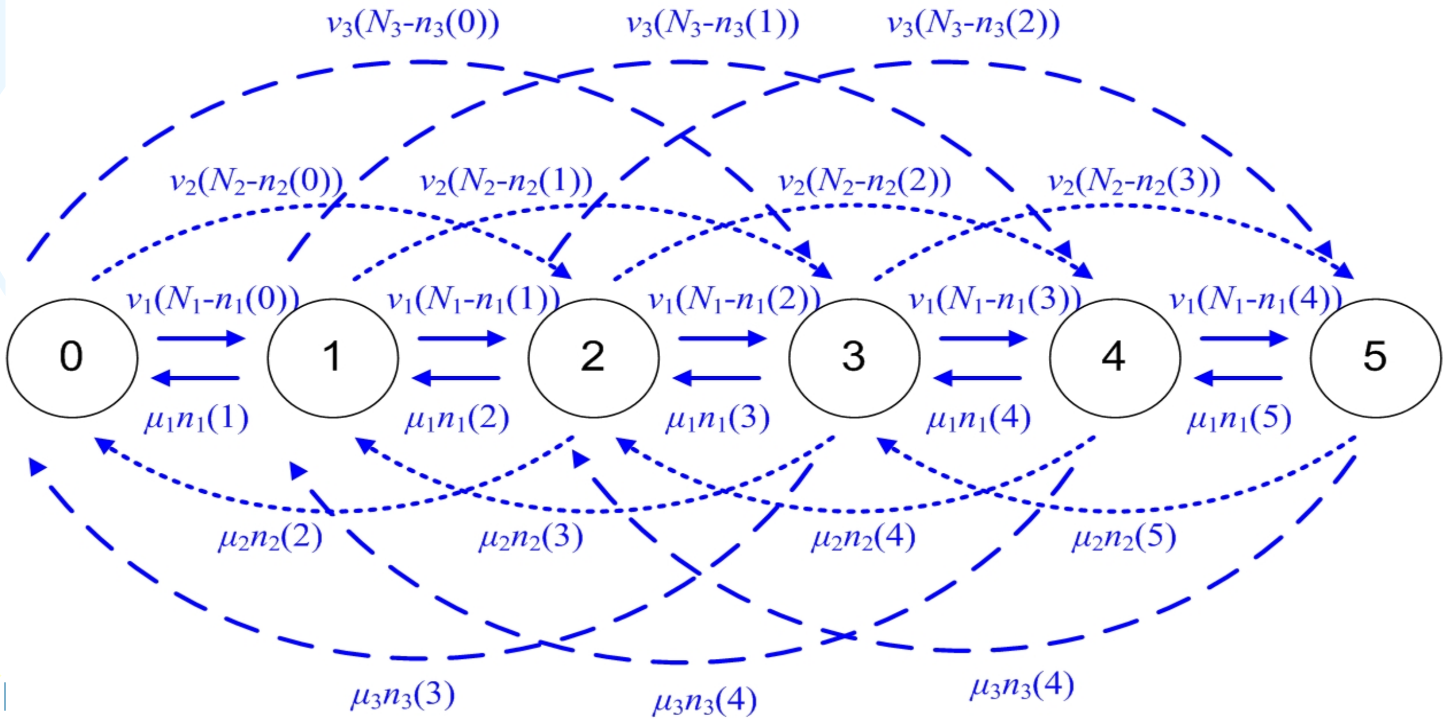
Example

$K = 3$

$b_1 = 1$

$b_2 = 2$

$b_3 = 3$



EnMLM – The Analytical Model (cont.)

Stamatelos & Hayes, Computer Communications 1994

Link occupancy distribution – Recursive formula

$$q(j) = \begin{cases} 1 & \text{for} \quad j = 0 \\ \frac{1}{j} \sum_{k=1}^{K} (N_k - n_k + 1)\alpha_k b_k q(j - b_k) & \text{for} \quad j = 1, \ldots, C \\ 0 & \text{otherwise} \end{cases}$$

Time congestion probability: $$P_{b_k} = \sum_{j=C-b_k+1}^{C} G^{-1} q(j)$$

For $K = 1 \rightarrow$ $$P_{b1} = \frac{\binom{N}{C}(\alpha_1)^C}{\sum_{i=0}^{C} \binom{N}{i}(\alpha_1)^i}$$ Engset formula (1918)

For $N_k \rightarrow \infty$, q(j) results in Kaufman/Roberts recursion (EMLM)

EnMLM – State Space Determination

The problem

- In calculating the q(j)'s
- The link occupancy j (macro-state)

  ⇔ single state (not valid in many cases)

<u>Example:</u>

C = 5 b.u.

K = 3 service-classes

$N_1=N_2=N_3=$ 10 sources

$b_1=$ 3 b.u. (per call)

$b_2=$ 2 b.u. (per call)

$b_3=$ 1 b.u. (per call)

$a_1=a_2=a_3=$ 0.1 erl (per idle source)

$$q(4)=\frac{1}{4}\sum_{k=1}^{K}(N_k-n_k+1)a_k b_k q(4-b_k)$$

?

single state			macro state
n_1	n_2	n_3	j
0	0	0	0
0	0	1	1
0	0	2	2
0	0	3	3
0	0	4	4
0	0	5	5
0	1	0	2
0	1	1	3
0	1	2	4
0	1	3	5
0	2	0	4
0	2	1	5
1	0	0	3
1	0	1	4
1	0	2	5
1	1	0	5

EnMLM – State Space Determination (cont.1)

The solution

Theorem:
Two stochastic systems with the same state space and the same parameters K, N_k, a_k are equivalent – they have the same Blocking States

Lemma:
Modify only the b_k's so that the resultant link occupancy per state is unique.

Example
By choosing b_1=16, b_2=12 and b_3=5 an equivalent system results with unique link occupancy per state, j_{eq} and capacity C=29.

State space: n_1, n_2, n_3, j — Blocking states: B_1, B_2, B_3

n_1	n_2	n_3	j	B_1	B_2	B_3	j_{eq}
0	0	0	0				0
0	0	1	1				5
0	0	2	2				10
0	0	3	3	🕐			15
0	0	4	4	🕐	🕐		20
0	0	5	5	🕐	🕐	🕐	25
0	1	0	2				12
0	1	1	3	🕐			17
0	1	2	4	🕐	🕐		22
0	1	3	5	🕐	🕐	🕐	27
0	2	0	4	🕐	🕐		24
0	2	1	5	🕐	🕐	🕐	**29**
1	0	0	3	🕐			16
1	0	1	4	🕐	🕐		21
1	0	2	5	🕐	🕐	🕐	26
1	1	0	5	🕐	🕐	🕐	28

The Single Retry Model for finite population (f-SRM)

Stamatelos & Koukoulidis, IEEE/ACM Trans. on Networking 1997

~~Local Balance~~ ⇒ ~~Product Form~~ Solution ⇒ $\approx P_{bk}$

Assumptions – Approximations

- **Local Balance**
- **When $j \le C- b_k + b_{kr}$ (migration space) then $y_{kr}(j) = 0$ (Migration approximation, M.A.)**

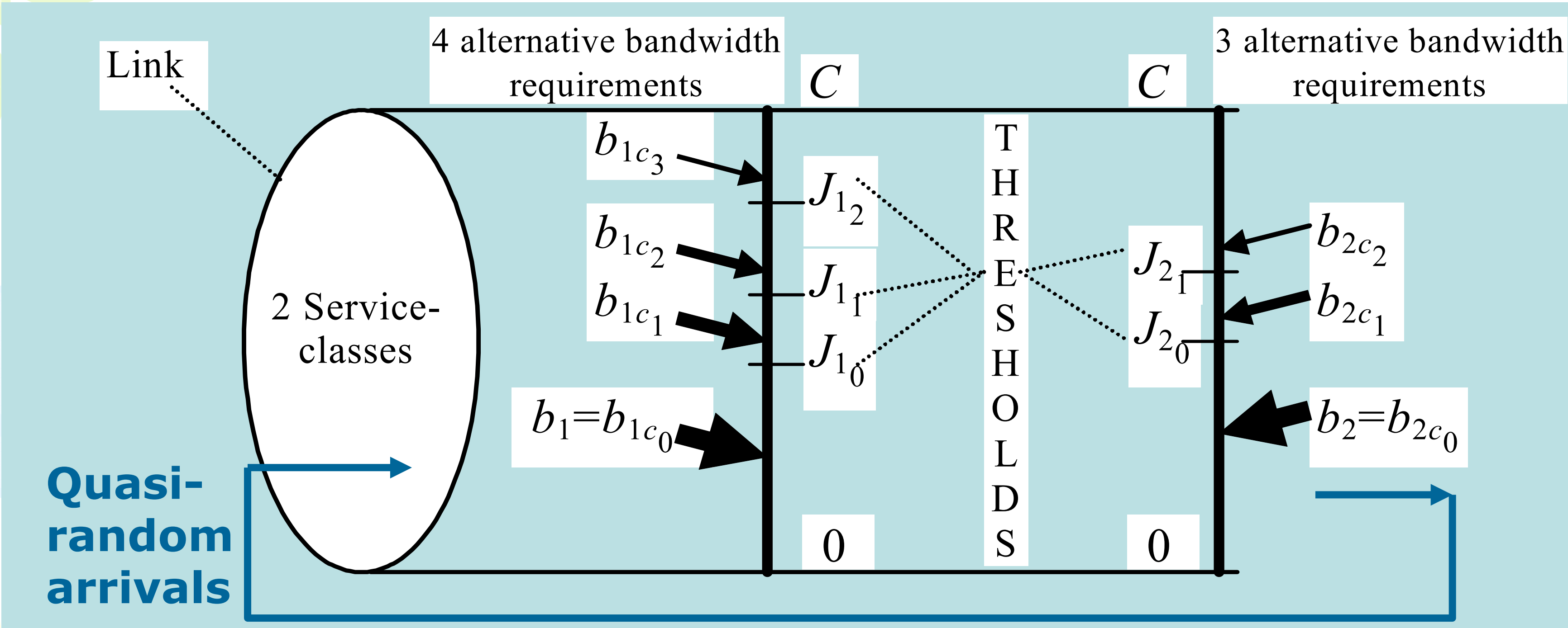
$$q(j) = \begin{cases} 1 \text{ for } j = 0 \\ \frac{1}{j}\left(\underbrace{\sum_{k=1}^{K} (N_k - n_k + 1) a_k b_k q(j - b_k)}_{\text{EnMLM}} + \underbrace{\sum_{k=1}^{K} (N_k - (n_k + n_{kr}) + 1) a_{kr} b_{kr} \gamma_k(j) q(j - b_{kr})}_{\text{calls with } b_{kr}} \right) & \text{for } j = 1,\dots,C \\ 0 \text{ otherwise} \end{cases}$$

$a_{kr} = v_{kr}\mu_{kr}^{-1}$, $\gamma_k(j) = 1$ when $j > C - b_k + b_{kr}$ otherwise $\gamma_k(j) = 0$

For $N_k \to \infty$ ⇒ the Single Retry Model (for random traffic)

Time Congestion Probability: $P_{b_k} = \sum_{j=C-b_{kr}+1}^{C} G^{-1} q(j)$ where $G = \sum_{j=0}^{C} q(j)$

The Connection Dependent Threshold Model for finite population (f-CDTM)



ON **Constant bit rate (stream traffic) while in service**

When $N_k \to \infty$ the f-CDTM results in CDTM (for random traffic)

~~Local Balance~~ ⇨ ~~Product Form Solution~~ ⇨ $\approx P_{bk}$

f-CDTM – The Analytical Model

Moscholios et al., Performance Evaluation 2005

Assumptions - Approximations

1) Local Balance
2) Migration approximation, M.A. ($\delta_{kcs}(j)$)
3) Upward approximation, U.A. ($\delta_{\kappa}(j)$)

$$
q(j)=\begin{cases} 1 \text{ for } j=0 \\ \frac{1}{j}\left(\sum_{k=1}^{K}(N_k - n_k + 1)\alpha_k b_k \delta_k(j) q(j-b_k) + \right. \\ \left. + \sum_{k=1}^{K}\sum_{s=1}^{S(k)} (N_k - (n_k + n_{kc_1} + \ldots + n_{kc_s} + \ldots + n_{kc_{S(k)}}) + 1)\alpha_{kc_s} b_{kc_s} \delta_{kc_s}(j) q(j-b_{kc_s})\right) \text{ for } j=1,\ldots,C \\ 0 \text{ otherwise} \end{cases}
$$

$$
\delta_k(j)=\begin{cases} 1 & (\text{if } 1 \le j \le J_{k0} + b_k \text{ and } b_{kc_s} > 0) \text{ or } (\text{if } 1 \le j \le C \text{ and } b_{kc_s} = 0) \\ 0 & \text{otherwise} \end{cases}
$$

U.A

$$
a_{kc_s} = v_{kc_s} \mu_{kc_s}^{-1}
$$

$$
\delta_{kc_s}(j)=\begin{cases} 1 & \text{if } J_{ks} + b_{kc_s} \ge j > J_{ks-1} + b_{kc_s} \text{ and } b_{kc_s} > 0 \\ 0 & \text{otherwise} \end{cases}
$$

M.A

Time Congestion Probability: $P_{b_k} = \sum_{j=C-b_{kc_{S(k)}}+1}^{C} G^{-1} q(j)$ where $G = \sum_{j=0}^{C} q(j)$

f-CDTM – State Space Determination

- **A Good Approximation - Without equivalent system!**

$$n_k(j) \approx y_k(j)$$

The parameters $n_k(j)$ can be approximated by the average number of service-class k calls in state j, $y_k(j)$, assuming infinite population for each service-class (i.e. from the corresponding CDTM)

Glabowski & Stasiak, Proc. MMB&PGTS 2004
Moscholios et al., MEDJCN 2007

Numerical example: f-CDTM versus CDTM

Σ	$N_1 = N_2 = 12$ (f-CDTM)		$N_1 = N_2 = \infty$ (CDTM)	
	P_{b1c2} (%)	P_{b2c1} (%)	P_{b1c2} (%)	P_{b2c1} (%)
1	1.96	1.07	4.49	2.48
2	2.78	1.52	6.70	3.65
3	3.76	2.05	9.39	5.10
4	4.90	2.66	12.55	6.74
5	6.19	3.34	16.06	8.62
6	7.63	4.09	19.84	10.65

The Generalized f-CDTM where random and quasi-random traffic coexist

Moscholios et al., Performance Evaluation 2005

- **K_{fin} service-classes of finite sources (quasi-random input).**
- **K_{inf} service-classes of infinite sources (random – Poisson input).**

Link occupancy distribution

$$q(j)=\begin{cases} 1 \quad \text{for } j=0 \\ \frac{1}{j}\sum_{k\in K_{fin}}(N_k-n_k+1)\alpha_k b_k\delta_k(j)G(j-b_k)+\frac{1}{j}\sum_{k\in K_{fin}}\sum_{t=1}^{T}(N_k-(n_k+n_{kc_1}+\ldots+n_{kc_t}+\ldots+n_{kc_T})+1)a_{kc_t}b_{kc_t}\delta_{kc_t}(j)q(j-b_{kc_t}) \\ +\frac{1}{j}\sum_{k\in K_{inf}}\alpha_k b_k\delta_k(j)G(j-b_k)+\frac{1}{j}\sum_{k\in K_{inf}}\sum_{t=1}^{T}a_{kc_t}b_{kc_t}\delta_{kc_t}(j)q(j-b_{kc_t}) \text{ for } j=1,\ldots,C \\ 0 \quad \text{otherwise} \end{cases}$$

Where:

$\delta_k(j) = 1$ when $1 \le j \le C$ and $b_{kc} = 0$, or, when $j \le J_{kt}+b_k$ and $b_{kc} > 0$, otherwise $\delta_k(j) = 0$.

$\delta_{kc_t}(j) = 1$ when $J_{k_t}+b_{kc_t} \ge j > J_{k_t}-1+b_{kc_t}$ otherwise $\delta_{kc_t}(j) = 0$.

STRUCTURE – Where We Are

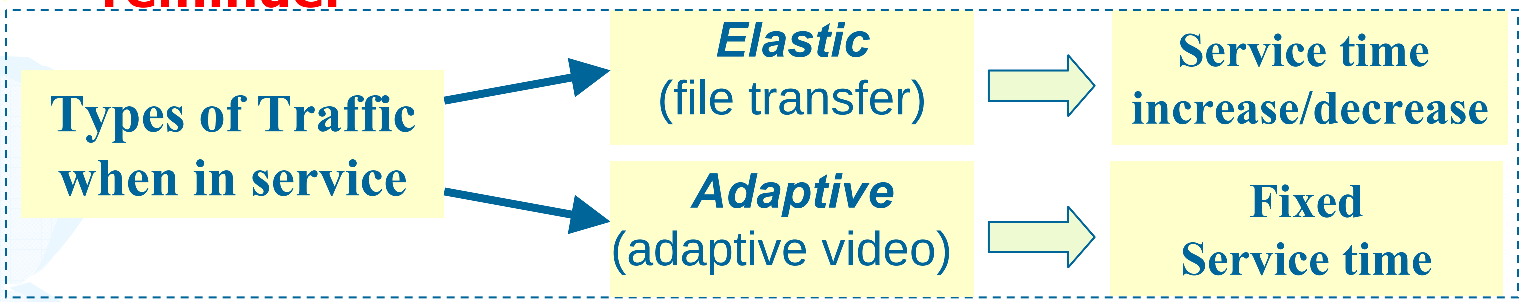
- (A) Random Traffic
  - (A1) Constant-bit-rate/stream traffic
  - (A2) Elastic Traffic while in service
- **(B) Quasi-random Traffic**
  - (B1) Constant-bit-rate/stream traffic
  - **(B2) Elastic Traffic while in service**
- **(C) Batched Poisson Traffic**
- **(D) ON-OFF Traffic**
  - **(D1) Poisson arrivals**
  - **(D2) Quasi-random arrivals**
  - **(D3) Batched Poisson arrivals**

We are here!

(B) Quasi-random Traffic

(B2) *Quasi-random arriving calls with either fixed or elastic bandwidth requirements upon arrival, and elastic bandwidth while in service.*

reminder



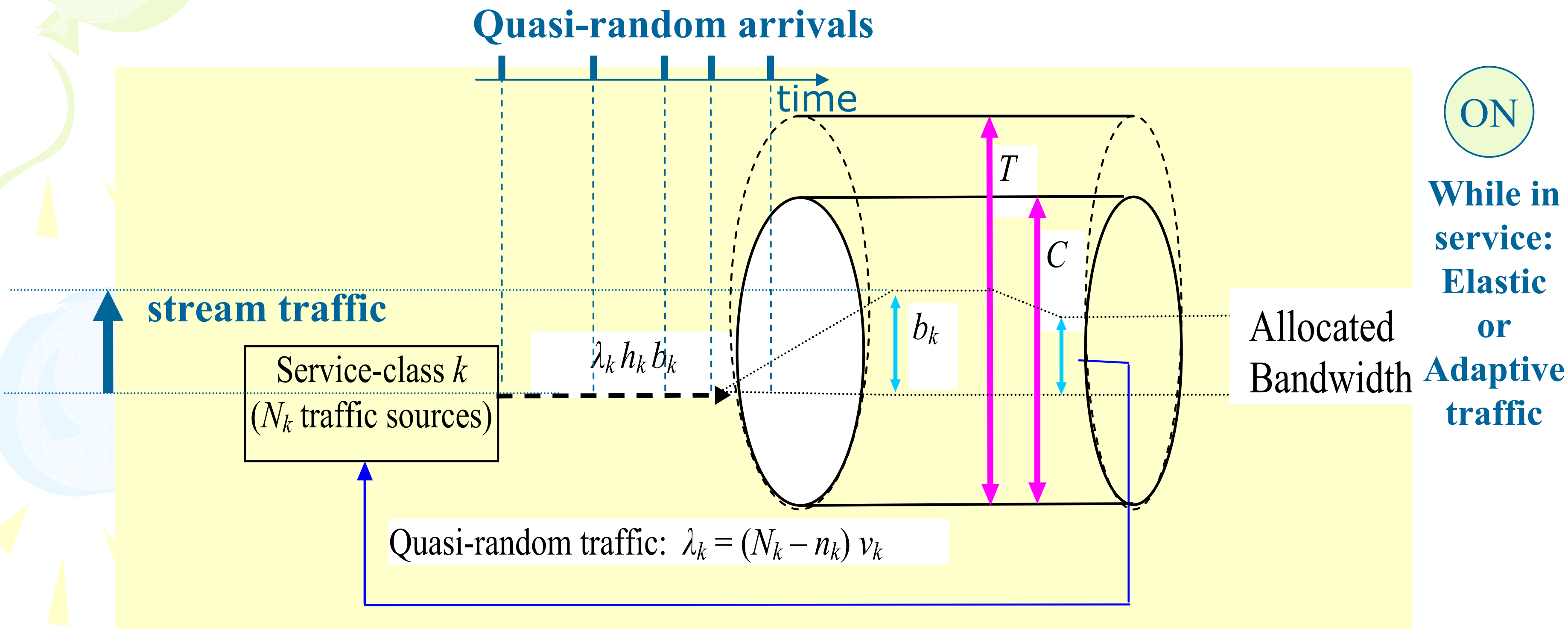
State of the art

- **The Extended Engset Multi-rate Loss Model (E-EnMLM)** *1997*

Furthermore

- **The Extended Connection Dependent Threshold Model for finite population (Ef-CDTM)** *2007*

The Extended Engset Multi-rate Loss Model (E-EnMLM)



h_k : holding (service) time of service-class k calls

If compression: "Bandwidth * Service-time" ⇒ constant ⇒ elastic traffic

j : total bandwidth demand ($0 \le j \le T$)

T : maximum total bandwidth demand ($T \ge C$)

s : real bandwidth allocation ($0 \le s \le C$)

E-EnMLM – The analytical model

Stamatelos & Koukoulidis, IEEE/ACM Trans. Networking 1997

Link occupancy distribution

$$q(j) = \begin{cases} 1 & \text{for } j = 0 \\ \dfrac{1}{\min(C,j)} \displaystyle\sum_{k \in K_e} (N_k - n_k + 1)\alpha_k b_k q(j - b_k) + \\ + \dfrac{1}{j} \displaystyle\sum_{k \in K_a} (N_k - n_k + 1)\alpha_k b_k q(j - b_k) & \text{for } j = 1,\dots,T \\ 0 & \text{otherwise} \end{cases}$$

un-normalized

$$G = \sum_{j=0}^{T} q(j)$$

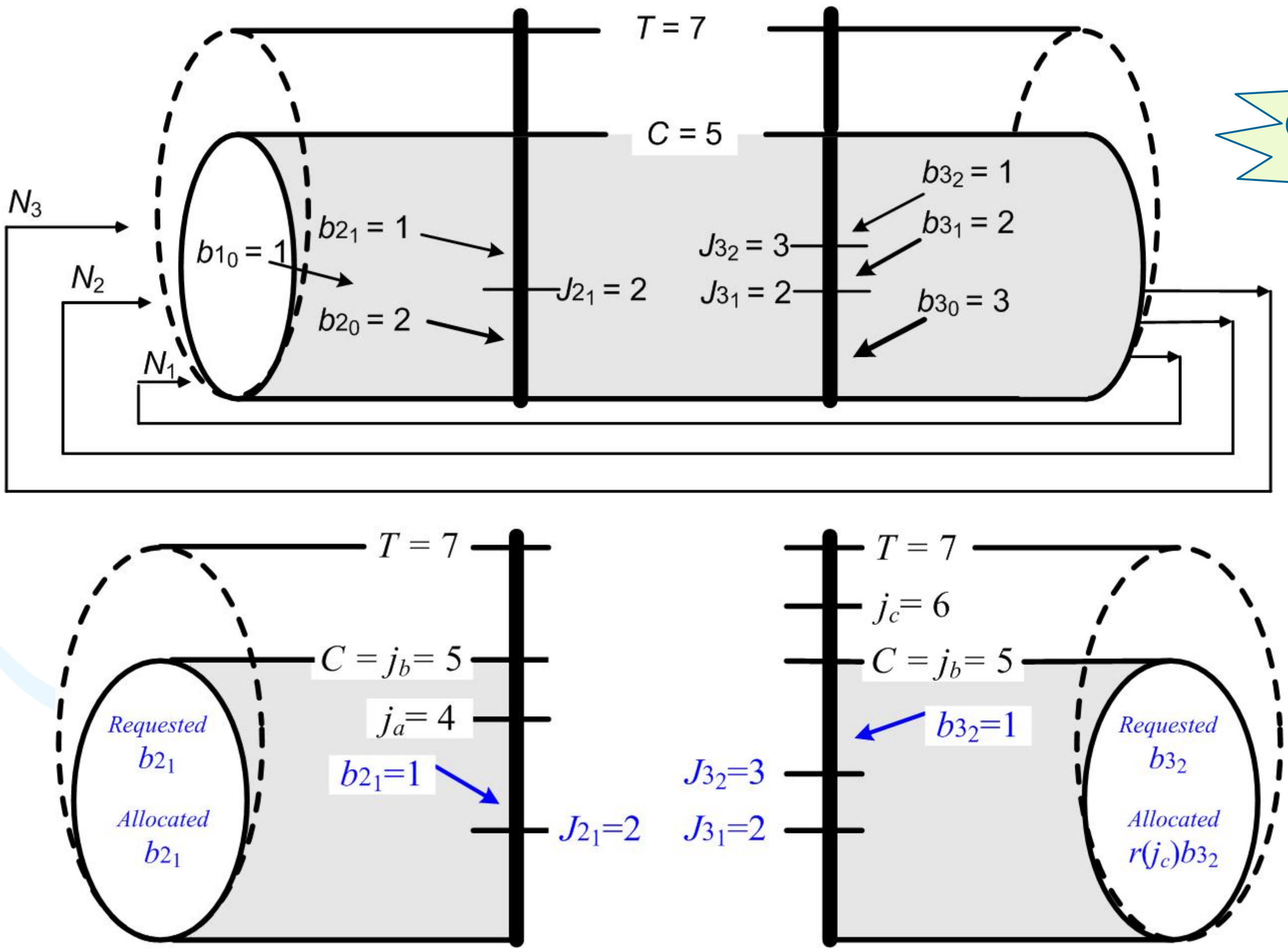
Time Congestion Probability

$$P_{b_k} = \sum_{j=T-b_k+1}^{T} G^{-1} q(j)$$

Link Utilization

$$U = \sum_{j=1}^{C} j G^{-1} q(j) + \sum_{j=C+1}^{T} G^{-1} C q(j)$$

The Extended Connection Dependent Threshold Model for finite population (Ef-CDTM)



example

Compression rate=C/j= 5/6

Ef-CDTM – The analytical model

Vassilakis et al., IEICE Trans. Commun. 2008

Link occupancy distribution

$$q(j)=\begin{cases}1 & \text{for } j=0\\ \frac{1}{\min(C,j)}\sum_{k\in K_e}\sum_{l=0}^{S_k}(N_k-\sum_{l=0}^{S_k}n_{k_l}+1)\,\alpha_{k_l}\,b_{k_l}\,\delta_{k_l}(j)\,q(j-b_{k_l})+ & \\ +\frac{1}{j}\sum_{k\in K_a}\sum_{l=0}^{S_k}(N_k-\sum_{l=0}^{S_k}n_{k_l}+1)\,\alpha_{k_l}\,b_{k_l}\,\delta_{k_l}(j)\,q(j-b_{k_l}) & \text{for } j=1,\ldots,T\\ 0 & \text{otherwise}\end{cases}$$

un-normalized

$$G=\sum_{j=0}^{T}q(j)$$

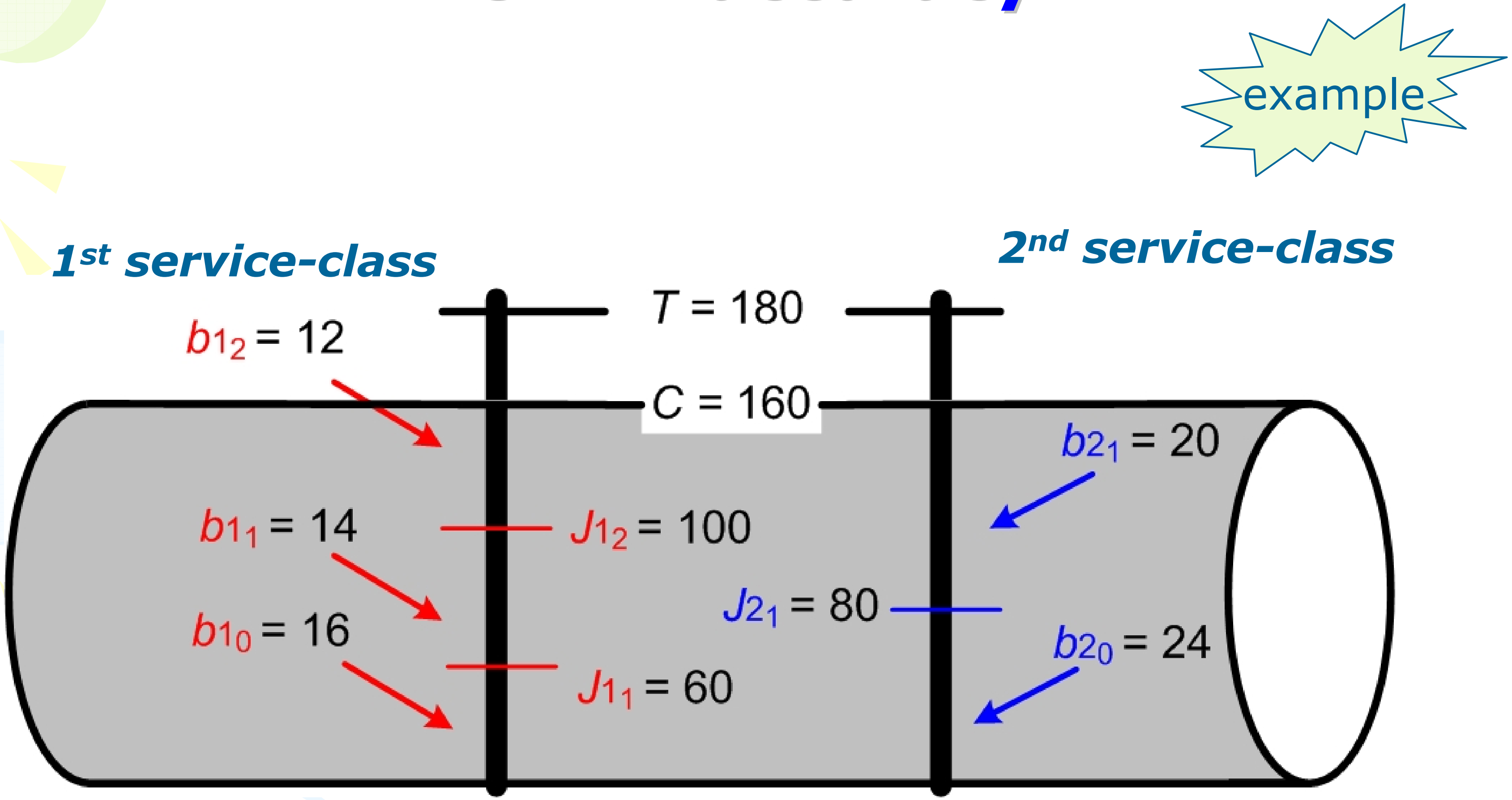
Time Congestion Probability

$$P_{b_k}=\sum_{j=T-b_{k_{S_k}}+1}^{T}G^{-1}q(j)$$

Link Utilization

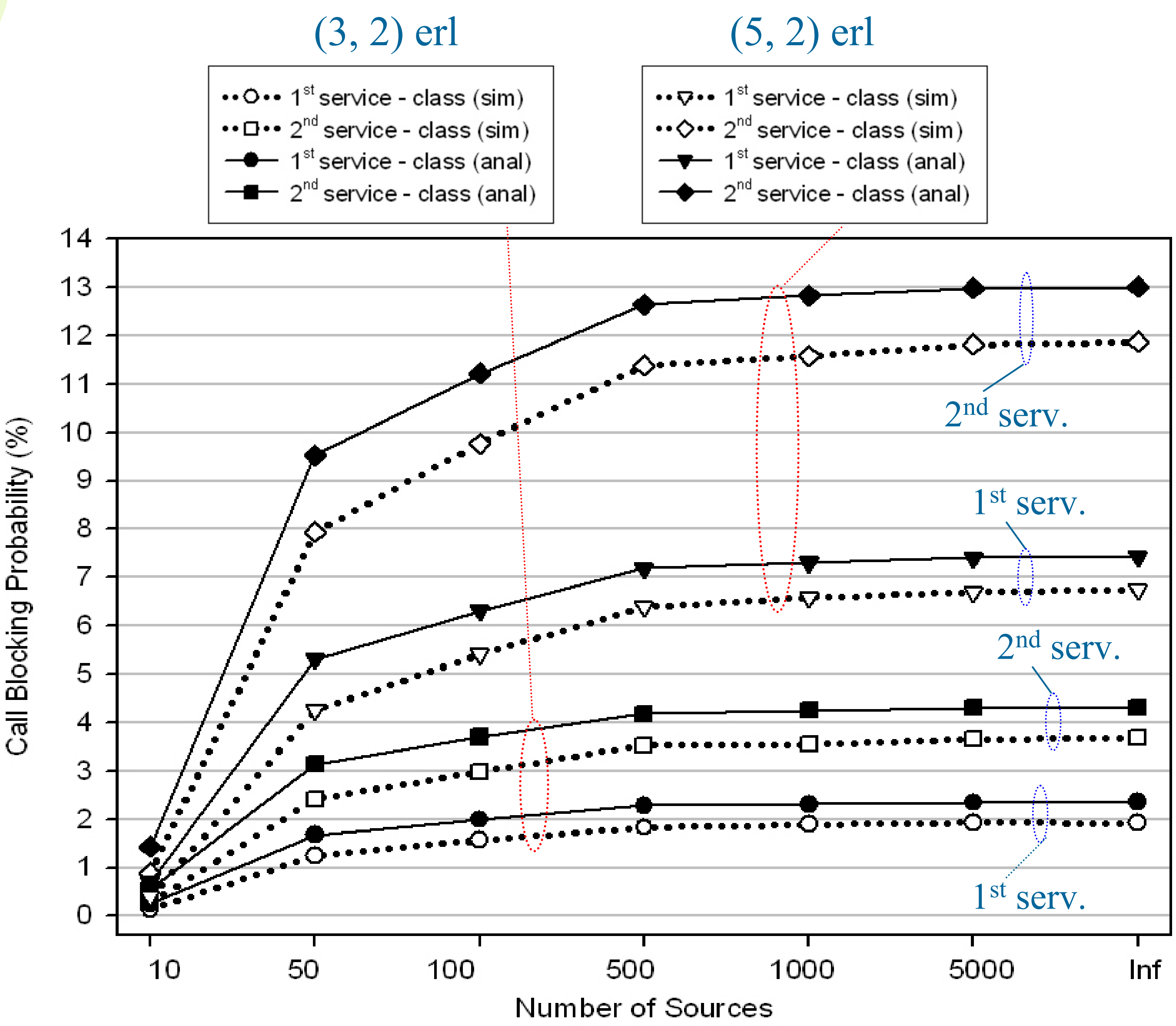
$$U=\sum_{j=1}^{C}jG^{-1}q(j)+\sum_{j=C+1}^{T}G^{-1}C\,q(j)$$

Ef-CDTM accuracy

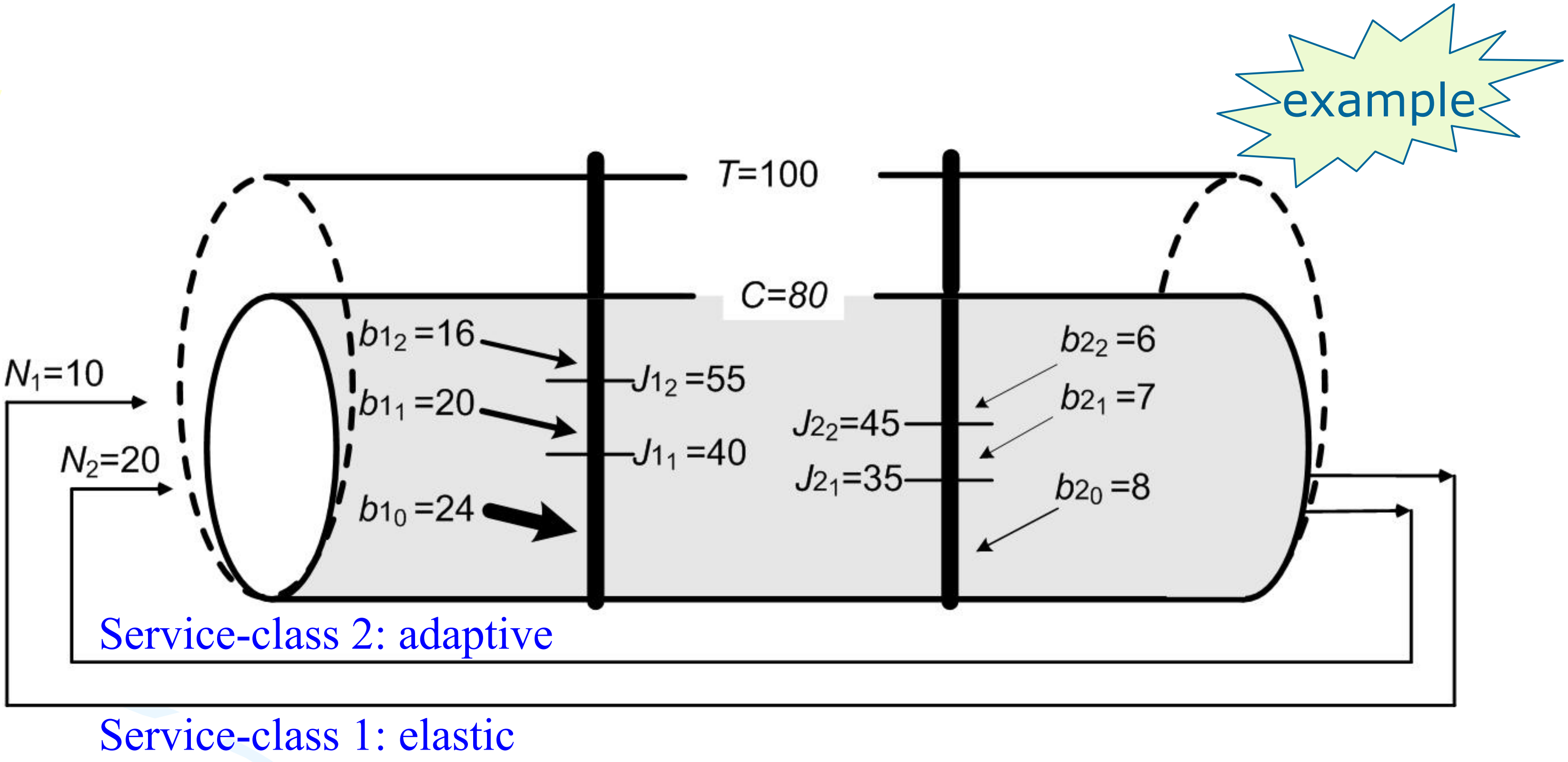


For example 1b.u. = 64 Kbps

Ef-CDTM accuracy (cont.)

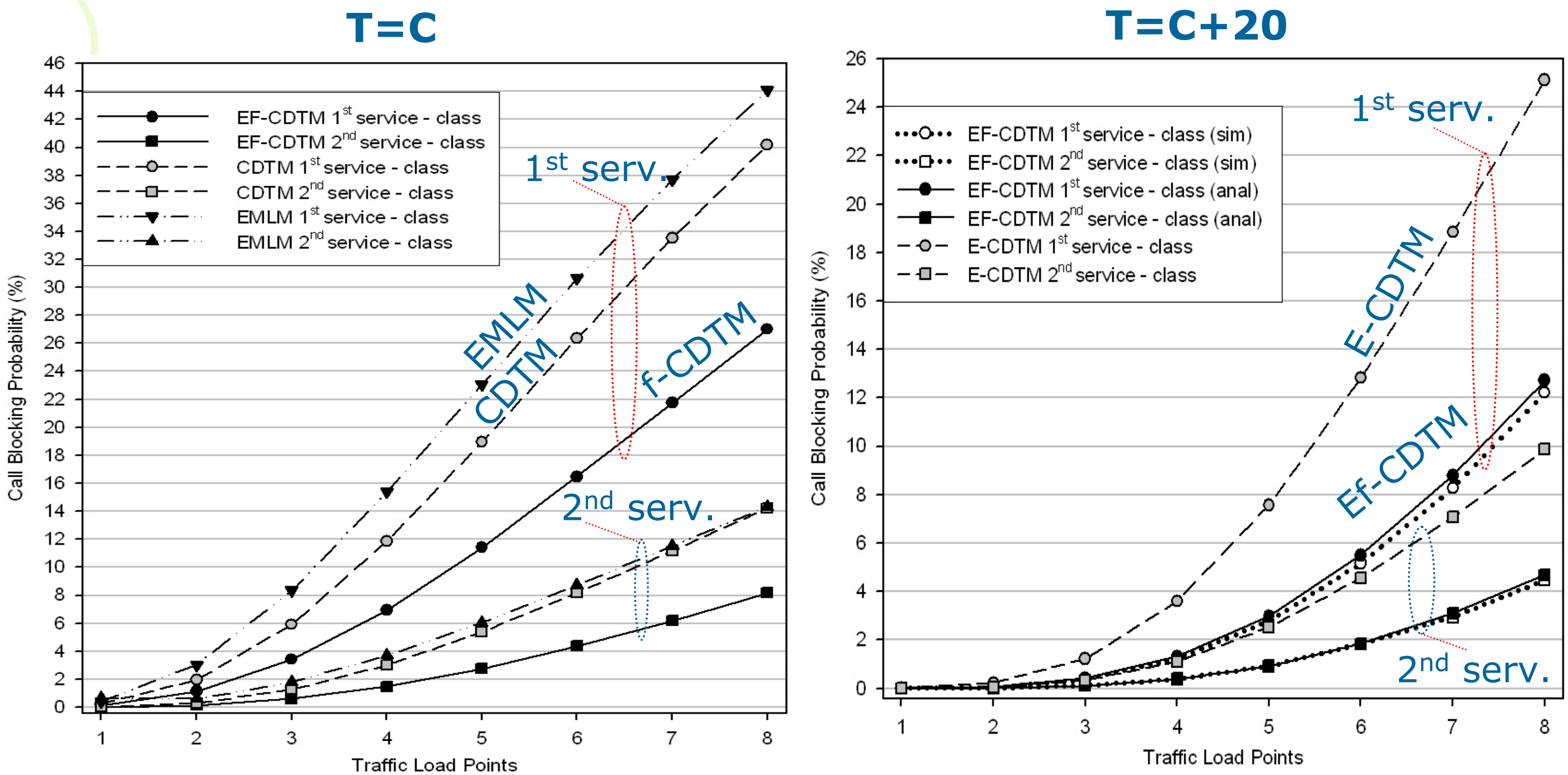


Ef-CDTM comparison with other models: EMLM, CDTM, E-CDTM



Offered Traffic-Load per idle source = 0.025 erl
Consequently, it increases by 0.025 erl

Ef-CDTM comparison with other models: EMLM, CDTM, E-CDTM (cont.)



Ef-CDTM ⟺ f-CDTM

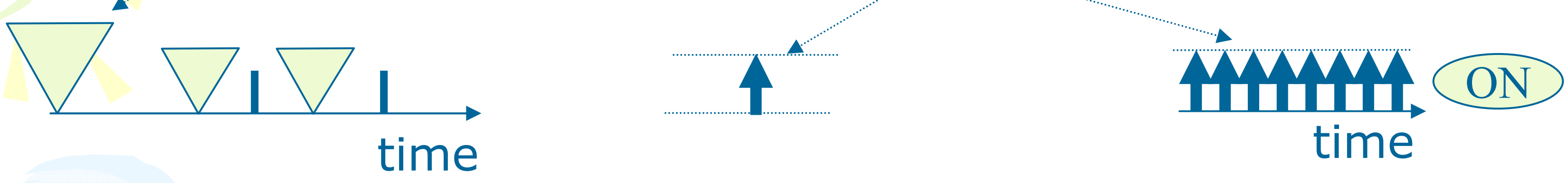
STRUCTURE – Where We Are

- (A) Random Traffic
  - (A1) Constant-bit-rate/stream traffic
  - (A2) Elastic Traffic while in service
- (B) Quasi-random Traffic
  - (B1) Constant-bit-rate/stream traffic
  - (B2) Elastic Traffic while in service
- **(C) Batched Poisson Traffic**
  - **(C1) Constant-bit-rate/stream traffic**
  - (C2) Elastic Traffic while in service

We are here!

(C) Batched Poisson Traffic

(C1) *Batched Poisson arriving calls with fixed bandwidth requirements and continuous use of the assigned bandwidth (constant-bit-rate/stream traffic) while in service.*



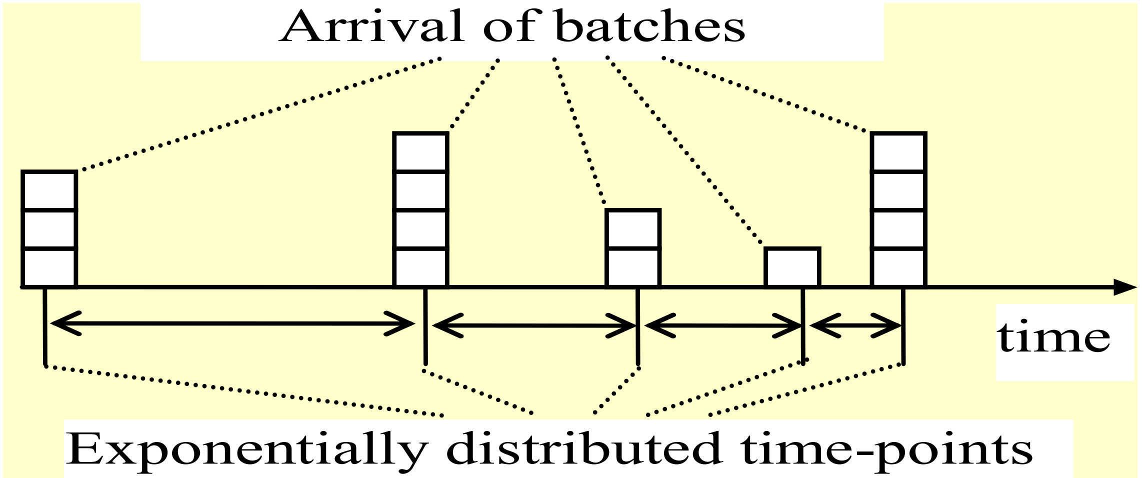
State of the art

- **The Batched Poisson Erlang Multirate Loss Model (BP-EMLM)** *1996*

Furthermore

- **The Batched Poisson Erlang Multirate Loss Model under the Bandwidth Reservation Policy** *2010*

Batched Poisson arrival process



λ_k **batch arrival rate**

λ_k^{-1} **batch interarrival time (exponentially distributed).**

B_r^k **probability that there are *r* calls in an arriving batch of service-class *k***

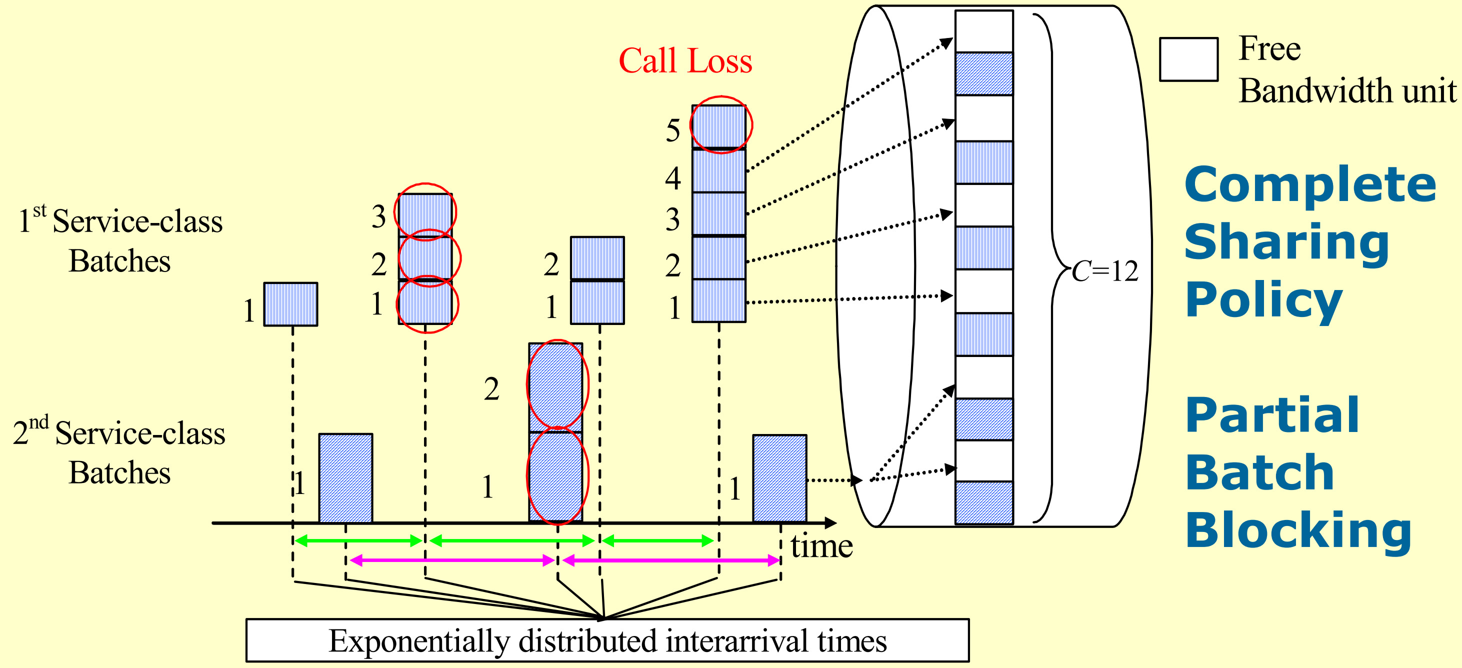
The Batched Poisson Erlang Multirate Loss Model (BP-EMLM)

$C = 12$

$K = 2$

$b_1 = 1$

$b_2 = 2$



Complete Sharing Policy

Partial Batch Blocking

Call Congestion Probabilities	>	Time Congestion Probabilities
$\frac{1+0+3+0}{5+2+3+1} = \frac{4}{11}$ 1st service-class		$\frac{0+0+1+0}{5+2+3+1} = \frac{1}{11}$ 1st service-class
$\frac{0+2+0}{1+2+1} = \frac{2}{4}$ 2nd service-class		$\frac{0+1+0}{1+2+1} = \frac{1}{4}$ 2nd service-class

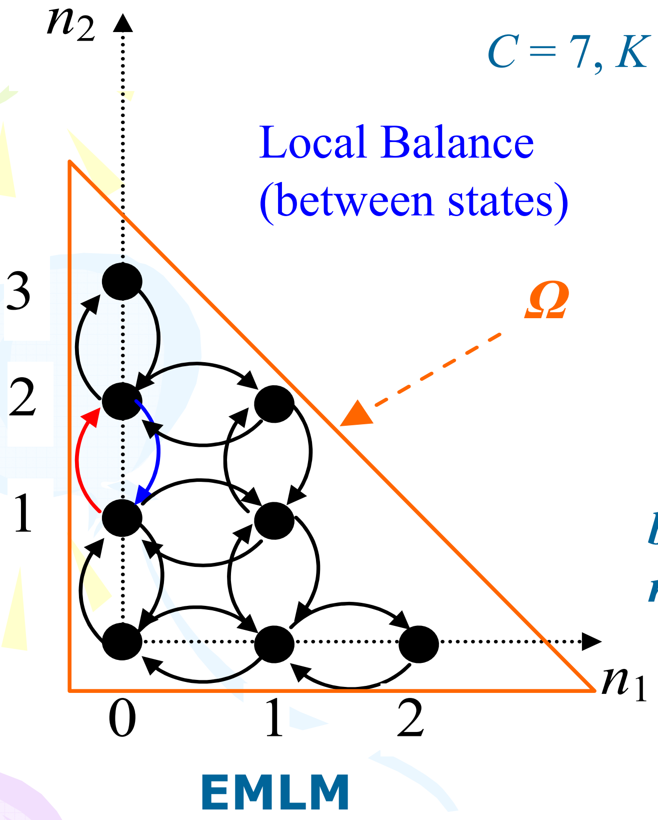
The proportion of arriving calls that find the system congested.

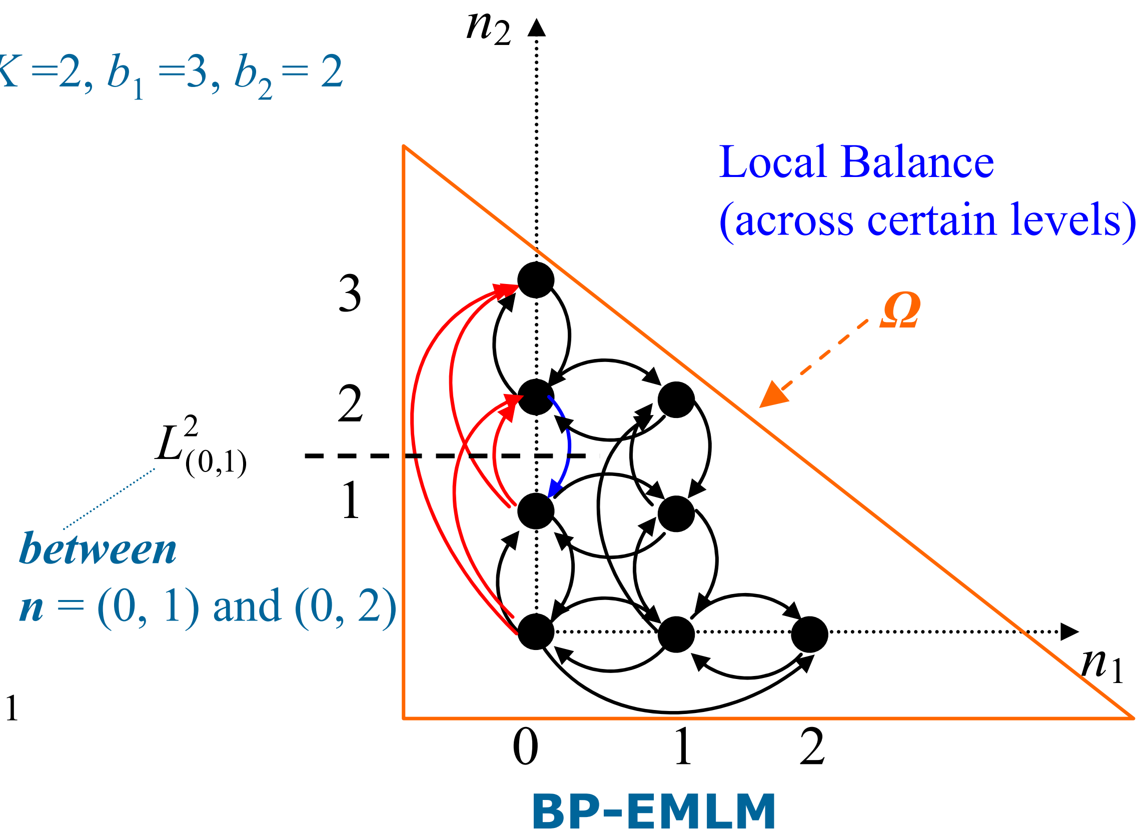
The proportion of time that the system is congested.

BP-EMLM Analysis

State Space – Local Balance

$C = 7, K = 2, b_1 = 3, b_2 = 2$





The level $L_{\boldsymbol{n}}^{k}$ separates the state-vector $\boldsymbol{n} = (n_1, n_2, \ldots, n_{k-1}, n_k, n_{k+1}, \ldots, n_K)$ from the state-vector $(n_1, n_2, \ldots, n_{k-1}, n_k + 1, n_{k+1}, \ldots, n_K)$, for service-class k.

BP-EMLM – The analytical Model

Kaufman, Rege, Performance Evaluation 1996

- C link capacity
- K service classes
- b_k bandwidth requirements (k=1,…,K)
- λ_k batch arrival rate
- μ_k service rate
- h_k $= \mu_k^{-1}$ service time (exponentially distributed).
- B_r^k probability that there are *r* calls in an arriving batch of service-class *k*
- j occupied link bandwidth
- $q(j)$ probability that j out of C bandwidth units are occupied

Link occupancy distribution

$$q(j) = \frac{1}{j} \sum_{k=1}^{K} \alpha_k b_k \sum_{l=1}^{\lfloor j/b_k \rfloor} \hat{B}_{l-1}^{k} q(j - l b_k)$$

where $\alpha_k = \lambda_k / \mu_k$ and

$$\hat{B}_l^k = \sum_{r=l+1}^{\infty} B_r^k$$ (the complementary batch size distribution)

BP-EMLM – The analytical Model (cont.)

Performance measures

$$E(n_k|j) = \frac{\alpha_k \sum_{l=1}^{\lfloor j/b_k \rfloor} \hat{B}_{l-1}^{k} \mathrm{q}(j - lb_k)}{\mathrm{q}(j)}$$

Average number of service-class k calls in state j

$$\overline{n}_k = \sum_{j=1}^{C} E(n_k|j)\mathrm{q}(j)$$

Average number of service-class k calls in the system

$$C_{b_k} = \frac{\alpha_k \hat{B}_k - \overline{n}_k}{\alpha_k \hat{B}_k}$$

Call congestion probability of service-class k

$$P_{b_k} = \sum_{j=C-b_k+1}^{C} G^{-1}\mathrm{q}(j)$$

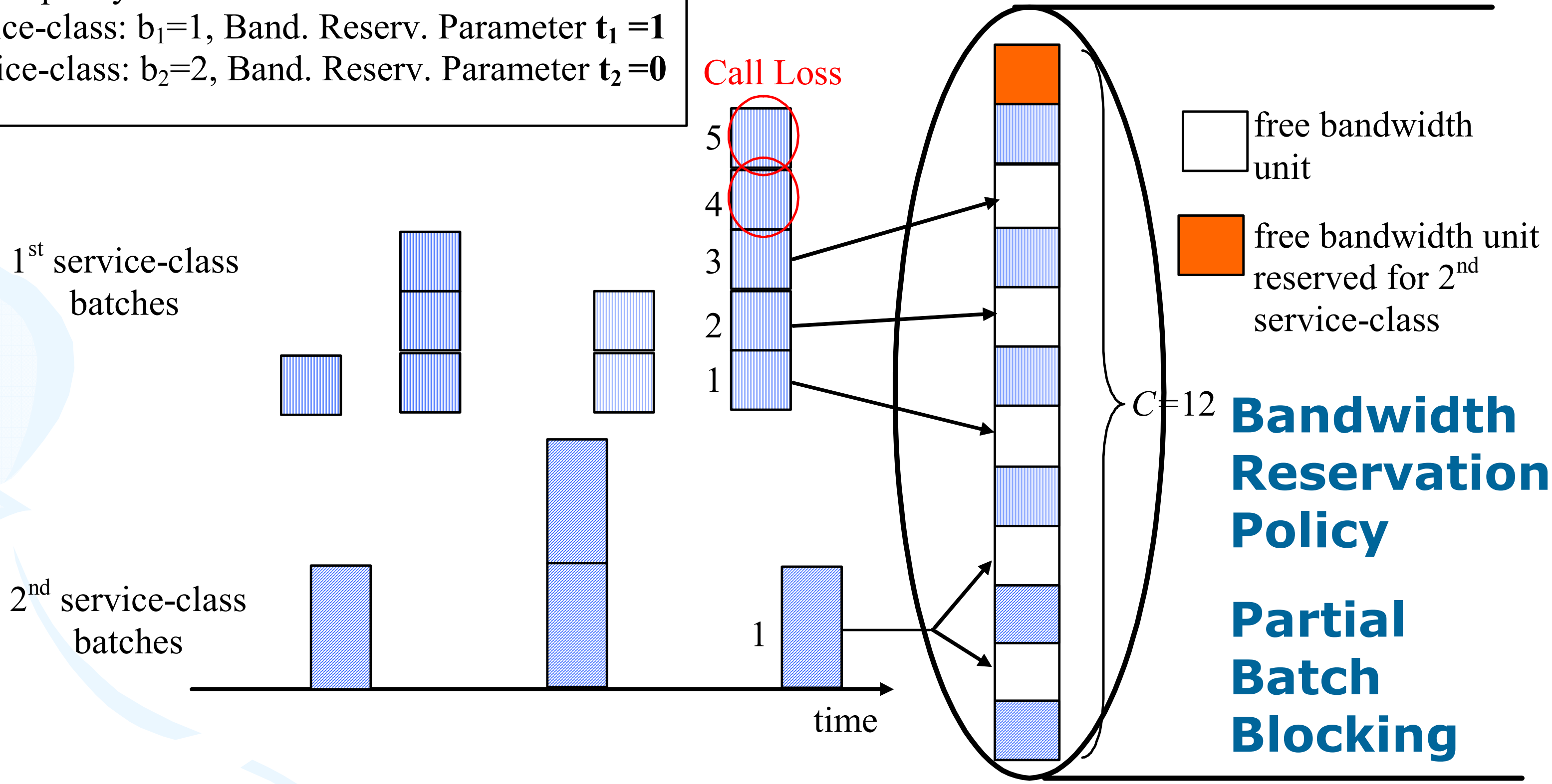
Time congestion probability of service-class k

$$U = \sum_{j=1}^{C} jq(j)$$

Link utilization

The BP-EMLM under Bandwidth Reservation Policy (BP-EMLM/BR)

Link of capacity C= 12 b. u.
1st service-class: b_1=1, Band. Reserv. Parameter $\mathbf{t_1}$ **=1**
2nd service-class: b_2=2, Band. Reserv. Parameter $\mathbf{t_2}$ **=0**



Bandwidth Reservation Policy

Partial Batch Blocking

A call of service-class k is accepted when $j + b_k \leq C - t_k$

BP-EMLM/BR – Roberts' Method

Assumption:

Calls of service-class k are assumed to be negligible when $j=C-t_k+1, C-t_k, \dots, C$

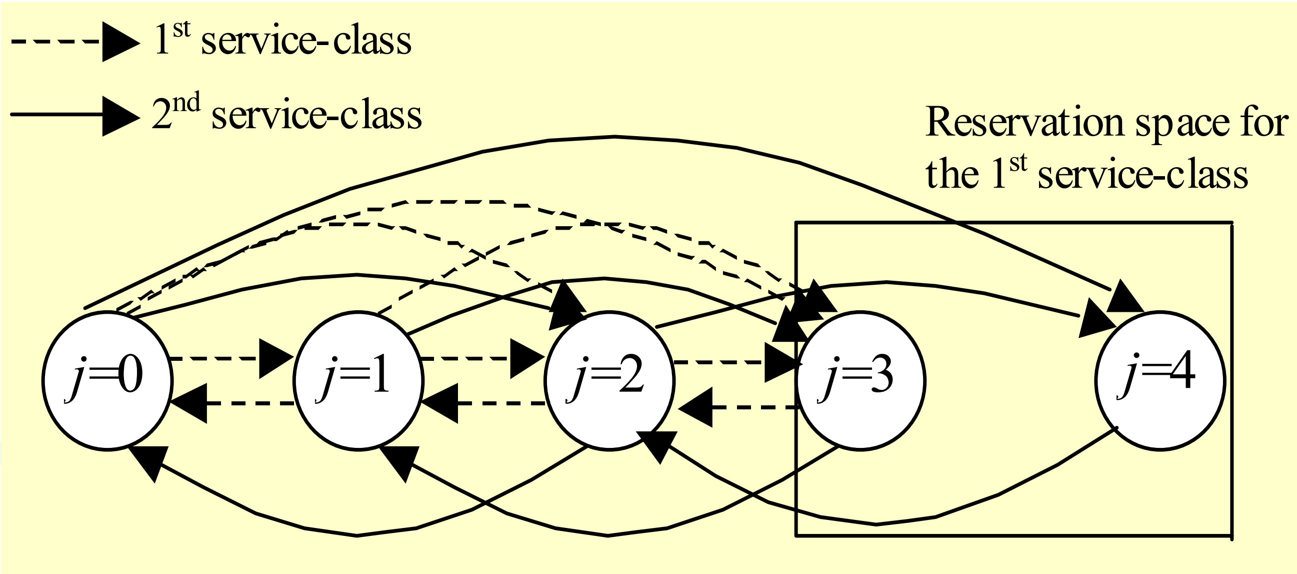
example

$C = 4$

$K = 2$

$b_1=1, t_1=1$

$b_2=2, t_2=0$



The reservation space of a service-class k includes the blocking states: $C-b_k-t_k+1, \dots, C$ e.g. for the 1st service-class, $j=3$ and 4.

BP-EMLM/BR – Roberts' Method (cont.)

Moscholios and Logothetis, Computer Communications, 2010

Link Occupancy Distribution

$$q(j) = \frac{1}{j}\sum_{k=1}^{K} \alpha_k D_k(j-b_k) \sum_{l=1}^{\lfloor j/b_k \rfloor} \hat{B}_{l-1}^{k} q(j-lb_k)$$

$$D_k(j-b_k) = \begin{cases} b_k & \text{when } j \le C - t_k \\ 0 & \text{when } j > C - t_k \end{cases}$$

Performance measures

$$E(n_k | j) = \begin{cases} \dfrac{\alpha_k \sum_{l=1}^{\lfloor j/b_k \rfloor} \hat{B}_{l-1}^{k} q(j-lb_k)}{q(j)} & \text{when } j \le C - t_k \\ 0 & \text{when } j > C - t_k \end{cases}$$

Average number of service-class k calls in state j

$$P_{b_k} = \sum_{j=C-b_k-t_k+1}^{C} G^{-1} q(j)$$

Time Congestion probability of service-class k

BP-EMLM/BR–Method of Stasiak & Glabowski (cont.)

$$E^*(n_k \mid j) = \begin{cases} \dfrac{\alpha_k \sum_{l=1}^{\lfloor j/b_k \rfloor} \hat{B}_{l-1}^k \mathrm{q}(j - lb_k)}{\mathrm{q}(j)} & \text{when} \quad j \le C - t_k \\ \sum_{i=1, i \ne k}^{K} E^*(n_k \mid j - b_i) w_{k,i}(j) & \text{when} \quad j > C - t_k \end{cases}$$

Average number of service-class k calls when j=C-t_k+1, C-t_k, …,C

$$where \quad w_{k,i}(j) = \frac{\alpha_i b_i}{\sum_{j=1, j \ne k}^{K} \alpha_j b_j} \qquad \hat{B}_l^k = \sum_{r=l+1}^{\infty} B_r^k$$

Link Occupancy Distribution

$$\mathrm{q}(j) = \frac{1}{j^*} \sum_{k=1}^{K} \alpha_k b_k \sum_{l=1}^{\lfloor j/b_k \rfloor} \hat{B}_{l-1}^k \mathrm{q}(j - lb_k)$$

$$j^* = \sum_{k=1}^{K} b_k E^*(n_k \mid j)$$

Numerical example: BP-EMLM – BP-EMLM/BR

$C = 100$ b.u.

$K = 3$

$b_1 = 1$ b.u., $t_1 = 15$ b.u.

$b_2 = 4$ b.u., $t_2 = 12$ b.u.

$b_3 = 16$ b.u., $t_3 = 0$ b.u.

$P_r(s_k = r) = (1 - \beta_k)\beta_k^{r-1}$ (geometric distribution of batch size s_k)

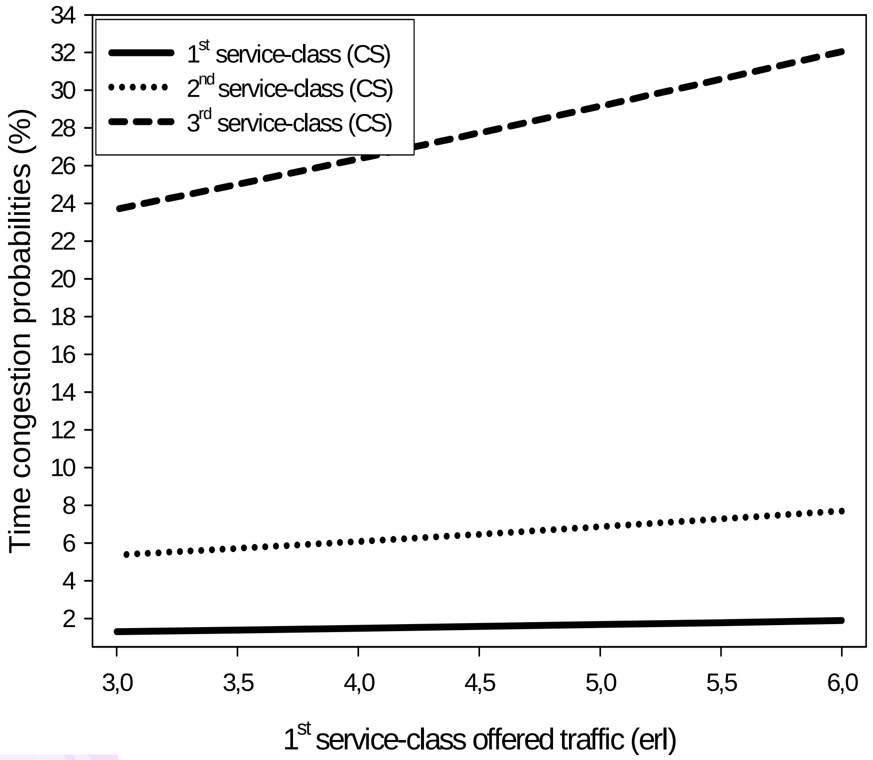
$\beta_1 = 0.75$, $\beta_2 = 0.5$, $\beta_3 = 0.2$ (note: average batch size is $1/(1-\beta_k)$)

$\mu^{-1}{}_1 = \mu^{-1}{}_2 = \mu^{-1}{}_3 = 1$ (exponentially distributed call service time)

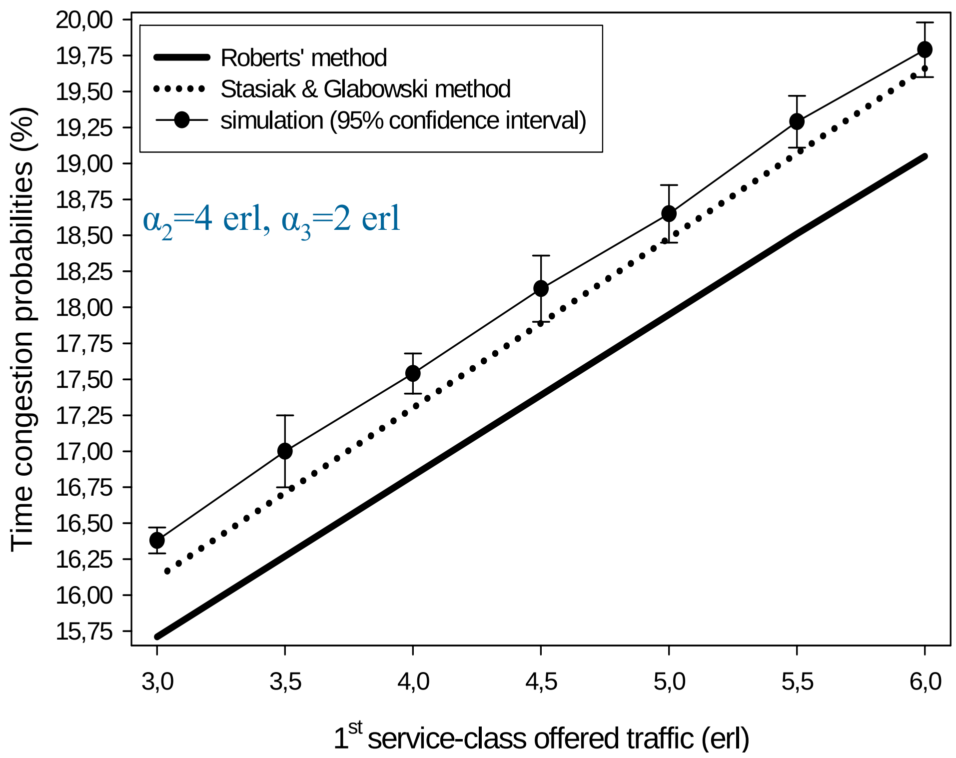
$\alpha_1 = 6$ erl, $\alpha_2 = 4$ erl, $\alpha_3 = 2$ erl (offered traffic)

Numerical example: BP-EMLM – BP-EMLM/BR (cont.1)

Time Congestion Probabilities



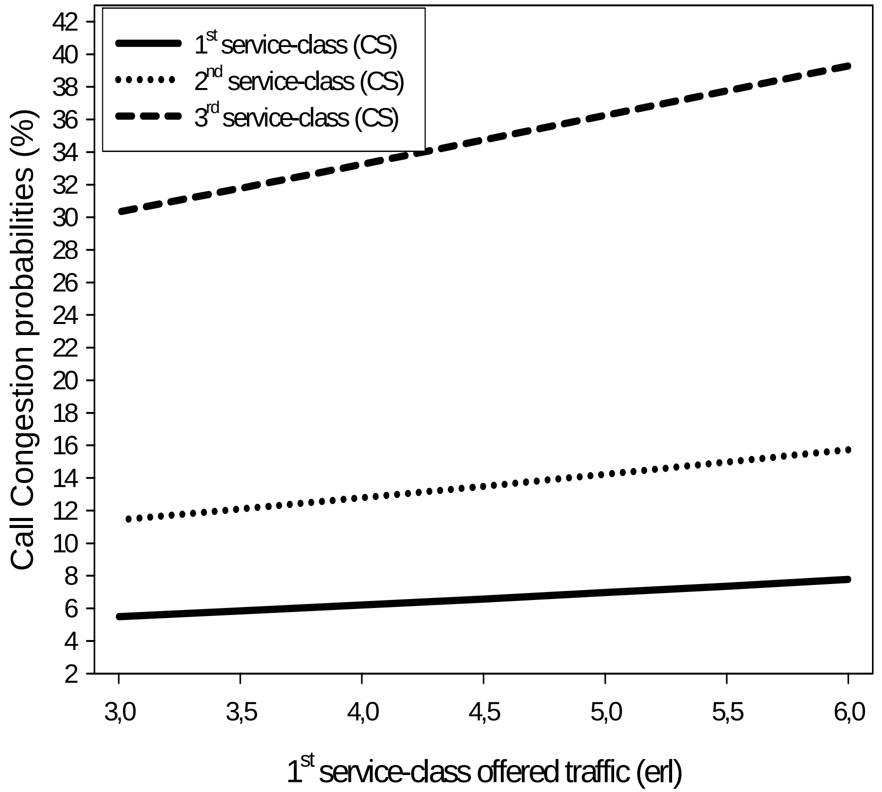
BP-EMLM

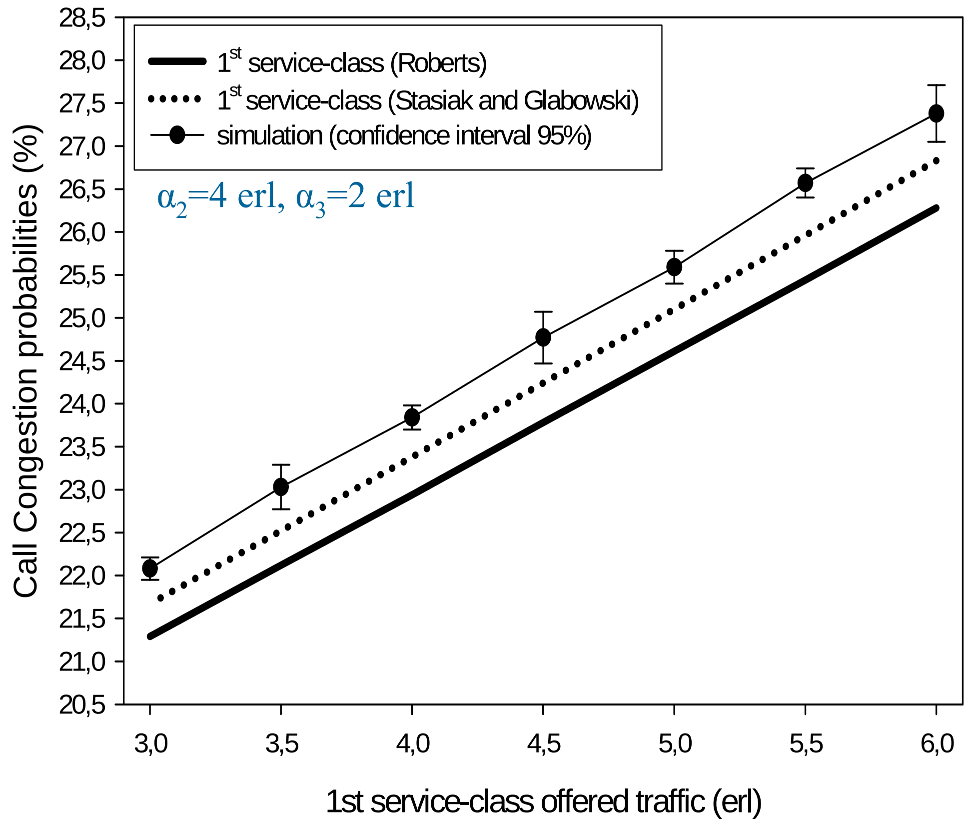


BP-EMLM/BR

Numerical example: BP-EMLM – BP-EMLM/BR (cont.2)

Call Congestion Probabilities
(higher than time congestion probabilities)





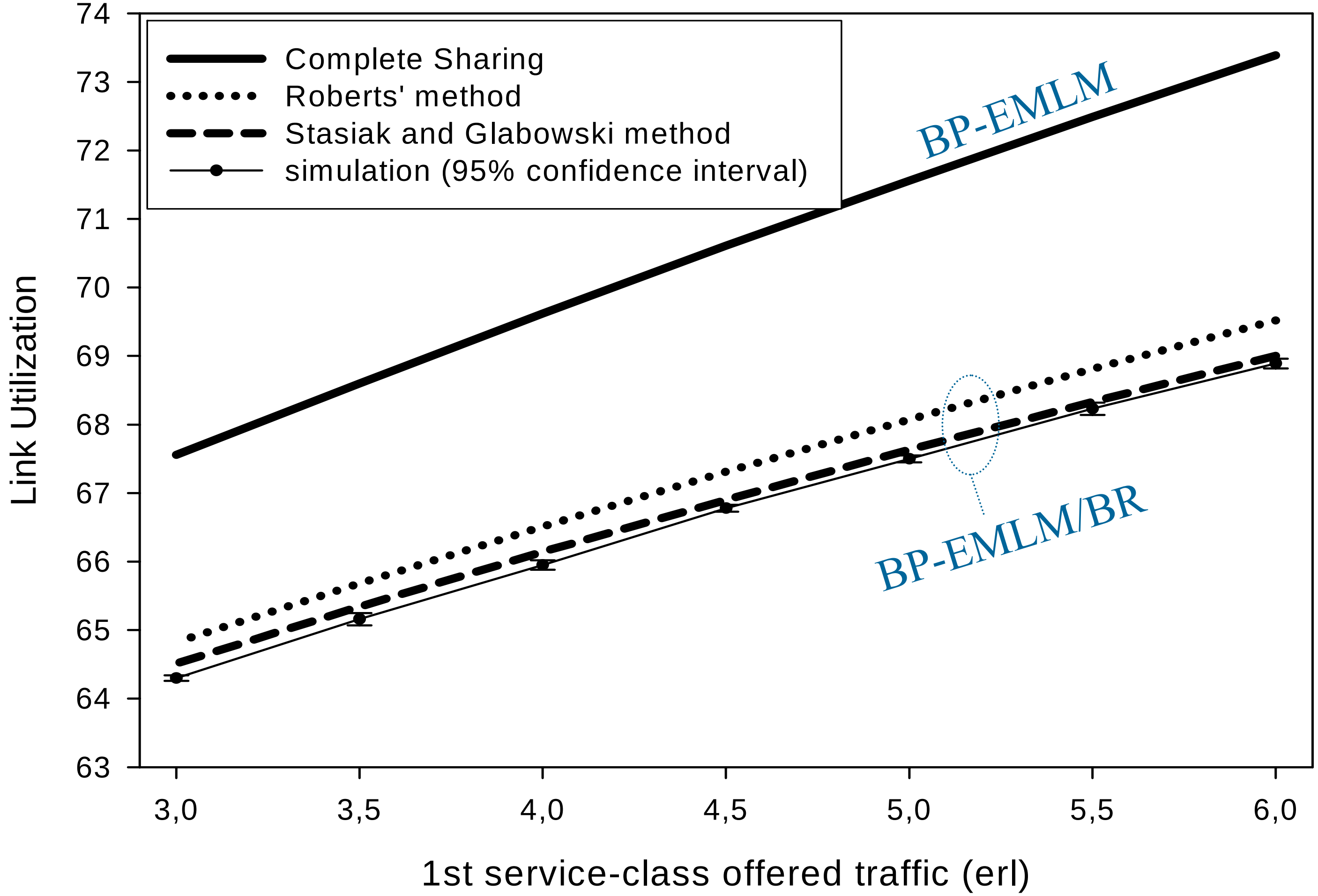
Numerical example: BP-EMLM – BP-EMLM/BR (cont.3)

Call congestion probabilities

	Roberts' method (%)			Method of S&G (%)			Simulation results (%)		
α_1	*1st class*	*2nd class*	*3rd class*	*1st class*	*2nd class*	*3rd class*	*1st class*	*2nd class*	*3rd class*
6.0	26.28	28.45	27.67	26.83	28.98	28.21	27.38 ±0.33	29.32 ±0.40	28.23 ±0.46
5.5	25.44	27.57	26.91	25.96	28.08	27.42	26.57 ±0.17	28.40 ±0.22	27.46 ±0.33
5.0	24.61	26.69	26.15	25.10	27.17	26.63	25.59 ±0.19	27.28 ±0.16	26.67 ±0.22
4.5	23.78	25.81	25.37	24.24	26.26	25.83	24.77 ±0.30	26.63 ±0.15	25.88 0.16
4.0	22.94	24.93	24.60	23.38	25.36	25.02	23.84 ±0.14	25.65 ±0.21	25.07 ±0.17
3.5	22.12	24.06	23.81	22.52	24.45	24.21	23.03 ±0.26	24.62 ±0.29	24.37 ±0.25
3.0	21.29	23.18	23.03	21.67	23.55	23.40	22.08 ±0.13	23.70 ±0.07	23.47 ±0.08

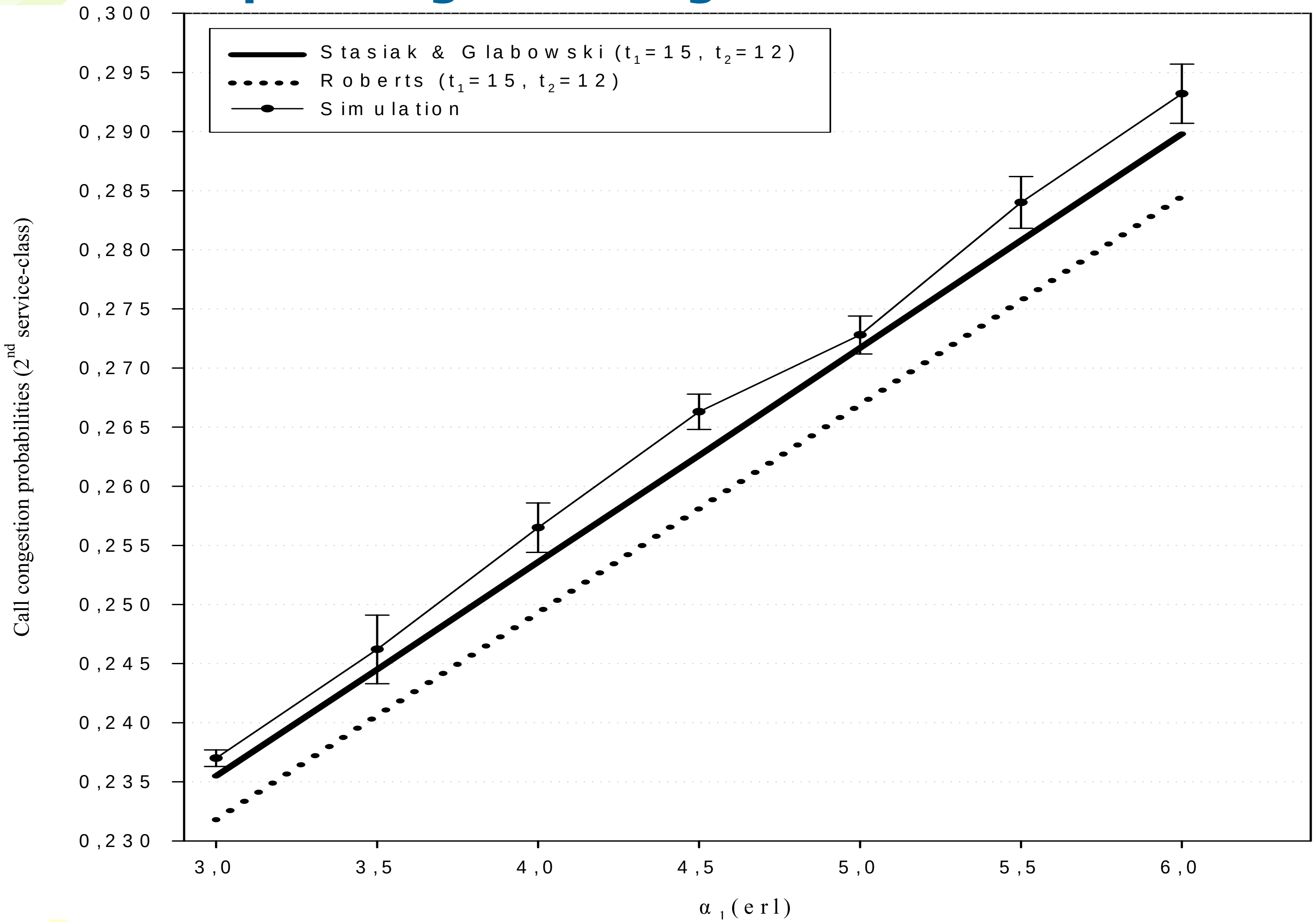
Numerical example: BP-EMLM – BP-EMLM/BR (cont.4)

Link Utilization (C= 100)



Numerical example: BP-EMLM – BP-EMLM/BR (cont.5)

Equalizing Call Congestion Probabilities



STRUCTURE – Where We Are

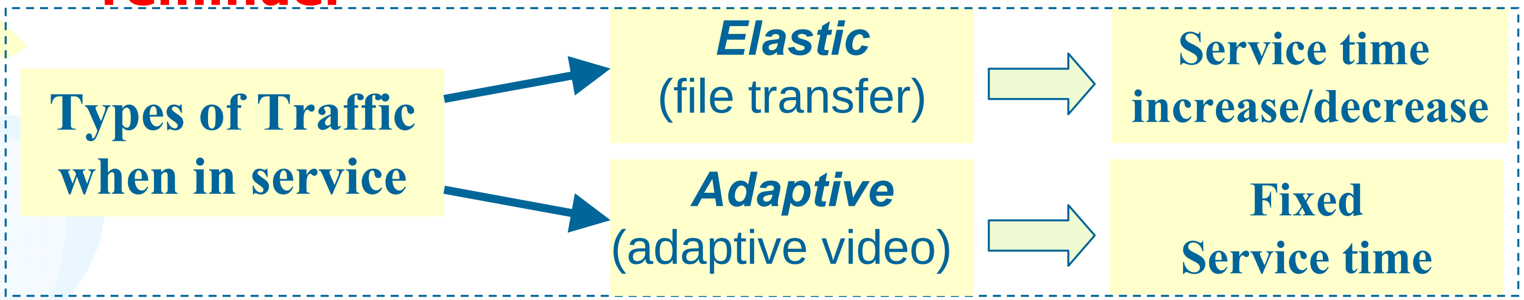
- (A) Random Traffic
  - (A1) Constant-bit-rate/stream traffic
  - (A2) Elastic Traffic while in service
- (B) Quasi-random Traffic
  - (B1) Constant-bit-rate/stream traffic
  - (B2) Elastic Traffic while in service
- **(C) Batched Poisson Traffic**
  - (C1) Constant-bit-rate/stream traffic
  - **(C2) Elastic Traffic while in service**

We are here!

(C) Batched Poisson Traffic

(C2) *Batched Poisson arriving calls with fixed bandwidth requirements upon arrival, and elastic bandwidth while in service.*

reminder



State of the art

- **The Batched Poisson Erlang Multirate Loss Model (BP-EMLM)** *1996*

Furthermore

- **The BP-EMLM supporting elastic and adaptive traffic under the BR policy** *2011, 2012*

The BP EMLM for elastic & adaptive traffic under the BR policy

Moscholios et. al (IEEE ICC 2012, Annals of Telecommunications 2012)

Link Occupancy Distribution

$$q(j)=\begin{cases}1 & \text{for } j=0\\ \dfrac{1}{\min(j,C)}\sum_{k=1}^{K_e}\alpha_k D_k(j-b_k)\sum_{l=1}^{\lfloor j/b_k\rfloor}\widehat{B}_{l-1}^{(k)}G(j-lb_k) & \\ +\dfrac{1}{j}\sum_{k=1}^{K_a}\alpha_k D_k(j-b_k)\sum_{l=1}^{\lfloor j/b_k\rfloor}\widehat{B}_{l-1}^{(k)}G(j-lb_k) & \text{for } j=1,\dots,T\\ 0 & \text{for } j<0\end{cases}$$

Elastic classes

Adaptive classes

where: $D_k(j-b_k)=\left\langle\begin{matrix} b_k & \text{for } j\le T-t_k\\ 0 & \text{for } j>T-t_k\end{matrix}\right\rangle$

The BP EMLM for elastic & adaptive traffic under the BR policy (cont.)

Performance Metrics

TC probability of service-class k

$$P_{b_k} = \sum_{j=C-b_k-t_k+1}^{C} G^{-1} q(j)$$

CC probability of service-class k

$$C_{b_k} = \sum_{j=0}^{C} G^{-1} q(j) \sum_{m=\left\lfloor \frac{C-j}{b_k} \right\rfloor + 1}^{\infty} B_m^{(k)}$$

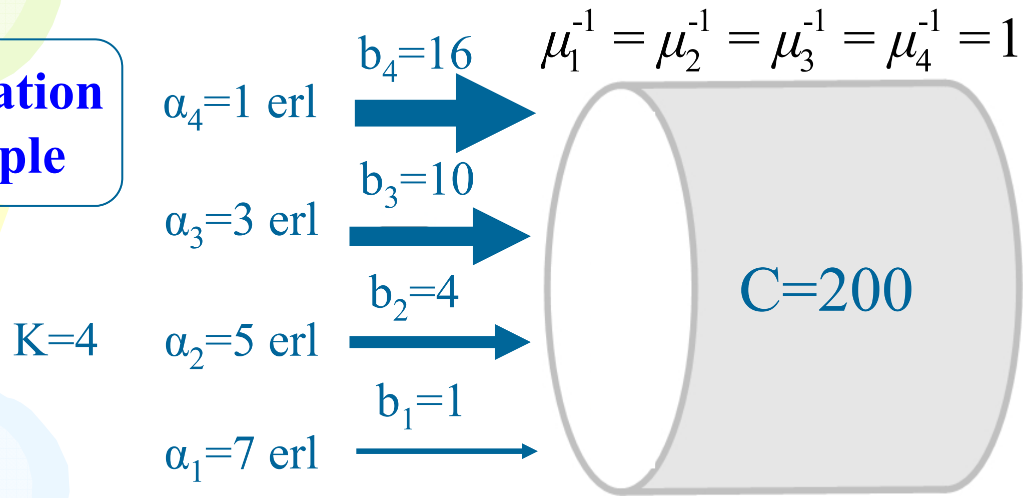
Link Utilization

$$U = \sum_{j=1}^{C} j G^{-1} q(j) + \sum_{j=C+1}^{T} C G^{-1} q(j)$$

- No Product Form Solution
- Approx. calculation of link occupancy distribution and all performance measures.

Numerical Results – Evaluation

Application example



Batch size, s_k:
Geometrically distributed,
$Pr(s_k=r)=(1-\beta_k)\,\beta_k^{r-1}$
$\beta_1=0.75$, $\beta_2=0.5$, $\beta_3=\beta_4=0.2$.

One set of BR parameters:
$t_1 = 15$, $t_2 = 12$, $t_3 = 6$, $t_4 = 0$ (TC equalization among calls of all service-classes).

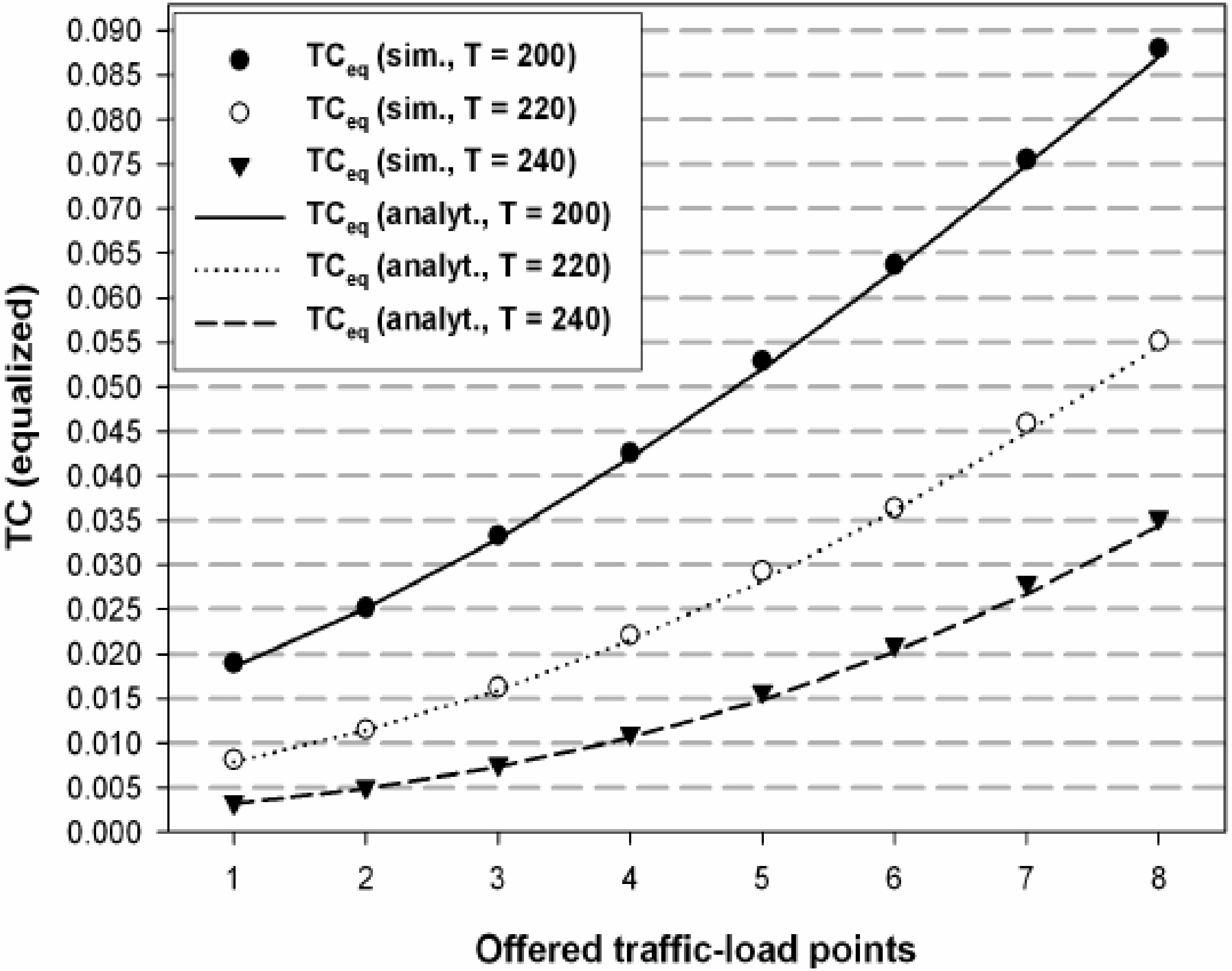
Three different values of T:

a) $T = C = 200$ b.u. (no bandwidth compression - results coincide with BP-EMLM/BR)

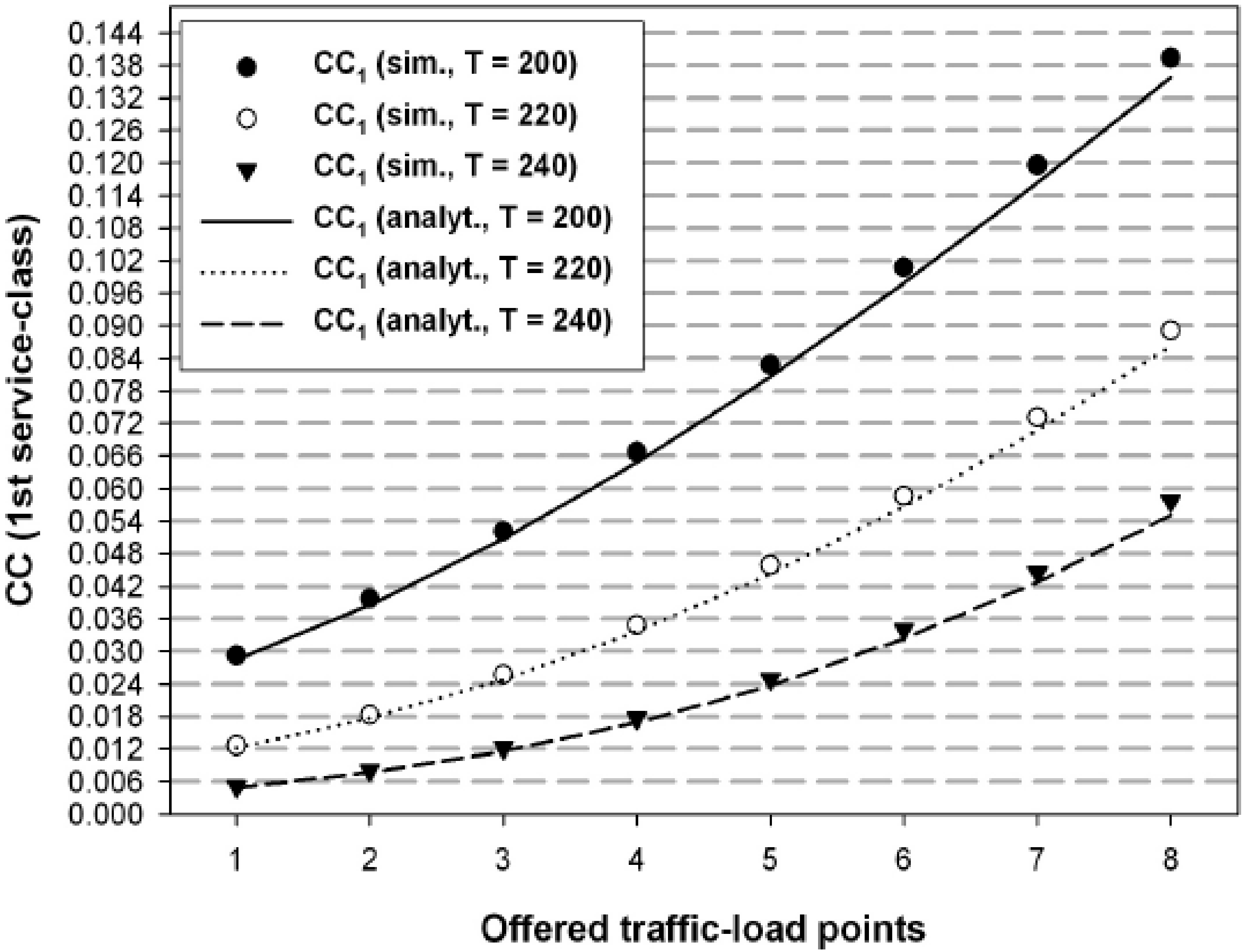
b) $T = 220$ b.u. (max compression factor $C/T = 200/220$) $b_1 = 1 \rightarrow b_{1min} = 0.91$

c) $T = 240$ b.u. (max compression factor $C/T = 200/240$) $b_1 = 1 \rightarrow b_{1min} = 0.83$

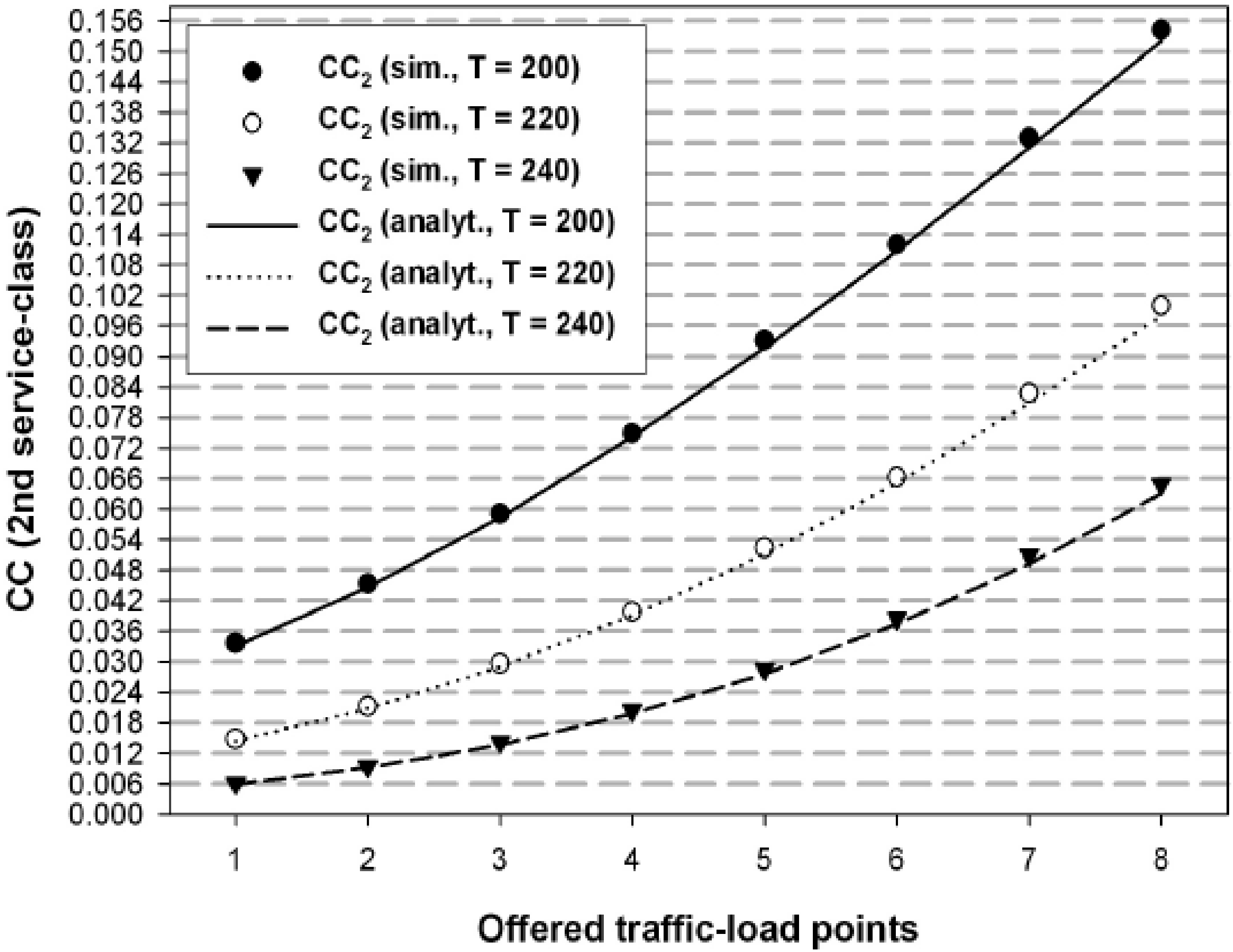
Numerical Results – Evaluation (cont.)



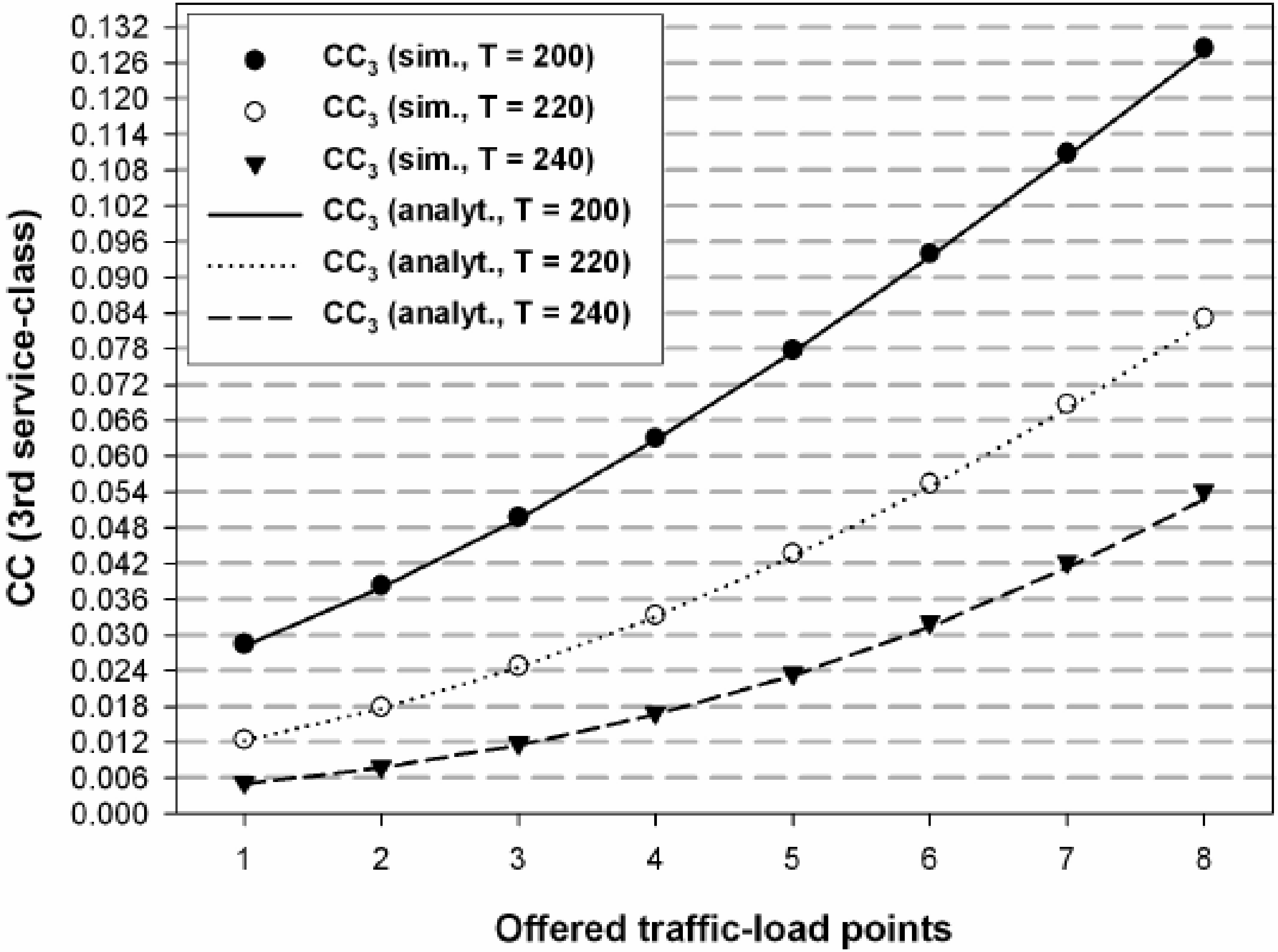
Numerical Results – Evaluation (cont.)



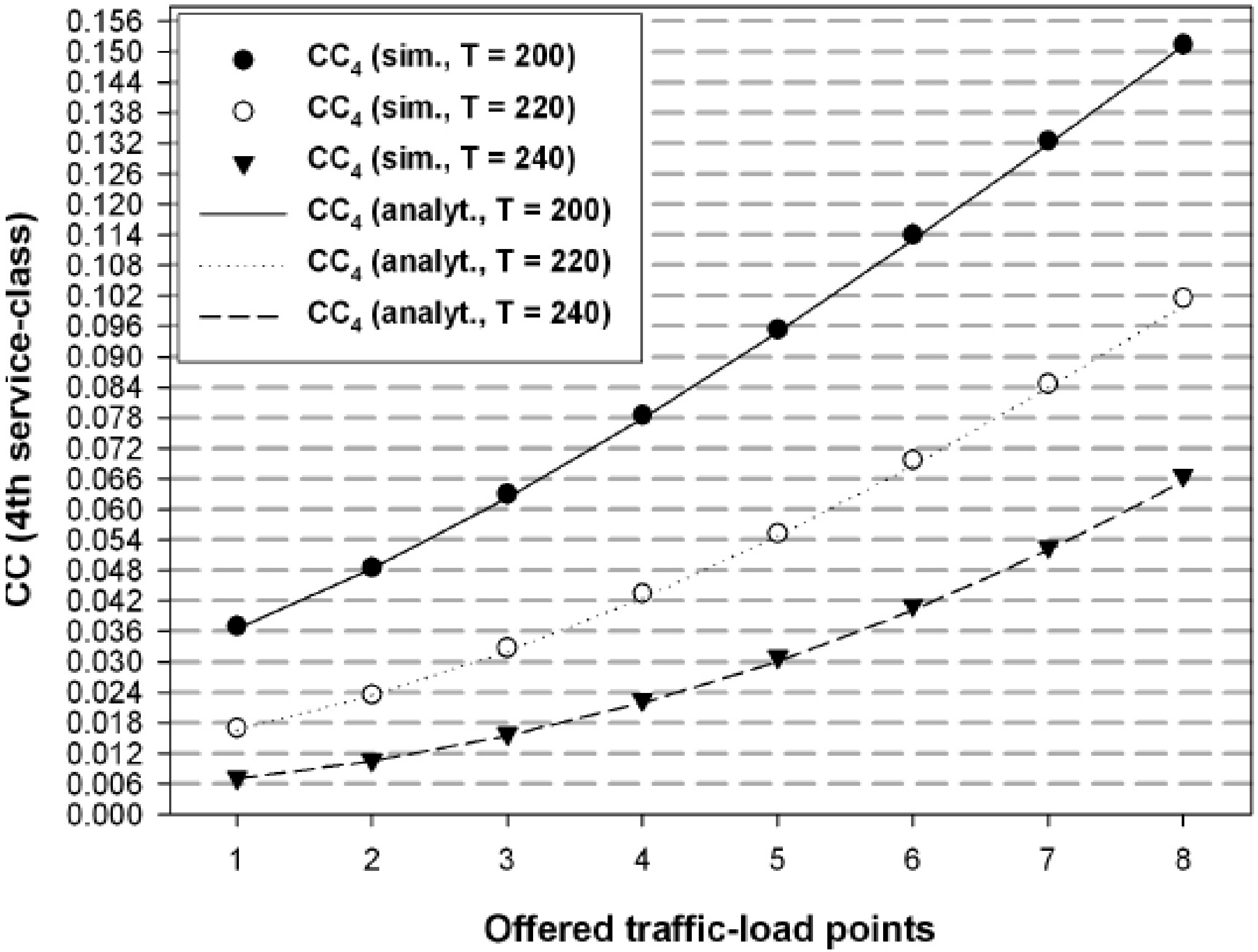
Numerical Results – Evaluation (cont.)



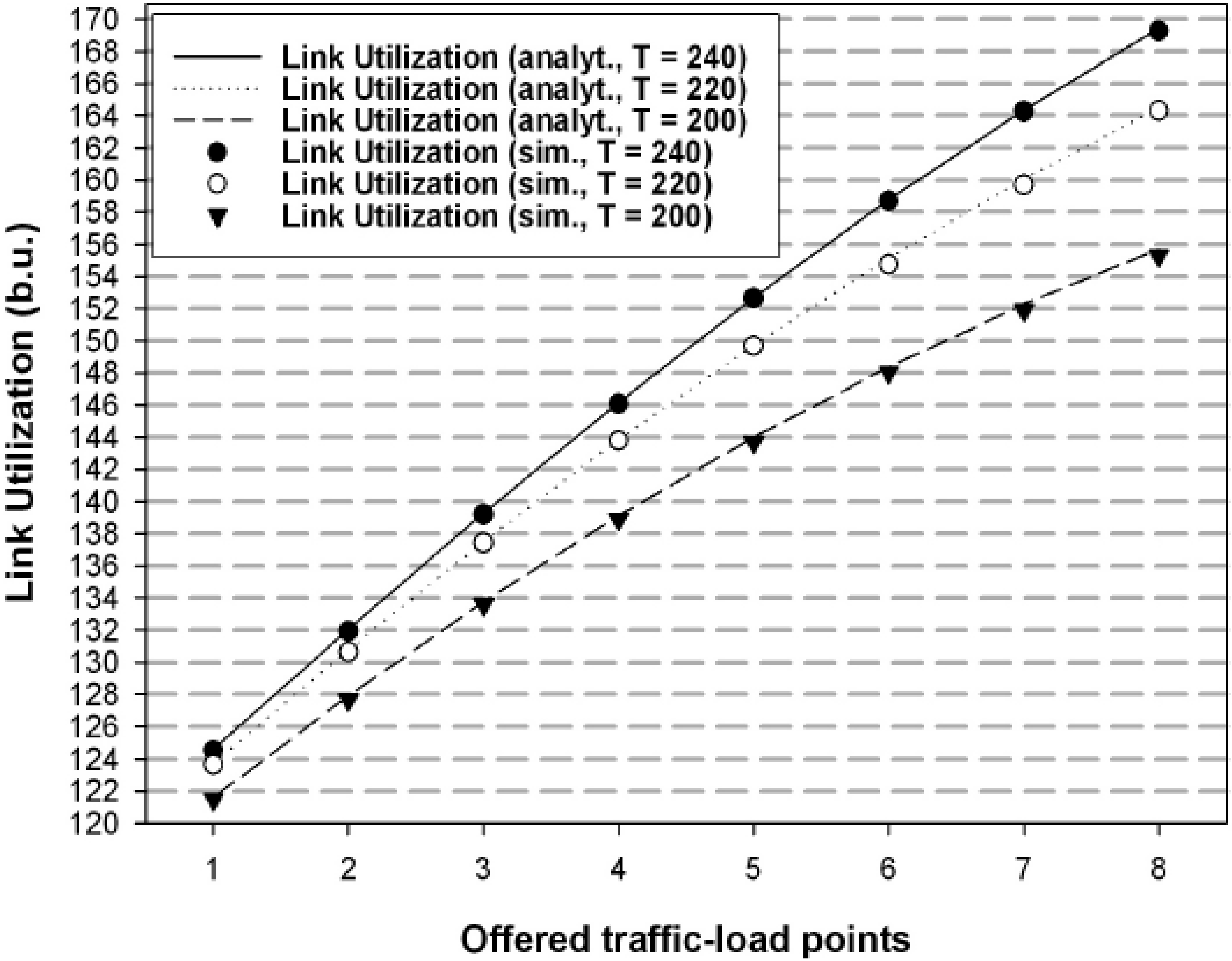
Numerical Results – Evaluation (cont.)



Numerical Results – Evaluation (cont.)



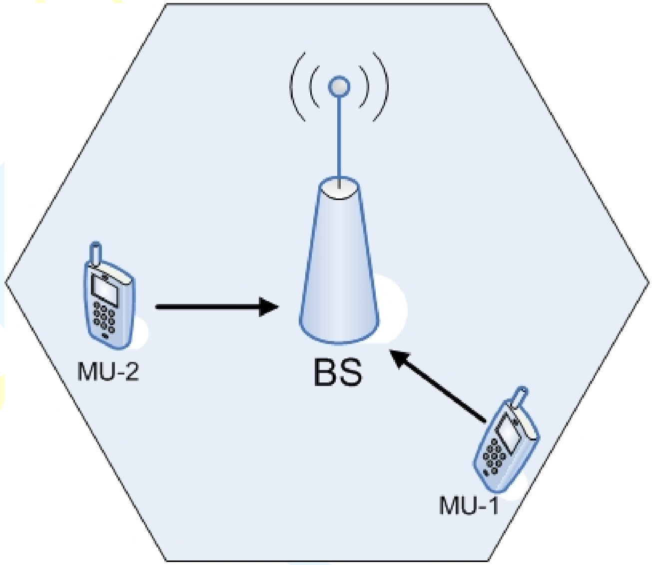
Numerical Results – Evaluation (cont.)



Introduction to W-CDMA

User Activity

Uplink: calls from the Mobile Users (MUs) to the Base Station (BS)



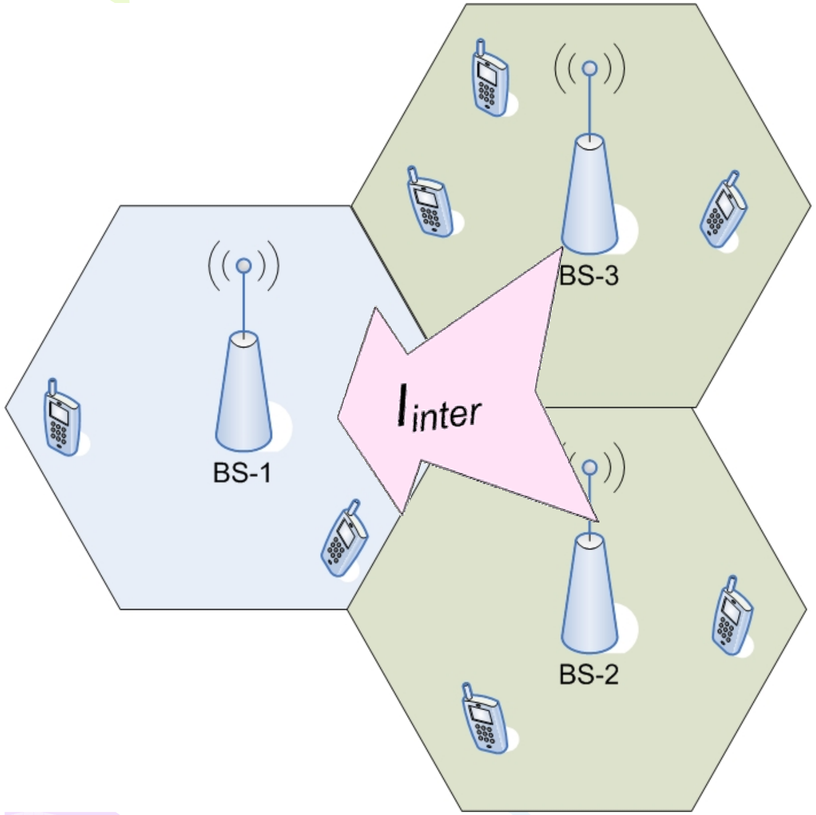
***K* service-classes** $(k=1,\ldots,K)$

- N_k : Number of traffic sources (MUs)
- R_k : Transmission bit rate
- $(E_b/N_0)_k$: Signal energy per bit divided by noise spectral density, required to meet a predefined Bit Error Rate (BER) parameter
- v_k : Activity factor

***User Activity*:** users alternate between transmitting and silent periods

- **Active users**: have a call in progress (occupy system resources)
- **Passive users**: are silent (do not occupy any system resources)

Introduction to W-CDMA
Interference & Call Admission Control



Interference

Intra-cell Interference (caused by users of the reference cell): I_{intra}

Inter-cell Interference (caused by users of the neighboring cells): I_{inter}

Existence of Thermal Noise: P_N

Need to preserve the QoS of in-service calls

Call Admission Control

can be based on the measurement of the Noise Rise

A new call is accepted if the

$$\text{NoiseRise}: \; NR = \frac{I_{total}}{P_N} = \frac{I_{intra} + I_{inter} + P_N}{P_N} \leq NR_{\max}$$

Wireless Erlang Multi-rate Loss Model (Wireless EMLM)

***The EMLM* is not suitable for W-CDMA Networks, since it does not take into account:**
1) User activity (active and silent periods)
2) Blocking due to inter-cell interference (soft blocking)

↓

Solution: **The Wireless EMLM**

D. Staehle and A. Mäder, "*An analytic approximation of the uplink capacity in a UMTS network with heterogeneous traffic*," in proc. 18th International Teletraffic Congress (ITC18), Sept. 2003.

Wireless EMLM

Cell Load, Load Factor and Local Blocking Probability

n = **Cell Load**: The ratio of the received power from all active users to the total received power

$$n = \frac{I_{intra} + I_{inter}}{I_{intra} + I_{inter} + P_N} = n_{intra} + n_{inter}$$

n_{intra}: *cell load from users of the reference cell*

n_{inter}: *cell load from users of the neighboring cells*

$$NR = \frac{I_{intra} + I_{inter} + P_N}{P_N}$$

$$n = \frac{NR - 1}{NR} \rightarrow n_{\max} = \frac{NR_{\max} - 1}{NR_{\max}}$$

Typical value, $n_{max} = 0.8$ (can be considered as the shared system resource)

L_k = **Load Factor**: can be seen as the bandwidth requirement of service-class k calls

$$L_k = \frac{(E_b / N_0)_k * R_k}{W + (E_b / N_0)_k * R_k}$$

R_k: Transmission bit rate

$(Eb/No)_k$: Bit error rate (BER) parameter

W = 3.84 Mcps: Chip rate of the W-CDMA carrier

β_k = **Local Blocking Probability**: The prob. that a new call is blocked when arriving at an instant with intra-cell load n_{intra}. It depends on the system occupied bandwidth as well as on the calls requirement

$$\beta_k(n_{intra}) = P(n_{intra} + n_{inter} + L_k > n_{\max})$$

Wireless EMLM

Intra-cell load and Inter-cell load

n_{intra}: Intra-cell load (cell load from users of the reference cell)

$$n_{intra} = \sum_{k=1}^{K} m_k L_k$$

where m_k is the number of active service-class *k* calls and

L_k is the load factor of service-class *k* calls

n_{inter}: Inter-cell load (cell load from users of the neighboring cells)

$$n_{inter} = (1 - n_{\max}) \frac{I_{inter}}{P_N}$$

where I_{inter} is modeled as a lognormal random variable, that is independent of the intra-cell interference, with mean $E[I_{inter}]$ and variance $Var[I_{inter}]$

Wireless EMLM

Bandwidth Discretization & Bandwidth Occupancy

g: *basic cell load unit* used for **Banwidth Discretization**

Bandwidth discretization is needed since the EMLM considers discrete state space

$$n \rightarrow j = \frac{n}{g}, \qquad n_{\max} \rightarrow C = \frac{n_{\max}}{g}$$

$$L_k \rightarrow b_k = \text{round}(\frac{L_k}{g})$$

Due to the existence of passive users a state j does not represent the total number of occupied b.u.

$\Lambda(c|j)$ = **Bandwidth Occupancy:** conditional probability that *c* b.u.are occupied in state *j*

Note that: c=0 all users are passive, c=j all users active while in the EMLM, c=j always

$$\Lambda(c \mid j) = \sum_{k=1}^{K} P_k(j)[v_k \Lambda(c - b_k \mid j - b_k) + (1 - v_k)\Lambda(c \mid j - b_k)],$$

for $j = 1, \ldots, j_{\max}$ and $c \le j$

where $\Lambda(0 \mid 0) = 1$ and $\Lambda(c \mid j) = 0$ for $c > j$

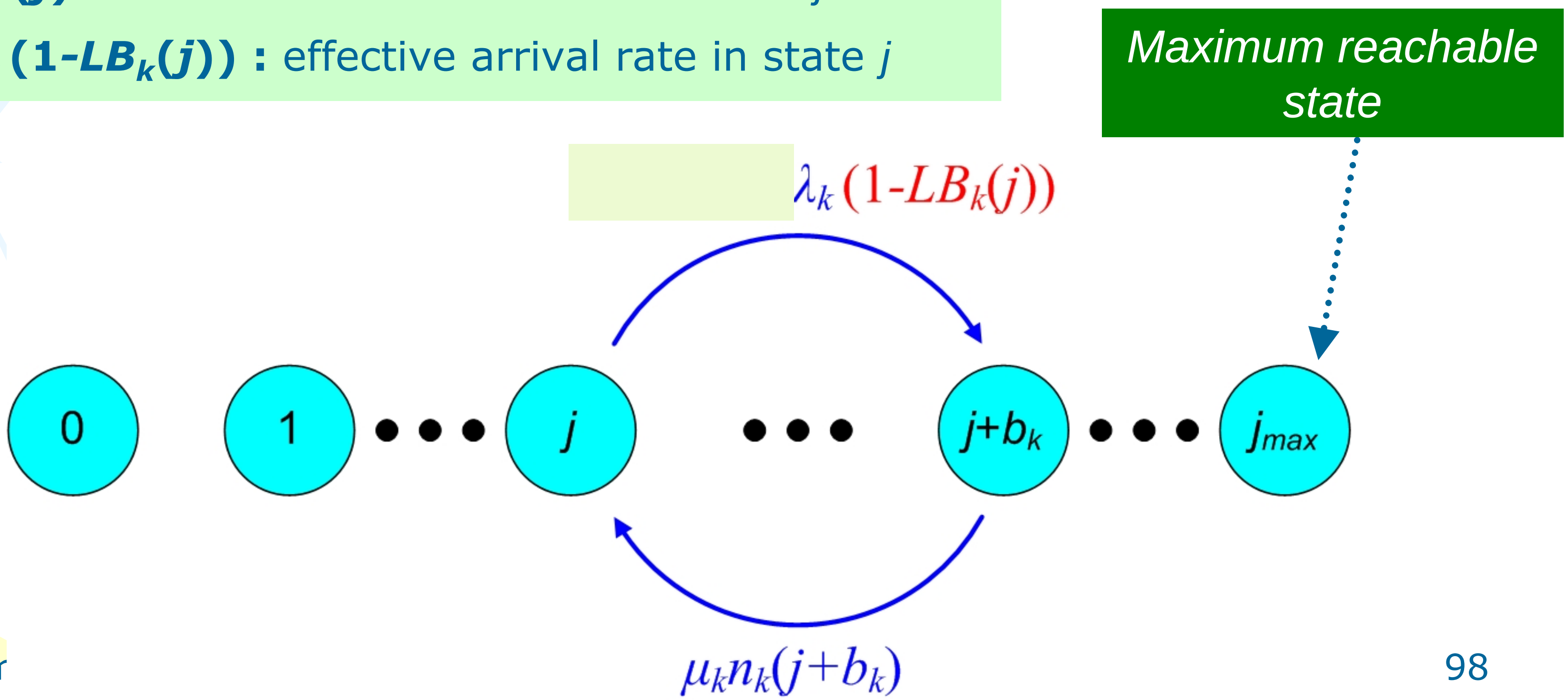
Wireless EMLM

Local Blocking Factor

Local Blocking Factor: due to the inter-cell interference blocking may occur in every state *j* with probability $LB_k(j)$

$$LB_k(j) = \sum_{c=0}^{j} \beta_k(c) \Lambda(c \mid j)$$

- λ_k **:** arrival rate (Poisson)
- μ_k **:** service rate
- $n_k(j)$**:** number of in-service calls in state *j*
- $\lambda_k(1-LB_k(j))$ **:** effective arrival rate in state *j*



Wireless EMLM

Call Blocking Probabilities Calculation

State Probabilities

$$\hat{q}(j) = \begin{cases} 1 & \text{for } j = 0 \\ \sum_{k=1}^{K} \alpha_k (1 - LB_k(j - b_k) b_k \hat{q}(j - b_k) & \text{for } j = 1, \ldots, j_{\max} \\ 0 & \text{otherwise} \end{cases}$$

$$q(j) = \frac{\hat{q}(j)}{\sum_{j=0}^{j_{\max}} \hat{q}(j)}$$

Bandwidth Share

$$P_k(j) = \frac{a_k (1 - LB_k(j - b_k) b_k q(j - b_k)}{j q(j)}$$

Call Blocking Probabilities

$$B_k = \sum_{j=0}^{j_{max}} q(j) LB_k(j)$$

Wireless Engset Multirate Loss Model

Vassilakis et. al (IEEE PIMRC 2007)

Due to the limited coverage area of a cell, it is certainly more realistic to consider that the number of mobile users, in a cell, is finite. This consideration is especially true in the case of microcells (small size cells).

In that case the Wireless EMLM should be replaced by the Wireless Engset Multirate Loss Model (Wireless EnMLM).

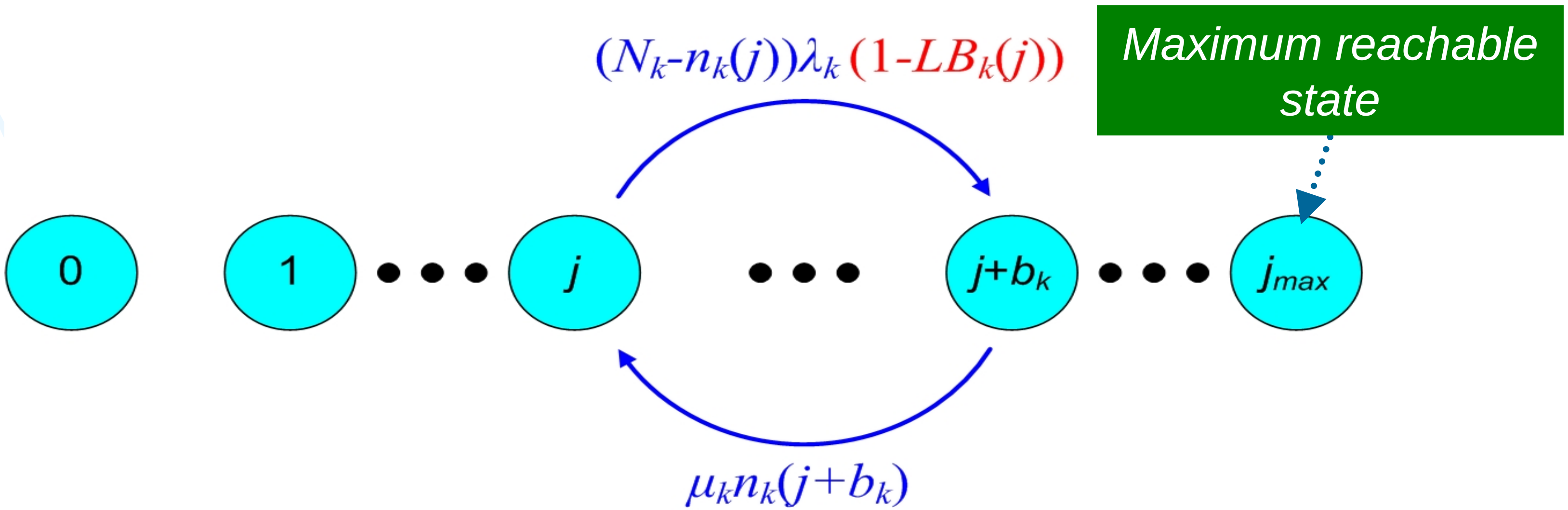
Wireless Engset Multirate Loss Model

Local Blocking Factor

Local Blocking Factor: due to the inter-cell interference blocking may occur in every state *j* with probability $LB_k(j)$

$$LB_k(j) = \sum_{c=0}^{j} \beta_k(c) \Lambda(c \mid j)$$

- **λ_k :** arrival rate from an idle source
- **μ_k :** service rate
- **N_k:** number of traffic sources (MUs)
- **$n_k(j)$:** number of in-service calls in state *j*
- **$(N_k - n_k(j))\lambda_k(1-LB_k(j))$:** effective arrival rate in state *j*



Wireless EnMLM

Call Blocking Probabilities Calculation

State Probabilities

$$\hat{q}(j) = \begin{cases} 1 & \text{for } j = 0 \\ \sum_{k=1}^{K} (N_k - n_k + 1)\alpha_k (1 - LB_k(j - b_k) b_k \hat{q}(j - b_k) & \text{for } j = 1, \ldots, j_{\max} \\ 0 & \text{otherwise} \end{cases}$$

$$q(j) = \frac{\hat{q}(j)}{\sum_{j=0}^{j_{\max}} \hat{q}(j)}$$

Bandwidth Share

$$P_k(j) = \frac{(N_k - n_k + 1)a_k (1 - LB_k(j - b_k) b_k q(j - b_k)}{j q(j)}$$

Call Blocking Probabilities

$$B_k = \sum_{j=0}^{j_{max}} q(j) LB_k(j)$$

Evaluation – Application Example

We compare:

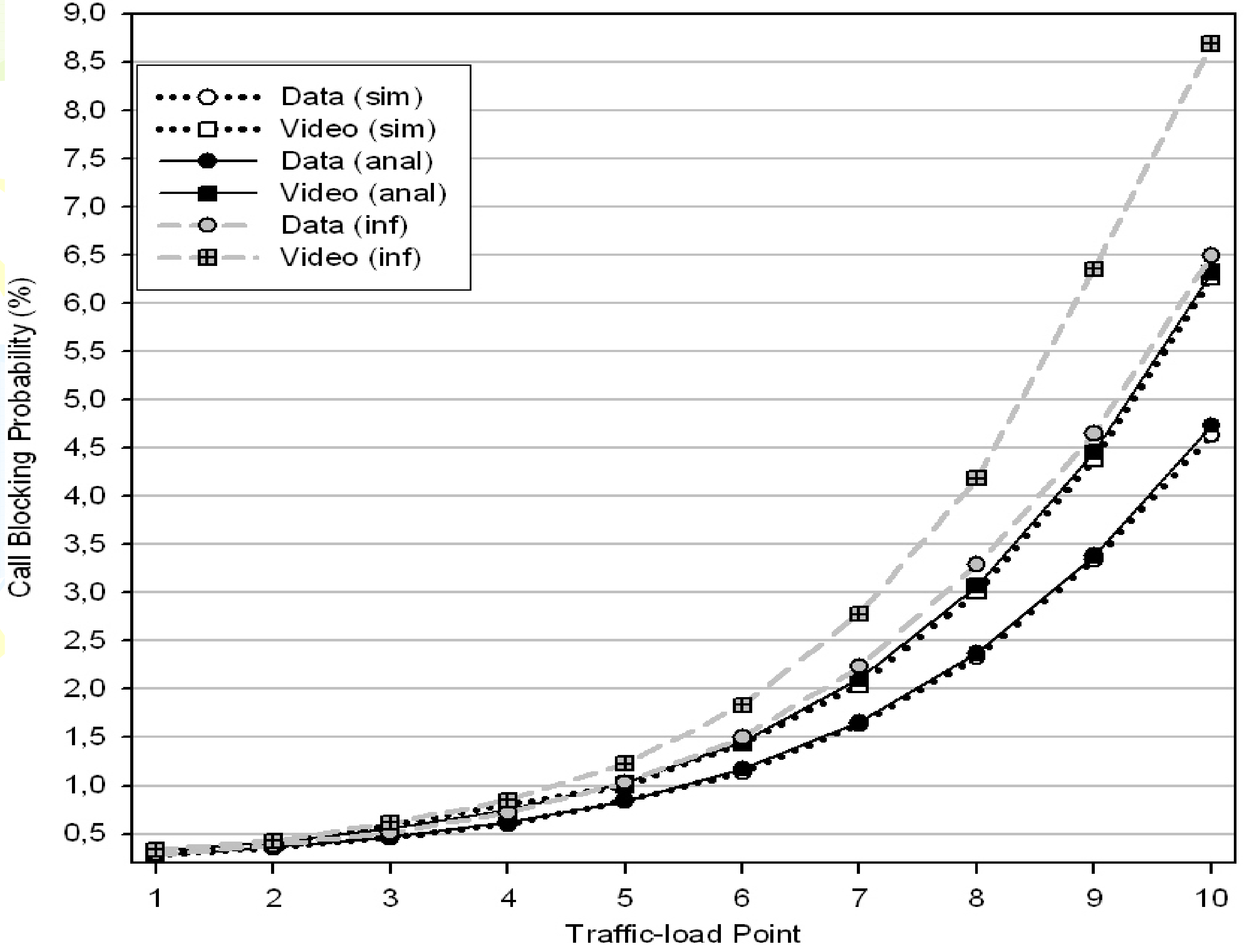
a) **Analytical to Simulation CBP results of the Wireless-EnMLM**

b) **The Wireless-EnMLM to the Wireless-EMLM (infinite source model)**

	Data	**Video**
Transmission rates (Kbps)	R_1=64	R_2= 144
Activity factor	v_1=1.0	v_2=0.3
BER parameter (dB)	$(E_b/N_0)_1$=4	$(E_b/N_0)_2$=3
Inter-cell Interference	E[I_{inter}] = 2*10^{-18} mW and CV[I_{inter}] = 1	

Traffic load point	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Number of sources (N_1=N_2)	10	20	30	40	50	60	70	80	90	100
Offered traffic for Data (erl)	1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0	9.0	10.0
Offered traffic for Video (erl)	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0

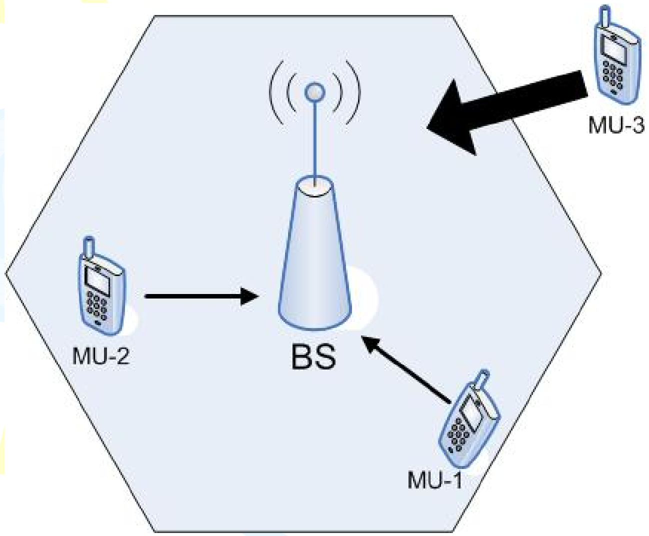
Evaluation – Application Example (cont.)



The Wireless EMLM including Handoff traffic (WH-EMLM)

Vassilakis et. al (IARIA AICT 2008)

Uplink: calls from the Mobile Users (MUs) to the Base Station (BS)



Types of Calls

- ***New Calls***
- ***Handoff Calls***

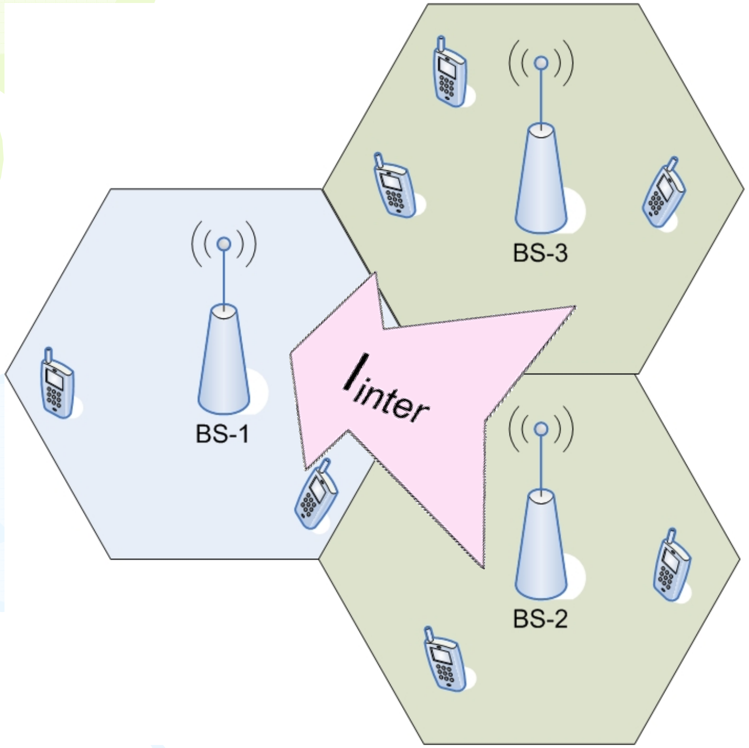
Calls of a single service-class

- R : Transmission bit rate
- (E_b/N_0) : Bit error rate (BER) parameter
- v : Activity factor

User Activity: users alternate between transmitting and silent periods

- **Active users**: have a call in progress (occupy system resources)
- **Passive users**: are silent (do not occupy any system resources)

The WH-EMLM
Interference & Call Admission Control



Interference

Intra-cell Interference: I_{intra}

Inter-cell Interference: I_{inter}

Thermal Noise: P_N

Need to preserve the QoS of in-service calls

Call Admission Control

NoiseRise: $NR = \frac{I_{total}}{P_N} = \frac{I_{intra} + I_{inter} + P_N}{P_N} \le NR_{\max}$

A New call is accepted if

$NR \le NR_{\max,N}$

$NR_{\max,N} < NR_{\max,H}$

A Handoff call is accepted if

$NR \le NR_{\max,H}$

The WH-EMLM

Cell Load, Load Factor and Local Blocking Probability

n = **Cell Load**: The ratio of the received power from all active users to the total received power

$$n = \frac{I_{intra} + I_{inter}}{I_{intra} + I_{inter} + P_N} = n_{intra} + n_{inter}$$

n_{intra}: *cell load from users of the reference cell*

n_{inter}: *cell load from users of the neighboring cells*

$$NR = \frac{I_{intra} + I_{inter} + P_N}{P_N}$$

$$n = \frac{NR - 1}{NR}$$

$$n_{\max,N} = \frac{NR_{\max,N} - 1}{NR_{\max,N}}$$

$$n_{\max,H} = \frac{NR_{\max,H} - 1}{NR_{\max,H}}$$

We use *Cell Load* instead of *Noise Rise* for the CAC

L= **Load Factor**: call bandwidth requirement

$$L = \frac{(E_b / N_0) * R}{W + (E_b / N_0) * R}$$

W = 3.84 Mcps: Chip rate of the W-CDMA carrier

β = **New Call & Handoff Call Local Blocking Probability**: The prob. that a new call (or a handoff call) is blocked when upon arrival the intra-cell load is n_{intra}.

$$\beta_{\mathrm{N}}(n_{intra}) = P(n_{intra} + n_{inter} + L > n_{\max,\mathrm{N}}) \qquad \beta_{\mathrm{H}}(n_{intra}) = P(n_{intra} + n_{inter} + L > n_{\max,\mathrm{H}})$$

The WH-EMLM

Bandwidth Discretization & Bandwidth Occupancy

In order to describe the system by a Markov Chain we express all parameters with integer values.

g: *basic cell load unit* used for **Resource Discretization**

$$n \longrightarrow j = \frac{n}{g}, \quad n_{\max} \longrightarrow C = \frac{n_{\max}}{g}$$

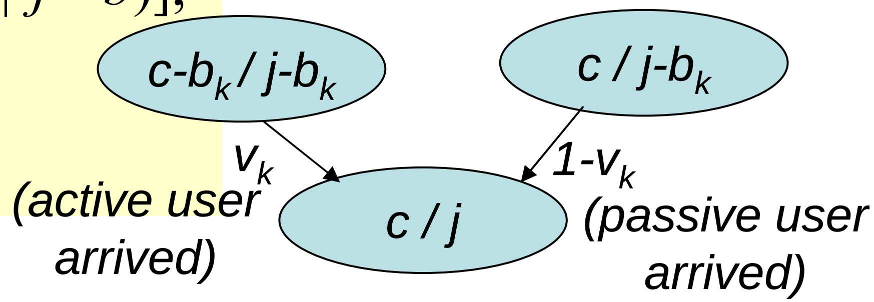
$$L \longrightarrow b = \text{round}(\frac{L}{g})$$

$\Lambda(c|j)$ = **Resource Occupancy:** conditional probability that *c* resources are occupied in state *j*

$$\Lambda(c \mid j) = P(j)[v\Lambda(c-b \mid j-b) + (1-v)\Lambda(c \mid j-b)],$$

for $j = 1, \ldots, j_{\max}$ and $c \le j$

where $\Lambda(0 \mid 0) = 1$ and $\Lambda(c \mid j) = 0$ for $c > j$



The WH-EMLM
Local Blocking Factor

Local Blocking Factor: due to the inter-cell interference blocking may occur in every state *j* with probability $LB(j)$

New Calls

- λ_N : mean arrival rate of new calls (Poisson process)
- μ_N : mean service rate of a new call
- $Y_N(j)$: number of in-service calls in state *j*
- $\lambda_N(j) = \lambda_N(1-LB_N(j))$: effective arrival rate in *j*

$$LB_N(j) = \sum_{c=0}^{j} \beta_N(c)\Lambda(c \mid j)$$

Handoff Calls

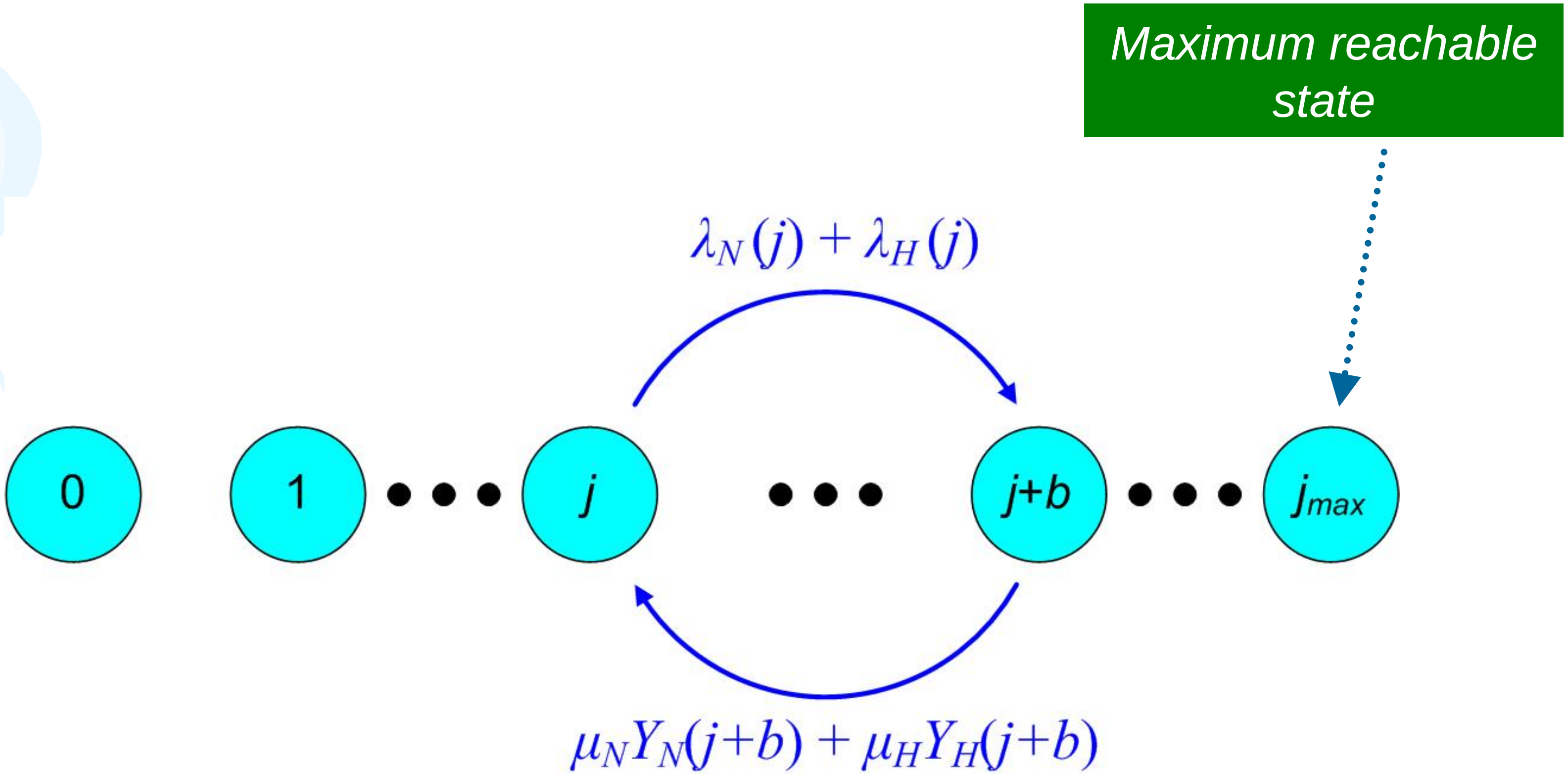
- λ_H : mean arrival rate of handoff calls (Poisson)
- μ_H : mean service rate of handoff calls
- $Y_H(j)$: number of in-service handoff calls in state *j*
- $\lambda_H(j) = \lambda_H(1-LB_H(j))$: effective arrival rate in *j*

$$LB_H(j) = \sum_{c=0}^{j} \beta_H(c)\Lambda(c \mid j)$$

$$\mu_H > \mu_N$$

The WH-EMLM
State Transition Diagram

- s_N : Number of New Calls
- s_H : Number of Handoff Calls
- $j = (s_H + s_N)\, b$: occupied bandwidth (system state)



The WH-EMLM
Call Blocking Probabilities Calculation

State Probabilities

$$\hat{q}(j) = \begin{cases} 1 & \text{for } j = 0 \\ \frac{1}{j}\alpha_N (1 - LB_N(j-b)) b \hat{q}(j-b) + \\ \frac{1}{j}\alpha_H (1 - LB_H(j-b)) b \hat{q}(j-b) & \text{for } j = 1, \ldots, j_{\max} \\ 0 & \text{otherwise} \end{cases}$$

$$q(j) = \frac{\hat{q}(j)}{\sum_{j=0}^{j_{\max}} \hat{q}(j)}$$

Call Blocking Probabilities

$$B_N = \sum_{j=0}^{j_{max}} q(j) LB_N(j)$$

$$B_H = \sum_{j=0}^{j_{max}} q(j) LB_H(j)$$

The WH-EMLM

Generalization to K Service-Classes

State Probabilities

$$\hat{q}(j) = \begin{cases} 1 & \text{for } j = 0 \\ \frac{1}{j}\sum_{k=1}^{K} \alpha_{N,k}(1 - LB_{N,k}(j - b_k))b_k\,\hat{q}(j - b) + \\ \frac{1}{j}\sum_{k=1}^{K} \alpha_{H,k}(1 - LB_{H,k}(j - b_k))b_k\,\hat{q}(j - b_k) & \text{for } j = 1, \ldots, j_{\max} \\ 0 & \text{otherwise} \end{cases}$$

$$q(j) = \frac{\hat{q}(j)}{\sum_{j=0}^{j_{\max}} \hat{q}(j)}$$

Bandwidth Share

$$P_{H,k}(j) = \frac{a_{H,k}(1 - LB_{H,k}(j - b_k))b_k\, q(j - b_k)}{jq(j)}$$

$$P_{N,k}(j) = \frac{a_{N,k}(1 - LB_{N,k}(j - b_k))b_k\, q(j - b_k)}{jq(j)}$$

Example:
$b_1=2$
$b_2=1$

j=5
($1*b_1$ + $3*b_2$)

$P_{H,1}$ (5)= 2/5 and $P_{H,2}$ (5)= 3/5

Call Blocking Probabilities

$$B_{N,k} = \sum_{j=0}^{j_{max}} q(j) LB_{N,k}(j)$$

$$B_{H,k} = \sum_{j=0}^{j_{max}} q(j) LB_{H,k}(j)$$

Evaluation – Application Example

We compare Analytical to Simulation CBP results

	Data	Video
Transmission rates (Kbps)	R_1=144	R_2= 384
Activity factor	v_1=0.7	v_2=0.6
BER parameter (dB)	$(E_b/N_0)_1$=3	$(E_b/N_0)_2$=4
Inter-cell Interference	$E[I_{inter}] = 2*10^{-18}$ mW and $CV[I_{inter}] = 1$	

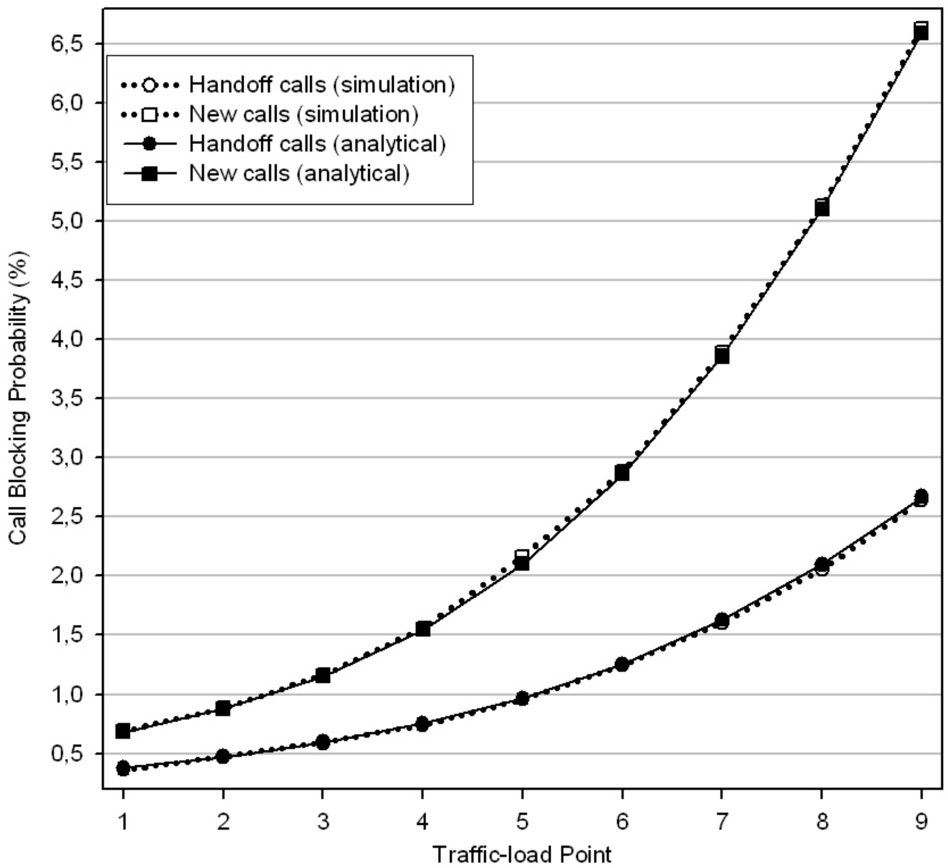
Traffic load point	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	
New call Offered traffic for Data, (erl)	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0	
Handoff Call Offered traffic for Data (erl)	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	
New call Offered traffic for Video (erl)	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	
Handoff Call Offered traffic for Video (erl)	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5	

Evaluation – Application Example (cont.)

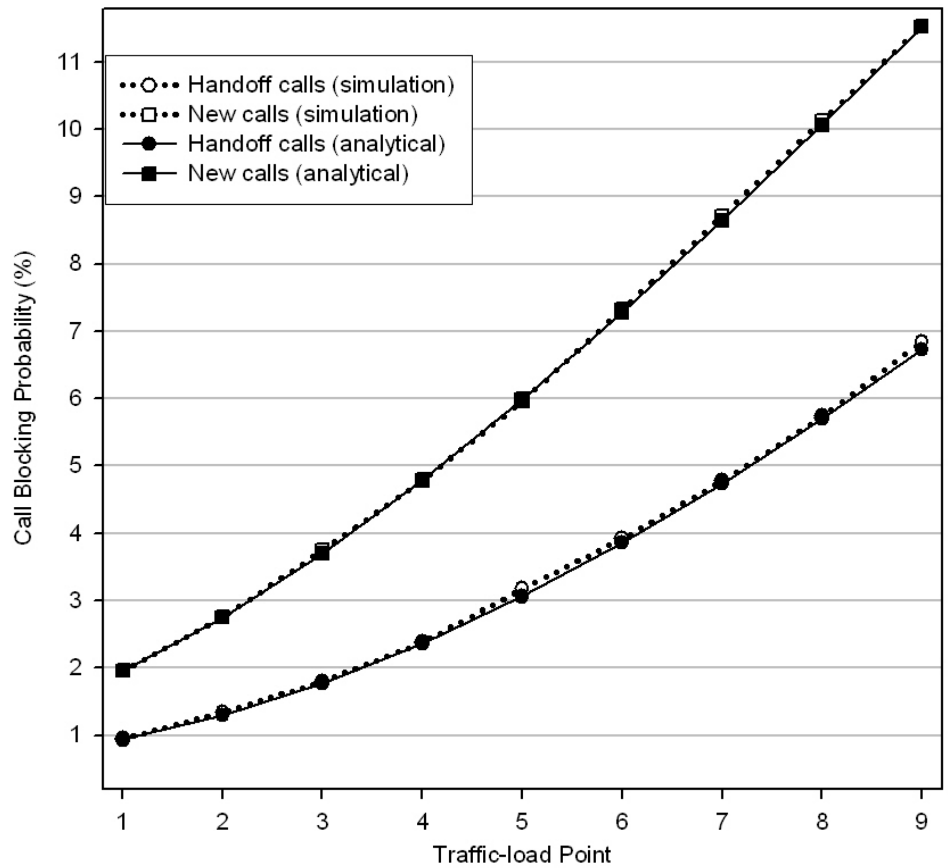
$$n_{\max,N} = 0.7$$

$$n_{\max,H} = 0.8$$

Data



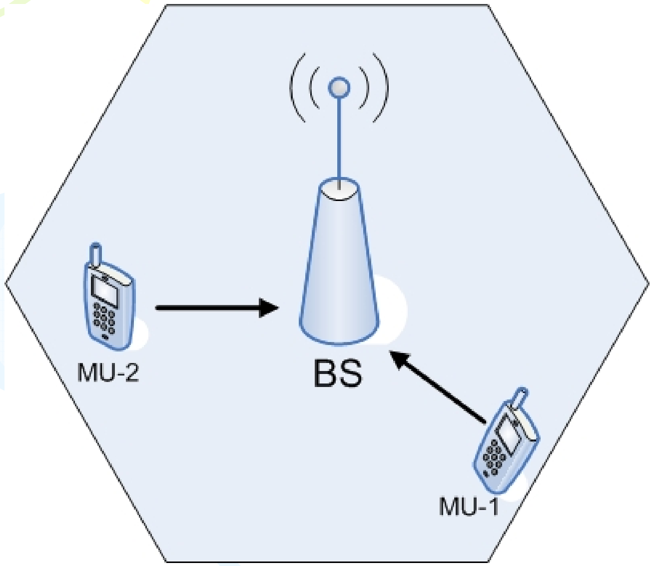
Video



The Wireless finite CDTM

Vassilakis et. al (IEEE ICC 2008)

Uplink: calls from the Mobile Users (MUs) to the Base Station (BS)



Types of Services

Stream (real-time video)

Elastic (file transfer)

K Service-Classes

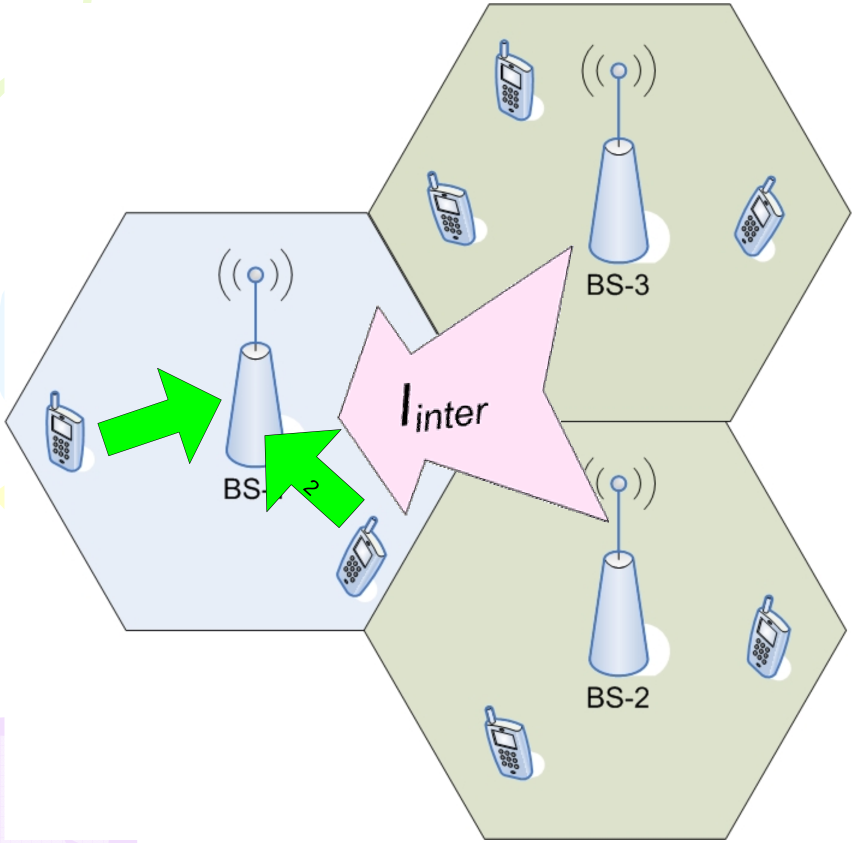
S_k (k=1,…,K) QoS levels (l=1,…, S_k)

- $R_{k,l}$: Transmission bit rate
- $(Eb/No)_{k,l}$: Bit error rate (BER) parameter

User Activity: users alternate between transmitting and silent periods

- **Active users**: have a call in progress (occupy system resources)
- **Passive users**: are silent (do not occupy any system resources)

The Wireless finite CDTM
Interference & Call Admission Control



Interference

Intra-cell Interference: I_{intra}

Inter-cell Interference: I_{inter}

Thermal Noise: P_N

Need to preserve the QoS of in-service calls

↓

Call Admission Control

$$\text{NoiseRise}: \ NR = \frac{I_{total}}{P_N} = \frac{I_{intra} + I_{inter} + P_N}{P_N} \leq NR_{\max}$$

The Wireless finite CDTM

Cell Load, Load Factor and Local Blocking Probability

n ≡ **Cell Load**: Shared system bandwidth/resource

$$n = \frac{I_{intra} + I_{inter}}{I_{intra} + I_{inter} + P_N} = n_{intra} + n_{inter} \qquad n = \frac{NR - 1}{NR} \;\longrightarrow\; n_{max} = \frac{NR_{max} - 1}{NR_{max}}$$

$$NR = \frac{I_{intra} + I_{inter} + P_N}{P_N}$$

We use *Cell Load* (instead of *Noise Rise*) for the CAC

$L_{k,l}$ = **Load Factor**: call resource requirement

$$L_{k,l} = \frac{(E_b / N_0)_{k,l} * R_{k,l}}{W + (E_b / N_0)_{k,l} * R_{k,l}}$$

$R_{k,l}$: Transmission bit rate

$(Eb/No)_{k,l}$: Bit error rate (BER) parameter

W = 3.84 Mcps: Chip rate (bit rate of the spreading signal)

$\beta_{k,l}$ = **Local Blocking Probability**: depends on the system occupied resources as well as on the calls requirement

$$\beta_{k,l}(n_{intra}) = P(n_{intra} + n_{inter} + L_{k,l} > n_{max})$$

(NEW CAC CRITERION)

The Wireless finite CDTM

Resource Discretization & Resource Occupancy

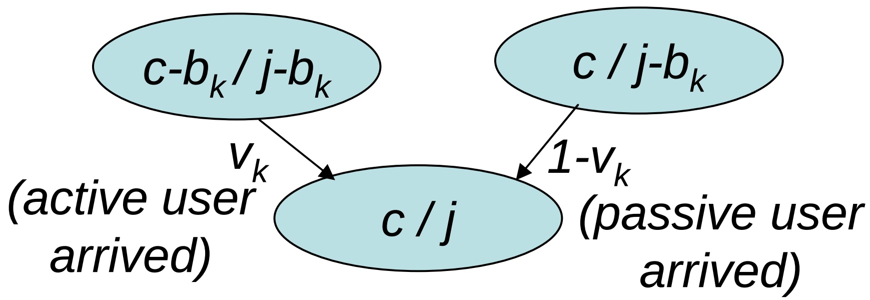
g: *basic cell load unit* used for **Resource Discretization**

$$n \longrightarrow j = \frac{n}{g}$$

$$n_{\max} \longrightarrow C = \frac{n_{\max}}{g}$$

$$L_{k,l} \longrightarrow b_{k,l} = \text{round}\left(\frac{L_{k,l}}{g}\right)$$

$\Lambda(c \mid j)$ = **Resource Occupancy:** conditional probability that *c* resources are occupied in state *j*



$$\Lambda(c \mid j) = \sum_{k=1}^{K} \sum_{l=1}^{S_k} P_{k,l}(j)[v_k \Lambda(c - b_{k,l} \mid j - b_{k,l}) + (1 - v_k)\Lambda(c \mid j - b_{k,l})],$$

for $j = 1, \ldots, j_{\max}$ and $c \leq j$

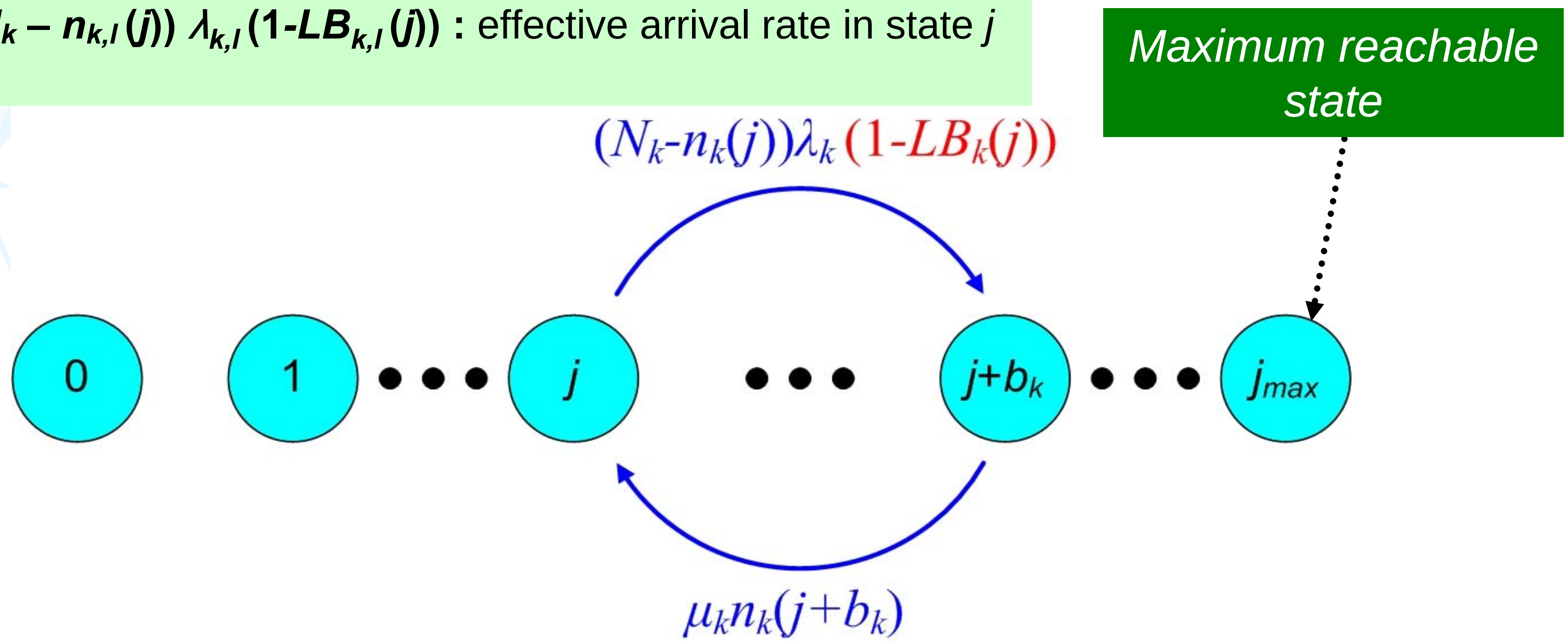
where $\Lambda(0 \mid 0) = 1$ and $\Lambda(c \mid j) = 0$ for $c > j$

The Wireless finite CDTM
Local blocking factor

Local Blocking Factor: due to the inter-cell interference. Blocking may occur in every state *j* with probability $LB_{k,l}(j)$

$$LB_{k,l}(j) = \sum_{c=0}^{j} \beta_{k,l}(c)\Lambda(c \mid j)$$

- $\lambda_{k,l}$: arrival rate from an idle source
- $\mu_{k,l}$: service rate
- $n_{k,l}(j)$: number of in-service calls in state *j*
- $(N_k - n_{k,l}(j))\,\lambda_{k,l}\,(1-LB_{k,l}(j))$: effective arrival rate in state *j*



The Wireless finite CDTM

Call blocking probabilities calculation

Un-normalized State Probabilities

$$\hat{q}(j) = \begin{cases} 1 & \text{for } j = 0 \\ \sum_{k=1}^{K} \sum_{l=0}^{S_k} (N_k - \sum_{l=0}^{S_k} n_{k,l}(j) + 1) A_{k,l}(j) \hat{q}(j - b_{k,l}) & \text{for } j = 1, \ldots, j_{\max} \\ 0 & \text{otherwise} \end{cases}$$

$$A_{k,l}(j) = \alpha_{k,l}(1 - LB_{k,l}(j - b_{k,l}) b_{k,l} \delta_{k,l}(j)$$

$$n_{k,l}(j) \approx \frac{a_{k,l}(j) q(j - b_{k,l})(1 - LB_{k,l}(j - b_{k,l}))}{q(j)}$$

Normalization

$$q(j) = \frac{\hat{q}(j)}{\sum_{j=0}^{j_{\max}} \hat{q}(j)}$$

The Wireless finite CDTM
Call blocking probabilities

Performance Metrics

Bandwidth Share

$$P_{k,l}(j) = \frac{(N_k - \sum_{l=0}^{S_k} n_{k,l}(j) + 1)\, A_{k,l}(j) q(j - b_{k,l})}{j q(j)}$$

Call Blocking Probabilities

$$B_k = \sum_{j=0}^{j_{max}} q(j) \sum_{l=1}^{S_k} \omega_{k,l}(j) LB_k(j)$$

$$\omega_{k,1}(j) = \begin{cases} 1 & \text{when } j \le J_{k,1} \\ 0 & \text{otherwise} \end{cases}$$

$$\omega_{k,l}(j) = \begin{cases} 1 & \text{when } J_{k,l} < j \le J_{k,l+1} \\ 0 & \text{otherwise} \end{cases}, \text{ for } l > 1$$

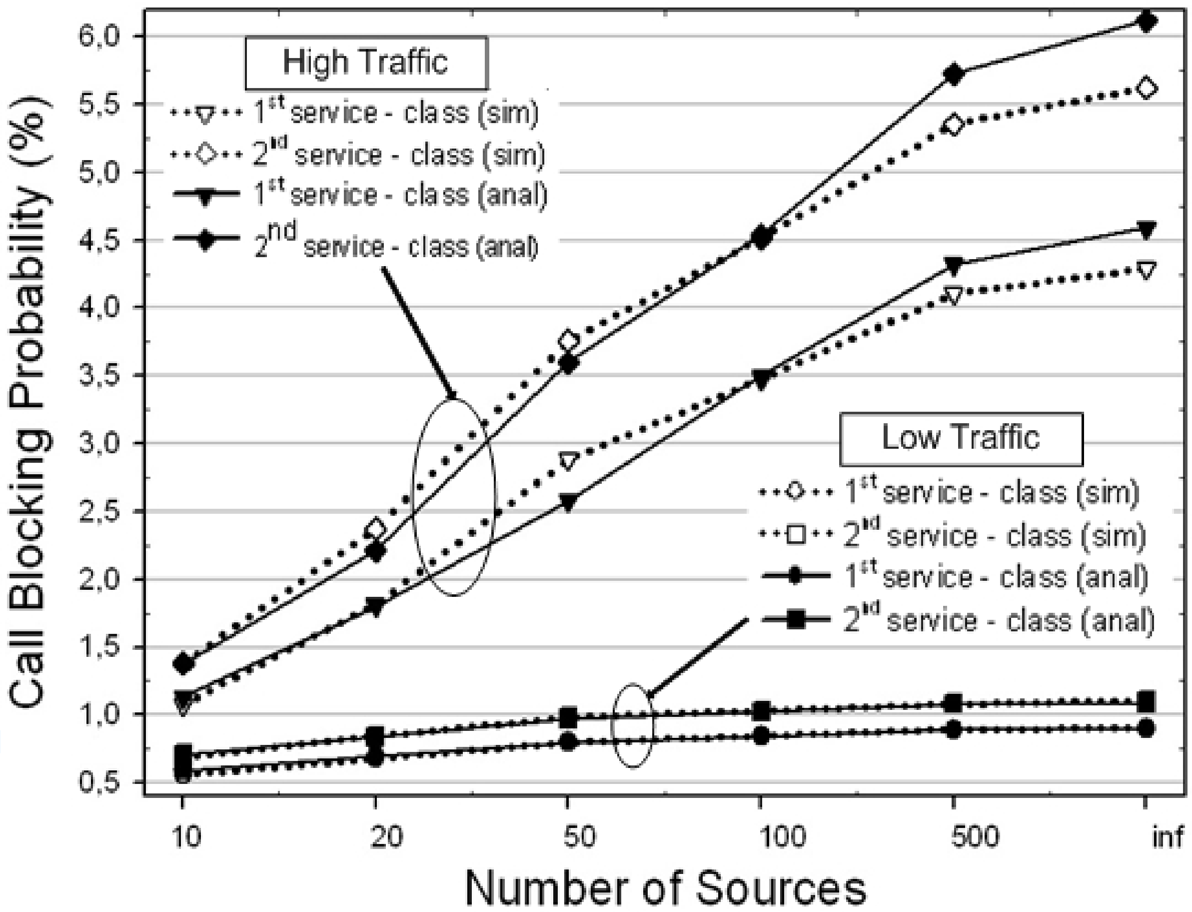
Evaluation – 1st Application Example

Characteristics of the Service-classes

Service-class	Data	Video
Type	Elastic	Elastic
Transmission rate (Kbps)	$R_{1,1}$=64 and $R_{1,2}$=32	$R_{2,1}$=144, $R_{2,2}$=128 and $R_{2,3}$= 112
Thresholds	$J_{1,1}$= 0.6	$J_{2,1}$= 0.4 and $J_{2,2}$= 0.6
Activity factor	v_1=1.0	v_2=0.7
BER parameter (dB)	$(E_b/N_0)_1$=4	$(E_b/N_0)_2$=3

Evaluation – 1st Application Example (cont.)

We compare *Analytical* to *Simulation* results



Low Traffic: $N_1\alpha_1$ = 4 *erl*, $N_2\alpha_2$ = 1 *erl*
High Traffic: $N_1\alpha_1$ = 8 *erl*, $N_2\alpha_2$ = 2 *erl*

Evaluation – 2nd Application Example

Characteristics of the Service-classes

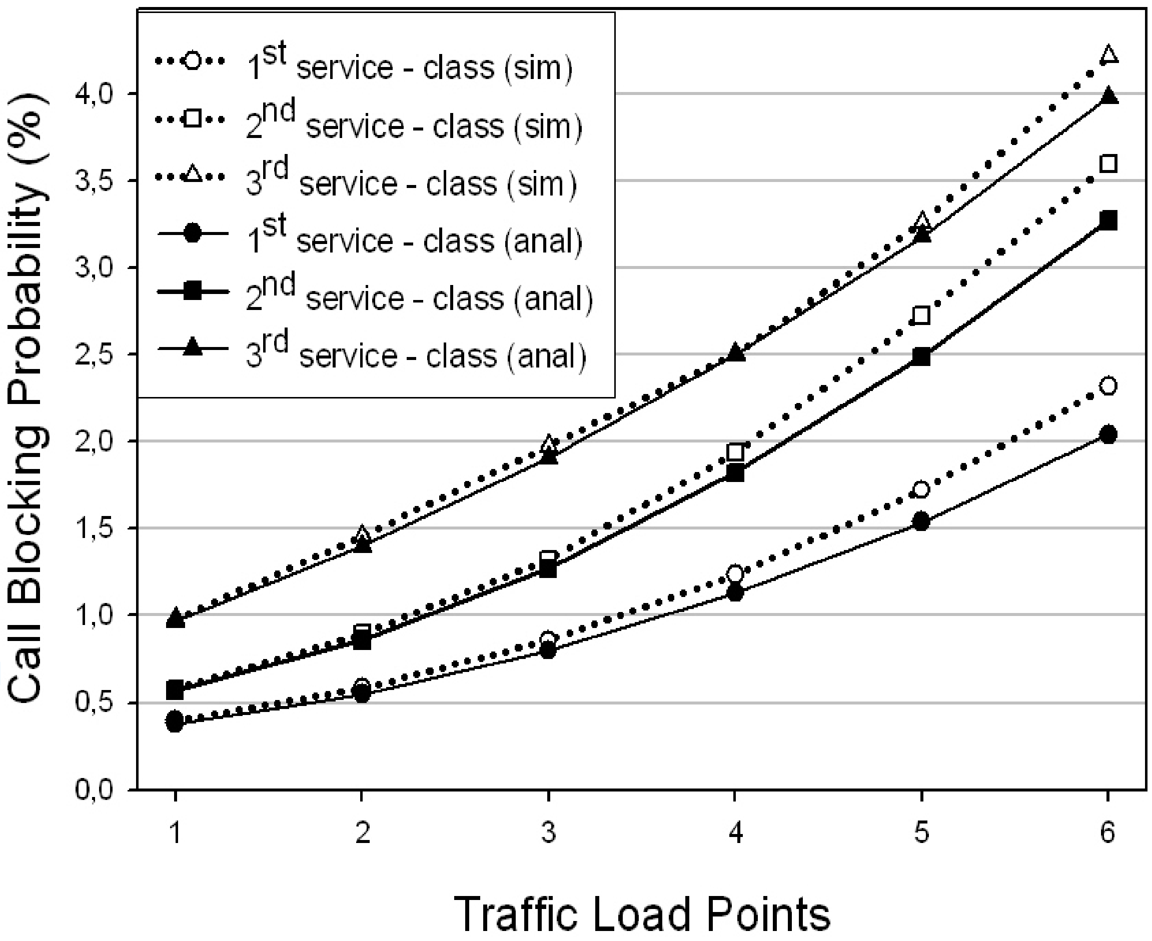
Service-class	Voice	Data	Video
Type	Stream	Elastic	Elastic
Transmission rate (Kbps)	$R_{1,1}$=12.2	$R_{2,1}$=128 and $R_{2,2}$=64	$R_{3,1}$=384, $R_{3,2}$=144 and $R_{3,3}$=128
Thresholds	-	$J_{2,1}$= 0.6	$J_{3,1}$= 0.4 and $J_{3,2}$= 0.6
Activity factor	v_1=0.5	v_2=1.0	v_3=0.7
BLER parameter (dB)	$(E_b/N_0)_1$=5	$(E_b/N_0)_2$=4	$(E_b/N_0)_3$=3
Number of sources	N_1=100	N_2=50	N_3=10

Offered traffic-load

Traffic load point:		**1**	**2**	**3**	**4**	**5**	**6**
Offered traffic-load **(erl)**	**Voice**	4.0	6.0	8.0	10.0	12.0	14.0
	Data	1.0	1.4	1.8	2.2	2.6	3.0
	Video	0.1	0.2	0.3	0.4	0.5	0.6

Evaluation – 2nd Application Example (cont.)

We compare *Analytical* to *Simulation* results



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Thank You !